

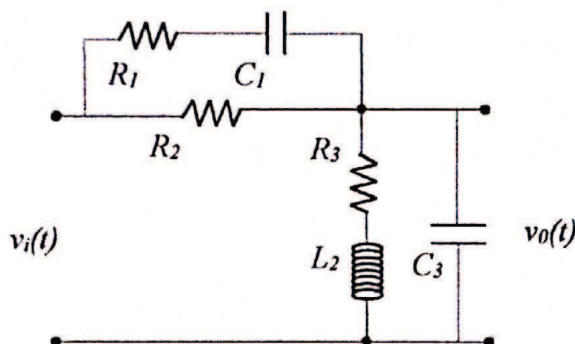


TEORÍA DE CIRCUITOS I

Facultad Regional De Buenos Aires. Departamento de Electrónica

Primer Examen Parcial Integrador. 8 de julio de 2017

Teniendo en cuenta el siguiente *circuito eléctrico*, se pide resolver los primeros 5 incisos del examen integrador, a saber:



1. Realice un *lugar geométrico* de la *impedancia total* del circuito, teniendo en cuenta para este inciso que $C_3 = 0$. La única variable del circuito es la frecuencia angular de la excitación $v_i(t) = V_p \sin(\omega t + \theta_v)$, con $0 \leq \omega < \infty$. Tenga en cuenta en su gráfico cualitativo que debe presentarse al menos una frecuencia de resonancia en el circuito.
2. Verifique analíticamente lo obtenido en el inciso anterior, determinando la/s posibles *frecuencia/s angular/es* para las cuales el circuito presenta un factor de potencia unitario, así también, como la relación entre los componentes circuitales para que ello ocurra. Considere para sus cálculos son $R_1 = 10\Omega$, $R_2 = 1M\Omega$, $R_3 = 1K\Omega$, $L_2 = 10\text{ mHy}$, $C_1 = 100\text{ }\mu\text{F}$
3. Calcule las *ecuaciones de estado* del circuito eléctrico completo, considerando como entrada la tensión aplicada $v_i(t)$ y como vector de salida $Y(t)$ la tensión de salida $v_o(t)$, la corriente en la resistencia R_2 , la corriente del capacitor C_1 , la corriente en el inductor L_2 y la caída de potencial en L_2 . El modelo obtenido debe contener las matrices de estado A , B , C y D .
4. Se aplica al circuito una señal senoidal $v_i(t) = V_0 + V_p \sin(\omega t + \theta_v)$, siendo $V_0 = 10\text{ V}$, $V_p = 100\sqrt{2}\text{ V}$, $\omega = 1000\text{ r/s}$ y $\theta_v = 30^\circ$. Los componentes del circuito son $R_1 = 10\Omega$, $R_2 = 1M\Omega$, $R_3 = 1K\Omega$, $L_2 = 10\text{ mHy}$, $C_1 = 100\text{ }\mu\text{F}$ y $C_3 = 1\text{ }\mu\text{F}$.
 - a) Calcule la *potencia total disponible* en el circuito.
 - b) Calcule la *potencia activa y reactiva total* del circuito.
 - c) ¿La *potencia de deformación* es nula? Si no es así, calcule su valor.
5. Calcule la *ecuación diferencial* que relaciona la tensión de entrada $v_i(t)$ con la tensión de salida $v_o(t)$, considerando en este inciso que $L_2 = 0$. El resultado quedará en función de los componentes circuitales, es decir R_1 , R_2 , R_3 , C_1 y C_3 . Si la excitación de esta ecuación diferencial fuera $v_i(t) = Vu(t)$, considerando condiciones iniciales nulas, calcule los valores de $i_{R_2}(0)$, $i_{R_3}(0)$, $i_{R_1}(t \rightarrow \infty)$ y $v_o(t \rightarrow \infty)$.
6. A partir del siguiente *modelo de estado*, se pide *reconstruir el circuito eléctrico* que le dio origen:

$$\begin{cases} \dot{X}(t) = AX(t) + Bv_i(t) \\ v_o(t) = CX(t) + Dv_i(t) \end{cases}, \text{ siendo } A = \begin{bmatrix} \frac{-1}{RC} & \frac{-1}{C} \\ \frac{1}{L} & 0 \end{bmatrix}, B = \begin{bmatrix} \frac{1}{RC} \\ 0 \end{bmatrix}, C = [-1 \quad 0] \text{ y } D = [1].$$

