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Random Move Tabu Search for Freight Proportion Allocation Problem

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Abstract

We study a freight proportion allocation problem (FPAP), which is a kind of transportation problem faced by MG, one of the world's leading grocery retailers. MG has a large quantity of freight for carriers to ship to Europe. During the process of freight allocation, the shipper must consider three constraints, which are minimum quantity commitment (MQC), quantity limit per carrier and cost balance among sales divisions. With these constraints, the FPAP becomes computationally intractable. By incorporating random move subroutine, we devised a special tabu search procedure to solve this problem. Different from classical tabu search who usually runs in the feasible regions, random move tabu search enables the search process to enter into infeasible regions and visit disjointed feasible regions. Extensive experiments have been conducted to measure the performance of our proposed tabu search and CPLEX solver and have shown that the random move tabu search behaves better.

1 Introduction

MG is one of the top 5 largest grocery retailers worldwide and its purchasing department (refer to purchasing department as shipper) located in Hong Kong annually procures products with turnover exceeding 2 billion euro, about 64000 orders and 45000 TEUs, from China and other Asian countries, to satisfy the demands of its sales divisions distributed in over twenty European countries. Although all sales divisions as well as the shipper are members of MG, they are financially independent, i.e., the stuffs receive bonus commensurate with the profits and performance of their respective divisions. Thus, the shipper treats all sales divisions as independent buyers. Restricted by MG, the shipper can not reap benefits from the transportation process and commission fees are the only source of its profits. Therefore, information regarding transportation is allowed to be accessed by all participating buyers.

At the beginning of its fiscal year, the shipper forecasts the coming annual volume associated with each loading-discharge port pair (refer to loading-discharge port pair as shipping lane) and receives rates and minimum quantity commitments (MQC) through the Request-for-Quotation (RFP) process from candidate far-ocean forwarding carriers. MQC is a common restriction emerging from real practice which leads the transportation problem to computational intractability[6]. It is motivated by the U.S. Federal Maritime Commission, which stipulates that the total volume of freight allocated to each carrier either none or at least as large as a fixed minimum quantity.

Assume there is a set of shipping lanes $J = \{j : 1 \leq j \leq n\}$ and each lane $j \in J$ has forecasting demand d_j. Allocating freight of each lane to several carriers can increase the reliability and flexibility of the shipper's service and so the shipper likes the allocation patterns such as (carrier $A \leftarrow 40\% * d_j$, carrier $B \leftarrow 40\% * d_j$, carrier $C \leftarrow 20\% * d_j$) or (carrier $A \leftarrow 30\% * d_j$, carrier $B \leftarrow 30\% * d_j$, carrier $C \leftarrow 30\% * d_j$, carrier $D \leftarrow 10\% * d_j$). This requirement can be justified by following reasoning: for lane j, many deliveries during the whole year constitute the quantity d_j. If only one carrier is available, it is of great possibility that someday when one delivery arrives at the loading port, the target carrier happens to have no space left or this delivery

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misses the departing date of the designated carrier. In these situations, this delivery has to wait until next shipping date of that carrier, which may result in the delivery delay since span of one carrier's two consecutive shipping dates is usually a long period. However, if there are other alternative carriers, this delivery can be handled much soon. Moreover, unpredictable low-quality performance of some carrier has limited influence on the service level of lanes when more carriers join in. To help the shipper achieve its preferable allocation pattern, the mechanism we adopt is setting a quantity limit for each participating carrier and as a result, the minimum number of carriers serving each lane can be guaranteed. For instance, for the lanes requiring at least 4 carriers we can set a rule "each carrier can get at most $30\% * d_j$"; for the lanes requiring at least 3 carriers, we use the rule "each carrier can get at most $40\% * d_j$", etc. Consequently, one quantity limit constraint "each carrier can get at most $p_j * d_j$, where $p_j \in [0, 1]$", is incorporated in our model and the parameter p_j can be determined by the shipper according to its practical needs.

We assume there exists one-one mapping relationship between discharge ports and sales divisions. Freight proportion allocation problem (FPAP) can be regarded as a two-stage process: (1) select a carrier combination from candidate carriers; (2) allocate freight of all lanes to the selected carriers. For any selected carrier combination, we can easily find the lowest rate of each lane and the theoretically lowest total transportation cost for each sales division. The theoretically lowest cost is calculated on the basis of the selected carrier combination and the assumption that the shipper forwards all freight at the lowest rates found in selected carriers. Intuitively, it is ideal to allocate all freight of each lane to the carrier whose rate is the lowest, but doing so violates the quantity limit constraint and the resulting allocation may not satisfy the MQC constraint. Therefore, it is highly possible that the actual total transportation cost paid by some sales divisions are higher than their theoretically lowest costs. Considering MQC and quantity limit constraint, the situation shown in Table 1 may occur in which buyer A gets actual cost 10% more than its theoretically lowest cost, whereas buyer B need pay 20% more. If these two buyers know all information regarding rates of selected carriers and allocation results, buyer B is most likely to complain the unfairness of this allocation. We can balance the actual costs among sales divisions by imposing a balancing constraint like "the actual cost gained by all sales divisions must be at most 15% more than their corresponding theoretically lowest cost " and then the following result shown in Table 2 may be generated. The percentage we used in balancing constraint is called *balancing factor*, represented by α.

