An Improved Tabu Search for Economic Dispatch With Multiple Minima

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Abstract—This paper develops an improved tabu search algorithm (ITS) for economic dispatch (ED) with noncontinuous and nonsmooth cost functions. ITS employs a flexible memory system to avoid the entrapment in a local minimum and developed the ideal of "distance" to the fitness to accelerate optimization. The new approach extends simple tabu search algorithm (STS) to real valued optimization problem and applies parallelism to weaken the dependence of the convergence rate of modified tabu search algorithm (MTS) on the initial condition. Effectiveness of the method was compared with many conventional methods. Results show that the proposed algorithm can provide accurate solutions with reasonable performance and has a great potential for other applications in the power system.

Index Terms—A move, adaptive progressing scheme, aspiration criteria, economic dispatch (ED), evolutionary programming (EP), improved tabu search (ITS), modified tabu search (MTS), recombination, simple tabu search (STS), tabu restrictions.

I. INTRODUCTION

ECONOMIC dispatch (ED) is an important function in the power system operation. For simplicity, the generator cost functions were mostly approximated by piecewise linear functions. Practically, opterating conditions of the cost function tend to be better segmented as piecewise quadratic functions which Lin and Viviani [1] applied Lagrangian function to solve. For units with nonmonotonically increasing incremental cost curves, the conventional process either ignores or flattens out these portions, which could induce inaccurate results.

Dynamic programming (DP) is one method to solve the nonconvex ED problem [2], [3], but solutions of the "local optimality" might be found when avoiding the problem of "curse of dimensionality." The Hopfield neural network models have been employed to study ED [4], [5], however, these methods require tremendous time for training. Stochastic searching algorithms, such as the simulated annealing (SA) [6], the genetic algorithms (GA) [7] and evolutionary programming (EP) [8], were developed to solve the highly nonlinear ED problem without restrictions on the shape of fuel cost functions. These applications, however, involved a large number of iterations and were susceptible to the related control parameters.

Simple tabu search algorithm (STS) introduced by Glover [9], [10] has been successfully applied to a number of integer opti-

mization problems [11]–[13]. STS avoids cycling by storing the information of the past from the search. Recently, modified tabu search (MTS) has been developed to study real valued optimization [14], [15]. It usually reaches local minima since a single candidate solution is used to generate offsprings [16].

The theory of tabu search has been well documented [17]. In this paper, some techniques have been developed in ITS to make tabu search more practicable in real valued systems.

II. FUNDAMENTAL THEORIES

The economic dispatch problem can be modeled by

$$\text{minimizing cost} = \text{Min} \sum_{i \in \Omega} C_i(P_i) \tag{1}$$

$$\text{subject to} \sum_{i \in \Omega} P_i = D + P_{loss} \tag{2}$$

$$\underline{P_i} < P_i < \overline{P_i} \tag{3}$$

with

$$P_{loss} = \sum_i \sum_j P_i B_{ij} P_j \tag{4}$$

$C_i(P_i)$ is the generation cost of power P_i for unit i. Ω is the set of all units for dispatch and B_{ij} is the transmission loss coefficient. $\underline{P_i}$ and $\overline{P_i}$ are high and low limits.

STS [9], [10] differs from the well-known hill climbing local search techniques in the sense that it does not become trapped in local optimal solutions. Fundamental concept of STS involves *individual*, *population* and *generation* [9]. If v_k is the trial vector up to iteration k, Δv_k is a *move*, then $v_{k+1} = v_k + \Delta v_k$ is the trial vector at iteration $k + 1$. The set of all possible moves out of a current solution is called the *set of candidate moves*. One could operate with a subset of it.

A. Tabu Restrictions

There are certain conditions imposed on moves, which make some of them forbidden. These forbidden moves are known as tabu. A *tabu list* will be formed to record these moves. A new trial vector satisfies tabu restriction is classified as tabu. The dimension of the tabu list is called the *tabu list size*. Let v_{tabu} be any vector in the tabu list and d_{tabu} the *tabu distance*, for extending STS to the real-valued optimization problem, a tabu restriction in MTS [15] can be expressed as

$$|(v_k + \Delta v_k) - v_{tabu}| < d_{tabu} \tag{5}$$

B. Aspiration Criteria

Aspiration criteria can override tabu restrictions. That is, if a certain move is forbidden, the aspiration criteria, when satisfied, can reactivate this move. Let v_k^* be the best current solution, aspiration criteria can be expressed as

$$|(v_k + \Delta v_k) - v_{tabu}| < d_{tabu}$$

with

$$\text{cost}(v_k + \Delta v_k) < \text{cost}(v_k^*) \qquad (6)$$

III. DEVELOPMENT OF ITS ALGORITHM

The real-valued variables to be determined in the ITS algorithm are represented as a trial N-dimension vector "v" that is associated with a cost function $\text{cost}(v) : R^N \to R$. Each vector v is an individual to be evolved.