1) (1.5 puntos)	2) (1.5 puntos)	3) (2.0 puntos)	4) (2.0 puntos)	5) (2.0 puntos)	6) (1.0 puntos)
Apellido y Nombres:				Nota:	

Primer examen Parcial de
Teoría de Circuitos I

3) en: V_i

$$Y(t) = [V_0(t) \ i_{R2} \ i_{C1} \ i_{L2} \ V_{L2}]^T$$

$$V_0 = V_{C3} = I_{R3} + V_{L2}$$

$$\begin{cases} \dot{X}(t) = A X(t) + B V_i(t) \\ Y(t) = C X(t) + D V_i(t) \end{cases}$$

$$i_{R1} = i_{C1}$$

$$i_{R3} = i_{L2}$$

$$X_1 = V_{C1}, X_2 = i_{L2}, X_3 = V_{C3}$$

$$i_{R1} + i_{R2} = i_{R3} + i_{C3}$$

$$i_{C1} + i_{R2} = i_{L2} + i_{C3}$$

$$i_{C1} + i_{R2} = i_{L2} + i_{C3}$$

$$i_{R1} + \frac{(V_i - V_{C3})}{R_2} = i_{L2} + C_3 \frac{dV_{C3}}{dt}$$

$$V_i = V_{R2} + V_0$$

$$V_i = V_{R2} + V_{C3}$$

$$= V_{R2} + V_{R3} + V_{L2}$$

$$\frac{(V_{R2} - V_{C1})}{R_1} + \frac{V_i}{R_2} - \frac{V_{C3}}{R_2} = i_{L2} + C_3 \frac{dV_{C3}}{dt}$$

$$\frac{(V_i - V_{C3})}{R_1} - \frac{V_{C1}}{R_1} + \frac{V_i}{R_2} - \frac{V_{C3}}{R_2} = i_{L2} + C_3 \frac{dV_{C3}}{dt}$$

$$\left(\frac{V_i}{R_1} - \frac{X_3}{R_1} - \frac{X_1}{R_1} + \frac{V_i}{R_2} - \frac{X_3}{R_2} \right) = \dot{X}_2 + C_3 \dot{X}_3$$

$$V_i \left(\frac{1}{R_1} + \frac{1}{R_2} \right) - X_3 \left(\frac{1}{R_1} + \frac{1}{R_2} \right) - \frac{X_1}{R_1} - X_2 = C_3 \dot{X}_3$$

$$\dot{X}_3 = V_i \left(\frac{1}{R_1 C_3} + \frac{1}{R_2 C_3} \right) - X_3 \left(\frac{1}{R_1 C_3} + \frac{1}{R_2 C_3} \right) - \frac{X_1}{R_1 C_3} - \frac{X_2}{C_3}$$

$$X_2 = i_{L2} = \frac{1}{L_2} \int V_{L2} dt$$

$$L_2 \frac{di_{L2}}{dt} = V_{C3} - V_{R3} = V_{C3} - i_{R3} R_3 = V_{C3} - i_{L2} R_3$$

