Desvío de la suma:

$$Y = X_A + X_B$$

$$y_1 = x_{A1} + x_{B1}$$
$$y_2 = x_{A2} + x_{B2}$$
$$\dots$$
$$y_n = x_{An} + x_{Bn}$$

$$\bar{y} = \bar{x}_A + \bar{x}_B$$

$$s^2(y) = \frac{1}{n-1}\sum_{i=1}^{n}\left(y_i - \bar{y}\right)^2$$

$$s^2(y) = \frac{1}{n-1}\sum_{i=1}^{n}\left[\left(x_{Ai} + x_{Bi}\right) - \left(\bar{x}_A + \bar{x}_B\right)\right]^2$$

$$s^2(y) = \frac{1}{n-1}\sum_{i=1}^{n}\left[\left(x_{Ai} - \bar{x}_A\right) + \left(x_{Bi} - \bar{x}_B\right)\right]^2$$

$$s^2(y) = \frac{1}{n-1}\sum_{i=1}^{n}\left[\left(x_{Ai} - \bar{x}_A\right)^2 + \left(x_{Bi} - \bar{x}_B\right)^2 + 2\cdot\left(x_{Ai} - \bar{x}_A\right)\cdot\left(x_{Bi} - \bar{x}_B\right)\right]$$

$$s^2(y) = \frac{1}{n-1}\sum_{i=1}^{n}\left(x_{Ai} - \bar{x}_A\right)^2 + \frac{1}{n-1}\sum_{i=1}^{n}\left(x_{Bi} - \bar{x}_B\right)^2 + \frac{1}{n-1}\sum_{i=1}^{n}\left[2\cdot\left(x_{Ai} - \bar{x}_A\right)\cdot\left(x_{Bi} - \bar{x}_B\right)\right]$$

$$\boxed{s^2(y) = s^2(x_A) + s^2(x_B) + 2\cdot s(x_A, x_B)}$$

Si se tratan de variables independientes (no correlacionadas) la covarianza tiende a 0, entonces:

$$\boxed{s(y) = \sqrt{s^2(x_A) + s^2(x_B)}}$$

Desvío de la media:

$$Y = \frac{1}{m}\sum_{i=1}^{m} X_i$$

$$y_1 = \frac{x_1 + x_2 + ... + x_m}{m}$$

$$y_2 = \frac{x_2 + x_3 + ... + x_{1+m}}{m}$$

...

$$y_n = \frac{x_n + x_{n+1} + ... + x_{n-1+m}}{m}$$

$$\bar{y} = \bar{x}$$

$$s^2(y) = \frac{1}{n-1}\sum_{i=1}^{n}\left(y_i - \bar{y}\right)^2$$

$$s^2(y) = \frac{1}{n-1}\sum_{i=1}^{n}\left[\left(\frac{x_i + x_{i+1} + ... + x_{i-1+m}}{m}\right) - \bar{x}\right]^2$$

$$s^2(y) = \frac{1}{n-1}\sum_{i=1}^{n}\left[\left(\frac{x_i - \bar{x}}{m}\right) + \left(\frac{x_{i+1} - \bar{x}}{m}\right) + ... + \left(\frac{x_{i-1+m} - \bar{x}}{m}\right)\right]^2$$

$$s^2(y) = \frac{1}{(n-1)\cdot m^2}\sum_{i=1}^{n}\left[\left(x_i - \bar{x}\right) + \left(x_{i+1} - \bar{x}\right) + ... + \left(x_{i-1+m} - \bar{x}\right)\right]^2$$

$$s^2(y) = \frac{1}{(n-1)\cdot m^2}\sum_{i=1}^{n}\left[\left(x_i - \bar{x}\right)^2 + \left(x_{i+1} - \bar{x}\right)^2 + ... + \left(x_{i-1+m} - \bar{x}\right)^2 + 2\cdot\left(x_i - \bar{x}\right)\cdot\left(x_{i+1} - \bar{x}\right) + ... + 2\cdot\left(x_{i-1+m-1} - \bar{x}\right)\cdot\left(x_{i-1+m} - \bar{x}\right)\right]$$

