Computational Mathematics - Assignment 2

Owen Burke, 15316452

March 15, 2019

Question 4.23

Write a user-defined MATLAB function that decomposes an n x n matrix [A] into a lower triangular matrix [L] and an upper triangular matrix [U] (such that [A] = [L][U]) using the Gauss elimination method (without pivoting). For the function name and arguments, use [L,U] = LUdecompGauss(A), where the input argument A is the matrix to be decomposed and the output arguments L and U are the corresponding upper and lower triangular matrices. Use LUdecompGauss to determine the LU decomposition of the following matrix:

$$\begin{bmatrix} 4 & -1 & 3 & 2 \\ -8 & 0 & -3 & -3.5 \\ 2 & -3.5 & 10 & 3.75 \\ -8 & -4 & 1 & -0.5 \end{bmatrix}$$

```
function [L,U] = LUdecompGauss(A)
% LUdecompGauss - decomposes an n x n matrix [A] into a lower triangular
%                 matrix [L] and an upper triangular matrix [U] (such that
%                 [A] = [L][U]) using the Gauss elimination method (without
%                 pivoting).
%
% Explanation : Obtains the upper triangular matrix by the gauss
% elimination method
%                 Step 1: Obtain the pivot coefficient for the current set of
%                 equations. Elements of the current column are eliminated by
%                 subtracting the (pivot equation * the multiplier) from them.
%                 The multipler is found by dividing the current coefficient
%                 by the pivot coefficient
%
%                 Step 2: This is then done for the next set of equations
%                 after eliminating the column and picking the new
%                 pivot coefficient
%
%                 Step 3: During the process of finding the upper matrix, we
```

```
%                 find the multipliers which are used to make the lower
%                 matrix
%
    [m,n] = size(A);
    upper = A;              %initialise the upper matrix
    lower = zeros(n,n);     %initialise the lower matrix
    for i = 1:n
        lower(i,i) = 1;     %Set the diagonals of the lower matrix to all 1's
    end
    for j = 2:m
        pivCoef = upper(j-1,j-1);   %get the pivot coefficient for the current set of equations
        for i = j:m
            mult = upper(i,j-1)/pivCoef;    %find the current multipler
            lower(i,j-1) = mult;            %assign the multipler to the lower matrix
            subEq = upper(j-1,:)*mult;      %multiply the pivot equation by the multipler
            upper(i,:) = upper(i,:) - subEq; %subtract the equation above from the current equation
        end
    end
    L = lower;  %return the lower matrix
    U = upper;  %return the upper matrix

end
```

