Mobile Robot Global Path Planning Using Hybrid Modified Simulated Annealing Optimization Algorithm

Yuming Liang, Lihong Xu
School of Electronics and Information Engineering, Tongji University
Room 304,o= Ongji University Science and Technology Garden 2nd building,
No. 67 Chifeng road, Yangpu District, Shanghai 201804, China
086-021-65980590
{nc21.lym, xulhk}@163.com

ABSTRACT

Global path planning for mobile robot using simulated annealing algorithm is investigated in this paper. In view of the slow convergence speed of the conventional simulated annealing algorithm, a modified simulated annealing algorithm is presented, and a hybrid algorithm based on the modified simulated annealing algorithm and conjugate direction method is proposed. On each temperature, conjugate direction method is utilized for searching local optimal solution firstly, then the modified simulated annealing algorithm is employed to move off local optimal solution, and then the temperature is updated; these operations are repeated until a termination criterion is satisfied. Experimental results indicate that the proposed algorithm has better performance than simulated annealing algorithm and conjugate direction method in term of both solution quality and computational time, and thus it is a viable approach to mobile robot global path planning.

Categories and Subject Descriptors

F.2.0 [**Theory of Computation**]: ANALYSIS OF ALGORITHMS AND PROBLEM COMPLEXITY – *General.* I.2.8 [**Computing Methodologies**]: ARTIFICIAL INTELLIGENCE – *Problem Solving, Control Methods, and Search.* I.2.9 [**Computing Methodologies**]: ARTIFICIAL INTELLIGENCE – *Robotics*. I.6.6 [**Computing Methodologies**]: SIMULATION AND MODELING – *Simulation Output Analysis.*

General Terms: Algorithms, Documentation, Experimentation, Performance, Theory.

Keywords: Modified Simulated Annealing Algorithm; Conjugate Direction Method; Mobile Robot; Global Path Planning.

1. INTRODUCTION

Path planning for mobile robot is a complicated and NP-Complete problem [1]. It is typically formulated as follows: given a robot and a description of an environment, plan a path between two specified locations which is collision-free and satisfies certain optimization criterion [2]. The core of mobile robot path planning algorithms is the shortest path algorithm, which is one research hotspot of computer science and geography information science. The shortest path algorithm includes graph theory, heuristic reconnaissance, dynamic programming, neural network, and so on. Path planning is one indispensable part for mobile robot navigation technology, it reflects the interactive ability of mobile robot motion under environment, and is the foundation and safety for mobile robot to complete the task of navigation.

Simulated annealing algorithm (SAA) [3-5] [10] [11], one method proposed recent years, solves large-scale combination optimization problems; especially it is the general effective approximate method to solve the NP-Complete problems. To compare with past approximate methods, SAA has the advantages of simple structure, efficient operation and less sensitivity to initial condition, etc.. But its obvious disadvantage is that the convergence speed is slow. While modified simulated annealing algorithm (MSAA) [6] [7], which based on the graph searching technique, can delete redundant points those generated by SAA and shorten computational time. It is not only to preserve the global optimization trait of SAA, but also to enhance the convergence speed. Since MSAA is based on SAA, its result is still global optimal, and the optimal solution obtained by MSAA is better than that by SAA.

A hybrid algorithm based conjugate direction method and MSAA for mobile robot global optimal path planning is proposed in this paper, and a shortest collision-free path from the initial point to the target one in an environment with obstacles is planned. This method is parallelism, simple computation, and it can be easy expanded from two-dimensional space to multi-dimensional space and does not have the problem of combination explosion. For combination optimization problems, MSAA can move off local optimal value and obtains global optimal solution, and conjugate direction method is definite local optimization algorithm. The hybrid algorithm can make up for the deficiencies of conjugate direction method and MSAA. Experimental results indicate that the proposed algorithm has better performance than simulated annealing algorithm and conjugate direction method in term of both solution quality and computational time, and thus it is a viable approach to mobile robot global path planning.