$$L_2 \dot{X}_2 = X_3 - X_2 R_3$$

$$X_1 = V_{C1} \Rightarrow C_1 \frac{dV_{C1}}{dt} = i_{R1}$$

$$\dot{X}_2 = -X_2 \frac{R_3}{L_2} + \frac{X_3}{L_2}$$

$$C_1 \frac{dV_{C1}}{dt} = \frac{(V_i - V_{C3} - V_{C1})}{R_1}$$

$$C_1 \dot{X}_1 = \frac{V_i}{R_1} - \frac{X_3}{R_1} - \frac{X_1}{R_1}$$

=

$$\dot{X}_1 = \frac{V_i}{R_1 C_1} - \frac{X_3}{R_1 C_1} - \frac{X_1}{R_1 C_1}$$

⇒

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -1/R_1 C_1 & 0 & -1/R_1 C_1 \\ 0 & -R_3/L_2 & 1/L_2 \\ -1/R_1 C_3 & -1/C_3 & \frac{1}{R_1 C_3} - \frac{1}{R_1 C_3} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1/R_1 C_1 \\ 0 \\ \frac{1}{R_1 C_3} + \frac{1}{R_1 C_3} \end{bmatrix} V_i(t)$$

$$V_o = V_{C3} = x_3$$

$$\begin{bmatrix} V_o(t) \\ i_{R2} \\ i_{C1} \\ i_{L2} \\ V_{L2} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & -1/R_2 \\ -1/R_1 & 0 & -1/R_1 \\ 0 & 1 & 0 \\ 0 & -R_3 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$+ \begin{bmatrix} 0 \\ 1/R_2 \\ -1/R_1 \\ 0 \\ 0 \end{bmatrix} V_i(t)$$

$$A_{P2} = \frac{V_i - V_{C3}}{R_2}$$

$$i_{C1} = i_{R1} = \frac{V_i - V_{C3} - V_{C1}}{R_1}$$

$$i_{L2} = x_2$$

$$V_{L2} = V_{C3} - V_{R3} = V_{C3} - A_{P2} R_3$$

$$\begin{cases} V_o = x_3 \\ A_{P2} = -\frac{V_{C3}}{R_2} + \frac{V_i}{R_2} \end{cases}$$

$$i_{C1} = -\frac{V_{C1}}{R_1} - \frac{V_{C3}}{R_1} - \frac{V_i}{R_1}$$

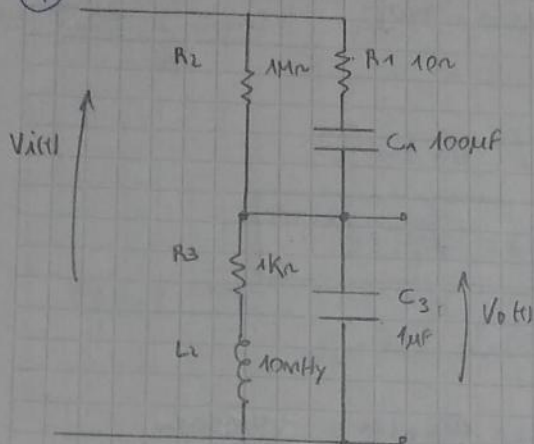
$$i_{L2} = x_2$$

$$V_{L2} = V_{C3} - i_{L2} R_3$$

Człote
Federico

MEGA 2/4

(4)



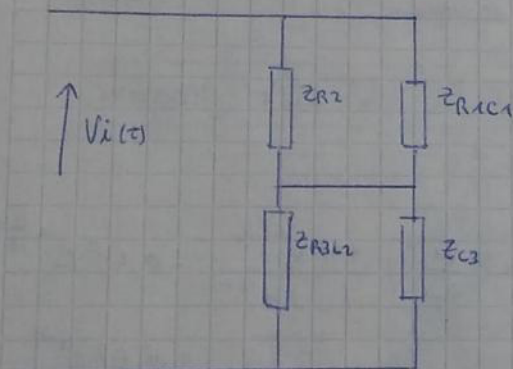
$$V_i(t) = 10V + 100\sqrt{2}V \sin(1000t + \pi/6)$$

$$\omega_{C1} = 0,1$$

$$\omega_{C3} = \frac{1}{1000}$$

$$\omega_{L2} = 10$$

→ Cálculo impedancias: ($p_{R2} = 100\sqrt{2} \sin(1000t + \pi/6) V$)



