CS3081: Computational Maths

Owen Burke (15316452) : Assignment Three

1 4.26 Write a user-defined MATLAB function that calculates the infinity norm of any matrix. For the func tion name and arguments use N = InfinityNorm (A), where A is the matrix, and N is the value of the norm. Use the function for calculating the infinity norm of:

(*a*) The matrix $A = \begin{bmatrix} -2 & 1 & 0 \\ 1 & -2 & 1 \\ 0 & 1 & -1.5 \end{bmatrix}$.   (*b*) The matrix $B = \begin{bmatrix} 4 & -1 & 0 & 1 & 0 \\ -1 & 4 & -1 & 0 & 1 \\ 0 & -1 & 4 & -1 & 0 \\ 1 & 0 & -1 & 4 & -1 \\ 0 & 1 & 0 & -1 & 4 \end{bmatrix}$.

The infinity norm of a matrix is simply given as the maximum of the sums of the rows of the matrix where the values used are the absolute values of the elements in the matrix.(See the matlab code below)

```
function N = InfinityNorm(A)
%Description
%   returns the infinity norm of the given matrix A. (returns N)
%Explanation
%   The infinity norm is simply the maximum of the sums of each row, where
%   the elements used are the absolute values of the elements in the matrix
    [m,n] = size(A);     % gets the size
    largestSum = 0; % set the maximum sum so far as 0
    for i = 1:m
        currentSum = 0; % set currentSum as 0
        for j = 1:n
            currentVal = abs(A(i,j));   % get the absolute value of the element
            currentSum = currentSum + currentVal;   % add to current sum
        end
        if currentSum > largestSum     % check if sum is larger than current max
            largestSum = currentSum;
        end
```

```
    end
    N = largestSum; % return largest sum (infinity norm)
```

The following images show the answers to the question, shown in matlab.

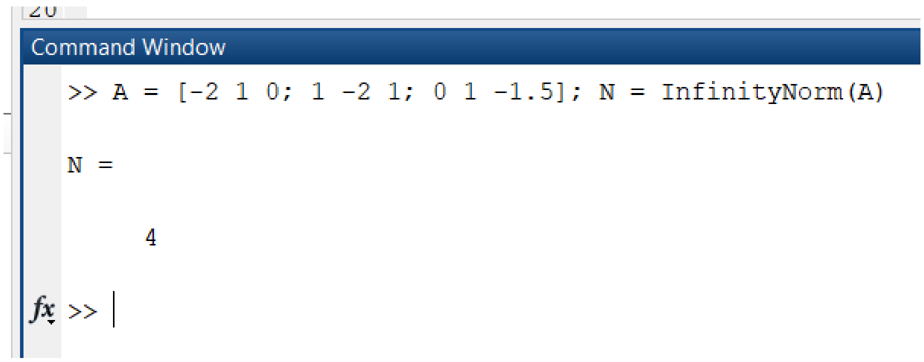


Figure 1: InfinityNorm for a

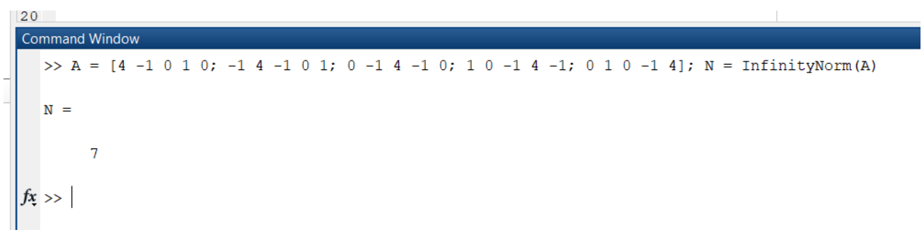


Figure 2: InfinityNorm for a

2 The power generated by a windmill varies with the wind speed. In an experiment, the following five measurements were obtained:

Wind Speed (mph)	14	22	30	38	46
Electric Power (W)	320	490	540	500	480

Determine the fourth-order polynomial in the Lagrange form that passes through the points. Use the polynomial to calculate the power at a wind speed of 26 mph.