After comparing Table 1 with Table 2, we can find that the total cost in the latter table is slightly larger than that in the former one, which means that in order to balance the costs, we may sacrifice the overall cost. Cost balancing constraint could not make all buyers absolutely faire, but it does effectively get rid of the situations that some buyers pay unreasonably high transportation costs.

	theoretically lowest cost	actual cost	
Division A	1000	1100	10%
Division B	500	600	20%

Table 1. Example of Unbalanced Allocation

	theoretically lowest cost	actual cost	
Division A	1000	1150	15%
Division$B$	500	570	14%

Table 2. Example of Balanced Allocation

2 Literature Review

There has been much discussion on the quantity commitment in supply contracts [9] [12] [2] [3], which is very similar to the MQC constraint in our work. In commitment-purchase contracts the unit price of some product is based on the committed quantity of product or total dollar amount of purchases from the supplier. These contracts are commonly applied in a stochastic environment and as a result they reduce both the flexibility for the buyer and uncertainty for the supplier. In recent years, in-depth investigation on procurement of transportation services with MQC has been conducted by Lim *et al*. One constraint that total quantity of cargoes delivered by each carrier must be none or at least as large as a fixed minimum quantity is incorporated in the traditional transportation problem, which makes the new problem computationally intractable, and several schemes were devised to seek high-quality allocation plans [6]. Lim *et al.* [7] further extended previous work by considering an fixed selection cost associated with each carrier, and for the extended problem they have shown that not only is finding an optimum solution $\mathcal{NP}$-hard in the strong sense, but finding a feasible solution is $\mathcal{NP}$-hard.

The transportation problems with MQC studied in previous works mainly focus on how to minimize the total transportation cost, however in real business the shipper is willing to make their buyers happy by treating them fairly and is sensitive to the response of their buyers because, generally speaking, the sales loss caused by dissatisfaction of customers is much more than the savings of transportation cost. Taking fairness of allocation into account, we are the first to introduce the concept of cost balancing constraint, which can prevent the actual cost of some buyer from deviating its theoretically lowest cost too much.

Random move tabu search implemented in our work borrowed some ideas from the works [8][11][1][10]. The tabu search processes developed in above articles have a common characteristic that at each iteration, one permissible and feasible solution can be always found in the neighborhood of current solution. If we use classical tabu search to solve our problem, at some iteration there may not exist permissible and feasible solution to be chosen as next stop. In order to deal with this special situation, we devised a random move subroutine and incorporated it into classical tabu search process.

3 Model Formulation

The purchasing department buys transportation service from m candidate carriers, indexed by $I = \{i : 1 \le i \le m\}$ for lane set $J = \{j : 1 \le j \le n\}$. And there are s sales divisions (also s discharge ports), indexed by $S = \{s : 1 \le k \le s\}$ and $D_k \subseteq J$ $(1 \le k \le s)$ represents the lanes whose discharge ports correspond to sales division k. M_i is the requested MQC of carrier i and $r_{i,j}$ is the shipping rate of forwarding freight of lane j by carrier i. If we have determined the carrier combination, we can construct a new transportation network shown in Figure 1.

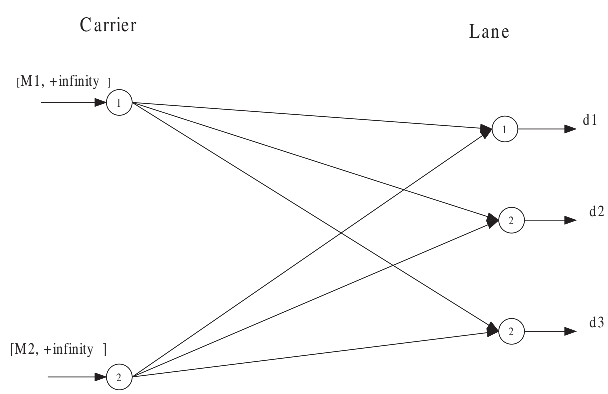

Figure 1. Carrier-lane Transportation Network

We use binary decision variable y_i for each carrier and assign $y_i = 1$ if the carrier i is selected and $y_i = 0$ otherwise. Letting decision variables $x_{i,j,}$ represent the quantity of freight transported through arc "$i \to j$", we have the following mixed integer programming (MIP) model for FPAP:

$$\textbf{MIP:}\ z = \min \sum_{i \in I} \sum_{j \in J} r_{i,j} x_{i,j} \tag{1}$$

$$\text{s.t.} \sum_{i \in I} x_{i,j} = d_j,\ for\ j \in J \tag{2}$$

$$M_i y_i \le \sum_{j \in J} x_{i,j} \le y_i \sum_{j \in J} d_j,\ for\ i \in I \tag{3}$$

$$\sum_{j \in D_k} \sum_{i \in I} r_{i,j} x_{i,j} \le (1+\alpha) \sum_{j \in D_k} \{d_j r_{i_j,j} + (1 - y_{i_j})M\},\ for\ k \in S,\ i_j \in I \tag{4}$$

$$0 \le x_{i,j} \le p_j d_j,\ x_{i,j} \in R,\ for\ i \in I,\ j \in J$$

$$y_i \in \{0,1\},\ for\ i \in I$$

where M is a large number. It can be seen that the objective (1) minimizes the total transportation cost; since for each lane j, the total volume allocated to all carriers is capped by d_j, constraints (2) apply; constraints (3) ensure each selected carrier can get at least M_i freight; constraints (4) balance the costs among sales divisions and guarantee the actual cost of each sales division is at most $100\alpha\%$ more than its theoretically lowest cost. We use following example to explain constraints (4) in detail. Suppose there are only two candidate carriers and D_1 contains two shipping lanes (1 and 2), we can get complete constraints (4) for sales division 1 as follows:

1. $\sum_{j \in D_1} \sum_{i \in I} r_{i,j} x_{i,j} \le (1+\alpha)\{d_1 r_{1,1} + d_2 r_{1,2} + (1-y_1)M\}$
2. $\sum_{j \in D_1} \sum_{i \in I} r_{i,j} x_{i,j} \le (1+\alpha)\{d_1 r_{1,1} + d_2 r_{2,2} + (1-y_1)M + (1-y_2)M\}$
3. $\sum_{j \in D_1} \sum_{i \in I} r_{i,j} x_{i,j} \le (1+\alpha)\{d_1 r_{2,1} + d_2 r_{2,2} + (1-y_2)M\}$
4. $\sum_{j \in D_1} \sum_{i \in I} r_{i,j} x_{i,j} \le (1+\alpha)\{d_1 r_{2,1} + d_2 r_{1,2} + (1-y_1)M + (1-y_2)M\}$

Recall that we have defined the theoretically lowest cost on the basis of selected carrier combination rather than all candidate carriers. If carrier 1 is not contracted with, i.e., $y_1 = 0$, all items containing y_1, such as item $1, 2, 4$, become redundant due to the existence of M and carrier 1 will not be considered in calculating the theoretically lowest cost of each sales division. If both of carrier 1 and 2 are selected and assume the theoretically lowest cost of sales division 1 is $d_1 r_{1,1} + d_2 r_{1,2} = \min\{d_1 r_{1,1} + d_2 r_{1,2}, d_1 r_{1,1} + d_2 r_{2,2}, d_1 r_{2,1} + d_2 r_{2,2}, d_1 r_{2,1} + d_2 r_{1,2}\}$, item $2, 3, 4$ become redundant. From this example, it is not difficult to find that as the size of problem grows, the number of constraints multiplies explosively.

Theorem: The FPA problem is $\mathcal{NP}$-complete in the strong sense.

Proof: According to the proof technique by restriction [4], if we can show the given problem Π contains a known $\mathcal{NP}$-complete in the strong sense problem Π' as a special case, so is Π $\mathcal{NP}$-complete in the strong sense. To obtain the restricted problem Π', we make M_i for all carriers equal to M' and relax the cost balancing and quantity limit constraints. The restricted problem Π' can be described as model RP:

$$\textbf{RP=} \min \sum_{j \in J} \sum_{i \in I} r_{i,j} x_{i,j} \tag{5}$$

$$\text{s.t.} \sum_{i \in I} x_{i,j} = d_j,\ for\ j \in J \tag{6}$$

$$M' y_i \leq \sum_{j \in J} x_{i,j} \leq y_i \sum_{j \in J} d_j,\ for\ i \in I \tag{7}$$

$$x_{i,j} \geq 0,\ x_{i,j} \in R,\ for\ i \in I,\ j \in J$$

$$y_i \in \{0,1\},\ for\ i \in I$$

Since problem Π' has proved to be $\mathcal{NP}$-complete in the strong sense by a reduction from **Cover By 3-Set (X3C)** problem in [6], we can directly derive that FPA problem is $\mathcal{NP}$-complete in the strong sense. $\square$