$$Z_{R1C1} = Z_{C1} + R_1 = -j\frac{1}{\omega_{C1}} + R_1$$

$$Z_{R1C1} = \left(-j\frac{1}{0,1} + 10\right) \Omega$$

$$Z_{R1C1} = (10 - 10j) \Omega = 10\sqrt{2} e^{-j\pi/4} \Omega$$

$$Z_{R2} = R_2 = 1M\Omega$$

$$\frac{1}{Z_{p1}} = \frac{1}{R_2} + \frac{1}{Z_{R1C1}} \Rightarrow \frac{R_2 \cdot Z_{R1C1}}{R_2 + Z_{R1C1}} = Z_{p1}$$

$$Z_{p1} = \frac{1M\Omega \cdot 10\sqrt{2} e^{-j\pi/4} \Omega}{(1M\Omega + (10 - 10j) \Omega)} = \frac{1 \cdot 10\sqrt{2} e^{-j\pi/4} \Omega^2}{1 \cdot 10^0 \Omega}$$

$$Z_{p1} \approx 10\sqrt{2} e^{-j\pi/4} \Omega = (10 - 10j) \Omega$$

$$Z_{C3} = Z_{C3} = -j\frac{1}{\omega_{C3}} = -j\frac{1}{1000} \Omega =$$

$$Z_{R3L2} = R_3 + Z_{L2} = R_3 + j\omega_{L2}$$

$$Z_{R3L2} = (1000 + j10) \Omega \approx 1000,25 e^{j0,57^\circ}$$

$$Z_{p2} = \frac{\left(-\frac{1}{\omega_{C3}}\right)j \cdot (1000 + j10)}{-j\frac{1}{\omega_{C3}} + 1000 + j10}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1/R_1 & 0 & -1/R_1 \\ 0 & -R_3/L_2 & 1/L_2 \\ -1/R_3 & -1/C_2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1/R_1 \\ 0 \\ 0 \end{bmatrix} V_{in}(t)$$

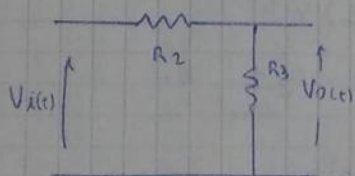
$$Z_{p2} = \frac{\left(-j \frac{1}{1000}\right)(1000 + 10j)}{j\left(10 - \frac{1}{1000}\right) + 1000} = \frac{(10000 - 110^6j)}{1000 - 990j} \approx \frac{505 - 509j}{710,68} \approx 710 e^{-i\pi/4} [2]$$

$$\Rightarrow Z_T = Z_{p1} + Z_{p2} \Rightarrow Z_T = (10 - 10j)\Omega + (505 - 509j)\Omega$$

$$Z_T = (515 - 519j)\Omega \approx 724,8 e^{-i\pi/4}$$

$$V_0 = 10V, \omega = 0$$

→ el circuito para continua: $\Rightarrow R_T = R_2 + R_3 = 1k\Omega + 1k\Omega = 1001000\Omega$



$$I_0 = \frac{V_0}{R_T} = \frac{10}{1001000} = 9,9\mu A \approx 10\mu A$$

$$\text{Luego: } P_0 = V_0 I_0 = 10V \cdot 10\mu A = 100\mu W$$

$$\omega = 1000$$

$$V_1 = 100V, Z_T = (515 - 519j) \Rightarrow \bar{I}_1 = \frac{V_1}{Z_T} = \frac{100 e^{i\pi/6}}{724,8 e^{-i\pi/4}} = 0,138 e^{i\frac{5}{12}\pi} = 0,138 e^{i75^\circ}$$

$$I_1 = 0,138 A$$

↑
coef. de 1º primer armónico (eficaz)

