Tabu Search Metaheuristics for Global Optimization of Electromagnetic Problems

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Abstract — **This paper presents a Tabu Search based strategy which has been applied to optimize the objective function associated to the design of a solenoids system. For this kind of problem, Tabu Search strategies are able to find the global minimum with considerably better performance than that obtained using a Simulated Annealing algorithm.**

Algorithms used to perform the self-tuning of Tabu Search parameters have also been developed, achieving interesting improvements in terms of computing time and quality of the solutions. Such algorithms are an original development of Reactive Tabu Search and they are based on the past history of the search.

Tabu Search strategies have also been applied in continuous optimization of the same problem with encouraging results.

Index of terms — **Magnetic fields, solenoids, optimization methods, minmax methods, tabu search algorithm, meta-heuristic methods.**

I. Introduction

The optimal design of electromagnetic structures using global optimization techniques has attracted a large number of researchers in the last years. This is due to the fact that the objective functions for this kind of problem are generally non linear, ill-conditioned and "classical" optimization algorithms may lead to poor results.

In this paper an optimization strategy based on the Tabu Search (TS) meta-heuristics is proposed. The TS algorithm guides the search for the optimal solution making use of flexible memory, allowing to take the search history into consideration [1-3].

The proposed TS optimization procedure is applied to a discrete electromagnetic problem whose cost function is very difficult to minimize and that may be considered a valid test bed for optimization algorithms. Furthermore, the number of variables (four) and their ranges allow an exhaustive search for the global minimum so that it is possible to compare the performances of different techniques [4].

Considering that real variables are often continuous in nature, an extension of the TS algorithm is proposed, where continuous variables are discretised into sub-ranges. Using the TS algorithm, it is possible to link to each design variable the corresponding sub-interval and to apply TS strategy in this finite alphabet. When a good solution is found, monodimensional optimization strategies allow finding the best value of each variable inside its corresponding sub-interval [5,6].

A good choice of the most important optimization parameters has a great impact on the performance of the procedure. For this reason a metaheuristic strategy to optimize such parameters both in the continuous optimization and in the discrete one is presented. This is automatically performed on the basis of the past history of the search. The resulting algorithm is an original implementation of Reactive Tabu Search (RTS) [7].

II. Test Problem

The design of solenoidal systems able to generate magnetic field with high level of homogeneity is a very important topic in many electromagnetic problems.

The magnetic field homogeneity is usually used as the objective cost to be minimized by means of suitable optimization procedures. Generally, this objective function has several local minima and non differentiable points. On the other hand, if no ferromagnetic parts are close to the magnet, it is possible to use integral formulas instead of numerical routines. This feature allows applying optimization techniques that require a large number of cost function evaluations in a reasonable time. In this paper, a benchmark test, directly derived from a real design in the field of Caesium frequency standard, is performed [4]. The flux density required by such applications is quite low (89 mT), but the constraints on inhomogeneity are very stringent because it must be lower than 100 p.p.m. in a cylindrical zone of 8 mm in diameter and 400 mm in axial length, whose track A-B is depicted in Fig.1.

The axial cross-section of a system of five solenoids able to comply with the above mentioned constraints is shown in Fig. 1. The main magnet is 700 mm long with an inner radius of 33.5 mm and thickness of 25 mm, the other four solenoids (compensating magnets) are put towards the end zone of the main coil and symmetrically with respect to its center. Actually, due to the cylindrical symmetry of the system, only

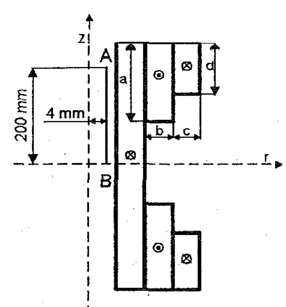

Fig. 1. Solenoid system used as test problem. Letters indicate design variables.

Manuscript received November 3, 1997.
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the design variables of two compensating coils have to be optimized, namely *a, b, c* and *d* in Fig. 1. Both the main coil and the compensating ones are connected in series and the best results may be obtained if the current flows as shown in Fig. 1.

The cost function f is defined as the maximum discrepancy between the axial component of the magnetic field **B**, evaluated in the point B in Fig.1, and the same component uniformly sampled on the line A-B.

$$f = \max_{i=1,..N} \frac{|B_z(r_B, z_i) - B_z(r_B, z_B)|}{B_z(r_B, z_B)} \quad (1)$$

The value of the magnetic field **B** has been calculated into N=31 points equally spaced on the line A-B depicted in Fig.1.

