Assigning Frequencies in GSM Networks*

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Abstract. Mobile communication is a key technology in today's information age. Despite the ongoing improvements in equipment design, interference remains a limiting factor for the use of radio communication. The author investigates in his PhD thesis how to largely prevent interference in GSM networks by carefully assigning the available frequencies to the installed base stations. The topic is addressed from two directions: first, new algorithms are presented to compute "good" frequency assignments fast; second, a novel approach, based on semidefinite programming, is employed to provide lower bounds for the amount of unavoidable interference.

The proposed new methods for automatic frequency planning are compared in terms of running times and effectiveness in computational experiments using instances from practice. For most of the heuristics the running time behavior is suited for interactive planning, and they provide good assignments from a practical point of view. Several of these methods are successfully employed by the German GSM operator E-Plus Mobilfunk GmbH & Co. KG.

The best lower bounds on the amount of unavoidable (co-channel) interference are presently obtained from solving semidefinite programs. These programs arise as nonpolyhedral relaxation of a minimum k-partition problem on complete graphs. The success of this approach is underpinned by revealing structural relations between the solution set of the semidefinite program and a polytope associated with an integer linear programming formulation of the minimum k-partition problem. Comparable relations are not known to hold for any polynomial time solvable polyhedral relaxation of the minimum k-partition problem. The application described is among the first of semidefinite programming to large industrial problems in combinatorial optimization.

1 Introduction

The General System for Mobile communication or, for short, GSM is nowadays the predominant technology for mobile communication. More than half a billion people in over 150 countries use GSM for mobile telephony and for exchanging short text messages (SMS). Within a decade, GSM has grown from a costly service used by few professionals to a mass market with penetration rates higher than 70 % in Finland and Iceland. In some countries, the mobile phone subscribers already outnumber the fixed-line telephone subscriptions.

The mobile communication relies on a radio link between the user's mobile phone and some stationary base station, which is part of a GSM operator's infrastructure, see Fig. 1. Currently, a base station typically serves three

* This presentation is based on the Ph.D. thesis of the author [5].

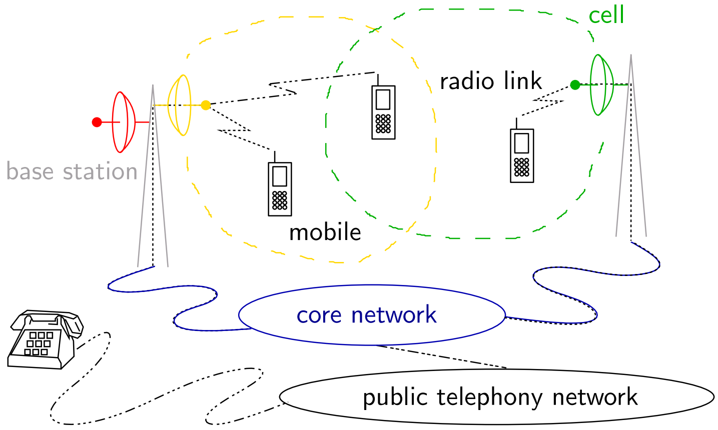

Fig. 1. GSM bits and pieces

different areas (cells) with up to 6 transmitters. Each transmitter uses a frequency slot of 200 kHz, called channel, to handle at most 6–8 users in parallel via time multiplexing (time division multiple access, TDMA). Nearby transmitters have to use different channels (frequency division multiple access, FDMA). As with all forms of radio communication, the limited radio spectrum is a bottleneck. National regulation authorities usually license between 60–120 channels of radio bandwidth to GSM operators.

An operator has to reuse his channels multiple times to operate the several tens of thousands transmitters, which are typically installed in a network. Each radio link, however, requires a signal of sufficient strength which, at the same time, is not suffering too severely from interference by other signals, see Fig. 2. Significant interference may be caused by transmitters using the same channel (co-channel) or an adjacent channel.