Luego

$$V_{ef} = \sqrt{(100)^2 + (0,138)^2}$$

$$V_{ef} = \sqrt{(10)^2 + (100)^2}$$

$$V_{ef} = 105 \checkmark$$

$$I_{ef} = \sqrt{(10\mu)^2 + (0,138)^2}$$

$$I_{ef} = 0,138 A$$

$$P_s = 14,5 \text{ W.A}$$

↑
potencia aparente

Potencia activa

$$P = P_0 + V_1 I_1 \cos(\varphi) \quad , \varphi = -\pi/4$$

$$P = 100 \mu W + 100 \cdot 0,138 \cdot \frac{\sqrt{2}}{2} (W)$$

$$\boxed{P \approx 9,76 W} \quad 9,75817$$

Potencia reactiva

$$P_q = V_1 I_1 \sin(\varphi)$$

$$P_q = 100 \cdot 0,138 \cdot \sin(-\pi/4)$$

$$\boxed{P_q \approx -9,76 VAR} \quad 9,75807$$

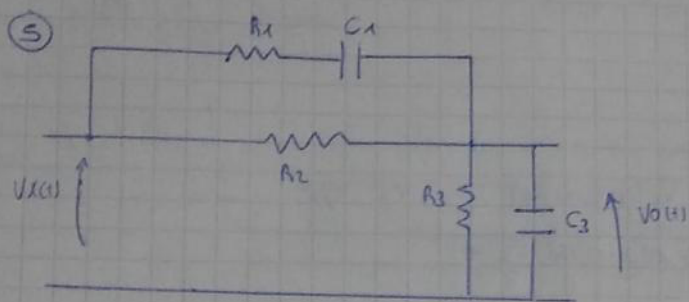
Potencia de deformación

$$P_d = \sqrt{P_s^2 - P_q^2 - P^2}$$

$$P_d = \sqrt{(14,5)^2 - (9,75817)^2 - (9,75807)^2}$$

$$\boxed{P_d = 4,45 VAd}$$

4,45

$$\Gamma_{\sqrt{2}} \quad \Gamma_{-1/2}$$
$$Z_p = 1, 1 \quad 1, 1$$


$$A_{R1} + A_{R2} = A_{R3} + A_{C3}$$

$$V_L \left(\frac{1}{R_1} + \frac{1}{R_2} \right) + V_L \cdot \frac{1}{R_2} = V_O C_3 + V_O \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{C_3}{R_1 C_1} \right) + V_O \left(\frac{1}{R_1 C_1 R_3} + \frac{1}{R_2} \right)$$

$$V_i \left(\frac{R_1 + R_2}{R_1 R_2} \right) + V_i \frac{1}{R_2} = V_o C_3 + V_o \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{C_3}{R_1 C_1} \right) + V_o \left(\frac{R_2 R_3 C_4 + 1}{R_1 R_2 R_3 C_3} \right)$$

$$(b) \quad (1) \quad \dot{X}_1 = -\frac{X_1}{RC} - \frac{X_2}{C} + \frac{V_i}{RC}, \quad \dot{X}_2 = \frac{X_1}{L} \quad (2)$$

$$(3) \quad V_o(t) = -X_1 + V_i(t)$$

de (1):

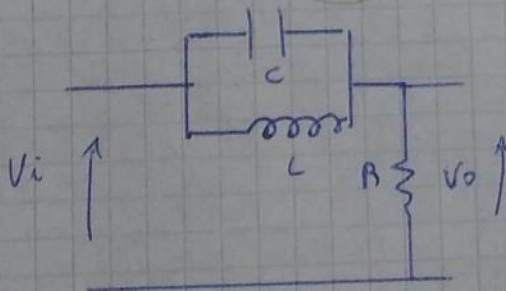
$$C \cdot \dot{X}_1 = -\frac{X_1}{R} - X_2 + \frac{V_i}{R}$$

$$\text{Si } X_1 = V_c, \quad X_2 = i_L$$

$$C \cdot \frac{dV_c}{dt} = -\frac{V_c}{R} - i_L + \frac{V_i}{R}$$

$$i_L + \frac{V_c}{R} + i_c = \frac{V_i}{R}$$

$$i_L + i_c = \frac{(V_i - V_c)}{R} \quad i_R$$

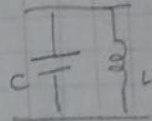


de (2):

