Tabla de Propiedades y algunas Transformadas de Fourier

	$f(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} F(\omega) e^{j\omega t} d\omega$	$F(\omega) = \int_{-\infty}^{+\infty} f(t) e^{-j\omega t} dt$
1	$a_1 f_1(t) + a_2 f_2(t)$	$a_1 F_1(\omega) + a_2 F_2(\omega)$
2	$f(at),\ a \neq 0$	$\frac{1}{\lvert a \rvert} F\left(\frac{\omega}{a}\right)$
3	$f(t \mp t_0)$	$e^{\mp j\omega t_0} F(\omega)$
4	$e^{\pm j\omega_0 t} f(t)$	$F(\omega \mp \omega_0)$
5	$f(t) \cdot cos(\omega_0 t)$	$\frac{1}{2} F(\omega - \omega_0) + \frac{1}{2} F(\omega + \omega_0)$
6	$f(t) \cdot sen(\omega_0 t)$	$\frac{1}{2j} F(\omega - \omega_0) - \frac{1}{2j} F(\omega + \omega_0)$
7	$F(t)$	$2\pi f(-\omega)$
8	$\frac{d^n f(t)}{dt^n},\ n \in \aleph$	$(j\omega)^n F(\omega)$
9	$\int_{-\infty}^{t} f(t')dt'$	$\frac{1}{j\omega} F(\omega) + \pi F(0)\delta(\omega)$
10	$(-jt)^n f(t),\ n \in \aleph$	$\frac{d^n F(\omega)}{d\omega^n}$
11	$\frac{f(t)}{-jt}$	$\int_{-\infty}^{\omega} F(\omega')d\omega'$
12	$f_1(t) * f_2(t)$	$F_1(\omega) \cdot F_2(\omega)$
13	$f_1(t) \cdot f_2(t)$	$\frac{1}{2\pi} F_1(\omega) * F_2(\omega)$
14	$\int_{-\infty}^{\infty} f_1(t) \overline{f}_2(t) dt$	$\frac{1}{2\pi} \int_{-\infty}^{\infty} F_1(\omega) \cdot \overline{F}_2(\omega) d\omega$
15	$e^{-at} u(t)$	$\frac{1}{a + j\omega}$
16	$e^{-a\lvert t \rvert}$	$\frac{2a}{a^2 + \omega^2}$
17	$e^{-at^2},\ a \neq 0$	$\sqrt{\frac{\pi}{a}} e^{-\frac{\omega^2}{4a}}$
18	$A \cdot p_{2T}(t)$	$2ATsinc(\omega T)$
19	$\Delta(t) = \begin{cases} A\left(1 - \frac{\lvert t \rvert}{T}\right), & \lvert t \rvert < T \\ 0, & \lvert t \rvert > T \end{cases}$	$ATsinc^2\left(\frac{\omega T}{2}\right)$
20	$\frac{t^{n-1}}{(n-1)!} e^{-at} u(t)$	$\frac{1}{(j\omega + a)^n}$
21	$e^{-at}\ sen\,\omega_0 t \cdot u(t)$	$\frac{\omega_0}{(a + j\omega)^2 + \omega_0^2}$
22	$e^{-at}\ cos\,\omega_0 t \cdot u(t)$	$\frac{a + j\omega}{(a + j\omega)^2 + \omega_0^2}$
23	$k\delta(t)$	k
24	k	$2\pi k\delta(\omega)$
25	$sgn(t)$	$\frac{2}{j\omega}$
26	$u(t)$	$\frac{1}{j\omega} + \pi\delta(\omega)$
27	$cos\,\omega_0 t$	$\pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$
28	$sen\,\omega_0 t$	$j\pi[\delta(\omega + \omega_0) - \delta(\omega - \omega_0)]$
29	$e^{\pm j\omega_0 t}$	$2\pi\delta(\omega \mp \omega_0)$
30	$\sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t},\ C_n = \frac{1}{T}\int_{-T/2}^{T/2} f(t) e^{-jn\omega_0 t} dt,\ \omega_0 = \frac{2\pi}{T}$	$2\pi \sum_{n=-\infty}^{\infty} C_n \delta(\omega - n\omega_0)$
31	$\sum_{n=-\infty}^{\infty} \delta(t - nT_S)$	$\omega_S \sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_S),\ \omega_S = \frac{2\pi}{T_S}$
32	$t^n,\ n \in \aleph$	$2\pi j^n \frac{d^n \delta(\omega)}{d\omega^n}$