Given points :
$(X_1, Y_1) = (14, 320)$
$(X_2, Y_2) = (22, 490)$
$(X_3, Y_3) = (30, 540)$
$(X_4, Y_4) = (38, 500)$
$(X_5, Y_5) = (46, 480)$

And given that we know the form of a lagrange polynomial that passes through five points is given as

$$y = f(x) = \frac{(X-X_2)(X-X_3)(X-X_4)(X-X_5)}{(X_1-X_2)(X_1-X_3)(X_1-X_4)(X_1-X_5)}Y_1 + \frac{(X-X_1)(X-X_3)(X-X_4)(X-X_5)}{(X_2-X_1)(X_2-X_3)(X_2-X_4)(X_2-X_5)}Y_2$$
$$+ \frac{(X-X_1)(X-X_2)(X-X_4)(X-X_5)}{(X_3-X_1)(X_3-X_2)(X_3-X_4)(X_3-X_5)}Y_3 + \frac{(X-X_1)(X-X_2)(X-X_3)(X-X_5)}{(X_4-X_1)(X_4-X_2)(X_4-X_3)(X_4-X_5)}Y_4 + \frac{(X-X_1)(X-X_2)(X-X_3)(X-X_4)}{(X_5-X_1)(X_5-X_2)(X_5-X_3)(X_5-X_4)}Y_5$$

The Y_1 part of the above equation (given the points from the question) gives

$$\left(\frac{(X-22)(X-30)(X-38)(X-46)}{(14-22)(14-30)(14-38)(14-46)}\right)(320)$$

which gives

$$\left(\frac{x^4-136x^3+6776x^2-146336x+1153680}{98304}\right)(320)$$

The Y_2 part of the above equation (given the points from the question) gives

$$\left(\frac{(X-14)(X-30)(X-38)(X-46)}{(22-14)(22-30)(22-38)(22-46)}\right)(490)$$

which gives

$$\left(\frac{x^4-128x^3+5864x^2-112192x+734160}{-24576}\right)(490)$$

The Y_3 part of the above equation (given the points from the question) gives

$$(\frac{(X-14)(X-22)(X-38)(X-46)}{(30-14)(30-22)(30-38)(30-46)})(540)$$

which gives

$$(\frac{x^4-120x^3+5080x^2-88800x+538384}{16384})(540)$$

The Y_4 part of the above equation (given the points from the question) gives

$$(\frac{(X-14)(X-22)(X-30)(X-46)}{(38-14)(38-22)(38-30)(38-46)})(500)$$

which gives

$$(\frac{x^4-112x^3+4424x^2-73088x+425040}{-24576})(500)$$

The Y_5 part of the above equation (given the points from the question) gives

$$(\frac{(X-14)(X-22)(X-30)(X-38)}{(46-14)(46-22)(46-30)(46-38)})(480)$$

which gives

$$(\frac{x^4-104x^3+3896x^2-61984x+351120}{98304})(480)$$

All this gives :

$$\mathrm{y} = \mathrm{f(x)} = (\frac{x^4-136x^3+6776x^2-146336x+1153680}{98304})(320) + (\frac{x^4-128x^3+5864x^2-112192x+734160}{-24576})(490)$$
$$+ (\frac{x^4-120x^3+5080x^2-88800x+538384}{16384})(540) + (\frac{x^4-112x^3+4424x^2-73088x+425040}{-24576})(500)$$
$$+ (\frac{x^4-104x^3+3896x^2-61984x+351120}{98304})(480)$$

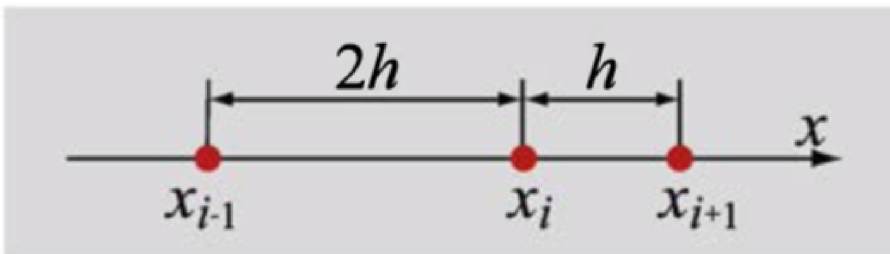
For a wind speed of 26 mph, simply substitute 26 in for x, giving :

$\mathrm{f}(26) = -12\frac{1}{2} + 229\frac{11}{16} + 379\frac{11}{16} - 78\frac{1}{8} + 11\frac{1}{4}$

giving

$\mathrm{f}(26) = 530$ W

3 Derive a finite difference approximation formula for f"(x_i) using three points x_{i-1}, x, and x_{i+1}, where the spacing is such that $x_i - x_{i-1} = 2h$ and $x_{i+1} - x_i = h$



From the above diagram and from the taylor series, we can obtain the following equations :