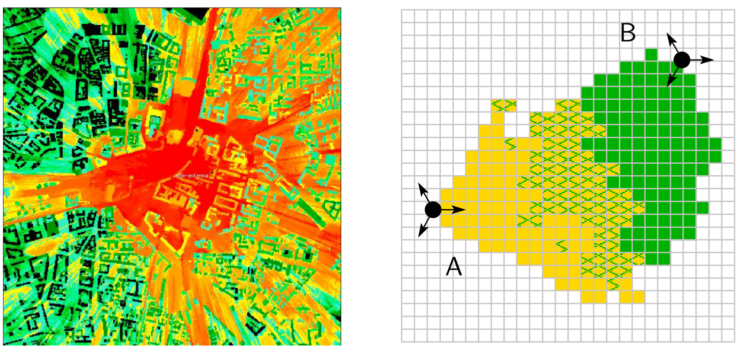

Fig. 2. Field strength and interference: **(a)** inhomogeneous decay of a signal's field strength (path loss) in an urban environment (by courtesy of E-Plus); **(b)** estimation of interference in terms of affected cell area

The reuse of channels is therefore limited, and frequency planning turns into a key issue in fully exploiting the available radio spectrum. Notice that it is customary to use the word frequency as a synonym for channel in this context. By avoiding interference, frequency planning has a significant impact on the quantity as well as on the quality of the radio communication services.

Frequency assignment is usually performed at the end of a chain of planning activities. The placement of the base station as well as the selection and configuration of their antennas are the basis for delivering the desired network coverage. The subsequent decisions on how many transmitters to operate in each cell build the foundation for the desired network capacity. The final step of assigning the frequencies "merely" has to ensure that the coverage and capacity goals can be met: namely, by providing each transmitter with a frequency that is (locally) at most moderately interfered.

The Ph.D. thesis [5] addresses several topics, ranging from the technical background of the GSM frequency planning problem (Chap. 2) over alternative mathematical models (Chap. 3) and heuristic planning methods (Chaps. 4, 5) to quality assessments for the generated frequency plans (Chaps. 6–8). An overview is given in the following. The theory (Chaps. 7, 8) underlying the computation of unavoidable interference for the quality evaluation, however, is not addressed here.

Much of this work is related to a cooperation between the ZIB and the German GSM 1800 network operator E-Plus Mobilfunk GmbH & Co. KG. The focus of the cooperation was primarily on fast frequency planning heuristics for the use in the regular radio planning process at E-Plus. New planning methods were developed at ZIB and integrated into E-Plus' software environment. In 1997, the new software was first used successfully in practice. A series of extensions have meanwhile been implemented.

2 Optimization Model for GSM Frequency Assignment

The frequency planning problem sketched above can be formalized as a combinatorial minimization problem. An undirected graph $G = (V, E)$ is defined together with vertex- and edge-labelings. A vertex is introduced for each transmitter (demand for one frequency). An edge is introduced whenever there is an interdependency between the corresponding transmitters.

The edge-labelings record three types of interdependencies. The *separation* label $d(vw)$ is the minimum required difference of the channels assigned to v and w. Typical values are (0,) 1, 2, 3. The *co- and adjacent channel interference* labels $c^{co}(vw)$, $c^{ad}(vw)$ record how much interference is incurred in case the same channel, respectively adjacent channels are assigned to v and w. Interference is normalized to values between 0 and 1.

Each vertex label A_v specifies the set of *available channels* for the transmitter v. These sets are often genuine subsets of the frequency spectrum C licensed to an operator. Such restrictions arise, for example, along national borders, where a cross-border coordination of the channel use is necessary to prevent (strong) interference between bordering networks. The licensed spectrum itself is usually contiguous.

A *frequency assignment* or simply an *assignment* is a function $y\colon V \to C$. An assignment is *feasible* if every carrier $v \in V$ is assigned an available channel and all separation requirements are met, that is, if

$$y(v) \in A_v \qquad \forall v \in V\ , \tag{1}$$
$$|y(v) - y(w)| \geq d(vw) \qquad \forall vw \in E\ . \tag{2}$$

Finding a feasible assignment is closely related to coloring a graph. Frequency assignment is a generalization of *list colorings* and related to *T-colorings* and *list T-colorings* of graphs [5, Chap. 3]. Drawing on this connection, it is easily shown that finding any feasible frequency assignment is $\mathcal{NP}$-complete in general.

In practice, not just some feasible assignment is of interest, but assignments that minimize the sum of co- and adjacent channel interferences are in demand. The corresponding optimization problem

$$\min_{y\ \text{feasible}} \sum_{\substack{vw \in E:\\ y(v)=y(w)}} c^{co}(vw) \quad + \sum_{\substack{vw \in E:\\ |y(v)-y(w)|=1}} c^{ad}(vw) \tag{FAP}$$

is called the *frequency assignment problem.*

This model has proven useful and is largely accepted among researchers and practitioners. From a computational complexity point of view, however, optimal solutions are even very hard to approximate, see [5, Chap. 3] for details. Further models of practical relevance or of theoretical interest are discussed in [1, 4–7, 15].

