Using simulated annealing to solve the p-Hub Median Problem

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Abstract *Locating hub facilities is important in different types of transportation and communication networks. The p-Hub Median Problem (p-HMP) addresses a class of hub location problems in which all hubs are interconnected and each non-hub node is assigned to a single hub. The hubs are uncapacitated, and their number p is initially determined. Introduces an Artificial Intelligence (AI) heuristic called simulated annealing to solve the p-HMP. The results are compared against another AI heuristic, namely Tabu Search, and against two other non-AI heuristics. A real world data set of airline passenger flow in the USA, and randomly generated data sets are used for computational testing. The results confirm that AI heuristic approaches to the p-HMP outperform non-AI heuristic approaches on solution quality.*

Introduction

Hub and spoke (H&S) network designs for distribution purposes are used in some form or another in companies such as Northwest Airlines, United Airlines, Federal Express, UPS, Norfolk Southern, and Yellow Freight. The key in an H&S design is to consolidate traffic from different origin cities and send it via one or more hubs to different destination cities. US airlines utilized the H&S design following the Airline Deregulation Act in 1978 to remain competitive by cutting costs and increasing flexibility through offering more city-to-city travel alternatives.

Implementing H&S designs to improve the efficiency of a distribution network has not been limited to passenger airlines. The express package delivery firm Federal Express has used the H&S design for a long time (Chestler, 1985; Chan and Ponder, 1976). A key advantage of implementing H&S designs in non-passenger airline applications is that packages are not sensitive to the number of stops they make at hubs, unlike human beings. The trucking industry is also benefiting from H&S designs (Lumsden *et al.*, 1999; Don *et al.*, 1995) to improve the operation of the distribution function. Unlike airlines, the primary benefit of hubs is not to increase the number of city pairs served but to reduce the driver tour length, which in turn would reduce annual driver turnover rates and the accompanying hiring and training costs (Don *et al.*, 1995).

Using H&S designs has become increasingly important in the context of supply chain management (Abdinnour-Helm, 1999). Improving the efficiency of the distribution function by cutting time and cost will contribute to reducing the total cycle time and total cost of the supply chain, hence improving

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customer satisfaction. The current CEO of Wal-Mart, H. Lee Scott, had served for 16 years in the retailer's vast and complicated logistics operations before becoming CEO. The rise of Mr Scott highlights the importance to Wal-Mart of its supply chain infrastructure, a hub-and-spoke network of distribution centers surrounded by stores not more than a day's truck drive away. "The network, which Mr. Scott helped build, has made Wal-Mart an earnings juggernaut, able to speed deliveries and cut costs, even as sales and store number have soared" (Nelson, 1999). Clearly, in this case, the logistics process at Wal-Mart is considered a strategic activity (Day, 1998).

The challenge to academics then is to develop efficient solution procedures that would transform a point-to-point network of interacting nodes to an H&S network. Certain nodes in the network should be chosen as hubs (consolidation points) and the rest of the nodes become spokes. The p-Hub Median Problem (p-HMP) is an H&S network design that assumes the following:

- the number of hubs p is determined a priori;
- there is no limit on the number of spokes assigned to a hub;
- each spoke is assigned to a single hub; and
- all hubs are interconnected.

O'Kelly (1987) was the first to formulate the p-HMP as a quadratic integer programming problem. He showed that the problem is NP-hard, and proposed two enumeration-based heuristics to solve it. Since then, most of the focus in the literature has been on developing heuristic approaches to solve the p-HMP. Klincewicz (1991) proposed two heuristics, one based on single and double exchanges of spoke assignments and the other based on clustering. Klincewicz (1992) also developed a Tabu Search heuristic and a greedy randomized adaptive search heuristic (GRASP). Campbell (1996) introduced the p-HMP-M, where M stands for multiple allocation, i.e. a spoke may be assigned to more than one hub. He showed that the solution for the p-HMP-M provides a lower bound for p-HMP, and developed two heuristic approaches to solve the p-HMP by starting with the solution to the p-HMP-M first. The heuristic approaches were called MAXFLO and ALLFLO. Skorin-Kapov and Skorin-Kapov (1994) introduced a Tabu Search heuristic to solve the p-HMP, in which the two decision levels (choosing the p hubs and assigning the spokes to the hubs) are made simultaneously. Skorin-Kapov *et al.* (1996) reformulated the p-HMP as a new mixed integer program and obtained optimal solutions by solving a relaxed version of the mixed integer program. Ernst and Krishnamoorthy (1996) presented a new linear programming formulation to the p-HMP, and used a simulated annealing heuristic to obtain upper bounds for an LP based branch and bound. Smith *et al.* (1996) used neural networks to solve the quadratic integer formulation of the p-HMP.