2. Mathematical Modeling of Mobile Robot Global Path Planning

Mobile robot global path planning problem can be equivalent to constraint optimization problem. Namely, the objective function is to plan a path and the constraint condition is to avoid

Permission to make digital or hard copies of all or part of this work for personal or classroom use is granted without fee provided that copies are not made or distributed for profit or commercial advantage and that copies bear this notice and the full citation on the first page. To copy otherwise, or republish, to post on servers or to redistribute to lists, requires prior specific permission and/or a fee.
GEC'09, June 12-14, 2009, Shanghai, China.
Copyright 2009 ACM 978-1-60558-326-6/09/06...$5.00.

obstacles. The mathematical model [8] of mobile robot global path planning is expressed as follows:

$$\begin{aligned} &\min \quad f(X) \quad X \in R^n \\ &s.t. \quad g_i(X) \le 0 \quad i = 1,2,3,\cdots,p \end{aligned} \qquad (1)$$

In view of judging the convergence speed of constraint optimization algorithm is difficult relative to that of non-constraint optimization algorithms and the research and progress status of the former are far inferior to the latter, the constraint optimization problem is transformed to non-constraint optimization problem through sequential non-constraint minimization technique (SUMT), namely, to optimize an energy function $\boldsymbol{E}$, which consists of the path length function $\boldsymbol{E}_l$ and collision penalty function $\boldsymbol{E}_c$ [9].

$\boldsymbol{E}_l$ is defined as follows:

$$E_l = \sum_{i=1}^{N-1} L_i^2 = \sum_{i=1}^{N-1}[(x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2] \qquad (2)$$

where N is the number of path points, L_i is the length of the i^{th} line, and (x_i, y_i) is the i^{th} point's coordinate.

$\boldsymbol{E}_c$ is defined as follows:

$$E_c = \sum_{i=1}^{N}\sum_{k=1}^{K} C_i^k \qquad (3)$$

where N is the number of path points, K is the number of obstacles, C_i^k represents the collision penalty function of the i^{th} point $P(x_i, y_i)$ with the k^{th} obstacle.

Assuming the energy function of the path to be $\boldsymbol{E}$, μ_l and μ_c to be weighted factors, and $\mu_l + \mu_c = 1$, then

$$E = \mu_l E_l + \mu_c E_c \qquad (4)$$

Therefore, to solve formula (1) is to solve the formula as follows.

$$\min \quad E(X \in R^n) \qquad (5)$$

To solve the formula (5) is to find the minimum solution of the energy function $\boldsymbol{E}$ in its solution space, namely, the path is shortest and collision-free. Essentially, the formula (5) is a polynomial consisting of the vectors $[x_i, y_i]^T$, to optimize it is to search the minimum $\boldsymbol{E}$ that causes the polynomial to be minimum. Because $\boldsymbol{E}$ is the function of each path point, to move each path point towards the direction that the energy is reduced, the path points $[x_i, y_i]^T$ those minimizes $\boldsymbol{E}$ is obtained ultimately.

3. SIMULATED ANNEALING ALGORITHM AND ITS MODIFIED ALGORITHM

Simulated annealing algorithm (SAA) is from the metal solid annealing principle: to heat metal solid firstly, the internal granules of metal solid become disorder with temperature rising, and its internal energy augments; then to cool it slowly, the granules tend to be order and reach equilibrium state in each temperature; finally the granules reach the ground state on normal temperature, and the internal energy reduces to be minimum. Later SAA was used in combination optimization [10] [11] by Kirkpatrick and Cemy et al.. The so-called combination optimization is to find the optimal solution S^*, $S = \{S_1, S_2, \ldots, S_n\}$, and $\boldsymbol{E}(S_i)$ is the value of non-negative objective function corresponding to the state of S_i, $\forall S_i \in S$, satisfy $\boldsymbol{E}(S^*)=\min \boldsymbol{E}(S_i)$. The SAA program flowchart to solve the formula (5) is as shown in Figure 1.