MSc. Ing. Franco Martin Pessana
E-mail: fpessana@favaloro.edu.ar

Tabla de Propiedades y algunas Transformadas de Laplace

	$f(t)=\frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty}F(s)\cdot e^{st}ds,\ s=\sigma+j\omega$	$F(s)=\int_{0}^{+\infty}f(t)\cdot e^{-st}dt$
1	$a_1 f_1(t)+a_2 f_2(t)$	$a_1F_1(s)+a_2F_2(s)$
2	$f(at), a\neq 0$	$\frac{1}{a}F\left(\frac{s}{a}\right)$
3	$f(t-\tau)u(t-\tau)$	$e^{-\tau s}F(s)$
4	$e^{\pm at}f(t)$	$F(s\mp a)$
5	$f_1(t)*f_2(t)$	$F_1(s)\cdot F_2(s)$
6	$f_1(t)\cdot f_2(t)$	$\int_{c-j\infty}^{c+j\infty}F_1(\tau)F_2(s-\tau)d\tau$
7	$\frac{d^n f(t)}{dt^n}, n\in\aleph,\ t\geq 0$	$s^nF(s)-s^{n-1}f(0)-s^{n-2}f'(0)-\cdots-f^{(n-1)}(0)$
8	$\int_{-\infty}^{t}f(t')dt'$	$\frac{F(s)}{s}+\frac{\int_{-\infty}^{0}f(t)dt}{s}$
9	$(-t)^n f(t), n\in\aleph$	$\frac{d^nF(s)}{ds^n}$
10	$\frac{f(t)}{t}$	$\int_{s}^{\infty}F(u)du$
11	$f(t)=f(t+T)$	$\frac{1}{1-e^{-sT}}\int_{0}^{T}f(t)e^{-st}dt$
12	$f(0)$	$\lim_{s\to\infty}sF(s)$
13	$\lim_{t\to\infty}f(t)$	$\lim_{s\to 0}sF(s)$
14	$\lim_{t\to\infty}f^{(n-1)}(t)$	$\lim_{s\to 0}s^nF(s)$
15	$\sum_{k=1}^{n}\frac{P(\alpha_k)}{Q'(\alpha_k)}e^{\alpha_k t},\ \alpha_k\ /\ Q(\alpha_k)=0$	$\frac{P(s)}{Q(s)}, gr(P)<gr(Q)=n$
16	$e^{\pm at}u(t)$	$\frac{1}{s\mp a}$
17	$t^ne^{\pm at}u(t), n\in\aleph$	$\frac{n!}{(s\mp a)^{n+1}}$
18	$u(t)$	$\frac{1}{s}$
19	$e^{-a\|t\|}$	$\frac{2a}{a^2-s^2}$, con $F_b(s)=\int_{-\infty}^{+\infty}f(t)\cdot e^{-st}dt$
20	$(1-e^{-at})\cdot u(t)$	$\frac{a}{s(s+a)}$
21	$e^{-at}\ sen\,\omega_0 t\cdot u(t)$	$\frac{\omega_0}{(s+a)^2+\omega_0^2}$
22	$e^{-at}\cos\omega_0 t\cdot u(t)$	$\frac{s+a}{(s+a)^2+\omega_0^2}$
23	$\cos\omega_0 t\cdot u(t)$	$\frac{s}{s^2+\omega_0^2}$
24	$sen\,\omega_0 t\cdot u(t)$	$\frac{\omega_0}{s^2+\omega_0^2}$
25	$k\delta(t)$	k
26	$\frac{d^n\delta(t)}{dt^n}$	s^n
27	$\sum_{n=-\infty}^{\infty}\delta(t-nT)$	$\frac{1}{1-e^{-sT}}$
28	$\cosh\omega_0 t\cdot u(t)$	$\frac{s}{s^2-\omega_0^2}$
29	$senh\,\omega_0 t\cdot u(t)$	$\frac{\omega_0}{s^2-\omega_0^2}$