There are classes of hub location problems other than the p-HMP. They were formulated and described by Campbell (1994a). One such problem is the Uncapacitated Hub Location Problem (UHP), in which the number of hubs is not pre-determined. Heuristics for the UHP were developed by O'Kelly (1992), Aykin (1995), Abdinnour-Helm and Venkataramanan (1998), and Abdinnour-Helm (1998). An optimal approach was introduced by Abdinnour-Helm and Venkataramanan (1998) and by Aykin (1995). A literature survey of the different solution approaches to the different classes of hub location problems is also available (Campbell, 1994b).

In this paper we develop a simulated annealing heuristic to design H&S networks for the p-HMP. Simulated annealing is an Artificial Intelligence (AI) technique (Glover and Greenberg, 1989). We then compare the results of the simulated annealing heuristic to three other heuristics from the literature for the p-HMP:

- an AI heuristic called Tabu Search (Skorin-Kapov and Skorin-Kapov 1994); and
- two non-AI heuristics called MAXFLO and ALLFLO (Campbell, 1996).

This provides a comparison of the performance of two AI approaches against two non-AI approaches. All the heuristics share the fact that the assignments of spokes to hubs is not necessarily made based on proximity, i.e. when the hubs are determined, each spoke is not necessarily assigned to the closest hub. Some heuristics in the literature have assigned spokes to the closest hubs, which does not necessarily guarantee an optimal solution. Finally, the implementation of simulated annealing in this paper is different from the one done by Ernst and Krishnamoorthy (1996). Even though the results of their implementation are better than those obtained in this study, we offer more details of the implementation and a thorough analysis of AI versus non-AI approaches. Tabu Search yields the best results to date in the AI category of heuristics. Similarly, MAXFLO and ALLFLO yield the best results to date in the non-AI category of heuristics. Finally, it is not uncommon in the literature to have two different implementations of the same technique to the same problem. See, for example, Klincewicz (1992) and Skorin-Kapov and Skorin-Kapov (1994).

Overview of simulated annealing

Simulated annealing (SA) is one of five Artificial Intelligence (AI) techniques (Glover and Greenberg, 1989). The other four techniques are Genetic Algorithms, Neural Networks, Tabu Search (TS), and Target Analysis. Companies have turned to a wide variety of natural models for new approaches to problem solving (Naik, 1996). The term annealing in physics describes the process in which a solid is heated to a high temperature T_0, where the atoms randomly arrange themselves in liquid phase, and then it is cooled gradually according to a cooling schedule $\{T_0 > T_1 > T_2 > T_3 \ldots T_f\}$, where T_f is the freezing point. At T_f the solid is said to be in its lowest energy state (ground state). The amount of time spent at each temperature level during the annealing process must be long to allow the system to reach thermal equilibrium (steady state). If cooling is done quickly (rapid quenching), undesirable random fluctuations of the atoms get frozen into the material, thereby making the attainment of the ground state impossible. Metropolis *et al.* (1953) developed a simple Monte Carlo approach to simulate the behavior of a collection of atoms in achieving thermal equilibrium at a given temperature level. The idea is to start from a current configuration of the atoms and apply a small randomly generated perturbation. If this results in a lower energy state, the process is repeated using the new state. If, however, the result of the perturbation is a higher energy state, then the new state is accepted with a certain probability.

Kirkpatrick *et al.* (1983) were the first to apply the idea of annealing to combinatorial optimization problems. Since then, the method has been referred to as simulated annealing. The analogy between a physical system and an optimization problem was given by Johnson *et al.* (1989) and is presented in Table I.

In the analogy, the different states of the substance correspond to different feasible solutions to the combinatorial optimization problem, and the energy of the system corresponds to the function to be minimized. Simulated annealing offers a strategy that is similar to iterative improvement techniques with one major advantage: iterative improvement techniques only allow moves that decrease the cost function (downhill moves), whereas simulated annealing allows moves that increase the cost function (uphill moves) in a controlled manner (cooling schedule). This idea makes it possible to leave a local minima and potentially fall into a more promising downhill path. The notion of a temperature parameter is to control the acceptance of uphill moves. At the beginning (high temperatures), most uphill moves are accepted to permit an aggressive search of the configuration space. As the search proceeds (temperature drops), the solution should be close to a near-optimal solution and so fewer uphill moves are allowed.

When the simulated annealing approach is applied to the design of an H&S network, a feasible solution corresponds to a set of hubs with the rest of the nodes assigned to one of the hubs. The cost of this feasible solution is the sum of the transportation costs between the hub-to-hub links and the hub-to-spoke links. The temperature parameter of SA is a function of the acceptable cost difference between two feasible solutions. When SA is started, a big cost difference is generally accepted but, as the number of iterations increases, a smaller cost difference is accepted. This temperature analogy allows SA to accept a worse feasible solution in the hope of arriving at a better solution (lower total transportation cost).