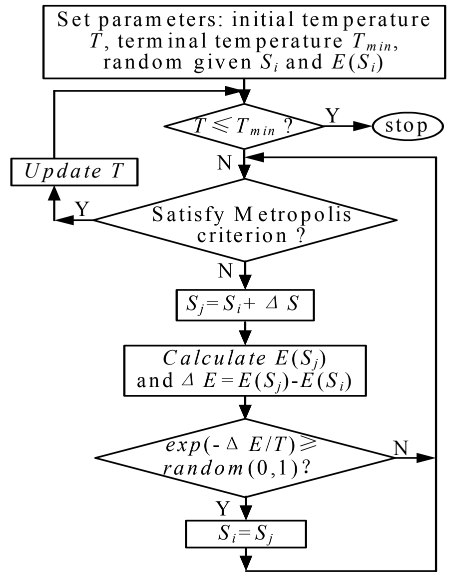

Figure 1. Flowchart of SAA.

Theoretically speaking, the serial state points generated by SAA on the condition of temperature descending, can be regarded as a series of Markov chain [12]. Repeating the operation of Metropolis behaviors many times on certain temperature, the distribution rule of the objective function value will satisfy Boltzmann distribution. When system temperature dropping is much slowly, Boltzmann distribution will tend to be uniform distribution, i.e., to generate infinite length of Markov chain under a certain condition, SAA can guarantee to restrain in global optimal state points with probability 1.0 [13]. Under certain temperature, so long as computational time is long enough, namely the length of Markov chain is long enough, the function value of initial points will be lower than that of final points with great probability, and global optimal solution is obtained. Therefore, the optimization process of SAA is a quite long, this is a great disadvantage of SAA.

Modified simulated annealing algorithm (MSAA), which based on the graph searching technique, firstly deletes the points those are generated in the process of annealing operation and the points those do not participate in annealing operation, and then generates path points again. In this way, it can decrease the number of redundant path points and shorten computational time. It is not only to preserve the global optimization trait of SAA, but also to enhance convergence speed. Since MSAA based on SAA, its result is still global optimal, and the optimal path obtained by MSAA is better than that of SAA.

To expound the basic idea of MSAA, the results that compared with SAA and MSAA are as shown in Figure 2(a) and Figure 2(b). From Figure 2(a) and Figure 2(b), it can be seen obviously that, the optimal path points from b to o in Figure 2(a) generated by SAA, take effect for generating other optimal path points in optimizational process; but these optimal points do not play positive role for obtaining the final optimal path points. Therefore, these redundant points can be deleted before optimization, and the result is obtained as shown in Figure 2(b), which is better than that in Figure 2(a).

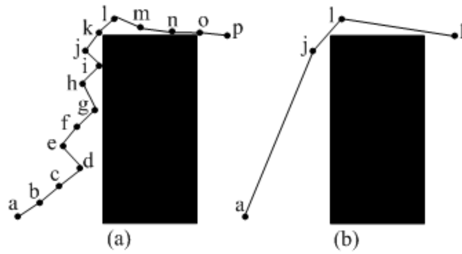

Figure 2. (a) Optimized path by using SAA,
(b) Optimized path by using MSAA.

The realization process of MSAA is as follows:

step1. Set parameters: initial temperature T_0, terminal temperature T_{min}, random given start point S_0 and goal S_n, point out initial searching direction $S_0 \rightarrow S_n$;

step2. Calculate path point S_i along the negative gradient direction;

step3. If S_i is not inside obstacle, go to step5; otherwise go to next step;

step4. Deviate from obstacle area by using golden section method;

step5. Perform SAA;

step6. Save S_i as the optimal solution, i=i+1;

step7. If i<n, go to step2; otherwise stop the algorithm and output the result.