MSc. Ing. Franco Martin Pessana
E-mail: fpessana@favaloro.edu.ar

Tabla de Propiedades de la Transformada Z

Propiedades Comunes para TZB y TZU		
#	$f_{[n]} = \dfrac{1}{2\pi j}\oint_C F(z) z^{n-1} dz\ ,\ n \in Z$	$F_b(z) = \sum_{n=-\infty}^{\infty} f_{[n]} z^{-n}\ ;\ F(z) = \sum_{n=0}^{\infty} f_{[n]} z^{-n}$
1	$a_1 f_{1[n]} + a_2 f_{2[n]}$	$a_1 F_1(z) + a_2 F_2(z)$
2	$\overline{f}_{[n]}$	$\overline{F}(\overline{z})$
3	$Re\{f_{[n]}\}$	$\frac{1}{2}\left[F(z) + \overline{F}(\overline{z})\right]$
4	$Im\{f_{[n]}\}$	$\frac{1}{2j}\left[F(z) - \overline{F}(\overline{z})\right]$
5	$n^k f_{[n]}$	$(-1)^k \left(z\dfrac{d}{dz}\right)^k F(z),\ k \in \aleph \cup \{0\}$
6	$a^{\pm n} f_{[n]},\ a \neq 0$	$F\left(a^{\mp 1} z\right)$
7	$\{f_{[n]}\} * \{g_{[n]}\}$	$F(z) \cdot G(z)$
8	$f_{[n]} = \dfrac{1}{2\pi j}\oint_\gamma F(z) z^{n-1} dz,\ n \in Z$	$F(z)$
9	$f_{[nT_S]} = \dfrac{T_S}{2\pi}\int_{-\pi/T_S}^{\pi/T_S} F(e^{j\omega T_S}) \cdot e^{jn\omega T_S} d\omega$	$F(z)$
10	$f_{[n]} \cdot g_{[n]}$	$\dfrac{1}{2\pi j}\oint_\gamma \dfrac{1}{\omega} F(\omega) \cdot G\left(\dfrac{z}{\omega}\right) d\omega$
Propiedades Exclusivas de TZB		
#	$f_{[n]} = \dfrac{1}{2\pi j}\oint_C F(z) z^{n-1} dz\ ,\ n \in Z$	$F_b(z) = \sum_{n=-\infty}^{\infty} f_{[n]} z^{-n}$
1	$f_{[n+a]}$	$z^a F(z)$
2	$f_{[n-a]}$	$z^{-a} F(z)$
3	$f_{[-n]}$	$F\left(\dfrac{1}{z}\right)$
4	$\dfrac{f_{[n]}}{n+a}$	$-z^a \int z^{-1-a} F(z) dz$
5	$\overline{f}_{[-n]}$	$\overline{F}\left(\dfrac{1}{\overline{z}}\right)$
Propiedades Exclusivas de TZU		
#	$f_{[n]} = \dfrac{1}{2\pi j}\oint_C F(z) z^{n-1} dz\ ,\ n \in \aleph \cup \{0\}$	$F(z) = \sum_{n=0}^{\infty} f_{[n]} z^{-n}$
1	$f_{[n+a]}$	$z^a\left[F(z) - f_{[0]} - z^{-1} f_{[1]} - \cdots - z^{-a+1} f_{[a-1]}\right],\ a \in \aleph$
2	$f_{[n-a]}$	$z^{-a} F(z),\ a \in \aleph$
3	$f_{[n-a]}$	$z^{-a} F(z) + f_{[-a]} + z^{-1} f_{[-a+1]} + z^{-2} f_{[-a+2]} + \cdots + z^{-a+1} f_{[-1]}$
4	$f_{[0]}$	$\lim_{z \to \infty} F(z)$
5	$\lim_{n \to \infty} f_{[n]}$	$\lim_{z \to 1} \left(1 - z^{-1}\right) F(z)$
6	$\nabla f_{[n]}$	$\dfrac{z-1}{z} F(z)$
7	$\nabla^m f_{[n]}$	$\left(\dfrac{z-1}{z}\right)^m F(z)$
8	$\Delta f_{[n]}$	$(z-1) F(z) - z f_{[0]}$
9	$\Delta^m f_{[n]}$	$(z-1)^m F(z) - z \sum_{k=0}^{m-1} (z-1)^{m-k-1} \Delta^k f_{[0]}$
10	$\sum_{k=0}^{n} f_{[k]}$	$\dfrac{z}{z-1} F(z)$
11	$\dfrac{f_{[n]}}{n},\ n \in \aleph$	$\int_z^{\infty} \dfrac{F(z')}{z'} dz' + \lim_{n \to \infty} \dfrac{f_{[n]}}{n}$