Setting zero to α is equivalent to allocating all freight to the carriers associated with the lowest rates, but as stated before, doing so usually violates quantity limit and MQC constraints. Smaller α is better off balancing costs, however it not only may increase the overall transportation cost greatly, but also may lead model MIP to infeasibility. There must exist a minimal α, denoted by α', which is reached if MIP is infeasible in interval $[0, \alpha')$ and feasible in interval $[\alpha', +\infty]$. In this work, we do not study how to choose a reasonable α and only investigate how to solve the given model MIP.

4 Solution Procedure

Tabu Search (TS) is the best way so far we can find to handle large-scale cases of model MIP. Tabu search as a meta-heuristic method for solving combinatorial optimization problems, which is generally attributed to Fred Glover[5], proceeds iteratively from one solution x to another solution x' in the neighborhood space with the aid of tabu search memory. Solution x can be visited several times, but neighborhood spaces may be different each time because the memory changes as the tabu search progresses. Tabu search memory is managed by creating one or several tabu lists, which contain the historical information of solutions that have been accessed recently. These lists keep a set of solutions as tabu to avoid repetition of solutions visited previously. Possibly, some tabu solutions might be the best solution so far and might not have been visited. To reach these best solutions, aspiration criteria are introduced to override the tabu state of those solutions. One of the commonly used aspiration criteria is to allow moving to solutions which are better than the currently best known solution.

There are two states for carriers in our tabu search process, i.e.,

$$I_1 = \{i | y_i = 1,\ i \in I\} \text{ and } I_0 = \{i | y_i = 0,\ i \in I\}$$

and $I_1 \cup I_0 = I$. If values of binary variables, y_i, are determined, model MIP becomes $MIP(I_1)$, which is given by:

$$\textbf{MIP}(I_1)\textbf{:}\ z = \min \sum_{i \in I_1} \sum_{j \in J} r_{i,j} x_{i,j} \tag{8}$$

$$\text{s.t.} \sum_{i \in I_1} x_{i,j} = d_j,\ for\ j \in J \tag{9}$$

$$M_i \leq \sum_{j \in J} x_{i,j},\ for\ i \in I_1 \tag{10}$$

$$\sum_{j \in D_k} \sum_{i \in I_1} r_{i,j} x_{i,j} \leq (1+\alpha) \sum_{j \in D_k} \{d_j \min\{r_{i_j,j} | i_j \in I_1\}\},\ for\ k \in S \tag{11}$$

$$0 \leq x_{i,j} \leq p_j d_j,\ x_{i,j} \in R,\ for\ i \in I_1,\ j \in J$$

We can see that $MIP(I_1)$ is a linear programming model and can be solved efficiently using simplex method. We call (I_1, I_0) feasible carrier partition of model MIP if $MIP(I_1)$ has feasible solutions. Given partition (I_1, I_0), if $MIP(I_1)$ with $\alpha = +\infty$ has feasible solutions, there always exists a minimal α which makes $MIP(I_1)$ feasible. Based on each carrier partition (I_1, I_0), we can derive two corresponding models, $LP1(I_1)$ and $LP2(I_1)$, which are used in our random move tabu search process and shown by:

$$\textbf{LP1}(I_1)\textbf{:}\ \lambda = \min \sum_{k \in S} \lambda_k \tag{12}$$

$$\text{s.t.} \sum_{i \in I_1} x_{i,j} = d_j,\ for\ j \in J \tag{13}$$

$$M_i \leq \sum_{j \in J} x_{i,j},\ for\ i \in I_1 \tag{14}$$

$$\lambda_k \geq \sum_{j \in D_k} \sum_{i \in I_1} r_{i,j} x_{i,j} - (1+\alpha) \sum_{j \in D_k} \{d_j \min\{r_{i_j,j} | i_j \in I_1\}\},\ for\ k \in S \tag{15}$$

$$0 \leq x_{i,j} \leq p_j d_j,\ x_{i,j} \in R,\ for\ i \in I_1,\ j \in J$$

$$\lambda_k \geq 0,\ \lambda_k \in R,\ for\ k \in S$$

$$\textbf{LP2}(I_1)\textbf{:}\ \omega = \min \sum_{i \in I_1} \omega_i \tag{16}$$

$$\text{s.t.} \sum_{i \in I_1} x_{i,j} = d_j,\ for\ j \in J \tag{17}$$

$$\omega_i \geq M_i - \sum_{j \in J} x_{i,j},\ for\ i \in I_1 \tag{18}$$

$$0 \leq x_{i,j} \leq p_j d_j,\ x_{i,j} \in R,\ for\ i \in I_1,\ j \in J$$

$$\omega_i \geq 0,\ \omega_i \in R,\ for\ i \in I_1$$

It can be easily observed that the relationship among models $MIP(I_1)$, $LP1(I_1)$ and $LP2(I_1)$ is: if model $MIP(I_1)$ has feasible solutions, the optimal objective value of model $LP1(I_1)$ must be zero; if model $LP1(I_1)$ has feasible solutions, the optimal objective value of model $LP2(I_1)$ must be zero.