$$L \dot{X}_2 = X_1$$

↓

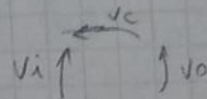
$$L \frac{di_L}{dt} = V_c = V_L$$



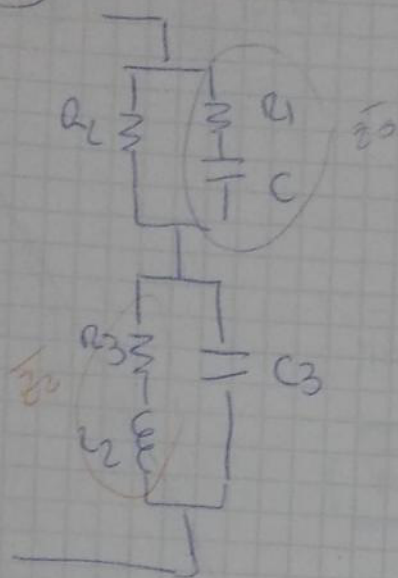
de (3)

$$V_o(t) = -V_c + V_i$$

$$V_i = V_o + V_c$$



4



$$u(t) = U_0 + U_p \sin(\omega t + \theta_U)$$

\downarrow \downarrow
 $10V$ $100\sqrt{2}V$

$$\omega = 1000 \text{ rad/s} \quad \theta_U = 30^\circ$$

$$R_1 = 10 \Omega$$

$$R_2 = 1 \cdot 10^6 \Omega$$

$$R_3 = 1k\Omega$$

$$L_2 = 10 \cdot 10^{-3} \text{ H}$$

$$C_1 = 100 \cdot 10^{-6} \text{ F}$$

$$C_3 = 1 \cdot 10^{-6} \text{ F}$$

2) $P_S = \vec{V} \cdot \vec{I}^*$

$$\underline{Y_{C3}} = j\omega C = j 1000 \cdot 1 \cdot 10^{-6} = j 1 \cdot 10^{-3}$$

$$\underline{Z_S} = R_1 + j \frac{1}{\omega C} = 10 \Omega - j \frac{1}{1000 \cdot 100 \cdot 10^{-6}}$$

$$\underline{Z_S} = 10 \Omega - j 10 = 9,85 e^{-j45^\circ}$$

$$\underline{Z_{R2}} = R_2 = 1 \cdot 10^6 \Omega$$

$$\underline{Y_1} = \frac{1}{R_2} + \frac{1}{\underline{Z_S}} = \frac{1}{1 \cdot 10^6 \Omega} + \frac{1}{9,85 e^{-j45^\circ}}$$

$$\underline{Y_1} = 1 \cdot 10^{-6} + 0,10 e^{j45^\circ}$$

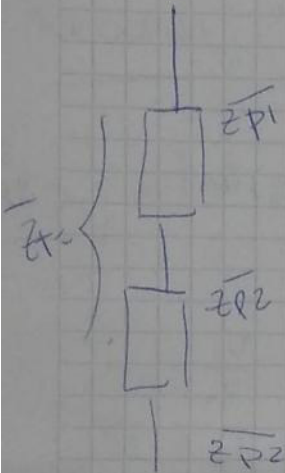
$\underbrace{0,07 + j0,07}$

$$\underline{Y_1} = (70 \cdot 10^{-3} + j 0,07) \Omega$$

$$\underline{Z_2} = R_3 + j\omega L = 1 \cdot 10^3 \Omega + j 1000 \cdot 10 \cdot 10^{-3} \text{ H}$$

$$\underline{Z_L} = 1 \cdot 10^3 \Omega + j 10 = 1 \cdot 10^3 e^{j0,57^\circ}$$

$$\begin{aligned}\bar{Y}_2 &= \frac{1}{\bar{Z}_2} + \bar{Y}_3 = \frac{1}{10^3 e^{j0,57^\circ}} + j 10^{-3} \\ &= 10^{-3} e^{-j0,57^\circ} + j 10^{-3} \\ &= 0,99 \cdot 10^{-3} - j 9,9 \cdot 10^{-6} + j 10^{-3} \\ \bar{Y}_2 &= 0,99 \cdot 10^{-3} - j 9,9 \cdot 10^{-4}\end{aligned}$$