MSc. Ing. Franco Martin Pessana
E-mail: fpessana@favaloro.edu.ar

Siendo: $\begin{cases}\nabla f_{[n]} = f_{[n]} - f_{[n-1]} \\ \Delta f_{[n]} = f_{[n+1]} - f_{[n]}\end{cases}$

Identidad de Parseval

$$\sum_{n=-\infty}^{\infty} f_{[n]}\bar{g}_{[n]} = \frac{1}{2\pi j}\oint_{\gamma} F(\omega)\cdot G\left(\frac{1}{\omega}\right)\omega^{-1}d\omega$$

$$\sum_{n=-\infty}^{\infty} \left|f_{[n]}\right|^2 = \frac{1}{2\pi j}\oint_{\gamma} F(\omega)\cdot F\left(\frac{1}{\omega}\right)\omega^{-1}d\omega$$

Pares de Transformadas Z Unilateral

#	$f_{[n]} = \frac{1}{2\pi j}\oint_C F(z)\cdot z^{n-1}dz\ ,\ n\in \aleph \cup \{0\}$	$F(z) = \sum_{n=0}^{\infty} f_{[n]}z^{-n}$	**Región de Convergencia**
1	$\delta[n]$	1	$\forall z\in \mathcal{Z}$
2	$\delta[n-m]$	z^{-m}	$\forall z\in \mathcal{Z}-\{0\}$
3	$u[n]$	$\frac{z}{z-1}$	$\lvert z\rvert > 1$
4	$n\cdot u[n]$	$\frac{z}{(z-1)^2}$	$\lvert z\rvert > 1$
5	$n^2\cdot u[n]$	$\frac{z(z+1)}{(z-1)^3}$	$\lvert z\rvert > 1$
6	$a^n\cdot u[n]$	$\frac{z}{z-a}$	$\lvert z\rvert > \lvert a\rvert$
7	$n\cdot a^{n-1}\cdot u[n]$	$\frac{z}{(z-a)^2}$	$\lvert z\rvert > \lvert a\rvert$
8	$(n+1)\cdot a^n\cdot u[n]$	$\frac{z^2}{(z-a)^2}$	$\lvert z\rvert > \lvert a\rvert$
9	$\frac{(n+1)(n+2)\cdots(n+m)}{m!}a^n\cdot u[n]$	$\frac{z^{m+1}}{(z-a)^{m+1}}$	$\lvert z\rvert > \lvert a\rvert$
10	$cos(\Omega_0 n)\cdot u[n]$	$\frac{z(z-\cos\Omega_0)}{z^2-2z\cos\Omega_0+1}$	$\lvert z\rvert > 1$
11	$sen(\Omega_0 n)\cdot u[n]$	$\frac{z\,sen\,\Omega_0}{z^2-2z\cos\Omega_0+1}$	$\lvert z\rvert > 1$
12	$a^n\cdot cos(\Omega_0 n)\cdot u[n]$	$\frac{z(z-a\cos\Omega_0)}{z^2-2az\cos\Omega_0+a^2}$	$\lvert z\rvert > \lvert a\rvert$
13	$a^n\cdot sen(\Omega_0 n)\cdot u[n]$	$\frac{az\,sen\,\Omega_0}{z^2-2az\cos\Omega_0+a^2}$	$\lvert z\rvert > \lvert a\rvert$
14	$e^{-anT}u[n]$	$\frac{z}{z-e^{-aT}}$	$\lvert z\rvert > e^{-aT}$
15	$e^{-anT}\cdot cos(n\omega_0 T)\cdot u[n]$	$\frac{z(z-e^{-aT}\cos\omega_0 T)}{z^2-2ze^{-aT}\cos\omega_0 T+e^{-2aT}}$	$\lvert z\rvert > e^{-aT}$
16	$e^{-anT}\cdot sen(n\omega_0 T)\cdot u[n]$	$\frac{ze^{-aT}\,sen\,\omega_0 T}{z^2-2ze^{-aT}\cos\omega_0 T+e^{-2aT}}$	$\lvert z\rvert > e^{-aT}$
17	$senh(\Omega_0 n)\cdot u[n]$	$\frac{z\,senh\,\Omega_0}{z^2-2z\cosh\Omega_0+1}$	$\lvert z\rvert > e^{-\Omega_0}$
18	$cosh(\Omega_0 n)\cdot u[n]$	$\frac{z(z-\cosh\Omega_0)}{z^2-2z\cosh\Omega_0+1}$	$\lvert z\rvert > e^{-\Omega_0}$