A. Initialization

Let $v_k = [P_1 \dots P_d \dots P_i \dots P_N]$ be a trial vector denoting the kth individual of a population, where N is the number of dispatchable units. Let u be uniformly distributed in (0,1) and PS the population size, the initial trial vector $v_k, k = 1 \dots PS$, is determined by setting its ith components P_i by

$$P_i = \underline{P_i} + u * (\overline{P_i} - \underline{P_i}). \qquad (7)$$

A dependent generating unit is arbitrarily selected from among the committed N units by

$$P_d = D + P_{loss} - \sum_{i \in \Omega, i \neq d} P_i. \qquad (8)$$

Calculate the cost function of each trial vector, sort in ascending and pick up the first $\tau_{\max}$ vectors to place in the original tabu list, where $\tau_{\max}$ is the maximum tabu list size.

B. Adaptive Progressing Scheme

An adaptive scheme is defined in this paper by

$$S(g+1) = \begin{cases} S(g) - S_\Delta; C_{\min}(g) = C_{\min}(g-1) \\ S(g); C_{\min}(g) < C_{\min}(g-1) \end{cases} \qquad (9)$$

and

$$S(g+1) = S_{\min}; \text{if } S(g) - S_\Delta < S_{\min}. \qquad (10)$$

The adaptive scheme was used by setting S equal to various parameters α, β and τ in later sessions. $C(g)$ is the cost at generation number g. S_Δ is the step size related to associate parameters. S depends on the number of generations and the complexity of the system.

Let d_0 be the initial value of tabu distance and γ a *drop factor*. For improving MTS, an adaptive tabu distance will be used in this paper by

$$d_{tabu} = d_0 * \gamma^g. \qquad (11)$$

A bigger d_{tabu} can provide diversification in the beginning and a smaller d_{tabu} provides intensification at the end for final searches. So the algorithm could intensify the search around an already elite small region without wasting invalid trips to improve performance. From extensive tests, a good drop factor is between 0.95 to 0.99 to solve this problem. The power g is used to control the tabu distance, which is the generation number recommended in the paper.

C. Mutation

Given $v_k = [P_1 \dots P_d \dots P_i \dots P_N]$ and $v_k' = [P_1' \dots P_d' \dots P_i' \dots P_N']$, $k = 1, \dots, N_m$, the old and mutated kth vectors, the ith element of v_k' is mutated by

$$P_i' = P_i + N(\mu, \sigma^2), \ i \neq d \qquad (12)$$

$$P_d' = D + P_{loss} - \sum_{i \in \Omega, i \neq d} P_i' \qquad (13)$$

where N_m is the number of mutated individuals randomly selected, $N(\mu, \sigma^2)$ is a Gaussian random variable with mean μ and variance σ^2, μ is set to 0 in general and σ^2 can be expressed by considering the cost ratio as

$$\sigma^2 = \frac{C_k}{C_{\max}} (\overline{P_i} - \underline{P_i}) \beta. \qquad (14)$$

β is an adaptive mutation scale with $S = \beta$, $S_\Delta = \beta_\Delta$ and $S_{\min} = \beta_{\min}$. If P_i' exceeds its limit, it will be set to the limit.

D. Recombination [18]

Recombination is a mechanism to generate a new vector v_k' by using parameters from two randomly selected old vectors v_{k1} and v_{k2}. It generates an individual, with certain qualities inherited from the parents. The recombination function can be found by

$$\begin{aligned} v_k' &= v_{k1} + u * (v_{k2} - v_{k1}) \\ &= [P_1' \dots P_d' \dots P_i' \dots P_N'], \ k = 1, \dots, N_r \end{aligned} \qquad (15)$$

with P_d' adjusted by (13) where u is a uniform random number between 0 and 1, and N_r is the number of recombined individuals.