$$\bar{Y}_2 = 1,4 \cdot 10^{-3} e^{-j45^\circ} \Omega$$

$$\bar{Z}_{p2} = \frac{1}{1,4 \cdot 10^{-3} e^{-j45^\circ}}$$

$$\bar{Z}_{p2} = 714,29 e^{j45^\circ} \Omega$$

$$\bar{Z}_{p2} = 505,1 + j 505,1$$

$$\bar{Y}_{p1} = (70 \cdot 10^{-3} + j 0,07) \Omega$$

$$\bar{Y}_{p1} = 0,099 e^{j45^\circ} \Omega$$

$$\bar{Z}_{p1} = \frac{1}{0,099 e^{j45^\circ}} \Omega$$

$$\bar{Z}_{p1} = 10 e^{-j45^\circ} \Omega$$

$$\bar{Z}_{p1} = 7,07 - j 7,07$$

$$\bar{Z}_T = \bar{Z}_{p1} + \bar{Z}_{p2} = 505,1 + j 505,1 + 7,07 - j 7,07$$

$$\bar{Z}_T = 512,17 + j 498,03 \Omega$$

$$\bar{Z} = (512,17 + j498,03) \Omega = 714,39 \angle 44,2^\circ$$

a)

$$P_S = \sqrt{\sum_{n=0}^{+\infty} U_n^2} \sqrt{\sum_{n=0}^{+\infty} I_n^2} = \sqrt{10 \cdot 100^2} \sqrt{0,02^2} = 14,21$$

$$\sum_{n=0}^{+\infty} U_n^2 = U_0^2 + U_1^2 = (10V)^2 + (100V)^2 = 10.100 V^2$$

$$\sum_{n=0}^{+\infty} I_n^2 = I_0^2 + I_1^2 = (0,014A)^2 + (0,14A)^2 = 0,02^2$$

$$\bar{Z} = \frac{\bar{V}}{\bar{I}} \Rightarrow \bar{I} = \frac{\bar{V}}{\bar{Z}} \Rightarrow \bar{I}_0 = \frac{V_0}{Z_1} \quad \bar{I}_1 = \frac{V_1}{Z_1}$$

$$\bar{I}_0 = \frac{10V}{714,39} = 0,014 A$$

$$\bar{I}_1 = \frac{100}{714,39} = 0,14 A$$

b)

$$\cos \theta_Z = \frac{P}{P_S} \Rightarrow P = 10 W$$

$$P_S = 14,21 \text{ VAR}$$

$$\theta_Z = 44,2^\circ$$

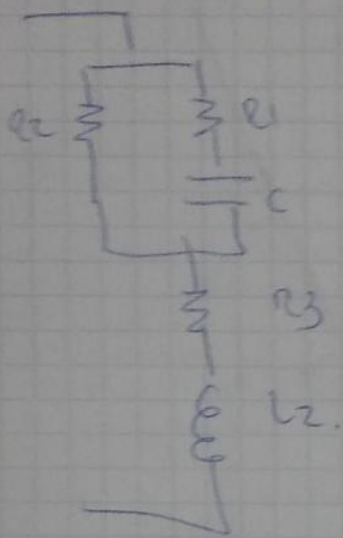
$$\sin \theta_Z = \frac{P_Q}{P_S} \Rightarrow P_Q = 10 \text{ VAR}$$