$f(x_{i+1}) = f(x_i) + f'(x_i)h + \frac{f''(x_i)}{2!}h^2 + \frac{f'''(x_i)}{3!}h^3 + \frac{f^{(4)}(\epsilon_1)}{4!}h^4$

and

$f(x_{i-1}) = f(x_i) - f'(x_i)2h + \frac{f''(x_i)}{2!}(2h)^2 - \frac{f'''(x_i)}{3!}(2h)^3 + \frac{f^{(4)}(\epsilon_2)}{4!}(2h)^4$

$f(x_{i-1})$ simplifies down to :

$f(x_{i-1}) = f(x_i) - 2f'(x_i)h + 4\frac{f''(x_i)}{2!}h^2 - 8\frac{f'''(x_i)}{3!}h^3 + 16\frac{f^{(4)}(\epsilon_2)}{4!}h^4$

In order to obtain our desired equation, we need to cancel unwanted terms. However, if we now add the two equations above, some terms won't cancel, so we multiply $f(x_{i+1})$ by 2, giving :

$2f(x_{i+1}) = 2f(x_i) + 2f'(x_i)h + 2\frac{f''(x_i)}{2!}h^2 + 2\frac{f'''(x_i)}{3!}h^3 + 2\frac{f^{(4)}(\epsilon_1)}{4!}h^4$

Now we get $f(x_{i-1}) + 2f(x_{i+1})$ as :
$f(x_{i-1}) + 2f(x_{i+1}) = 3f(x_i) + 3f''(x_i)h^2 + f'''(x_i)h^3 + 2\frac{f^{(4)}(\epsilon_1)}{4!}h^4 + 16\frac{f^{(4)}(\epsilon_2)}{4!}h^4$

We can now solve for $f''(x_i)$, as :

$f(x_{i-1}) + 2f(x_{i+1}) = 3f(x_i) + 3f''(x_i)h^2 + f'''(x_i)h^3 + 2\frac{f^{(4)}(\epsilon_1)}{4!}h^4 + 16\frac{f^{(4)}(\epsilon_2)}{4!}h^4$

f(x_{i-1}) + 2f(x_{i+1}) - 3f(x_i) = 3f$''$(x_i)h^2 + f$'''$(x_i)h^3 + $2\frac{f^{(4)}(\epsilon_1)}{4!}h^4 + 16\frac{f^{(4)}(\epsilon_2)}{4!}h^4$

Now divide across by $3h^2$ to isolate f$''$:

$\frac{f(x_{i-1})+2f(x_{i+1})-3f(x_i)}{3h^2} = f''(x_i) + \frac{f'''(x_i)h}{3} + (2\frac{f^{(4)}(\epsilon_1)}{4!}h^4)/(3h^2) + (16\frac{f^{(4)}(\epsilon_2)}{4!}h^4)/(3h^2)$

After truncating from f$'''$ and the higher terms, we get :

$f''(x_i) = \frac{f(x_{i-1})+2f(x_{i+1})-3f(x_i)}{3h^2} + O(h)$

4 The following data show the number of female and male physicians in the U.S. for various years (American Medical Association):

Year	1980	1990	2000	2002	2003	2006	2008
# males	413,395	511,227	618,182	638,182	646,493	665,647	677,807
# females	54,284	104,194	195,537	215,005	225,042	256,257	276,417

4.1 (a) Calculate the rate of change in the number of male and female physicians in 2006 by using the three point backward difference formula for the derivative, with unequally spaced points

4.2 (b) Use the result from part (a) and the three-point central difference formula for the derivative with unequally spaced points, Eq. (8.36), to calculate (predict) the number of male and female physicians in 2008.

(a)

By using the three point backward difference formula for the derivative, with unequally spaced points, Eq.(8.37) given as :

$$\mathrm{f'(x_{i+2})} = \frac{x_{\mathrm{i+2}}-x_{\mathrm{i+1}}}{(x_i-x_{\mathrm{i+1}})(x_i-x_{\mathrm{i+2}})}\mathrm{y}_i + \frac{x_{\mathrm{i+2}}-x_{\mathrm{i}}}{(x_{\mathrm{i+1}}-x_{\mathrm{i}})(x_{\mathrm{i+1}}-x_{\mathrm{i+2}})}\mathrm{y_{i+1}} + \frac{2x_{\mathrm{i+2}}-x_{\mathrm{i}}-x_{\mathrm{i+1}}}{(x_{\mathrm{i+2}}-x_{\mathrm{i}})(x_{\mathrm{i+2}}-x_{\mathrm{i+1}})}\mathrm{y_{i+2}}$$

and taking $\mathrm{x_{i+2}} = 2006$, $\mathrm{x_{i+1}} = 2003$ and $\mathrm{x_i} = 2002$, we simply sub in the corresponding y values for the men and women.