Expresiones útiles

$sen^2\,\theta + cos^2\,\theta = 1$

$cos^2\,\theta - sen^2\,\theta = cos\,2\theta$

$cos^2\,\theta = \frac{1}{2}(1+cos\,2\theta)$

$sen^2\,\theta = \frac{1}{2}(1-cos\,2\theta)$

$sen(\alpha\pm\beta) = sen\,\alpha\cdot cos\,\beta \pm cos\,\alpha\cdot sen\,\beta$

$cos(\alpha\pm\beta) = cos\,\alpha\cdot cos\,\beta \mp sen\,\alpha\cdot sen\,\beta$

$sen\,\alpha\cdot sen\,\beta = \frac{1}{2}cos(\alpha-\beta) - \frac{1}{2}cos(\alpha+\beta)$

$cos\,\alpha\cdot cos\,\beta = \frac{1}{2}cos(\alpha-\beta) + \frac{1}{2}cos(\alpha+\beta)$

$sen\,\alpha\cdot cos\,\beta = \frac{1}{2}sen(\alpha-\beta) + \frac{1}{2}sen(\alpha+\beta)$

MSc. Ing. Franco Martin Pessana
E-mail: fpessana@favaloro.edu.ar

Tabla de Propiedades Transformada de Fourier de una Secuencia (TFS)