4.1 Notations and Moves

To facilitate the discussion, we define some notations pertaining to our tabu search process as follows:

z the optimal objective value of model $MIP(I_1)$

z_0 the currently best known objective value of model MIP

k the current iteration number

k_0 the iteration number when better objective value is found

Δz_i^k the net objective value change of model MIP when one move switches the state of carrier i at iteration k

t_i iteration number at which carrier i switched state the last time

ξ^1 tabu tenure for currently selected carriers, i.e, carrier i is kept in I_1 for at least ξ^1 iterations after one move makes it become selected

ξ^0 tabu tenure for currently unselected carriers, i.e., carrier i is kept in I_0 for at least ξ^0 iterations after one move makes it become unselected

λ_i^k the optimal objective value of model $LP1(I_1)$ after the state of carrier i is changed at iteration k

ω_i^k the optimal objective value of model $LP2(I_1)$ after the state of carrier i is changed at iteration k

Each iteration in tabu search is actually a two-stage process: one is to determine the selected carrier set I_1 and the other is to solve problem $MIP(I_1)$. The state change of any carrier is defined to be a move, i.e., selecting a single carrier from either set and transferring it to the other one. Any successful move corresponds to one iteration and leads the current carrier partition to a new one. At iteration k, for instance, one move switching the state of carrier i alters the carrier partition from (I_1, I_0) to (I_1', I_0'). Suppose z and z' are optimal objective values of $MIP(I_1)$ and $MIP(I_1')$ respectively, we get $\Delta z_i^k = z' - z$. If switching state of carrier i results in infeasibility of $MIP(I_1')$, $LP1(I_1')$ or $LP2(I_1')$, we set $+\infty$ to Δz_i^k, λ_i^k or ω_i^k. Before a move is determined, the values of Δz_i^k,λ_i^k and ω_i^k for all carriers are calculated and to minimize the model, the move with smaller Δz_i^k is more attractive.

4.2 Framework and Step-by-step Procedure

Tabu search memory, also called recency based memory, is represented by vector $t \in R^n$, where t_i is one of elements in t associated with carrier i. At iteration k, if changing state of carrier $i \in I_1$ is determined to be the move, we delete carrier i from I_1, add it into I_0, and set $t_i = k$. As long as the state of carrier i is changed, the value of t_i is updated accordingly. z_0 is updated when a new solution with objective value smaller than current z_0 is found during the course of search. Each carrier must stay in I_1 at least ξ^1 iterations after it changes state from "selected" to "unselected" and stay in I_0 at least ξ^0 iterations after it changes state from "unselected" to "selected", unless the aspiration criterion is satisfied. At each iteration, we select a carrier $\bar{i}$ to switch state, such that

$$\Delta z_{\bar{i}}^k = \min\{\Delta z_i^k | i \in I\ \} \tag{19}$$

Then check the following tabu conditions:

$$k - t_{\bar{i}} > \xi^1,\ \text{ if }\ \bar{i} \in I_1\ \text{ or }\ k - t_{\bar{i}} > \xi^0,\ \text{ if }\ \bar{i} \in I_0 \tag{20}$$

If (20) is satisfied, the selected move is not a tabu and can be executed; otherwise the following aspiration criterion is checked:

$$z + \Delta z_{\bar{i}}^k < z_0 \tag{21}$$

i.e., whether the resulting optimal objective value is smaller than z_0. If this move is tabu but satisfies aspiration criterion (21), it is still permissible. If both (20) and (21) are violated, the move associated with carrier $\bar{i}$ is not allowed and we set $\Delta z_{\bar{i}}^k = +\infty$. Then, according to (19) again, we select another carrier and repeat previous process until a permissible move is found.

However, during the course of searching solution for this problem, tabu search is quite possible to encounter a situation that $\Delta z_i^k = +\infty$ for all carriers, which is resulted from three possibilities:

1. After adding some carrier $i \in I_0$ into I_1, the amount $\sum_{i \in I_1 \cup i} M_i$ may be larger than $\sum_{i \in J} d_j$. If it happens, the resulting $MIP(I_1 \cup i)$ must be infeasible and we set $\Delta z_i^k = +\infty$.