Concerning the computation of **B**, a computer code, based on the solution of the Urankar's integral formulas, has been employed [8-9]. In particular, since the environment surrounding the coils is air, the problem is linear and it can be faced with analytical procedures making use of the second kind complete integrals to integrate the Biot-Savart law [10].

In the discrete optimization problem, the variables b and c in Fig.1 are constrained to assume values from 1 to 30 mm with 1 mm steps, while the variables a and d may vary from 3 to 90 mm with 3 mm steps, with $d \leq a$. With these geometrical constraints, the total number of possible combinations is 418,500 but only 56 combinations of design variables are able to respect also the constraints on field homogeneity.

In continuous optimization the same constraints on the variables have to be complied with, but each design variable can assume all the real values within its range of variation.

The objective function of both discrete and continuous problems has several local minima, differing from the global one even by orders of magnitude, with very high variations and very narrow basins of attractions. This behaviour determines significant difficulties during optimization.

III. Optimisation Method

Tabu Search is a metaheuristic method that guides the search for the optimal solution making use of flexible memory systems which exploit the history of the search. TS consists of the systematic prohibition of some solutions to prevent cycling and to avoid the risk of trapping in local minima. New solutions are searched in the neighbourhood of the current one. The neighbourhood is defined as the set of the points reachable with a suitable sequence of local perturbations, starting from the current solution.

One of the most important features of TS is that a new configuration may be accepted even if the value of the cost function f is greater than that of the current solution. In this way it is possible to avoid being trapped in local minima.

Among all the visited solutions the best one is chosen. This strategy can lead to cycling on previously visited solutions. To prevent this effect, the algorithm marks as "tabu" certain moves for a number of iterations. To do this, a so-called tabu list T of length TT=|T|, named *tabu-tenure*, which can be fixed or variable, is introduced.

Some aspiration criteria which allow overriding of tabu status can be introduced if that move is still found to lead to a better cost with respect to the cost of the current optimum. This is a characteristic aspect of TS methods, whose main novelty is the use of flexible memory systems for taking advantage of the history of the search. The previously described memory is the so-called 'short term memory'. A second kind of memory called 'long term' can also be implemented.

Two main important long term memory concepts, which should be evaluated, are intensification and diversification strategies. Intensification strategies are based on the idea of encouraging move combinations and solution features historically found to be good. Diversification strategies, on the other hand, are designed to drive the search into new promising regions.

Summing up, the performance of a TS algorithm depends on the proper choice of the neighbourhood of a solution, on the number of iterations for which a move is kept as tabu, on the aspiration criteria, on the best combination of short and long term memory and on the best balances of intensification and diversification strategies. These choices are closely linked to the problem at hand and often require expensive "trial and error" processes. In order to achieve a general purpose algorithm able to reduce the computing time, a "reactive" scheme inspired by Battiti's work [7] has been implemented. According to this scheme the search history is employed both for tuning the algorithm parameters and for automatically balancing the diversification and intensification policy.

IV. Implementation

A. Generalities

Like many other optimization algorithms, TS is based on a sequence of moves that, starting from an initial admissible solution $\boldsymbol{x}_o$, takes to a new solution $\boldsymbol{x}_n$. In the discrete case the optimization variables $\boldsymbol{y}$ are constrained to assume a finite set of real values and a solution $\boldsymbol{x}$ is a vector of integers associated to each real variable.

Let y_{imax} and y_{imin} be the maximum and minimum of the i-th variable respectively, Δ_i the imposed value for the i-th variable variation and $m_i=(y_{imax} - y_{imin})/\Delta_i$ an integer that defines the number of different possible values that y_i can assume, then it is possible to identify y_i by means of an integer that varies within 1 and m_i. The real value of the variables that correspond to the elements of $\boldsymbol{y}$ may be achieved with the following expression: $y_i = \Delta_i \cdot x_i$. In the proposed optimization problem, taking into consideration the previously introduced constraints, m_i is equal to 30 for each variable *a, b, c* and *d*. A solution $\boldsymbol{x}$ is a vector of integers with four elements, each of them ranging from 1 to 30.

The set $\boldsymbol{D}$ of feasible solutions is defined as:

$$\boldsymbol{D} = \left\{ \boldsymbol{x} \,|\, x_i = 1,2,\ldots,m_i \text{ and } x_4 \leq x_1 \right\}.$$

The neighbourhood $N(\boldsymbol{x}_c)$ of the current solution $\boldsymbol{x}_c$ is defined as the set of all $\boldsymbol{x}$ achieved with an admissible move from x_c. An admissible move is the substitution of the current value x_c of the i-th variable with a new integer, x_{ni}, that is comprised in the range $[1, m_i]$ and is not equal to x_{ci}. Each time a move is performed, the current configuration x_c is entered in the tabu list T, which behaves like a first in-first out stack.