E. Determination of N_m and N_r

N_m and N_r must satisfy

$$N_m + N_r = PS \qquad (16)$$

with

$$0 \leqq N_m \leqq PS$$
$$0 \leqq N_r \leqq PS.$$

N_m and N_r were both initialized to 1/2 PS. Mutation is a diversification strategy and recombination is an intensification strategy to reinvestigate historically attractive regions thoroughly. With I a designated intensification number, if the best solution $C_{\min}(g)$ is from mutation and $C_{\min}(g-1) > C_{\min}(g)$, the number of mutated individuals in the next population will increase, that is

$$\begin{aligned} N_m(g+1) &= N_m(g) + I \\ N_r(g+1) &= PS - N_m(g+1). \end{aligned} \qquad (17)$$

Similarly, if the best solution $C_{\min}(g)$ is from recombination, we have

$$N_r(g+1) = N_r(g) + I$$
$$N_m(g+1) = PS - N_r(g+1). \tag{18}$$

On the other hand, for $C_{\min}(g-1) = C_{\min}(g)$, we have

$$N_m(g+1) = N_m(g) - I; \text{ if } N_m(g) = N_m(g-1) + I$$
$$N_r(g+1) = N_r(g) - I; \text{ if } N_r(g) = N_r(g-1) + I. \tag{19}$$

F. Evaluation and Selection

Update the solution and assign the rank of the calculated cost, RC_k, to each new vector, v'_k, $k = 1, \ldots, PS$. A best solution would gain the first number place, i.e., the highest rank $RC = 1$. A combined population with $2*PS$ individuals is formed with the old and new population. The concept of "distance" was added to the fitness function to prevent from being trapped in a local minimum. A far away point needs a higher rank to be selected, even the cost is slightly worse. The fitness score of the kth individual is expressed as

$$F_k = RC_k + \alpha * RD_k, \ k = 1, \ldots, 2*PS \tag{20}$$

where α is an adaptive decay scale with $S = \alpha$, $S_\Delta = \alpha_\Delta$ and $S_{\min} = \alpha_{\min}$. RD_k is the rank of D_k assigned to the kth individual ($RD_k = 1$ for the largest D_k), TS the tabu list size and D_k is the sum of distances from the individual to each solution vector in the tabu list and is given by

$$D_k = \sum_{t=1}^{TS} |v_k - v_{tabu,t}|. \tag{21}$$

Individuals will be ranked in descending according to their fitness scores by a sorting algorithm. The first PS individuals are transcribed along with their cost for the next generation. If the new population does not include the current best solution, the best solution must replace the last individual in the new population (Elitism).

G. Control of Tabu List

Tabu list is a finite length one-in one-or-more-out circular data structure, which records the solutions just visited and all the local optimal solutions visited so far. A new trial vector is placed on top of the list and the oldest trial vector is taken out of the list. An adaptive tabu list size $\tau(g)$ is given by setting $S = \tau$, $s_\Delta = \tau_\Delta$ and $S_{\min} = \tau_{\min}$, i.e., the tabu move will be kept tabu for a duration of τ moves. The tabu list in ITS is a recency memory in the earlier searching process, where most solutions are similar to each other, called the frequency vectors. ITS can concentrate in searching the region of nonfrequency memory to find the global optimum. The choice of tabu list size is critical. If the size is too small, cycling may occur in the searching process and the process often returns to the solution just visited. On the contrary, appealing moves may become forbidden and higher quality solutions can not be explored. Empirically, a suitable tabu list size is between five to 30 [13].

TABLE I
PARAMETER SETTING OF THE ITS

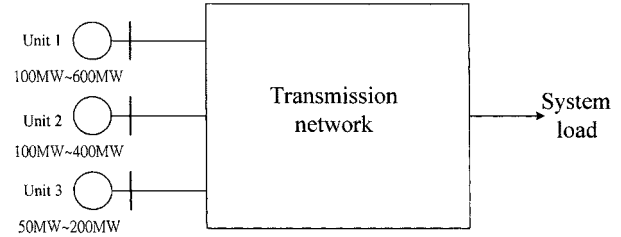| Parameters			3 unit			30 unit		
g_{max}			50			200		
PS			30			60		
β_{max}	β_Δ	β_{min}	0.5	0.025	0.005	0.5	0.025	0.005
α_{max}	α_Δ	α_{min}	0.6	0.025	0.005	0.6	0.025	0.005
τ_{max}	τ_Δ	τ_{min}	25	1	7	30	1	7