Table I. The analogy between physics and simulated annealing

Physical system	Optimization problem
State	Feasible solution
Energy	Cost
Ground state	Optimal solution
Rapid quenching	Local search
Careful annealing	Simulated annealing

Collins *et al.* (1988) provided an annotated bibliography of SA and Eglese (1990) gave a brief description of the different aspects of SA. Koulamas *et al.* (1994) did a survey of the applications of Simulated Annealing in production areas. Since 1994, the date of the survey, applications of simulated annealing continued to appear in areas that included job shop scheduling (Kolonko, 1999), facility layout (Chwif *et al.*, 1998), examination scheduling system (Thompson and Dowsland, 1998), resource-constrained projects (Gemmill and Tsai, 1997; Cho and Kim, 1997), scheduling (Park and Kim, 1997; Sier *et al.*, 1997), cell formation (Adil *et al.*, 1997), and cutting stock problem (Lai and Chan, 1997). An example of the application of SA in logistics is *Federal Express*, which built a bid-line generator to produce a complete set of legal, flyable lines for an airplane fleet using SA as a first step to find as many good bid lines as possible (Campbell *et al.*, 1997).

There are several examples of using other AI approaches in logistics. Santa Fe Railway developed an operating plan model that produces a weekly train timetable and assigns traffic to trains (Gorman, 1998). The model uses a combination of Tabu Search and Genetic Algorithms to successively search for better operating plans. Berry *et al.* (1998) described the use of Genetic Algorithms in designing complex distribution systems.

Implementation of simulated annealing to the p-HMP

Given the flow and the cost between each pair of nodes in a network with n nodes, the purpose of the p-Hub Median Problem (p-HMP) is to make p nodes as hubs and assign each of the remaining $n - p$ nodes in the network to a single hub at the minimum cost possible. The mathematical formulation of the p-HMP is as follows (O'Kelly, 1987):

Let:

$$X_{ik} = \left\{ \begin{array}{ll} 1 & \textit{if node } \mathrm{i} \textit{ is assigned to hub } \mathrm{k} \\ 0 & \textit{otherwise} \end{array} \right\}$$

$$X_{ii} = \left\{ \begin{array}{ll} 1 & \textit{if node } \mathrm{i} \textit{ is a hub} \\ 0 & \textit{otherwise} \end{array} \right\}$$

Minimize

$$\sum_i \sum_j W_{ij} \left(\sum_k X_{ik} C_{ik} + \sum_m X_{jm} C_{jm} + \alpha \sum_k \sum_m X_{ik} X_{jm} C_{km} \right) \tag{1}$$

S.T.

$$(n - p + 1) X_{jj} - \sum_i X_{ij} \geq 0 \tag{2}$$

$$\sum_j X_{ij} = 1 \tag{3}$$

$$\sum_j X_{jj} = p \tag{4}$$

The parameters W and C represent flow and cost respectively. The parameter $\propto$ reflects the discount factor on inter-hub flow. The objective function in constraint (1) minimizes the total transportation cost. Constraint (2) assures that a hub must be opened before an assignment is made to it. Constraint (3) assigns a node to only one hub. Constraint (4) generates the correct number of hubs.

Recently, O'Kelly and Bryan (1998) addressed the issue that using the same discount factor on all inter-hub links in the network, regardless of the differences in flows travelling across them, is an over-simplification in hub location models.

The two stages in solving the p-HMP involve selecting the hubs, and making the assignments to the hubs. The simulated annealing procedure will be applied to the assignment stage of the problem (referred to as ASA), and will be completely embedded within a simple procedure that chooses a different set of p hubs each time. The motivation to this approach stems from the fact that the complexity of the p-HMP is due to the assignment stage. Even if the p hubs are given, the problem is a special case of the NP-hard quadratic assignment problem (O'Kelly, 1987). Since simulated annealing proved to be an efficient heuristic to solve the quadratic assignment problem (Connolly, 1990), it seemed reasonable to pick an approach that finds good assignments for a given set of hubs.

When simulated annealing (SA) is used to solve optimization problems, it is customary to continue using the terminology of the physical process (see Table I). After looking at several examples from the literature, we decided to do the same and be consistent with what has been done so far in the literature. We used small letters for all the variables that are part of the algorithm and capital letters for all variables related to the SA approach. The Greek letters are consistent with those used in the hub location and simulated annealing literature. Finally, two groups of decisions have to be made when implementing SA (Johnson *et al.*, 1989): problem specific choices and generic choices.

The problem specific choices are:

(1) *solution* (S): A current solution in ASA consists of an assignment of the $n-p$ nodes to the p hubs.

(2) *neighbor* (S'): Given a current solution, a neighbor solution is generated by making a pairwise exchange, i.e. if nodes i and j are assigned to hubs k and m respectively in the current solution, then the neighbor solution results from reassigning node i to hub m and node j to hub k.

(3) *cost of a solution* $C(S)$: This calculates the actual cost of the current hub-and-spoke topology.