B. Improving strategies

1) Diversification strategy: A "frequency based" long term memory has been applied in order to improve TS capabilities. Every time a new vector x_n belonging to the $N(x_c)$ is examined, a counter $F(x_n)$ associated with this new vector, is increased. $F(x_n)$ indicates how many times the corresponding configuration has been evaluated and represents the frequency associated with a certain configuration x_n.

By using the $F(x_n)$ counter, it is possible to identify in $N(x_c)$ the best configuration x_b by verifying that the two following constraints,

a) x_n is not in the tabu list,

b) $f(x_n) + flag \cdot F(x_n) < f(x_b)$ where *flag* is a variable equal to 1 if $f(x_n) > f(x_c)$ and equal to 0 otherwise,

are complied with.

By so doing, a new solution x_n may be accepted as the best of $N(x_c)$, x_b, even if its cost function is greater than $f(x_c)$ when its frequency $F(x_b)$ is equal to zero. At the end of the research in $N(x_c)$, x_b is adopted as the new current solution x_c.

2) Intensification strategy: If the algorithm is not able to find a better optimum within a large number of iterations (e. g. 150), the exploration in a promising zone is intensified by restarting the search procedure from the last found optimum. The application of the b) condition permits to penalize moving along already visited configurations. This diversification strategy has a great impact on the algorithm due to the fact that it permits to explore new search paths.

3) Reactive Tabu Search (RTS): The TT may be formulated as a function of the $F(x_c)$ value. In this way it becomes a function of the long term memory as shown in [7]. The TT functional dependence on $F(x_c)$ is given by (2).

$$TT = TT \cdot (2 \cdot F(x_c) + 1) - 1 \qquad (2)$$

With the applications of (2), TT grows rapidly if $F(x_c)>0$ and decreases slowly if $F(x_c)=0$.

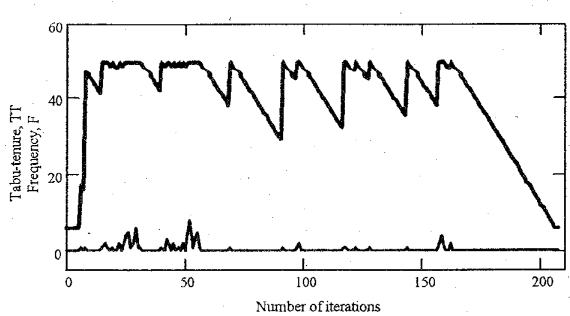

Fig. 2. Trend of tabu-tenure TT (bold line) and frequency F (plain line) vs. the number of iterations in reactive tabu search.

An upper and lower threshold for the TT has been imposed. The values imposed to such thresholds have a negligible impact on the RTS algorithm as it has been proven with the aid of a large number of tests.

Fig. 2 shows the trend of TT and frequency vs. the number of iterations.

C. Continuous optimization

In order to apply the previously described algorithm to an optimization problem dealing with continuous variables, the range of each variable x_i has been "encoded" subdividing it uniformly into a finite number, m_i, of sub-ranges.

Each solution y in the continuous may be associated with a vector of integers x as well as in the discrete optimization problem, due to the identification of each sub-range with an integer number. The choice of small sub-ranges permits to describe the whole range in a quasi-continuous fashion, but also increases the computational time. Past experiences [5,6] show that a choice of $m_i = 10$ is a good compromise.

In order to calculate the cost function for a solution given in terms of integers and representing the sub-intervals which its variables belong to, a mono-dimensional minimization with a Golden Search method performed after each move has been applied. To evaluate the moves, a random value in each sub-interval has been chosen.

A multidimensional simplex method of Nelder and Mead [11] is performed at the end of TS. This requires a considerable effort in terms of computational time, justified by the improvement of the found solutions.

V. Results and Discussion

In Table I the first 10 smallest values of the objective function achieved with an exhaustive search in the discrete case are reported. For each value of f the values of the design variables are also reported.

Table II shows the results achieved with the application of our implementation of TS and RTS in comparison with the results obtained with the application of Simulated Annealing (SA) [4].