3 Heuristic Planning Methods

The focus is on planning heuristics, capable of dealing with carrier networks of around 2000 transmitters in a few minutes on a modern PC or workstation. Such methods are well-suited for practical applications, with a particular emphasis on intermediate iterations in the planning cycle.

Seven heuristic planning methods are described [5, Chap. 4]: three greedy-type construction methods, T-Coloring, Dsatur with Costs, and Dual Greedy, as well as four improvement methods, Iterated 1-Opt, k-Opt, Vds, and Mcf. The performance of each heuristic (sometimes with augmentations) and its parameter interdependence are extensively analyzed [5, Chap. 5]. Most of the above methods are suited (in combination) for automatic frequency planning in practice. The Dual Greedy drops out, because it is slow and produces by far the poorest results.

The concerted acting of various combinations of them is studied on the basis of eleven realistic planning instances, which have been made available over the Internet [9]. Table 1 displays several characteristic parameters of the constraint graphs associated with the planning instances. Notice the high average degrees of the vertices, i. e., the large numbers of transmitters that may directly be affected by the frequency assignment to one transmitter. Notice also the large maximum clique numbers, which in most cases proves directly that an interference free frequency assignment is impossible.

Table 1. Characteristics of constraint graphs of realistic planning instances [9]

	$\lvert V(G)\rvert$	density $\varrho(G)$ in %	avg. degree	max. degree $\Delta(G)$	max. clique $\omega(G)$	$\lvert\{e \in E : d(e) \neq 0\}\rvert$	$\lvert\{e \in E : c^{co}(e) \neq 0\}\rvert$	$\lvert\{e \in E : c^{ad}(e) \neq 0\}\rvert$	spectrum size
K	267	56.57	151.0	238	69	1053	19111	996	50
B[0]	1886	13.59	256.4	779	81	7288	234479	4263	75
B[1]	1971	13.46	265.3	805	84	7996	253441	4825	75
B[2]	2214	13.50	299.0	916	93	10284	320684	6871	75
B[4]	2775	13.44	373.0	1133	120	16663	500805	12524	75
B[10]	4145	13.41	555.9	1704	174	38234	1113850	33548	75
Sie1	930	9.03	84.0	209	52	6039	33002	9911	75
Sie2	977	49.17	480.4	877	182	17761	216912	25615	43
Sie3	1623	9.18	149.1	519	78	23093	97861	15069	76
Sie4	2785	10.50	292.3	752	100	27964	379052	26445	39
Sw	310	8.29	25.7	94	21	3984	0	2075	3 + 49

4 Automatic Frequency Planning

In essence, the following observations are made [5, Chap. 5]. There is one particular strong combination of the fast heuristics presented: a self-tuning variant of the Dsatur with Costs start heuristic, combined with the Vds improvement heuristic. This combination achieves a decent balance between solution quality and running times.

The resulting assignments are usually not much worse than those obtained by the elaborate Threshold Accepting method [4, Section 4.2.5]. With respect to the maximum incurred co- and adjacent channel interference, they are even sometimes better. The precise running times of Threshold Accepting are not public, but they are roughly one order of magnitude higher than those of the fast heuristic combinations. In case yet faster methods are needed, a combination of a self-tuning variant of T-Coloring with Vds or, even faster, with Iterated 1-Opt may be attractive.

Interference plots are commonly used for frequency planning. They depict the (likely) occurrence of interference on the basis of the signal level predictions. Figure 3 contains two such plots, where the difference in dB between the serving sector's signal and the second strongest signal at the same frequency is color-coded. Clearly visible are the interference reductions achievable with the proposed methods in comparison to a formerly established, commercial routine. (This routine has meanwhile been replaced by the tool vendor.) In another example more than 96% of the interference could have been removed [5, Chap. 5].