The generic choices are usually referred to as the cooling schedule. This includes the following:

(1) initial temperature (T_0): The idea is to choose T_0, so that several cost increasing transitions are accepted at the beginning, i.e. $exp(-\Delta C/T_0) \cong 1$, where $\Delta C = C(S') - C(S)$. By setting an acceptance ratio χ (ratio

between the number of accepted transitions and the total number of transitions) close to 1, we get: $T_0 = \overline{\Delta C^{(+)}} / \ln(\chi)^{-1}$. The $\overline{\Delta C^{(+)}}$ represents the average increase in cost over a number of transitions that are randomly chosen.

(2) *discounting factor* $\alpha(0 < \alpha < 1)$: The temperature T is decreased using the rule: $T = \alpha^* T$, where α is a constant smaller than 1 and is usually between 0.8 and 0.99.

(3) *length of the Markov Chains* (L): This corresponds to the number of iterations at each T. There is a trade-off between large decreases in the temperature (small α) and small lengths of Markov Chains.

(4) *final temperature* (T_f): This is the zero temperature level. The procedure can be stopped when T_f is reached, or earlier if the procedure seems to be converging to a value that has not changed over a large number of iterations.

The above cooling schedule has been widely used and has been found effective on several problems. However, it is one of several types of cooling schedules that are available in the literature on simulated annealing (Collins *et al.*, 1988).

The heuristic to solve the p-HMP is described below, but first additional notation is given:

Notation:

ith: the number of iterations for exchanging a hub.

β: discount factor for the initial temperature T_0.

max: the maximum number of assignments made to a hub in the initial solution S.

aoptcst: assignment cost after a complete ASA.

hoptcst: the best cost found by the heuristic at any point in terms of both the choice of p hubs and the assignments to those p hubs.

$C(S)$: cost of solution S.

a, h: counters for the number of assignment exchanges and the number of hub exchanges respectively.

Heuristic-SA:

Step 1. Obtain the initial temperature T_0; set $hoptcst = \infty$.

Step 2. Determine the following parameters: L, ith, α, β.

Step 3. (*Initial solution, S*)
Choose p hubs randomly, and assign each of the remaining $n - p$ to the closest hub. Create dummy nodes and assign them to the other hubs, so that all hubs would have the same number of nodes, *max*, assigned to them. The initial solution is stored in S.

Step 4. (*Begin ASA*)
a) Set $a = 1$, $T = T_0$, $aoptcst = \infty$.
b) Calculate $C(S)$

Step 5. (*ASA routine*)
Do the following steps:
a) generate a neighbor S', and calculate $C(S')$.
b) if $C(S') < aoptcst < C(S)$, then set $aoptcst = C(S')$ and $S = S'$.
c) if $aoptcst < C(S') < C(S)$, then set $S = S'$.
d) if $aoptcst < C(S) < C(S')$, go to *step 6*.
e) go to *step 7*.

Step 6. (*Acceptance of a worse solution*)
a) generate a random number r that is uniformly distributed between zero and one.
b) calculate the acceptance probability ap, where $ap = exp((C(S') - C(S))/T)$.
c) if $r < ap$, then accept the move and set $S = S'$; otherwise retain the current solution S, and reject the move.

Step 7. (*Steady state check*)
Set $a = a + 1$; if $a < L$, go to *step 5*.

Step 8. (*End ASA check*)
If $T > T_f$, then set $T = \alpha^* T$, go to *step 5*;
otherwise go to *step 9*.

Step 9. (*Exchange hubs*)
a) if $aoptcst < hoptcst$,
then $hoptcst = aoptcst$ and set $S =$ the solution that corresponds to the value *aoptcst*;
otherwise ($aoptcst > hoptcst$), set $S =$ the solution that corresponds to the value *hoptcst*.
b) set $h = h + 1$.
c) if $h > ith$, go to *step 10*.
d) set $T_0 = \beta^* T_0$ to get the new initial temperature.
e) pick a *hub* and a *spoke* (assigned to *hub* and is not a dummy node) in S. Exchange the roles so that the *spoke* becomes the *new-hub* and the *hub* becomes a *spoke* assigned to this *new-hub*. All the assignments that were originally made to *hub* are now made to *new-hub*. Store the resulting solution in S.
f) go to *step 4*.

Step 10. (*Report results*)
Report the result of the best cost found by the heuristic, *hoptcst*, the choice of p hubs, and the assignments.

There is a need here to clarify some points about the heuristic:

(1) In *step 9*d), the initial temperature is decreased before starting a new ASA, because we assume that we are slowly approaching the optimal solution and so our acceptance of bad moves should decrease as *ith* increases. This is similar to the idea of decreasing the temperature level within the ASA routine. The value of β should be at least equal to α.

(2) Obviously, the number of iterations, *ith*, cannot be larger than the total number of possible initial temperatures achieved using the rule $T_0 = \beta^* T_0$ and the fact that $T_0 > \varepsilon$.

Computational testing

The SA heuristic was coded in Ansi C and was run on an IBM RISC System/6000, which operates under IBM's version of UNIX (AIX).