#	$\boldsymbol{f[n]} = \frac{1}{2\pi}\int_{-\pi}^{\pi} F\left(e^{j\omega}\right)e^{jn\omega}d\omega$	$\boldsymbol{F(e^{j\omega})} = \sum_{n=-\infty}^{\infty} f_{[n]}\cdot e^{-jn\omega}$
1	$a_1 f_{1[n]} + a_2 f_{2[n]}$	$a_1F_1(e^{j\omega}) + a_2F_2(e^{j\omega})$
2	$f[n \mp n_0]$	$F\left(e^{j\omega}\right)\cdot e^{\mp jn_0\omega}$
3	$f[n]\cdot e^{\pm jn\omega_0}$	$F\left(e^{j(\omega\mp\omega_0)}\right)$
4	$\overline{f}[n]$	$\overline{F}\left(e^{-j\omega}\right)$
5	$f[-n]$	$F\left(e^{-j\omega}\right)$
6	$\overline{f}[-n]$	$\overline{F}\left(e^{j\omega}\right)$
7	***Sobremuestrestreador:*** $f_{(L)}[n] = \begin{cases} f[n/L] & si \quad n = kL, k\in \mathrm{Z} \\ 0, & si \quad n \neq kL, k \in \mathrm{Z}\end{cases}$	$F\left(e^{j\omega L}\right)$
8	***Submuestreador:*** $g[n] = f[nM]$	$G\left(e^{j\omega}\right) = \frac{1}{M}\sum_{l=0}^{M-1} F\left(e^{j((\omega-2\pi l)/M)}\right)$
9	$x[n] * h[n]$	$X(e^{j\omega})\cdot H(e^{j\omega})$
10	$x[n]\cdot h[n]$	$\frac{1}{2\pi}\int_{-\pi}^{\pi} X\left(e^{j\theta}\right)\cdot H\left(e^{j(\omega-\theta)}\right)d\theta$
11	$f[n] - f[n-1]$	$\left(1-e^{-j\omega}\right)F\left(e^{j\omega}\right)$
12	$\sum_{k=-\infty}^{n} f[k]$	$\left(1-e^{-j\omega}\right)^{-1}F\left(e^{j\omega}\right)$
13	$nf[n]$	$j\,\frac{dF\left(e^{j\omega}\right)}{d\omega}$
14	$f[n] = \delta[n]$	$F\left(e^{j\omega}\right) = 1$
15	$f[n] = \delta[n-n_0]$	$F\left(e^{j\omega}\right) = e^{-jn_0\omega}$
16	$f[n] = \sum_{k=-\infty}^{\infty}\delta[n-kN]$	$F\left(e^{j\omega}\right) = \frac{2\pi}{N}\sum_{k=-\infty}^{\infty}\delta\left(\omega - \frac{2\pi k}{N}\right)$
17	$f[n] = e^{j\omega_0 n}$	$F\left(e^{j\omega}\right) = 2\pi\sum_{k=-\infty}^{\infty}\delta(\omega-\omega_0-2\pi k)$
18	***Serie Discreta de Fourier:*** $f[n] = \sum_{k=0}^{N-1} a_k e^{j\frac{2\pi nk}{N}}$	$F\left(e^{j\omega}\right) = 2\pi\sum_{k=-\infty}^{\infty} a_k\delta\left(\omega-\frac{2\pi k}{N}\right)$
19	***Relación de Parseval para señales aperiódicas:*** $E = \sum_{n=-\infty}^{\infty}\left\|f[n]\right\|^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi}\left\|F\left(e^{j\omega}\right)\right\|^2 d\omega$	

Tabla de Propiedades de Transformada Discreta de Fourier (TDF)

#	$\boldsymbol{f[n]} = \frac{1}{N}\sum_{k=o}^{N-1} F_{[k]}\cdot e^{j\frac{2\pi\cdot n\cdot k}{N}}$ ***;*** $n = 0,1,2,3,\cdots,N-1$	$\boldsymbol{F[k]} = \sum_{n=o}^{N-1} f_{[n]}\cdot e^{-j\frac{2\pi\cdot n\cdot k}{N}}$ ***;*** $k = 0,1,2,3,\cdots,N-1$
1	$a_1 f_{1[n]} + a_2 f_{2[n]}$	$a_1F_1[k] + a_2F_2[k]$
2	$f([n\mp n_0])_N$	$F[k]\cdot W_N^{\pm n_0 k}$ con $W_N = e^{-j\frac{2\pi}{N}}$
3	$f[n]\cdot W_N^{\pm k_0 n}$	$F([k\pm k_0])_N$ con $W_N = e^{-j\frac{2\pi}{N}}$
4	$x[n]\otimes h[n] = \sum_{l=0}^{N-1} x[l]\cdot h([n-l])_N$, con $n = 0,1,2,3,\cdots,N-1$	$X[k]\cdot H[k]$
5	$x[n]\cdot h[n]$	$\frac{1}{N}\sum_{l=0}^{N-1} X[l]\cdot H([k-l])_N$, con $k = 0,1,2,3,\cdots,N-1$
6	$\overline{f}[n]$	$\overline{F}([-k])_N$
7	$\overline{f}([-n])_N$	$\overline{F}[k]$

MSc. Ing. Franco Martin Pessana
E-mail: fpessana@favaloro.edu.ar