The men values give the following :

$$\mathrm{f'(2006)} = \frac{2006-2003}{(2002-2003)(2002-2006)}(638182) + \frac{2006-2002}{(2003-2002)(2003-2006)}(646493) + \frac{2(2006)-2002-2003}{(2006-2002)(2006-2003)}(665647) = 4939.916633$$

The women values give the following :

$$\mathrm{f'(2006)} = \frac{2006-2003}{(2002-2003)(2002-2006)}(215005) + \frac{2006-2002}{(2003-2002)(2003-2006)}(225042) + \frac{2(2006)-2002-2003}{(2006-2002)(2006-2003)}(256257) = 10681$$

(b)

By using the three point central difference formula for the derivative with unequally spaced points, Eq.(8.36) given as :

$$f'(x_{i+1}) = \frac{x_{i+1}-x_{i+2}}{(x_i-x_{i+1})(x_i-x_{i+2})}y_i + \frac{2x_{i+1}-x_i-x_{i+2}}{(x_{i+1}-x_i)(x_{i+1}-x_{i+2})}y_{i+1} + \frac{x_{i+1}-x_i}{(x_{i+2}-x_i)(x_{i+2}-x_{i+1})}y_{i+2}$$

and taking $x_{i+2} = 2008$, $x_{i+1} = 2006$ and $x_i = 2003$, we simply sub in the corresponding y values for the men and women and solve for the unknown to predict the number of physicians in 2008, as we know f'(2006) from part a.

The men values give the following :

$$4940 = \frac{2006-2008}{(2003-2006)(2003-2008)}(646493) + \frac{2(2006)-2003-2008}{(2006-2003)(2006-2008)}(665647) + \frac{2006-2003}{(2008-2003)(2008-2006)}(a)$$

This gives :

$4940 = (-86199\frac{1}{15}) + (-110941\frac{1}{6}) + \frac{3}{10}a$
$202080.2333 = \frac{3}{10}a$
$a = 673600\frac{7}{9}$ = predicted male physicians in 2008
error = $100 - (\frac{673600\frac{7}{9}}{677807} * 100) = 0.62056$

The women values give the following :

$$10681 = \frac{2006-2008}{(2003-2006)(2003-2008)}(225042) + \frac{2(2006)-2003-2008}{(2006-2003)(2006-2008)}(256257) + \frac{2006-2003}{(2008-2003)(2008-2006)}(a)$$

This gives :

$10681 = (-30005\frac{3}{5}) + (-42709\frac{1}{2}) + \frac{3}{10}a$
$83396\frac{1}{10} = \frac{3}{10}a$
$a = 277987$ = predicted female physicians in 2008
error = $100 - (\frac{276417}{277987} * 100) = 0.56477$

CS3081 Assignment 3

Name: Owen Burke

Student Number: 15316452

Question 1 (Problem 4.26)

(i) (a) = 4, (b)= 7
(ii) (a)=2.2, (b)=7
(iii) (a)=4, (b)=2.2
(iv) (a)=7, (b)=4

Your Answer ((i)-(iv)): (i)

Question 2 (Problem 6.13)

(i) 420W
(ii) 420KW
(iii) 530W
(iv) 580KW

Your Answer ((i)-(iv)): (iii)

Question 3 (Problem 8.7)

The truncation error is:

(i) O(h)
(ii) O(h^2)
(iii) O(h^3)
(iv) O(h^4)

Your Answer ((i)-(iv)) : (i)

Question 4 (Problem 8.9)

(i) f'_male(2006)=4965;
f'_female(2006)=10681;
Predicted_Males(2008)=673601;
Error_Males=0.62%;
Predicted_Females(2008)=277990;
Error_Females=0.58%

(ii) f'_male(2006)=4940;
f'_female(2006)=10681;
Predicted_Males(2008)=673601;
Error_Males=0.62%;
Predicted_Females(2008)=277987;
Error_Females=0.57%

(iii) f'_male(2006)=4940;
f'_female(2006)=10681;
Predicted_Males(2008)=673601;
Error_Males=0.68%;
Predicted_Females(2008)=277987;
Error_Females=0.42%

(iv) f'_male(2006)=4965;
f'_female(2006)=10670;
Predicted_Males(2008)=673601;
Error_Males=0.68%;
Predicted_Females(2008)=277987;
Error_Females=0.52%

Your Answer ((i)-(iv)): (ii)