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Upper and lower bounds for the fixed spectrum frequency assignment problem

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Abstract. A survey of the results described in the author's PhD thesis (Montemanni 2001) is presented. The thesis, which was supervised by Prof. Derek H. Smith and Dr. Stuart M. Allen, has been defended in January 2002 at the University of Glamorgan (U.K.).

The thesis proposes new heuristic algorithms, based on well-known meta-heuristic paradigms, and new lower bounding techniques, based on linear programming, for the fixed spectrum frequency assignment problem.

Key words: Radio frequency assignment, fixed spectrum, upper bounds, lower bounds

AMS classification: 90C27, 90C59,05C90, 90C05

1 Introduction

The author's PhD thesis studies the fixed spectrum frequency assignment problem, an important problem which arises in telecommunications, when a wireless network has to be established. The problem is modelled in terms of combinatorial optimization.

The main contributions of the thesis are some new heuristic algorithms based on meta-heuristic paradigms, summarised in Sect. 2, and some new lower bounding techniques, based on linear programming and outlined in Sect. 3.

The thesis is written in English and can be found at
http://www.glam.ac.uk/sot/doms/Research/Montemanni_thesis_disemina.pdf.

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The last decade has witnessed a dramatic increase in the importance of large scale radio networks, particularly in cellular mobile telephone applications. The corresponding growth in demand for frequencies has highlighted the importance of good network planning. The available radio spectrum is a limited resource, and frequency reuse must be implemented, in a way which balances the economies achieved in the use of spectrum with any consequent loss of quality in the network.

The frequency assignment problem involves the assignment of discrete channels (frequencies) to the transmitters of a radio network. A separation between the frequencies assigned to transmitters close to each other is required to avoid interference. Unnecessary separation causes an excess requirement for spectrum. Consequently good assignments minimise both interference and the spectrum required.

In the fixed spectrum frequency assignment problem the spectrum available is given and the target is to minimise the total interference of the system.

Interference is modelled through binary constraints, and consequently the problem can be represented by an undirected weighted graph.

2 Upper bounds

The thesis presents an adaptation to the fixed spectrum frequency assignment problem of two well-known meta-heuristic algorithms.

The first method is based on the simulated annealing paradigm, which is derived from thermodynamic principles. An analogy between the way that liquids freeze and crystalise, or metals cool and anneal, and the strategy followed to accept or reject cost-increasing solutions in our algorithm is created. The aim is to minimize the cost of the objective function in the same way liquids and metals reach their minimum energy states.

The second algorithm presented represents the first application to the problem of a modified version of the tabu search paradigm. The basic idea of the original method is to partially explore the search space of all feasible solutions by a sequence of moves. At each iteration, the move carried out is the most promising among those available. A tabu list forbids a set of moves at each iteration, aiming to help the algorithm to escape from local (but not global) minima. In the modified version presented here, the size of the tabu list is dynamically reduced while the computation proceeds. This allows the algorithm to select a promising region of the solution space and concentrate the search in it when the execution is close to its end. This is very important for problems like frequency assignment, where many equivalent solutions tend to be scattered around in different, well separated, regions of the solution space.

The improved quality of the solutions obtained is demonstrated using several sets of benchmark problems that have appeared in the literature. A further important advantage demonstrated in the thesis is that the scheme makes the algorithm more

robust, by easing the task of parameter tuning when a new class of problems has to be solved.

An efficient implementation, which uses ad-hoc data structures, also contributes to the very good performance of the algorithms. A detailed explanation of these data structures, together with a discussion of the advantages they guarantee, can be found in Montemanni and Smith (2001).

A description of the tabu search algorithm can be found in Montemanni et al. (2003a). This paper also presents the best assignments for some of the instances of the COST 259 test-bed, which are available at FAPweb (http://fap.zib.de), a web site entirely dedicated to frequency assignment problems.

3 Lower bounds

Some novel lower bounding techniques which, given a problem, work by combining lower bounds calculated for some of its clique-like subproblems are presented. The key idea is that it is quite easy to calculate tight lower bounds for problems represented by complete graphs (cliques).

The lower bounds for clique-like subproblems are produced by two different methods. The first is based on a linear program whose solution provides a lower bound for the penalty paid (or for the number of constraints violated) in the given clique-like subproblem. The second lower bound, which gives an estimate for the number of constraint violations, is provided by a closed formula.

The most effective method to generate estimates for general problems is based on the linear relaxation of an integer program, which is reinforced with some sets of inequalities derived from the lower bounds calculated on the clique-like subproblems. The results obtained are presented in Montemanni et al. (2001).

Some attempts to improve the quality of the lower bounding techniques, both for those working on general problems and for those developed for clique-like subproblems, are presented. In both the cases, new reinforcing inequalities are added to the respective linear programs, in order to improve the estimates provided.

The quality of lower bounds is improved by using some of these new ideas in conjunction with new theoretical results which permit the simplification of the linear relaxation on which our technique is based. As a consequence memory is saved and the computation is speeded. The results obtained are presented in Montemanni et al. (2003b).

The computational results of the lower bounding techniques are extremely satisfactory (75.9% of the upper bounds on average), particularly in light of the fact that no other general purpose lower bounding techniques exists in the literature (as far as we are aware).

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