Simultaneity, to delete the redundant path points generated by SAA, and to enhance the convergence speed, we amend the path with deleting redundant path points algorithm as follows:

step1. Take S_0 as the current path point S_i, viz. $S_i = S_0$;

step2. Order k=2, if S_{i+k} is the goal point S_n, exit; otherwise go to step3;

step3. If the path (S_i, S_{i+k}) is feasible, when S_{i+k} is the goal point S_n, exit; otherwise k=k+1, go to setp3; otherwise delete all the path points between S_i to S_{i+k-1}, i=i+k, go to setp3.

Here the feasible path is defined that any point on the path (S_i, S_{i+k}) is collision-free.

4. CONJUGATE DIRECTION METHOD

Definition: Suppose $\boldsymbol{H}$ to be n order positive definite matrix, $D_i^T H D_j = 0 \quad (i,j = 1,2,\cdots,m; i \neq j)$ $\boldsymbol{D}_1$, $\boldsymbol{D}_2$, $\boldsymbol{D}_3$, … , $\boldsymbol{D}_m$ are m nonzero vectors in the $\boldsymbol{E}_n$, if satisfy

（6）

call the vectors $\boldsymbol{D}_1$, $\boldsymbol{D}_2$, $\boldsymbol{D}_3$, … , $\boldsymbol{D}_m$ to be the conjugate vector group of matrix $\boldsymbol{H}$, or call vector group $\boldsymbol{D}_1$, $\boldsymbol{D}_2$, $\boldsymbol{D}_3$, … , $\boldsymbol{D}_m$ to be conjugate (direction) vector of matrix $\boldsymbol{H}$.

Analyzing the mathematical modeling of mobile robot path planning as shown in formula (5), it can be seen that E is the function of each path point $[x_i, y_i]^T$. Move each path point towards the direction that its energy is reduced, then the path that cause E to be minimum is obtained. Obviously formula (5) is a non-constraint minimization problem, and the objective function is a positive definite quadratic function. Conjugate direction method is based on definite searching strategy, and is quadratic convergence, namely starting from the initial path point at random, after limited steps it can achieve the minimum points of the object function [14]. The flowchart of conjugate direction method is as shown in Figure 3.

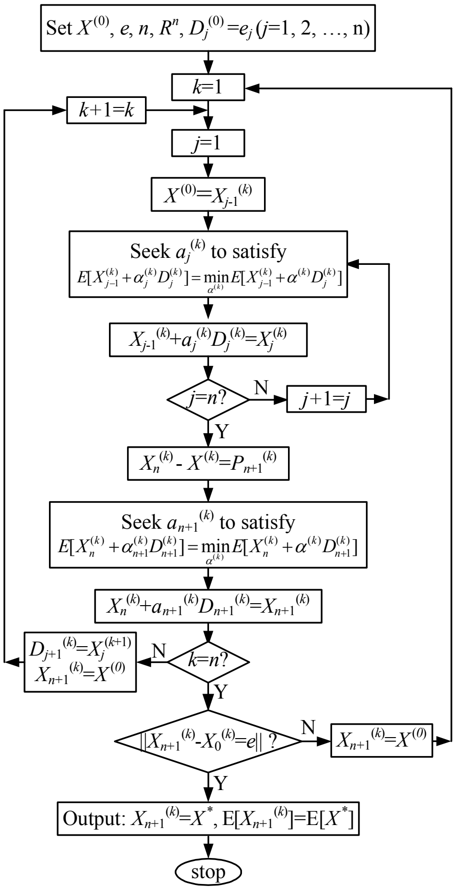

Figure 3. Flowchart of conjugate direction algorithm.