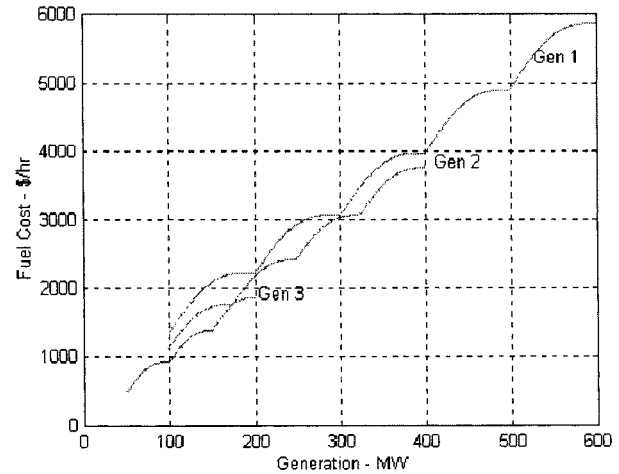Fig. 1. Schematic diagram of test system.

Fig. 2. Input-output (I/O) curve with valve point effects.

H. Stopping Criterion

If the maximum generation number is reached, stopping criterion will take effect. Otherwise, go to Step C for mutation to reiterate the whole process again.

IV. CASE STUDY

The ITS-based ED algorithm was implemented with C language on a 133-MHz Pentium computer with 32 MB RAM. Parameters of test examples are shown in Table I.

Test system of a multiple minima problem in Fig. 1 comprised three generating units, with fuel cost functions taking into account the valve-point effects by

$$C_i(P_i) + A_0 + A_1 P_i + A_2 P_i^2 + |e_i * \sin(f_i * (P_i - \underline{P_i}))| \tag{22}$$

The same multiple minima problem has been solved by the GA approach in [7] and EP approach in [8], where coefficients of the fuel cost functions were given and the golbal optimum had been found. Fig. 2 shows the ripple like input-output curves with valve-point and the classical Lagrange based algorithms ceased to be applicable.

TABLE II
COMPARISON OF ALL METHODS WITH 100 TRIAL TESTS

Methods	ITS	MTS	EP1	EP2	GA
Worst	8241.22	8382.78	8263.91	8251.84	8353.56
Average	8234.68	8261.14	8243.75	8241.11	8249.21
Best	8234.07	8234.07	8234.07	8234.07	8234.07
NGO$_{*1}$	92	26	37	39	34
NSO$_{*2}$	8	6	27	35	21

*1:NGO: number of times to reach global optimum ($8234.07)
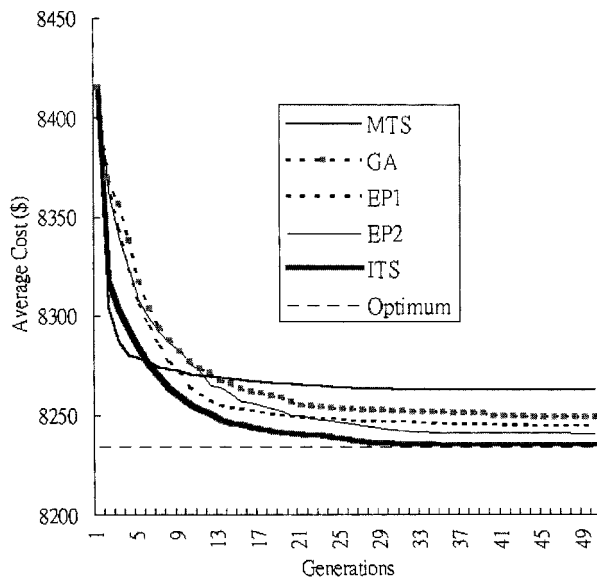*2:NSO: number of times to reach sub-optimum ($8241.22)

Fig. 3.   Average value of the best cost at each generation with 100 trial tests.

A. Convergence Test

The characteristics of ITS, MTS, EP1, EP2 and GA were compared. The same set of random initial solutions and the population size were used for all methods and the elitism was used to prevent from losing the best solution in each generation. MTS is similar to the method of [15] (with d_{tabu} in (11) used), EP1 used the constant mutation scale [8], EP2 used the adaptive mutation scale [19] and GA is similar to [7] with a dependent unit selected as in (8). 100 test runs were conducted for each method. Sample results are given in Table II. Every method could get the optimal solution.

However, the number of generations to converge is different and also different for different runs in the same method. Although a maximum $g_{\max} = 50$ is used in this paper for test comparison with [8], the average value reaches 99.9% accuracy with only 18 generations for ITS. Generally, the number of iterations without improving the current best solution could be used to set the maximum generation value. The average value of the best cost in each generation was calculated and shown in Fig. 3. Fig. 3 also shows that MTS converges fast. Convergent rates for other methods are similar.