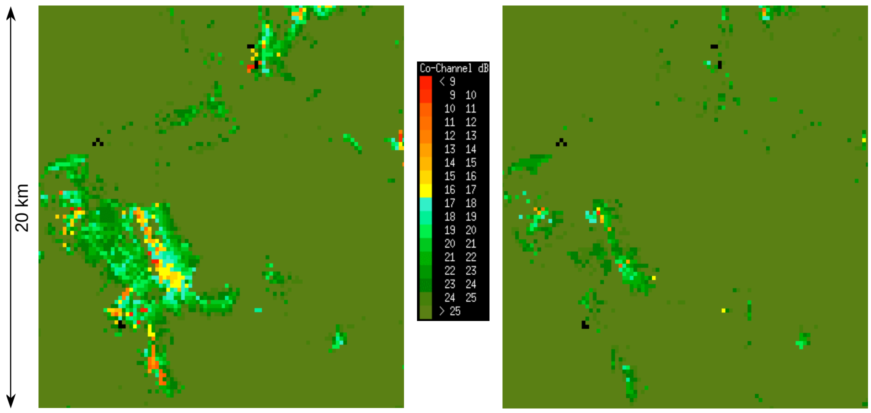

Fig. 3. Interference plots: improvements from optimization

In order to rigorously assess the quality of the plans, lower bounds on the amount of unavoidable interference are in demand. By far the best lower bounds on the *unavoidable co-channel interference* are currently obtained from semidefinite programming. Significant bounds are given for the five scenarios K, B[4], B[10], SIE2, and SIE4 [5, Chap. 6]. The reported bounds LB yield quality gaps, computed as $1 - LB/UB$, between the provably unavoidable co-channel interference and a "good" heuristic solution with co- and adjacent channel interference totaling to UB. The gaps are 50% for K, 77% for B[4], 63% for B[10], 53% for SIE2, and 66% for SIE4. From the application point of view, these gaps may not be satisfying. Nevertheless, they are the first noteworthy bounds on the gap for large realistic instances.

The link between the bound on unavoidable co-channel interference in frequency planning and semidefinite programming is a semidefinite relaxation of the well-known graph minimum k-partition problem [5, Chaps. 7, 8]. These problems are obtained by relaxing the original frequency planning problems as follows. Each vertex may receive any of the k frequencies in the available spectrum; all separation requirements are reduced to at most 1; and the adjacent channel interference is ignored. A lower bound for the optimal k-partition, and hence a bound on the unavoidable co-channel interference, is computed by solving the semidefinite relaxation of the k-partition instances. The dual semidefinite programming solvers [3, 10] are used for this purpose.

5 Conclusions

Planning the use of frequencies is a central task in managing a GSM network. It is a cornerstone for providing the desired grade and quality of service. Three planning situations are distinguished.

- In the *relaxed* situation, a new frequency assignment is to be generated for a large network region. Many frequencies are available, and the objective is to minimize interference, thus providing radio service at high quality.
- In the *congested* situation, again a new plan for large network portions is to be produced, but the number of available frequencies hardly allows to provide the desired grade of service (at the least accepted level of quality).
- In the *adaption* case, the assignment shall be adapted locally to changes in the network.

Each of these situations seems to call for different planning methods. For surveys directed towards the "congested" case, see [11, 12, 15]. The "adaption" case has hardly been addressed explicitly yet. The focus here is on the "relaxed" planning situation.

The goal was to design algorithms for generating frequency plans quickly that incur as little interference as possible. They are particularly attractive for interactive planning processes, where alternative plans are produced for tentative network changes. Heuristic methods of small theoretical running times were proposed. Their computational behavior was analyzed on eleven realistic, publicly available scenarios.

Several of these methods are successfully used at E-Plus. Better frequency assignments are obtained much quicker than through the previous planning process. The software is also incorporated into a commercial GSM radio network planning tool as the standard frequency planning component.

The development of new planning methods seems to slow down lately. The lacking demand from major European GSM operators might be a reason. Market saturation is approached and the need for network expansions decreases. A more fundamental reason may lie in the difficulty to provide reliable interference predictions. These are the basis for the frequency plan optimization. At the current stage, the quality of a frequency assignment may depend more on the field strength prediction model used for interference prediction than on which modern planning heuristics is applied [4].

GSM is a second generation, digital system for mobile communication. The upcoming Universal Mobile Telecommunication System (UMTS) is a third generation, offering transmission rates up to 384 kbps. UMTS uses a fundamentally different way to support multiple radio links in parallel (code division multiple access). Frequency planning is no longer necessary with UMTS. A high price has to be paid for this convenience, however: provisioning coverage and capacity are tightly coupled. Base stations affect each other much more with respect to coverage and capacity. This spawns a new line of research, focusing on dimensioning UMTS radio networks [2, 8, 13, 14].

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