5. HYBRID ALGORITHM BASED CONJUGATE DIRECTION METHOD AND MSAA

In this section, the hybrid algorithm based conjugate direction method and MSAA is proposed and expounded as follows: on each temperature, conjugate direction method is utilized for searching local optimal solution firstly, then the modified simulated annealing algorithm is employed to move off local optimal solution, and then the temperature is updated; these operations are repeated until a termination criterion is satisfied. Such conjugate direction algorithm and MSAA are carried out optimization computation alternately. Finally it is not only that the convergence speed is enhanced and the global optimal solution is obtained quickly, but also that the computational precision is enhanced. The flowchart of this hybrid algorithm is as show in Figure 4.

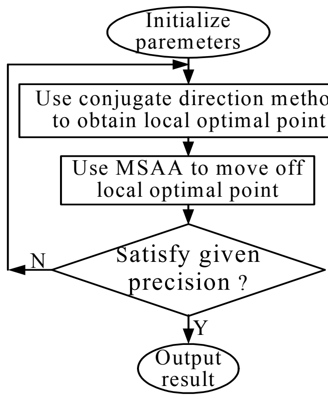

Figure 4. Hybrid algorithm's flowchart.

The realization process of hybrid algorithm is as follows:

step1: Set initial point, initial temperature T_0, terminal temperature T_{min}, and the number of searching step L;

step2: Start from the initial point, carry out conjugate direction method to obtain a local optimal point;

step3: Use MSAA to obtain a new point which is better than that obtained in step2;

step4: If the termination criterion is satisfied, stops; or else, go to step2.

6. SIMULATION RESULTS AND ANALYSIS

In this section, we use the hybrid algorithm to solve the optimization problems as formula (5). Here the temperature updating function which was proposed in [15] is adopted in this paper. The function is as follows:

$$T_k = T_0 / k^m \quad (k=1, 2, \ldots) \qquad (7)$$

Set parameters: the iterative precision ε=0.0001, initial temperature T_0 = 100°, terminal temperature T_{min} = 0.01°, the number of searching step L=1000, μ_l =0.4, m=3. And the condition of accepting new solution in MSAA is as follows:

$$\min[1, \exp(-\Delta E / T)] \geq random\ (0,1) \qquad (8)$$

Set the initial station S and end point G, and the initial path points are the serial points distributing uniformly on the S-G line.

To proof the proposed method to be feasibility, SAA, the proposed algorithm and conjugate direction method are carried out optimizing the path S-G respectively, and computer simulation results are obtained as shown in Figure 5 and Figure 6. At the same time, times of calling objective function and computational time to obtain global optimal path are given in Table 1.

In Table 1, starting from start point S, SAA called the objective function 3276 times, obtained the global optimal solution by spending 32.867 s to arrive at the end point G, and the path energy was 53.7424; the prposed algorithm called the objective function 289 times, obtained the global optimal solution by spending 4.253 s to arrive at the end point G, and the path energy was 53.7112; the conjugate direction method failed to arrive at the end point G. From Table 1, it can be seen that the global paths obtained by SAA and the proposed algorithm are similar; while the latter merely expends 4.253 s. Obviously the convergence speed of the proposed algorithm is better than that of SAA.

Table 1 Comparison with the convergence speed of three optimal algorithms

Algori-thm	Pat h	Times of calling objective function (s)	Expen d time (s)	Path energy
SAA	S-G	3276	32.867	53.7424
Hybrid algorithm	S-G	289	4.253	53.7112
Conjugate direction method	S-G	Fail to arrive at the end point G		

In conclusion, the proposed algorithm based conjugate direction method and MSAA, compromises the merits of conjugate direction method and MSAA, and includes stochastic search and deterministic search. It controls the search behavior with MSAA and guides the search direction with conjugate direction method, and enhances the convergence speed to a great extent. Experimental results indicate that the proposed algorithm has better performance than simulated annealing algorithm and conjugate direction method in term of both solution quality and computational time, and thus it is a viable approach to mobile robot global path planning.