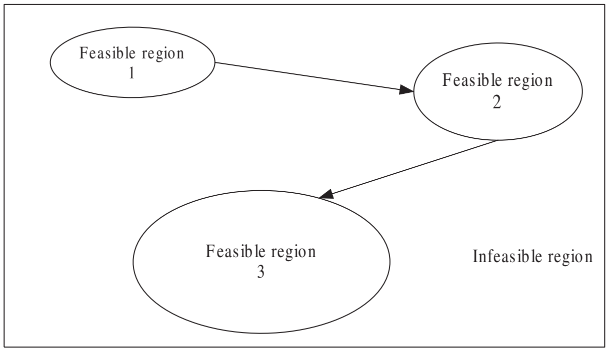

Figure 2. Solution Space of MIP

2. The move associated with carrier i does not satisfy both condition (20) and (21).

3. Switching the state of some carrier violates the cost balancing constraint. As mentioned before, if $MIP(I_1)$ with $\alpha = +\infty$ has feasible solutions, $MIP(I_1)$ must have a minimal α. After carrier i is deleted from I_1 (added into I_1), the minimal α of resulting model $MIP(I_1 - i)$ ($MIP(I_1 + i)$) may be either smaller or larger than that of model $MIP(I_1)$. Suppose at some iteration $MIP(I_1)$ with $\alpha = 0.1$ has feasible solutions. When we delete carrier i from I_1, the minimal α of $MIP(I_1 - i)$ may equal 0.11. If it happens, $MIP(I_1 - i)$ with $\alpha = 0.1$ must be infeasible and we set $\Delta z_i^k = +\infty$. Similarly, when we add a carrier into I_1, this situation also may occur.

Facing this special situation, the simplest way is to randomly select one carrier, but with this strategy the tabu search process may run into infeasible region and stay there for a long time, even may not jump out of it any more. To overcome this shortcoming, we developed a more effective way, called *random move* (See Algorithm 1) to assist the tabu search process. This random move subroutine is: randomly choose one non-tabu carrier and change its state based on the values of ω_i^k if $\lambda_i^k = +\infty$ for all carriers and based on λ_i^k instead otherwise; the carrier with smaller ω_i^k or λ_i^k has more probability to be chosen. Model MIP may have many disjointed feasible regions as Figure 2 shows. Random move subroutine, on one hand, permits the search process to enter into infeasible region, and on the other hand, it enforces the process to revisit feasible regions with more chance. Thus, with the aid of random move subroutine, our tabu search process has the ability to visit several disjointed feasible regions by crossing infeasible region within reasonable iterations.

After $\beta * m$ iterations, if no improvement on z_0 happens, i.e., $k - k_0 > \beta * m$, we stop the tabu search process. To expose more details of our tabu search procedure, a step-by-step description is presented as follows:

step 0. Starting from any carrier partition (I_1, I_0), solve model $MIP(I_1)$ to initialize z. Set $z_0 \leftarrow z$ and determine the values of tabu tenure ξ^1 and ξ^0. Let $t_i \leftarrow -\xi^1$ for all $i \in I_1$ and $t_i \leftarrow -\xi^0$ for all $i \in I_0$. Choose an appropriate value for β and set $k \leftarrow 1, k_0 \leftarrow 1$. Compute Δz_i^k, λ_i^k and ω_i^k for all carriers.

step 1. Select a carrier $\bar{i}$ according to (19). Check whether the move associated with carrier $\bar{i}$ is tabu according to (20). If it is tabu, continue to step 2; otherwise go to step 4.

step 2. Check whether the move satisfies the aspiration criterion (21). If (21) is satisfied, go to step 4; otherwise set $\Delta z_{\bar{i}}^k = +\infty$ and continue to step 3.

step 3. If $\Delta z_{\bar{i}}^k = +\infty$ for all $i \in I$, randomly choose carrier $\bar{i}$ by invoking random move subroutine and continue to step 4; otherwise, go to step 1.

step 4. If $\bar{i} \in I_1(I_0)$, delete $\bar{i}$ from $I_1(I_0)$, and put it into $I_0(I_1)$. Update $t_{\bar{i}} \leftarrow k$, $k \leftarrow k+1$, and $z \leftarrow z + \Delta z_{\bar{i}}^k$. If $z < z_0$, let $z_0 \leftarrow z$ and $k_0 \leftarrow k$. Continue to step 5.

step 5. Calculate Δz_i^k, λ_i^k and ω_i^k based on the new feasible carrier partition. If $k - k_0 \leq \beta * m$, go to step 1; otherwise, continue to step 6.

step 6. z_0 is the near-optimal solution for model MIP and end the tabu search procedure.