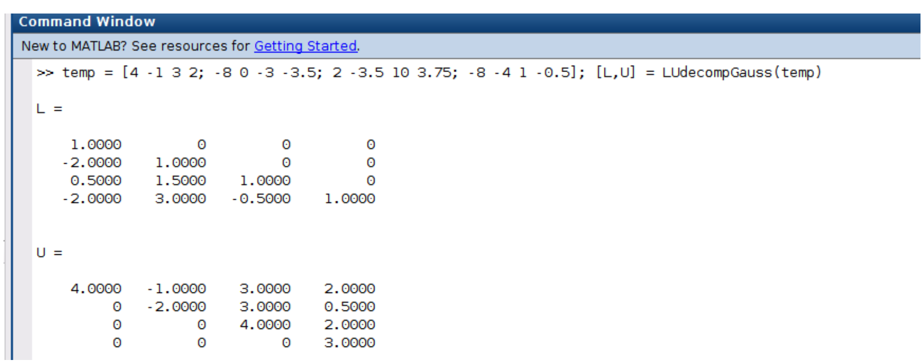


Figure 1: LUdecompGauss with matrix above

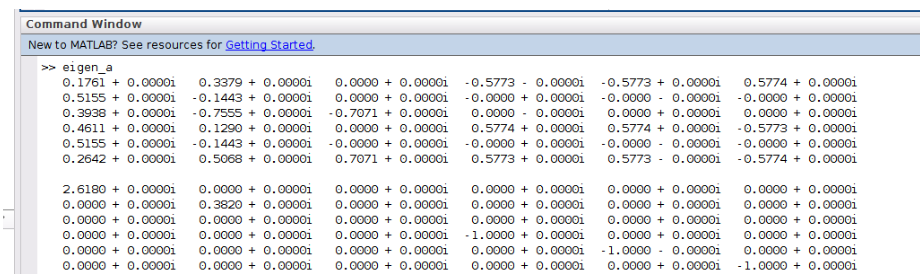
Question 5.17

A football conference has six teams. The outcome of the games is recorded in a binary fashion. For example, if team 1 defeats teams 5 and 6, then the equation $\mathrm{x}_1 = \mathrm{x}_5 + \mathrm{x}_6$ is written to indicate these results. At the end of the season, the wins and loses are tabulated in this fashion to produce the following ranking matrix:

$$\begin{bmatrix} x_\mathbf{1} \\ x_\mathbf{2} \\ x_\mathbf{3} \\ x_\mathbf{4} \\ x_\mathbf{5} \\ x_\mathbf{6} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_\mathbf{1} \\ x_\mathbf{2} \\ x_\mathbf{3} \\ x_\mathbf{4} \\ x_\mathbf{5} \\ x_\mathbf{6} \end{bmatrix}$$

(a) Find the eigenvalues and the corresponding eigenvectors of [A], using MATLAB's built-in function *eig*.

```
A = [0 0 0 1 0 0; 1 0 1 0 1 1; 0 1 0 0 1 0; 1 1 0 0 1 0;
    1 1 1 0 0 1; 1 0 0 0 1 0];
[v,e] = eig(A);
disp(v);
disp(e);
```



```
Command Window
New to MATLAB? See resources for Getting Started.
>> eigen_a
   0.1761 + 0.0000i   0.3379 + 0.0000i   0.0000 + 0.0000i  -0.5773 - 0.0000i  -0.5773 + 0.0000i   0.5774 + 0.0000i
   0.5155 + 0.0000i  -0.1443 + 0.0000i   0.0000 + 0.0000i  -0.0000 + 0.0000i  -0.0000 - 0.0000i  -0.0000 + 0.0000i
   0.3938 + 0.0000i  -0.7555 + 0.0000i  -0.7071 + 0.0000i   0.0000 - 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.4611 + 0.0000i   0.1290 + 0.0000i   0.0000 + 0.0000i   0.5774 + 0.0000i   0.5774 + 0.0000i  -0.5773 + 0.0000i
   0.5155 + 0.0000i  -0.1443 + 0.0000i  -0.0000 + 0.0000i  -0.0000 + 0.0000i  -0.0000 - 0.0000i  -0.0000 + 0.0000i
   0.2642 + 0.0000i   0.5068 + 0.0000i   0.7071 + 0.0000i   0.5773 + 0.0000i   0.5773 - 0.0000i  -0.5774 + 0.0000i

   2.6180 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.3820 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i  -1.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i  -1.0000 - 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i  -1.0000 + 0.0000i
```

Figure 2: eigenvalues and the corresponding eigenvectors of [A]

Eigenvalues and corresponding eigenvectors: *from the matlab execution of *eig* as seen above

$$\lambda = 2.6180, \begin{bmatrix} 0.1761 \\ 0.5155 \\ 0.3938 \\ 0.4611 \\ 0.5155 \\ 0.2642 \end{bmatrix}$$

$$\lambda = 0.3820, \begin{bmatrix} 0.3379 \\ -0.1443 \\ -0.7555 \\ 0.1290 \\ -0.1443 \\ 0.5068 \end{bmatrix}$$

$$\lambda = 0.0000, \begin{bmatrix} 0.0000 \\ 0.0000 \\ -0.7071 \\ 0.0000 \\ -0.0000 \\ 0.7071 \end{bmatrix}$$