The simulated annealing heuristic was tested using two types of data sets:

(1) A real world data set (O'Kelly, 1987), which has been consistently used in the literature. The data are based on airline passenger flow between 25 US cities in 1970, as evaluated by the Civil Aeronautics Board (the CAB data set). Testing of this data set in the literature has included the 25 city data set as well as subsets of it. In this study three data sets are considered: all 25 cities, a subset consisting of the first ten cities, and a subset consisting of the first 15 cities. For each data set, four levels of the discount factor α are considered: 1.0, 0.8, 0.6, and 0.4. For each data set and at each level of α, the number of hubs p is set to three values: 2, 3, and 4.

(2) Randomly generated data sets to study the performance of simulated annealing on a range of small to large problems. The data sets are generated using a random problem generator written in ANSI C. A symmetric flow is generated from a uniform distribution with a lower limit of 500 and an upper limit of 200,000. The limits correspond to the lowest and highest flows in the 25 CAB data set. The (x,y) locations for nodes are generated using a uniform distribution with a lower limit of zero and an upper limit of 2,000. The distances are then calculated using the coordinates, which guarantees that the triangular inequality holds. The distance between any pair of points is at least 40. Five data sets for each size (10, 12, 15, 20, 50, and 80) are generated. The discount factor α is set at 1.0, and the number of hubs p is considered at three levels: 2, 3, and 4.

Parameter choice for the AI heuristics

The two main parameters in the simulated annealing heuristic are: the factor α to discount the temperature at each level of the assignment simulated annealing routine (ASA), and the factor β to discount the initial temperature itself whenever a new hub exchange takes place. A study of 24 problems was conducted to determine the best combination for (α,β). Since the common values for discount factors range between 0.8 and 0.99, the following four combinations were considered: (0.8,0.8), (0.8,0.99), (0.99,0.8), and (0.99,0.99). For each combination, three problems of size 10 and three problems of size 15 (subsets of the CAB data set) were used. The three problems correspond to number of hubs $p = 2, 3, 4$. In all 24 problems, no discount on hub-to-hub links is assumed. The best combination was found to be $(\alpha, \beta) = (0.8, 0.99)$. The solutions in this category achieved the maximum average percentage deviation from the solutions obtained by complete enumeration. The combination (0.8,0.99) is logical, since a full simulated annealing is carried out only on the assignment part of the problem, and so a larger number of hub exchanges will guarantee better solutions than if conditions are the other way round.

The other parameters in the simulated annealing heuristic are *ith*, the number of hub exchanges, and L, the number of iterations at each temperature of the assignment simulated annealing (ASA) routine. After making several runs on the 25 node CAB data set, it was observed that $L = 5$ and $L = 10$ did not provide better results than $L = 1$. So the value $L = 1$ was chosen. A convenient value for *ith* is to make it equal to the number of iterations necessary until the initial temperature reaches zero. This provides consistency across the different problems, and avoids the need to predetermine a number of iterations that may turn out to be very small or very large.

Performance measures

For the CAB data sets, the results of the simulated annealing heuristic are compared against those obtained by Tabu Search (Skorin-Kapov and Skorin-Kapov, 1994). They are also compared against non-AI heuristics from the literature (Campbell, 1996) called MAXFLO and ALLFLO. The acronyms SA and TS are used to refer to the simulated annealing and the Tabu Search heuristics respectively. The main measure of performance will be the solution value. Comparison of time will not be made, since the heuristics were written in different programming languages and were run on different types of machines.

For all the randomly generated data sets, results of only the SA heuristic are reported. A conservative measure representing the percentage deviation from the lower bound is calculated. The lower bound is calculated as the initial variable cost (the sum of all the transportation costs on all direct links). Two time values are also measured: time to converge, and time to best. The stopping rule in simulated annealing is to stop whenever the initial temperature reaches zero. Since the SA heuristic is run for 40 iterations, the average time for the 40 iterations to achieve convergence is referred to as time to converge. Usually, however, the best solution is discovered early on in the procedure and before final convergence is declared. This is why the average time for the 40 iterations to get the best solution is also a measure of interest and is called time to best.

Results of the CAB data sets

The results are analyzed in a similar fashion to the study conducted by Campbell (1996). Tables II, III and IV report the results of SA, MAXFLO, ALLFLO and TS. The first half of each Table reports the actual values of the transportation costs for all the heuristics at each level of α, and for p values equal to 2, 3, and 4. The second half of each Table (columns 7 to 10) represents the percentage difference between the cost for each heuristic and the minimum cost for the four heuristics. The measure is called %dmin. Zero values of %dmin indicate a minimum cost solution and, since more than one heuristic

Table II. Transportation cost comparisons for $n = 10$: actual values and percentage difference from minimum cost (%dmin)