5 Computational Experiments

We conducted extensive experiments on randomly generated data. According to the numbers of loading ports, discharge ports and carriers, we generated six groups of test instances, which are (10, 10, 20), (20, 20, 20), (40, 40, 20),(10, 10, 40), (20, 20, 40), (40, 40, 40). Randomly assigned two loading ports to each discharge port and so each sales division had two shipping lanes. To simply the test instances, we set $M_i = 0.115 \sum_{j \in J} d_j$ for all carriers and $p_j = 0.4$ for all lanes. The demand d_j for all $j \in J$ were uniform random number on the interval $[30, 100]$. In order to generate rate $r_{i,j}$, we introduced a new notation $\bar{r_j}$, which was randomly chosen from a uniform distribution on interval $[50, 100]$, and we call it mean rate of lane j because $r_{i,j}$ were obtained randomly from a uniform distribution on interval $[0.7\bar{r_j}, 1.3\bar{r_j}]$. In our tabu search procedure the settings of parameters were: $\beta = 8$ for all instances, $\xi^1 = 4$ and $\xi^0 = 6$ for instances with 20 carriers, $\xi^1 = 4$ and $\xi^0 = 10$ for instances with 40 carriers. Set $\gamma = 0.1$

Algorithm 1 Random Move Subroutine

```
Set a value to γ
C′ ← {i|k − t_i > ξ^1 if i ∈ I_1 or k − k_i > ξ^0 if i ∈ I_0};
n = 0;
for i = 1 to |C′| do
    if i = |C′| then
        n = |C′|;
    else
        Generate a random number r on [0, 1];
        if r ≤ γ then
            n = i;
            Break;
        end if
    end if
end for
if λ_i^k = +∞ for all i ∈ C′ then
    Sort ω_i^k for i ∈ C′ increasingly;
    Choose the carrier with n-th smallest ω_i^k;
else
    Sort λ_i^k for i ∈ C′ increasingly;
    Choose the carrier with n-th smallest λ_i^k
end if
```

and use Algorithm 1 to implement the random move subroutine. By utilizing split-half search and tabu search, we have tried to explore the minimal α of each instance and the α we used for each test instance is a little larger than its minimal α we found. When running CPLEX solver, we set time limit $200s$ for group 1 to 3, $600s$ for group 4 and 5, and $1200s$ for group 6.

We used both ILOG CPLEX 9.0 and our random move tabu search to solve the model MIP with given α. All algorithms were coded in JAVA and executed on Intel P4 2.80GHz machine with 512MB memory. Computation times reported here are in CPU seconds on this computer.

Table 3 and 4 illustrate the results. From Table 5, it is not surprising to see that the time consumed by tabu search is much more less than that consumed by CPLEX for all instance groups. Except for instances in Group 1, CPLEX did not produce better solutions although it spent more time. For instances in Group 2, both of CPLEX and tabu search generated feasible solutions and their solution qualities are almost the same. For groups 3 and 4, CPLEX could not find a feasible solution for over half of the instances. As problem size grew as large as instances in group 5 and 6, CPLEX is unable to reach feasible solutions. Nevertheless, our tabu search generated feasible solutions for all instance through group 1 to 6. So we can conclude that for large-scale instances, our random move tabu search outperforms the CPLEX solver. In addition, all generated instances were based on the assumption that each sales division had two shipping lanes, but if each sales division owns more lanes, the number of constraints (4) will increase explosively. For example, the instance in group 5 with each sales division owning three lanes has over $40 \times 40 \times 40 \times 20 = 1280000$ constraints, which is hard to be handled by CPLEX. Contrarily, this kind of instances can be still efficiently processed by our tabu search procedure since the numbers of constraints (11) and sales divisions always keeps equal no matter how many lanes each sales division has.