$$\lambda = \text{-1.0000}, \begin{bmatrix} -0.5773 \\ -0.0000 \\ 0.0000 \\ 0.5774 \\ -0.0000 \\ 0.5773 \end{bmatrix}$$

$$\lambda = \text{-1.0000}, \begin{bmatrix} -0.5773 \\ -0.0000 \\ 0.0000 \\ 0.5774 \\ -0.0000 \\ 0.5773 \end{bmatrix}$$

$$\lambda = \text{-1.0000}, \begin{bmatrix} 0.5774 \\ -0.0000 \\ 0.0000 \\ -0.5773 \\ -0.0000 \\ -0.5774 \end{bmatrix}$$

(b) Find the eigenvector from part (a) whose entries are all real and of the same sign (it does not matter if they are all negative or all positive), and rank the teams from best (i.e., with most win) to worst (i.e., with fewest wins) based on the indices of the teams corresponding to the largest to the smallest entries in that eigenvector.

From part a), we can see that the eigenvector whose entries are all real and of the same sign is :

$$\begin{bmatrix} 0.1761 \\ 0.5155 \\ 0.3938 \\ 0.4611 \\ 0.5155 \\ 0.2642 \end{bmatrix}$$

Therefore, we can rank the teams from best to worst corresponding to the largest (0.5155) to the smallest (0.1761) entries in the eigenvector. Giving us :
Best (joint) : x_2, x_5
2$^{\mathbf{nd}}$ best : x_4
3$^{\mathbf{rd}}$ best : x_3
4$^{\mathbf{th}}$ best : x_6
Worst : x_1

NAME: **Owen Burke**

STUDENT NUMBER: **15316452**

Please indicate your answers by entering the option ((i), (ii), (iii) or (iv)) where asked. You should append the completed document as a pdf with your typewritten worked solutions including MATLAB code) and upload to Blackboard by Friday 22nd of March 2019.

Q 4.23

(i)

L =

1.5000	0	0	0
-2.0000	1.0000	0	0
0.5000	1.0000	1.5000	0
-2.0000	3.5000	-0.5000	1.0000

U =

4.0000	-1.0000	3.0000	2.0000
0	-1.0000	3.0000	0.5000
0	0	2.0000	1.0000
0	0	0	3.0000

(ii)

L =

1.0000	0	0	0
-2.0000	1.0000	0	0
0.5000	1.5000	1.0000	0
-2.0000	3.0000	-0.5000	1.0000

U =

4.0000	-1.0000	3.0000	2.0000
0	-2.0000	3.0000	0.5000
0	0	4.0000	2.0000
0	0	0	3.0000

(iii)

L =

1.5000	0	0	0
-2.0000	1.0000	0	0
0.5000	1.0000	1.0000	0
-2.0000	2.0000	-0.5000	1.0000

U =

3.0000	-1.5000	3.0000	2.0000
0	-2.0000	3.0000	0.5000
0	0	4.0000	2.5000
0	0	0	1.0000

(iv)

L =

1.5000	0	0	0
-2.0000	1.5000	0	0
0.5000	1.5000	1.5000	0
-2.0000	3.0000	-0.5000	1.5000

U =

4.0000	-1.0000	3.0000	2.0000
0	-2.0000	3.0000	0.5000
0	0	4.0000	2.0000
0	0	0	2.0000

Your Answer ((i) – (iv)): (ii)

Q 5.17

You need only to indicate the best team and the worst team (from teams 1 to 6).

Your Answers: **Best : X2 and X5 (joint)** **Worst : X1**

Q 6.3

(i) $b=4.6831\times10^{-8}, m=0.022, population(1985)=1014\ million$

(ii) $b=4.8932\times10^{-8}, m=0.022, population(1985)=1024\ million$

(iii) $b=4.6931\times10^{-8}, m=0.012, population(1985)=1038\ million$

(iv) $b=4.9932\times10^{-8}, m=0.014, population(1985)=1042\ million$

Your Answer ((i)-(iv)): This was removed from the assignment (see email)