						%dmin			
α	p	SA	MAXFLO	ALLFLO	TS	SA	MAXFLO	ALLFLO	TS
1	2	835.81	854.31	849.26	835.81	0.00	2.21	1.61	0.00
	3	776.68	808.91	784.59	776.68	0.00	4.15	1.02	0.00
	4	737.47	911.45	782.18	736.26	0.16	23.79	6.24	0.00
0.8	2	790.94	808.03	790.94	790.94	0.00	2.16	0.00	0.00
	3	716.98	743.66	717.48	716.98	0.00	3.72	0.07	0.00
	4	661.41	718.30	692.36	661.41	0.00	8.60	4.68	0.00
0.6	2	732.63	761.75	732.63	732.63	0.00	3.97	0.00	0.00
	3	643.89	650.38	643.89	643.89	0.00	1.01	0.00	0.00
	4	581.77	601.37	594.78	577.83	0.68	4.07	2.93	0.00
0.4	2	674.31	674.31	674.31	674.31	0.00	0.00	0.00	0.00
	3	572.24	583.27	567.91	567.91	0.76	2.70	0.00	0.00
	4	494.95	510.38	494.96	493.79	0.23	3.36	0.24	0.00

Table III. Transportation cost comparisons for $n = 15$: actual values and percentage difference from minimum cost (%dmin)

						%dmin			
α	p	SA	MAXFLO	ALLFLO	TS	SA	MAXFLO	ALLFLO	TS
1	2	1,221.92	1,300.89	1,268.91	1,221.92	0.00	6.46	3.85	0.00
	3	1,168.68	1,266.67	1,208.98	1,168.68	0.00	8.38	3.45	0.00
	4	1,119.38	1,208.23	NA	1,118.23	0.10	8.05	NA	0.00
0.8	2	1,190.77	1,261.15	1,213.67	1,190.77	0.00	5.91	1.92	0.00
	3	1,099.51	1,120.58	1,114.76	1,099.51	0.00	1.92	1.39	0.00
	4	1,043.30	1,053.64	1,037.71	1,026.52	1.63	2.64	1.09	0.00
0.6	2	1,157.07	1,143.97	1,143.97	1,143.97	1.15	0.00	0.00	0.00
	3	1,009.93	1,020.63	1,009.93	1,009.93	0.00	1.06	0.00	0.00
	4	910.21	931.11	918.87	910.21	0.00	2.30	0.95	0.00
0.4	2	1,062.63	1,062.63	1,062.63	1,062.63	0.00	0.00	0.00	0.00
	3	905.10	920.69	905.10	905.10	0.00	1.72	0.00	0.00
	4	790.23	796.33	779.71	779.71	1.35	2.13	0.00	0.00

may achieve a minimum cost for a specific problem, more than one zero may appear on any one line. The term NA under the ALLFLO column indicates that there is no available result because of the excessive computation requirement of ALLFLO for large values of α.

Tables V and VI summarize the results in Tables II, III and IV using the %dmin measure. Table V, includes only the results from SA, MAXFLO and TS on all the problems. Table VI includes the results from SA, MAXFLO, ALLFLO and TS only on problems for which ALLFLO had a result other than NA. For each heuristic, the following is reported: the number of problems for which %dmin is equal to zero; the percentage of problems for which %dmin is equal to zero; the maximum of all the %dmin values; the average of all the %dmin values; and the standard deviation of all the %dmin values. The TS heuristic performed best, because the transportation cost for all the problems (100

Table IV. Transportation cost comparisons for $n = 25$: actual values and percentage difference from minimum cost (%dmin)

α	p	SA	MAXFLO	ALLFLO	TS	%dmin SA	%dmin MAXFLO	%dmin ALLFLO	%dmin TS
1	2	1,359.19	1,368.96	NA	1,359.19	0.00	0.72	NA	0.00
	3	1,261.08	1,322.17	NA	1,256.63	0.35	5.22	NA	0.00
	4	1,216.68	1,272.69	NA	1,211.23	0.45	5.07	NA	0.00
0.8	2	1,294.08	1,294.08	1,294.08	1,294.08	0.00	0.00	0.00	0.00
	3	1,158.83	1,176.52	NA	1,158.83	0.00	1.53	NA	0.00
	4	1,087.66	1,090.90	NA	1,087.66	0.00	0.30	NA	0.00
0.6	2	1,201.21	1,201.21	1,201.21	1,201.21	0.00	0.00	0.00	0.00
	3	1,033.56	1,039.64	1,039.64	1,033.56	0.00	0.59	0.59	0.00
	4	939.21	939.21	939.21	939.21	0.00	0.00	0.00	0.00
0.4	2	1,101.63	1,108.33	1,101.63	1,101.63	0.00	0.61	0.00	0.00
	3	903.52	903.49	903.49	901.70	0.20	0.20	0.20	0.00
	4	787.51	794.89	794.49	787.51	0.00	0.94	0.89	0.00

percent) considered was the minimum. In fact a later study by Skorin-Kapov *et al.* (1996) has proved that all the TS heuristic solutions turned out to be equal to the optimal solutions. The SA heuristic came in second best with approximately 70 percent of the problems achieving the minimum cost value. The average of the %dmin values was very low (approximately 0.2), and so was the standard deviation (approximately 0.4), indicating a very low variation. The highest %dmin value was equal to 1.63. In comparison, MAXFLO on all problems had a low percentage of the problems achieving zero value (17 percent to 20 percent). Also, the average and standard deviation values were very high. ALLFLO achieved better results than MAXFLO but still not as good as SA. It is also important to remember that ALLFLO has the major drawback of requiring more CPU time than any of the other heuristics.