$$s^2(y) = \frac{1}{(n-1)\cdot m^2}\sum_{i=1}^{n}\left[\left(x_i - \bar{x}\right)^2 + \left(x_{i+1} - \bar{x}\right)^2 + ... + \left(x_{i-1+m} - \bar{x}\right)^2\right]$$

$$s^2(y) = \frac{1}{(n-1)\cdot m^2}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2 + \frac{1}{(n-1)\cdot m^2}\sum_{i=1}^{n}\left(x_{i+1} - \bar{x}\right)^2 + ... + \frac{1}{(n-1)\cdot m^2}\sum_{i=1}^{n}\left(x_{i-1+m} - \bar{x}\right)^2$$

$$s^2(y) = \frac{{s_1}^2(x)}{m^2} + \frac{{s_2}^2(x)}{m^2} + ... + \frac{{s_m}^2(x)}{m^2}$$

$$\boxed{s^2(y) = \frac{s^2(x)}{m}}$$

Teorema Central del Límite:

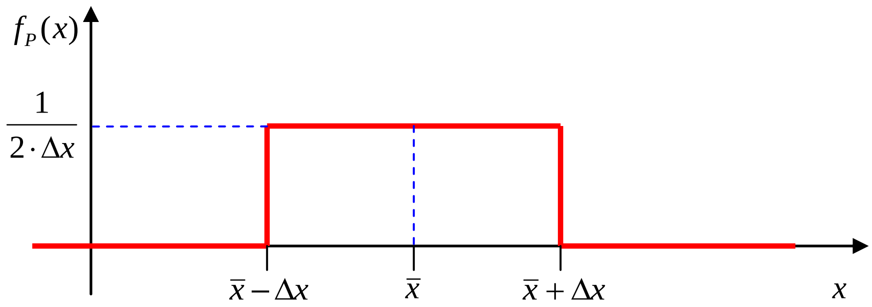
$$Y = \sum_{i=1}^{n} c_i \cdot X_i$$

$$E(Y) = \sum_{i=1}^{n} c_i \cdot E(X_i)$$

$$s^2(Y) = \sum_{i=1}^{n} c_i^{\,2} \cdot s^2(X_i)$$

Este teorema establece que la distribución de Y será aproximadamente normal, siempre que las X_i *sean independientes y* $s^2(Y)$ *sea mucho mayor que cualquier otro componente* $c_i^{\,2} \cdot s^2(X_i)$ *de una* X_i *cuya distribución no sea normal.*

Desvío Estándar de una función probabilidad cuadrada:



$$E(x)=\int_{-\infty}^{+\infty} x\cdot f_P(x)\cdot dx$$

$$E(x)=\int_{\bar{x}-\Delta x}^{\bar{x}+\Delta x} x\cdot\frac{1}{2\cdot\Delta x}\cdot dx=\frac{1}{2\cdot\Delta x}\cdot\int_{\bar{x}-\Delta x}^{\bar{x}+\Delta x} x\cdot dx=\frac{1}{2\cdot\Delta x}\cdot\left.\frac{x^2}{2}\right|_{\bar{x}-\Delta x}^{\bar{x}+\Delta x}=\frac{1}{2\cdot\Delta x}\cdot\frac{(\bar{x}+\Delta x)^2-(\bar{x}-\Delta x)^2}{2}=$$