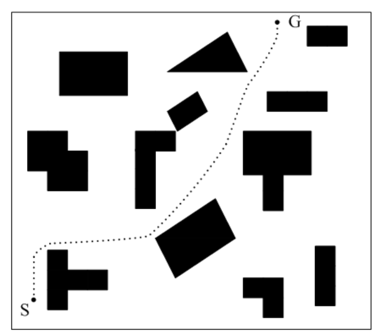

Figure 5. Optimized path by using

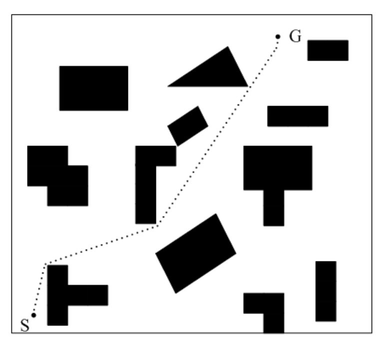

Figure 6. Optimized path by using hybrid algorithm.

7. CONCLUSION

In accordance with the problem of mobile robot global path planning, MSAA is adopted and a hybrid algorithm based conjugate direction method and MSAA is proposed in this paper. Computer simulation results demonstrate that the hybrid algorithm efficiently enhances the convergence speed and quickly completes the task of mobile robot path planning.

8. REFERENCES

[1] WANG Zhong-min, YAO Li-qing, ZHANG Han-song. 2003. Research for mobile robot path planning based on neural network. Journal of Tianjing University of Technology and Education, 13, l(Mar. 2003), 10-12.

[2] Jing Xiao, Zbigniew Michalewicz, Lixin Zhang, etc.. 1997. Adaptive Evolutionary Planner/navigator for Mobile Robots, IEEE Transactions on Evolutionary Computation, 1,1(Apr. 1997), 18-28.

[3] Kang Lishan, Xie yun, You Shiyong. 2000. Non-numerical parallel algorithm (1st volume) -- Simulated Annealing Algorithm. Beijing: Science Press.

[4] Chen L N，Aihara K. 1995. Chaotic simulated annealing by a neural network model with transient chaos. Neural Networks, 8, 6, 915-930.

[5] Wang Zhuopeng, Gao Guocheng, Yang Weiping. 1999. An Improved Fast Simulated Annealing Combinatorial Optimization. Systems Engineering Theory & Practice, 2, 5(Feb. 1999), 73-76.

[6] SHENG Guo-hua; CHEN Yu-jin. 2008. Modified Simulated Annealing Algorithm for TSP. Computer Knowledge and Technology. vol. 15, 1103-1104,1130.

[7] Wang Zhongmin, Yue Hong, Liu Jiyan. 2005. Path Planning for Mobile Robot Based on Modified Simulated Annealing Algorithm. Computer Engineering and Applications. vol. 19, 59-60,82.

[8] Chen Xiuning. 2000. Mechanical Optimization Design. Zhejiang: Zhejiang University Press.

[9] Sun Zengyi. 1997. Intelligent Control Theory and Technology. Beijing: TSINGHUA UNIVERSITY PRESS.

[10] Kirkpatrick S. 1983. Optimization by Simulated Annealing. Science, 220,4598 (May. 1983), 671-680.

[11] Romeijn H E. Smith R L. 1994. Simulated Annealing for Constrained Global Optimization. Journal of Global Optimization, 5, 2 (Sept. 1994), 101-124.

[12] Wang Ling. 2001. Intelligent Optimization Algorithm and Application. Beijing: TSINGHUA UNIVERSITY PRESS.

[13] Li Shiyong. 2004. Ant Colony Algorithm and Application. Harbin: HARBIN INSTIYUTE OF TECHNOLOGY PRESS.

[14] Liu Weixin. 1994. Machinical Optimization Design (2nd edition). Beijing: TSINGHUA UNIVERSITY PRESS, 1994.

[15] Yang Ruoli, Gu Jifa. 1997. An Effective Simulated Annealing for Global optimization. Systems Engineering Theory & Practice, 17, 5(May 1997), 29-35.