$$c) P_D^2 = P_S^2 - P^2 - P_Q^2$$

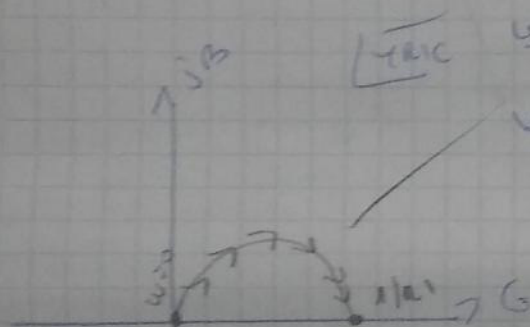
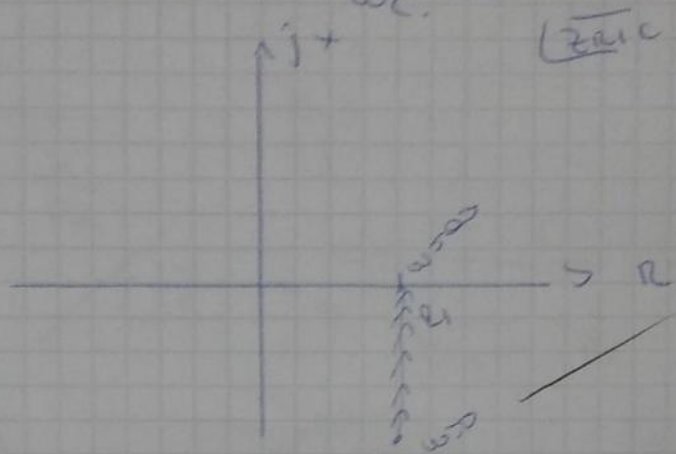
$$P_D^2 = 1,9241$$

$$P_D = 1,38$$

$$0 \leq \omega < \infty$$



$$\bar{z}_{RLC} = R_1 - j \frac{1}{\omega C}$$



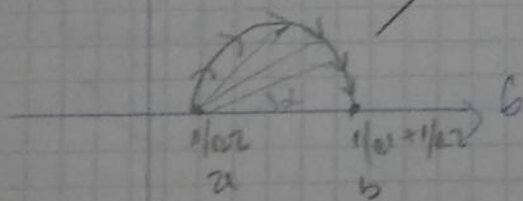
$$\bar{z} = R_1 - j\infty \quad \omega=0$$

$$\bar{z} = R_1 - j0 \quad \omega=\infty$$

$$\bar{y}_{RLC} = \frac{1}{R_1 - j \frac{1}{\omega C}}$$

$$\omega=0 \quad \bar{y} = \frac{1}{R_1 - j\infty}$$

$$\omega=\infty \quad \bar{y} = \frac{1}{R_1 - j0}$$



$$\bar{y}_{RLC} + \bar{y}_{RL} = \frac{1}{R_1 - j \frac{1}{\omega C}} + \frac{1}{R_2}$$

$$\omega=0 \quad \bar{y} = \frac{1}{R_1 - j\infty} + \frac{1}{R_2}$$

$$\omega=\infty \quad \bar{y} = \frac{1}{R_1 - j0} + \frac{1}{R_2}$$

$$\overline{Y}_{E1C} + \overline{Y}_{E2} = \frac{1}{R_1 - j\frac{1}{\omega C}} + \frac{1}{R_2}$$

$$\overline{Z} = \frac{1}{\frac{1}{R_1 - j\frac{1}{\omega C}} + \frac{1}{R_2}} + R_3$$

$$\begin{aligned} \omega=0 & \quad \frac{1}{R_1 - j\frac{1}{\omega C}} + \frac{1}{R_2} \\ \omega \rightarrow \infty & \quad \frac{1}{R_1 - j\frac{1}{\omega C}} + \frac{1}{R_2} \\ & \quad \frac{1}{R_1} + \frac{1}{R_2} + R_3 \end{aligned}$$

$$\begin{aligned} \omega=0 & \quad \frac{1}{R_1 - j\frac{1}{\omega C}} + \frac{1}{R_2} \\ & \quad \frac{1}{R_1 - j0} + \frac{1}{R_2} \\ & \quad \frac{1}{R_1} + \frac{1}{R_2} + R_3 \end{aligned}$$

$$\overline{Z}_{L2} = j\omega L_2$$