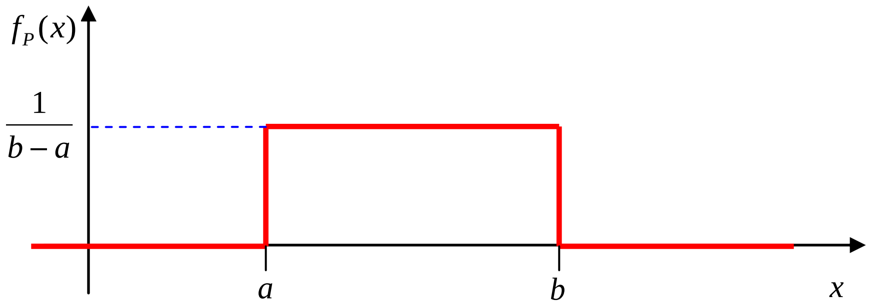
$$E(x)=\frac{1}{2\cdot\Delta x}\cdot\frac{\bar{x}^2+2\cdot\bar{x}\cdot\Delta x+\Delta x^2-\bar{x}^2+2\cdot\bar{x}\cdot\Delta x-\Delta x^2}{2}=\bar{x}$$

$$s^2(x)=\int_{-\infty}^{+\infty}(x-\bar{x})^2\cdot f_P(x)\cdot dx$$

$$s^2(x)=\int_{\bar{x}-\Delta x}^{\bar{x}+\Delta x}(x-\bar{x})^2\cdot\frac{1}{2\cdot\Delta x}\cdot dx=\frac{1}{2\cdot\Delta x}\cdot\int_{\bar{x}-\Delta x}^{\bar{x}+\Delta x}(x-\bar{x})^2\cdot dx=\frac{1}{2\cdot\Delta x}\cdot\left.\frac{(x-\bar{x})^3}{3}\right|_{\bar{x}-\Delta x}^{\bar{x}+\Delta x}=$$

$$s^2(x)=\frac{1}{2\cdot\Delta x}\cdot\frac{(\bar{x}+\Delta x-\bar{x})^3-(\bar{x}-\Delta x-\bar{x})^3}{3}=\frac{1}{2\cdot\Delta x}\cdot\frac{(\Delta x)^3-(-\Delta x)^3}{3}=\frac{1}{2\cdot\Delta x}\cdot\frac{2\cdot\Delta x^3}{3}=\frac{\Delta x^2}{3}$$

$$s(x)=\frac{\Delta x}{\sqrt{3}}$$



$$\bar{x} = \int_{-\infty}^{+\infty} x \cdot f_P(x) \cdot dx$$

$$\bar{x} = \int_a^b x \cdot \frac{1}{b-a} \cdot dx = \frac{1}{b-a} \cdot \int_a^b x \cdot dx = \frac{1}{b-a} \cdot \left. \frac{x^2}{2} \right|_a^b = \frac{1}{b-a} \cdot \frac{b^2 - a^2}{2} = \frac{1}{b-a} \cdot \frac{(b+a)\cdot(b-a)}{2}$$

$$\bar{x} = \frac{b+a}{2}$$

$$s^2(x) = \int_{-\infty}^{+\infty} (x-\bar{x})^2 \cdot f_P(x) \cdot dx$$

$$s^2(x) = \int_a^b (x-\bar{x})^2 \cdot \frac{1}{b-a} \cdot dx = \frac{1}{b-a} \cdot \int_a^b (x-\bar{x})^2 \cdot dx = \frac{1}{b-a} \cdot \left. \frac{(x-\bar{x})^3}{3} \right|_a^b =$$

$$s^2(x) = \frac{1}{b-a} \cdot \frac{(b-\bar{x})^3 - (a-\bar{x})^3}{3} = \frac{1}{b-a} \cdot \frac{\left(b - \frac{b+a}{2}\right)^3 - \left(a - \frac{b+a}{2}\right)^3}{3} = \frac{1}{b-a} \cdot \frac{\left(\frac{2b-b-a}{2}\right)^3 - \left(\frac{2a-b-a}{2}\right)^3}{3}$$

$$s^2(x) = \frac{1}{b-a} \cdot \frac{\left(\frac{b-a}{2}\right)^3 - \left(\frac{-b+a}{2}\right)^3}{3} = \frac{1}{b-a} \cdot \frac{(b-a)^3 - (-(b+a))^3}{24} = \frac{1}{b-a} \cdot \frac{(b-a)^3}{12} = \frac{(b-a)^2}{12}$$