Table V. Summary results of the %dmin values *excluding* ALLFLO

	SA	MAXFLO	TS
Number of problems with %dmin equal to zero	25	6	36
Percentage of problems with %dmin equal to zero	69.44	16.67	100.00
Maximum of all the %dmin values	1.63	23.79	0.00
Average of all the %dmin values	0.20	3.21	0.00
Standard deviation of all the %dmin values	0.41	4.33	0.00

Table VI. Summary results of the %dmin values *including* ALLFLO

	SA	MAXFLO	ALLFLO	TS
Number of problems with %dmin equal to zero	22	6	14	30
Percentage of problems with %dmin equal to zero	73.33	20.00	46.67	100.00
Maximum of all the %dmin values	1.63	23.79	6.24	0.00
Average of all the %dmin values	0.21	3.15	1.04	0.00
Standard deviation of all the %dmin values	0.44	4.58	1.62	0.00

It is also interesting to compare the results of the different heuristics by aggregating data for each level of α. For example, for SA we calculate the number of problems with %dmin value equal to zero for an α value of 1.0, then for an α value of 0.8, and so on. Figure 1 displays the results when the ALLFLO heuristic is excluded, and Figure 2 displays the results when all four heuristics are considered. Figure 1 shows that TS performed best, followed by SA, and then MAXFLO. Figure 2 shows that the ALLFLO results are better than the MAXFLO results, and they seem to improve more as the α value decreases. At the lowest values of 0.6 and 0.4 ALLFLO and SA gave similar results.

Results of the random data sets

It is more interesting to report only summary information for the random data sets. However, the detailed results of all the runs made can be obtained from the author. Table VII summarizes the averages of the three performance measures for each problem size and for values of p equal to 2, 3, and 4.

As the number of hubs increases, the percentage deviation from lower bound (%dlb) decreases. The average time to find the best solution (tbest) and the average time to achieve convergence (tconv) are below 0.5 and 1.5 CPU seconds

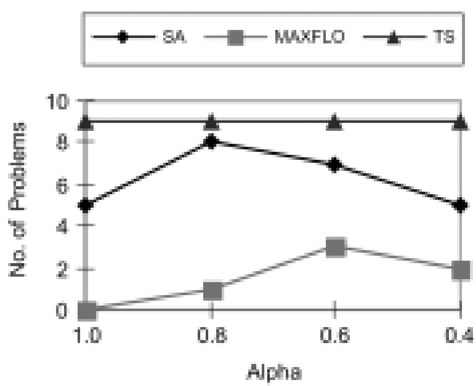

Figure 1.
CAB data: no. of problems with %dmin equal to zero at each level of α *excluding* ALLFLO

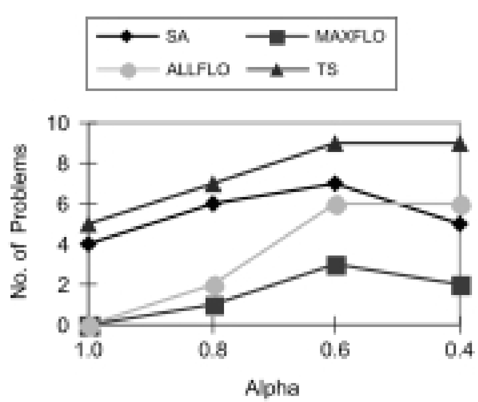

Figure 2.
CAB data: no. of problems with %dmin equal to zero at each level of α *including* ALLFLO

n	p	tbest	tconv	%dlb
10	2	0.03	0.73	0.33
	3	0.06	0.92	0.22
	4	0.07	0.92	16.45
12	2	0.06	0.78	0.33
	3	0.14	0.98	0.25
	4	0.21	1.02	0.20
15	2	0.20	0.94	0.37
	3	0.31	1.13	0.26
	4	0.40	1.25	0.21
20	2	0.47	1.34	0.37
	3	0.66	1.51	0.31
	4	0.75	1.69	0.27
50	2	1.52	4.25	0.42
	3	2.22	4.22	0.36
	4	3.27	5.11	0.33
80	2	3.47	9.26	0.43
	3	4.97	9.28	0.37
	4	5.96	9.45	0.34

Table VII. Summary results for the random data sets (times for tbest and tconv are in CPU seconds)

respectively for problem sizes 10, 12, and 15, and below 6.0 and 9.5 CPU seconds respectively for problem sizes 20, 50, and 80. This indicates that the time requirements for SA are acceptable even for large size problems.