B. Reliability Analysis

Table II shows that ITS can obtain the global optimal (GO) or suboptimal (SO) solution, other methods produce very different results and were very often trapped in the local optimum. MTS is easy to premature. EP2 with adaptive mutation scale behaves

TABLE III
PERFORMANCE COMPARISON WITH 100 TRIAL TESTS

Methods	ITS	MTS	EP1	EP2	GA
NCPG	1	0.18	1.05	1.06	1.49
NGC	18	13	18	21	20
NCC	18	2.34	18.9	22.3	29.8
NCO	18	7	12	13	10

NCPG: Normalized computational time per generations, the CPU time in ITS for each generation is about 0.16s.
NGC: Number of generations to converge, 99.9% the final average cost.
NCC: Normalized computational time to converge.
NCO: NCC of ITS reaching the average cost of every other method.

TABLE IV
TESTS OF VARIOUS INITIAL CONDITIONS

Methods	ITS	MTS	EP1	EP2	GA
GIC	100	86	96	100	92
BIC	48	2	4	5	3

TABLE V
COMPARISON OF VARIOUS RULES WITH 100 TRIAL TESTS

Case	Adaptive progressing scheme			Results	
	α	β	τ	NGO	NCPG
1	No	No	No	35	1.003
2	No	No	Yes	38	0.976
3	No	Yes	No	45	1.007
4	Yes	No	No	78	1.075
5 (ITS)	Yes	Yes	Yes	92	1

No: average value of max. and min. was used.

TABLE VI
COMPARISON OF CASE 1 AND CASE 2

Methods	ITS				lambda-iteration
	NCPG	NGC	NGO	Best	
Case 1	3.56	104	99	103487	103487
Case 2	3.69	132	85	103563	---------

better than EP1 with constant mutation scale. The results of GA are similar to EP1 and EP2, but the cost of some solutions was trapped near $8350 due to prematurity. ITS can reach GO 92% times and 8% times SO. EP and GA have similar results. MTS is the worst.

C. Performance Test

Table III shows that the average time of a search process in MTS only takes about one-fifth to one-eighth times the other methods. ITS takes almost the same time as EP. GA solves the real valued problem by encoding and decoding the decision variables and takes longer time. With the normalized computational time per generations (NCPG) of ITS being 1, test results are shown in Table III. ITS can obtain the same results with better performance than EP and GA.

D. Initialization Test

To evaluate the effect of various initial conditions, 100 good initial conditions (GIC) were created randomly, with distances to the optimum less than 100 MW. Another 100 bad initial conditions (BIC) were created randomly, with distances to the optimum greater than 150 MW. Table IV shows that most trials

can obtain global optimum with good initial conditions. However, ITS can still obtain about 50% global optimal solutions with BIC, other methods were mostly trapped in the local optimum.

E. Parameters Test

Five cases were run with different parameters enabled. Table V shows that τ enhances solution quality and performance, β improves solution quality, degrading the performance slightly and α provides a better chance to reach the global optimum.

F. The 30 Units Test

A real-sized 30-unit Taipower system was also tested with 7400 MW load. The second order cost curves were used in Case 1 where the lambda-iteration method was known to produce a global optimum. Results of 100 trials verified that ITS could produce the global optimum. In Case 2, three steam turbines were replaced by considering valve points in Fig. 2. Valve-point effects will produce slightly worse performance, however the lambda-iteration method ceases to be applicable. The results are shown in Table VI with NCPG, NGC and NGO defined before.

V. CONCLUSION

An ITS algorithm has been developed in solving the nonlinear multi-minimum economic dispatch problem where the classic Lagrange based algorithms failed to solve. Test results reveal that the worst and the best ITS-based solutions are very close, which can not be guaranteed by other methods.

MTS generates new population from one candidate and usually converges to a local optimum very quickly; for ITS, vectors with fitness score good enough would be used as candidates to create new solutions regardless of tabu restrictions (flexible aspiration criteria). It also forms a parallel structure and can weaken the dependence of the convergence rate on the initial solutions.

ITS adaptively regulates the tabu list size, the number of mutated and recombined individuals, so the performance can be improved. In addition, a gradually decreased decay scale can satisfy a successive statistic searching process by first using the diversification (bigger α) to explore more regions and then the intensification (smaller α) to exploit the neighborhood of an elite solution.

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