$$s(x) = \frac{(b-a)}{\sqrt{12}}$$

Sesgo en el estimador de la varianza $s_n^{\,2}$ *:*

$$\sigma^2 = E(X-\mu)^2 = E\left(X^2 - 2\cdot X\cdot\mu + \mu^2\right) = E\left(X^2\right) - E(2\cdot X\cdot\mu) + E\left(\mu^2\right) =$$
$$\sigma^2 = E\left(X^2\right) - 2\cdot\mu\cdot E(X) + \mu^2 = E\left(X^2\right) - 2\cdot\mu^2 + \mu^2 =$$
$$\sigma^2 = E\left(X^2\right) - \mu^2$$

$$s_n^{\,2} = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$$
$$s_n^{\,2} = \frac{1}{n}\sum_{i=1}^{n}\left(x_i^{\,2} - 2\cdot x_i\cdot\bar{x} + \bar{x}^2\right) = \frac{1}{n}\left[\sum_{i=1}^{n}x_i^{\,2} - \sum_{i=1}^{n}(2\cdot x_i\cdot\bar{x}) + \sum_{i=1}^{n}\bar{x}^2\right]$$
$$s_n^{\,2} = \frac{1}{n}\left(\sum_{i=1}^{n}x_i^{\,2} - 2\cdot\bar{x}\cdot\sum_{i=1}^{n}x_i + \sum_{i=1}^{n}\bar{x}^2\right) = \frac{1}{n}\left(\sum_{i=1}^{n}x_i^{\,2} - 2\cdot\bar{x}\cdot n\cdot\bar{x} + n\cdot\bar{x}^2\right)$$
$$s_n^{\,2} = \frac{1}{n}\left(\sum_{i=1}^{n}x_i^{\,2} - 2\cdot n\cdot\bar{x}^2 + n\cdot\bar{x}^2\right) = \frac{1}{n}\left(\sum_{i=1}^{n}x_i^{\,2} - n\cdot\bar{x}^2\right) == \frac{1}{n}\left(\sum_{i=1}^{n}x_i^{\,2}\right) - \bar{x}^2$$

$$E\left(s_n^{\,2}\right) = E\left[\frac{1}{n}\cdot\left(\sum_{i=1}^{n}x_i^{\,2} - n\cdot\bar{x}^2\right)\right]$$
$$E\left(s_n^{\,2}\right) = \frac{1}{n}\left[E\left(\sum_{i=1}^{n}x_i^{\,2} - n\cdot\bar{x}^2\right)\right] = \frac{1}{n}\left[E\left(\sum_{i=1}^{n}x_i^{\,2}\right) - E\left(n\cdot\bar{x}^2\right)\right]$$
$$E\left(s_n^{\,2}\right) = \frac{1}{n}\left[\sum_{i=1}^{n}E\left(x_i^{\,2}\right) - n\cdot E\left(\bar{x}^2\right)\right] = \frac{1}{n}\left[n\cdot E\left(x_i^{\,2}\right) - n\cdot E\left(\bar{x}^2\right)\right]$$
$$E\left(s_n^{\,2}\right) = E\left(x_i^{\,2}\right) - E\left(\bar{x}^2\right) = \left[\mu^2 + \sigma^2\right] - \left[\mu_{\bar{x}}^{\,2} + \sigma_{\bar{x}}^{\,2}\right]$$
$$E\left(s_n^{\,2}\right) = \left(\mu^2 + \sigma^2\right) - \left(\mu^2 + \frac{\sigma^2}{n}\right) == \mu^2 + \sigma^2 - \mu^2 - \frac{\sigma^2}{n}$$
$$E\left(s_n^{\,2}\right) = \sigma^2 - \frac{\sigma^2}{n} = \sigma^2\cdot\left(1 - \frac{1}{n}\right) =$$
$$E\left(s_n^{\,2}\right) = \sigma^2\cdot\underbrace{\left(\frac{n-1}{n}\right)}_{\textit{Sesgo}}$$