The results for the three measures of performance are displayed graphically. Figure 3 represents the percentage deviation from lower bound (%dlb) as a function of the number of nodes in the network. The Figure displays the stability of the SA heuristic, as the number of nodes increases. Figures 4 and 5 show the average times for finding the best solution (tbest) and the average

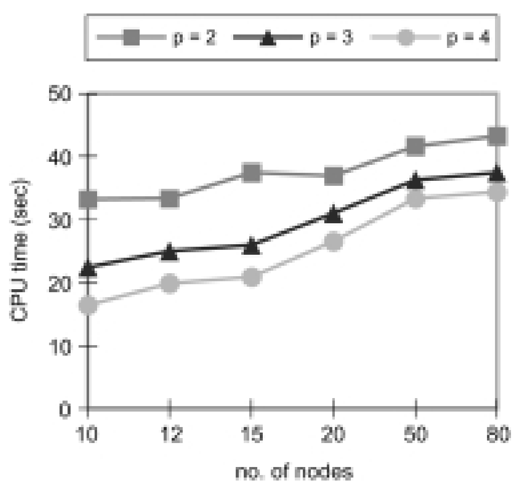

Figure 3. Random data: percentage deviation from lower bound (%dlb) as a function of the no. of nodes

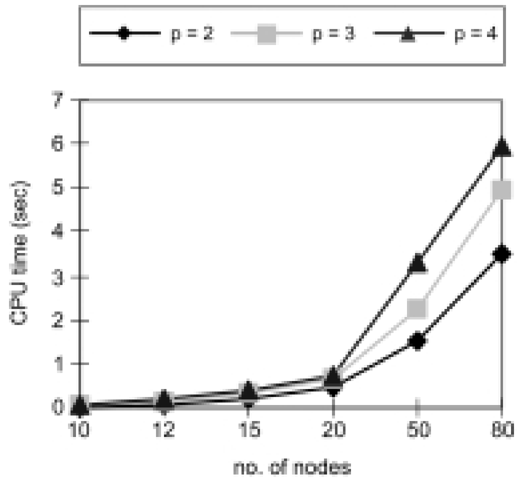

Figure 4.
Random data: CPU time to find the best solution (tbest) as a function of the no. of nodes

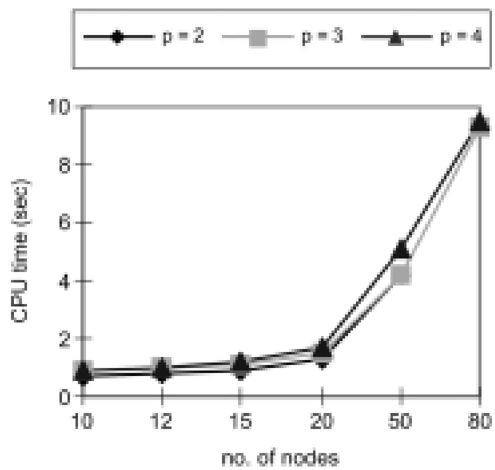

Figure 5.
Random data: CPU time to attain convergence (tconv) as a function of the no. of nodes

times for achieving convergence (tconv) for each value of p. The largest values of tbest (for problem size 80) are approximately 3.0, 5.0, and 6.0 CPU seconds for p, equal to 2, 3, and 4 respectively. The values for tconv are almost identical for the different values of p. The largest value is less than 10 CPU seconds.

Conclusions

In this paper we applied an Artificial Intelligence (AI) heuristic called simulated annealing (SA) to solve the p-Hub Median Problem (p-HMP). The p-HMP represents an important hub and spoke (H&S) network design that finds applications in several types of transportation and distribution networks. The performance of SA was compared with another AI heuristic called Tabu Search (TS) and two non-AI heuristics for the p-HMP. It can be concluded that the AI heuristic approaches to the p-HMP outperform the non-AI heuristic approaches in terms of solution value.

The results are significant to practitioners, since it means they can use AI approaches to design H&S networks for distributing their products quickly and at lower cost. The use of hubs implies that fewer routes are needed to connect the same number of points in a distribution network. This can lead to higher level frequencies between nodes (favoring a JIT production approach), better coverage of many outlying origins/destinations and low-traffic routes, and better driver retention due to reduced tour length. Similarly, the results are significant to academics, because more focus can be applied on AI approaches in developing heuristics for other types of H&S network designs. Also, use of results from AI approaches can be incorporated in developing optimal procedures. Designing optimal procedures is important for benchmarking the quality of the heuristics. Finally, in the context of supply chain management, the flow of information is as important as the flow of goods. AI approaches can be used in designing efficient H&S communication networks.

Future research will include the development of hybrid heuristics, which combine different types of AI approaches, to solve the p-HMP and other types of hub location problems. The performance of such hybrid heuristics will be compared with existing heuristics to find if improvements can be achieved.

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