	α	(M-L)/L	M(s)	(T-L)/L	T(s)	(M-T)/T
Group 1 (10-10-20)	0.07	5.9%	200	8.2%	10	-2.1%
	0.07	6.1%	184	8.9%	18	-2.6 %
	0.07	6.3%	153	10.8%	10	-4.0%
	0.07	5.9%	151	6.9%	15	-0.9%
	0.07	5.0%	32	6.2%	16	-1.1%
	0.07	6.8%	200	7.8%	9	-0.9%
	0.08	4.9%	24	6.0%	19	-1.1%
	0.08	6.8%	176	8.8%	15	-1.8%
	0.07	5.2%	83	7.2%	14	-1.9%
	0.07	4.9%	60	5.8%	11	-0.9%
Group 2 (20-20-20)	0.09	6.0%	200	5.9%	21	0.1%
	0.10	6.8%	200	7.4%	19	-0.5 %
	0.09	6.7%	200	7.3%	48	-0.5%
	0.10	8.2%	200	7.7%	27	0.5%
	0.10	6.4%	200	6.4%	92	0.0%
	0.11	7.4%	200	7.2%	51	0.2%
	0.10	5.7%	200	5.7%	38	0.0%
	0.10	7.4%	200	7.5%	48	-0.1%
	0.09	6.0%	200	6.0%	23	0.0%
	0.10	6.4%	200	6.5%	31	-0.1%
Group 3 (40-40-20)	0.10	N/A	200	9.6%	80	N/A
	0.11	6.7%	200	6.7%	47	0.0 %
	0.12	7.6%	200	8.3%	46	-0.7%
	0.11	N/A	200	9.3%	82	N/A
	0.12	N/A	200	7.7%	50	N/A
	0.11	6.7%	200	7.7%	126	-0.9%
	0.12	N/A	200	8.0%	73	N/A
	0.10	7.7%	200	7.2%	71	0.4%
	0.10	N/A	200	8.1%	83	N/A
	0.11	7.0%	200	7.0%	118	0.0%

M: the optimal objective value of model MIP solved by CPLEX 9.0
$M(s)$: time used to get M
T: the near optimal objective value of model MIP gained by Tabu Search
$T(s)$: time used to get T
L: the optimal objective value of MIP after relaxing y_i to be continuous
N/A: CPLEX did not access feasible solutions

Table 3. Result of solving instances with 20 carriers using Tabu Search and CPLEX 9.0

6 Conclusion

In this paper, we addressed a practical freight proportion allocation problem which is motivated by the experience of MG and proved it to be $\mathcal{NP}$-complete in the strong sense. Cost balancing constraint as a new concept was firstly introduced into our model and it improved the fairness of freight allocation. Classical tabu search can not handle our model well, so we devised an advanced tabu search with the aid of random move subroutine to solve MIP model. We implemented both random move tabu search and CPLEX

	α	(M-L)/L	M(s)	(T-L)/L	T(s)	(M-T)/T
	0.06	8.4%	600	7.5%	91	0.8%
	0.07	5.8%	491	6.1%	199	-0.3 %
	0.06	5.1%	310	6.4%	79	-1.3%
	0.06	N/A	600	11.3%	68	N/A
Group 4	0.06	N/A	600	14.8%	112	N/A
(10-10-40)	0.07	7.1%	600	8.5%	101	-1.4%
	0.06	N/A	600	13.2%	49	N/A
	0.06	6.1%	600	6.5%	76	-0.4%
	0.06	N/A	600	9.9%	70	N/A
	0.06	N/A	600	13.5%	97	N/A
	0.09	N/A	600	10.1%	189	N/A
	0.09	N/A	600	9.3%	164	N/A
	0.09	N/A	600	8.2%	212	N/A
	0.08	N/A	600	11.9%	144	N/A
Group 5	0.09	N/A	600	11.0%	125	N/A
(20-20-40)	0.09	N/A	600	9.8%	192	N/A
	0.09	N/A	600	10.0%	280	N/A
	0.09	N/A	600	7.3%	204	0.1%
	0.10	N/A	600	11.1%	239	N/A
	0.09	N/A	600	10.1%	149	N/A
	0.11	N/A	1200	11.4%	593	N/A
	0.10	N/A	1200	11.5%	394	N/A
	0.11	N/A	1200	11.3%	323	N/A
	0.11	N/A	1200	10.0%	423	N/A
Group 6	0.11	N/A	1200	9.1%	613	N/A
(40-40-40)	0.11	N/A	1200	11.5%	387	N/A
	0.11	N/A	1200	10.4%	699	N/A
	0.11	N/A	1200	10.4%	383	N/A
	0.10	N/A	1200	14.4%	348	N/A
	0.10	N/A	1200	13.0%	331	N/A

Table 4. Result of solving instances with 40 carriers using Tabu Search and CPLEX 9.0

Group	1	2	3	4	5	6
TAT(s)	13.6	39.7	77.6	94.3	189.8	449.4
TMT(s)	18.5	92	125.9	199.4	280.2	699.2
MAT(s)	≥ 126	≥ 200	≥ 200	≥ 560	≥ 600	≥ 1200

TAT=Tabu Average Time
TMT=Tabu Maximum Time
MAT= MIP Average Time

Table 5. Time used by Tabu Search and CPLEX

in JAVA, and extensive experiments showed that our tabu search outperformed the CPLEX solver.

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