TABLE I
The 10 Lowest Objective Function Values Achieved with an Exhaustive Search

Objective function	a (mm)	b (mm)	c (mm)	d (mm)
8.80433E-05	48	14	24	42
9.37770E-05	45	12	29	33
9.51439E-05	39	13	28	33
9.73087E-05	33	17	27	33
1.00310E-04	63	1	22	48
1.02585E-04	63	9	25	42
1.04501E-04	42	16	23	42
1.07329E-04	9	3	25	39
1.15456E-04	75	5	28	36
1.16406E-04	39	16	24	39

TABLE II
COMPARISON AMONG DIFFERENT ALGORITHMS:
%: PERCENTAGE OF OCCURRENCES OF FOUND MINIMA
N. FUNC. EVAL.: MEAN NUMBER OF OBJECTIVE FUNCTION EVALUATIONS ON 100 RUNS

Homogeneity	TS		RTS		SA	
	%	N. func. eval.	%	N. func. eval.	%	N. func. eval.
8.80433E-05	87	59736.8	86	33438.9	43	86689.4
9.37770E-05	9	72866.0	13	66455.9	12	88181.0
9.51439E-05	2	59014.0	1	84509.0	24	91937.7
9.73087E-05	1	138710.0	0	0.0	17	94448.1
1.00310E-04	1	99744.0	0	0.0	0	0.0
1.02585E-04	0	0.0	0	0.0	1	86801.0
1.04501E-04	0	0.0	0	0.0	1	89121.0
1.20925E-04	0	0.0	0	0.0	1	89121.0
1.21058E-04	0	0.0	0	0.0	1	87601.0

In order to evaluate the TS capability to solve the test problem, the procedure has been run 100 times, starting from random points in the domain of feasible configurations.

As it can be easily noticed in Table II, TS is able to find the lowest minimum of the f function with a percentage of success greater than that achievable with the SA algorithm even though a careful parameter tuning is required. The number of the objective function calls required with TS, even lower than SA, continues to be too large. With RTS it is possible to drastically reduce the number of evaluations keeping a high percentage of success. RTS seems to be the best tool to solve the optimization problem at hand: with the proposed methodology the number of function evaluations is lower than one half of those needed with SA and the percentage of success is double. In Fig. 3 the behaviour of RTS and SA is shown: the typical trend of the objective function for the two algorithms can be easily recognized.

The proposed optimization technique, based on RTS, has also been applied in continuous optimization. Fig. 4. shows the results achieved with 100 runs of continuos RTS. It has to be noticed that all the solutions found with the continuous optimization are able to comply with the homogeneity constraint. The best configuration obtained leads to a homogeneity value equal to 6.479E-5 (a=28.444 mm, b=17.792 mm , c=30 mm , d=28.443 mm).

It is worth noting that the objective function behaviour in discrete and continuous optimization is very different, but the proposed optimization technique is able to find very good solutions in a reasonable amount of time with high accuracy in both cases.

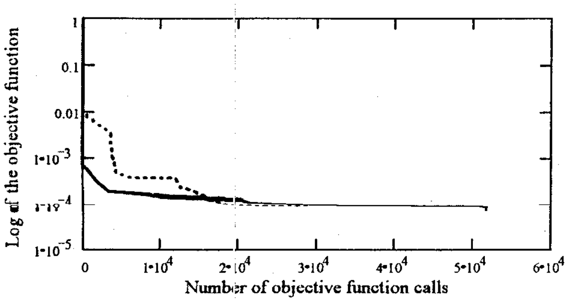

Fig. 3. Typical trend of the objective function for RTS (dashed lines) and SA (continuos line) algorithms.

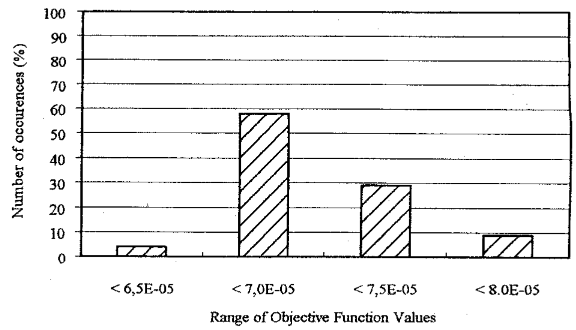

Fig. 4. Solutions (%) falling within the intervals specified in the x-axis.

CONCLUSIONS

TS has been successfully employed to solve an optimization problem of a magnetic structure characterized by stringent constraints on magnetic field homogeneity. The results have shown that TS is able to improve the performance of SA both for the computing time required and the accuracy of the solutions.

In the paper an RTS algorithm, able to self-adapt the algorithm parameters, is also proposed. RTS seems to be the best tool to solve hard optimization problems because it does not require a careful choice of the algorithm parameters.

The presented methodology, well suited for discrete optimization, has been also applied to the continuous one. The encouraging results achieved have demonstrated the validity of both the RTS algorithm and the discrete approach solving continuous problems.

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