Dynamic Channel Assignment with Cumulative Co-Channel Interference

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This paper studies the problem of centralized dynamic channel assignment (DCA) in wireless cellular systems under space and time-varying channel demand. The objective is to minimize the number of channels required to satisfy demand while also satisfying co-channel interference constraints. Cumulative co-channel interference constraints govern channel reuse, via a threshold decision criterion based on the carrier-to-interference ratio. The paper makes two contributions. First, it provides an empirical bound on the difference between the minimal number of channels required based only on geographic reuse distance versus the cumulative interference case in the context of linearly increasing demand. The bound is characterized using only the reuse distance. It is obtained with an Integer Programming (IP) based strategy that uses channel assignments for one demand state to assign channels for the next state. Geographic locality constraints are applied to limit reassignments. The impact of cumulative interference constraints is observed to be small for small geographic localities. Second, the paper presents a new, fast DCA heuristic that is based on the characteristic channel reuse patterns used by the IP-based strategy. The heuristic and IP-based method yield similar results for the zero blocking condition. The DCA heuristic is applied to the problem of estimating the blocking probabilities of call arrivals modeled by a two state discrete-time Markov chain and uniformly distributed holding times. The blocking performance for an ensemble of spatial load imbalance distributions is uniquely characterized using the heuristic and IP solutions.

I. Introduction

Emerging wireless communication systems will increasingly rely on smart systems and intelligent networks to optimize resources and maximize performance. Internet data services with highly variable application specific bandwidth requirements will represent a major traffic component on wireless networks. Protocols and algorithms that support bandwidth efficient distribution of resources for such applications are critical to the new generation of wireless systems. The adaptive allocation of wireless spectrum based on traffic characteristics and their performance requirements may be examined in the context of a Dynamic Channel Assignment (DCA) model.

The channel assignment problem has been examined in a number of studies [1, 2, 3, 4, 5] over the last three decades. Typically, each cell is associated with an interference region that is based on geographic distance. In some cases, a predefined channel compatibility matrix [6] specifies the required frequency separation between cells. Within this interference region, channel reuse is prohibited. Engineering approaches to this problem have addressed how the fixed channel assignment (FCA) policies applied to cellular networks may be adapted when cells experienced more calls than the number of available channels. Assignment strategies have involved channel borrowing schemes where channels from the richest neighboring cells are borrowed to minimize future call blocking probability. Chuang [5] and Anderson [3] discuss simulation studies of these algorithms and show that the number of search steps required impacts the time to solution. Modifications that reduce the number of search steps have also been considered [7]. These involve channel ordering schemes where the fixed-to-borrowable channel ratio is dynamically varied according to changing traffic conditions. A survey of fixed, dynamic and hybrid channel assignment schemes is provided by Katzela and Naghshineh [8].

Mathematical programming (MP) models for channel assignment find assignments through minimization of a cost function such as the allocated bandwidth under the constraint that channel reuse takes place above specified interference levels. Murphey *et al.* [9] provide a comprehensive survey of algorithmic approaches to the problem. Graph coloring and IP formulations have been used for graph-theoretic abstractions of the problem [10]. Mazzini and Mateus [11] have used a non-graph theoretic abstraction that integrates the problems of base-station location, topological network design and channel assignment. Results of a Lagrangian Relaxation compare favorably to those obtained using an IP solver. These algorithmic models have proven to be computationally taxing. Meta-heuristics of various types have been employed to improve the performance [12, 13, 14, 15, 16]. As in the case of engineering models, these algorithmic studies of DCA have typically applied frequency separation constraints using geographic distance for channel reuse. The more general cumulative interference constraint has been considered to a lesser extent. Capone and Trubian [13] and Gomes *et al.* [17] consider the cumulative effects, but their channel assignment is based on meta-heuristics.

An important consideration in wireless spectrum allocation is the efficient support of spatial and time varying channel demand functions. It is also important to evaluate the DCA gains achieved relative to FCA and the class of spatio-temporal patterns where such gains are significant. Argyropoulos *et al.* [18] showed that performance gains between FCA and DCA increase with the degree of spatial load imbalance. Everitt and Mansfield [4] assumed that the traffic in each cell was Gaussian distributed and showed that DCA is more resilient to traffic volatility than FCA. Fernando and Fapojuwo [19] considered uniformly distributed random demand across fixed size clusters in a hexagonal cellular grid pattern and calculated the bandwidth requirements using a Viterbi-like algorithm under the constraint of fixed co-channel and adjacent channel separation. Here the interference constraints were based solely on reuse distance. Cheng and Chang [20] proposed a distributed measurement based DCA and showed that on the time scales over which packet data traffic exhibits variations in required bandwidth, the signal-to-interference ratio can also significantly change.

The design of assignment algorithms in the aforementioned works was motivated by the existing policy where interference across spectral bands is controlled by limiting the maximum transmission power of mobile terminals. These models were typically suitable for teletraffic applications in the outdoor urban or rural propagation environments. New wireless technologies will operate in more complex environments where the spatio-temporal propagation patterns are less well understood. Recent reports by the FCC [21] propose the consideration of a more general cumulative interference measure, termed the interference temperature, for characterizing and managing newly allocated spectrum. These new channel metrics are expected to work in conjunction with emerging cognitive radio technologies that can search, acquire and release spectrum in a more adaptive framework.

In the present work, we consider the impact of both cumulative co-channel interference and spatio-temporal demand variations in the design of dynamic channel assignment algorithms. A mathematical programming approach using an IP solver is designed to provide upper bounds on channels required under linearly increasing demand with time. This instance of demand variation represents an upper bound envelope of more general stochastic demand variations that will also be considered. The IP model examines the effect on the number of channels required when using cumulative interference instead of just the reuse distance. The paper shows that solving the problem with cumulative interference constraints is at least as hard as solving an IP model whose co-channel interference constraints are based solely on reuse distance. This establishes theoretical hardness of the cumulative interference problem and provides a basis of comparison for the two different interference models.

A new, fast DCA heuristic is presented that utilizes some of the assignment properties exhibited by the IP solver. The performance of the heuristic is shown to be comparable to the IP solution. The heuristic is applied to evaluate the blocking performance of demands generated by on-off Markov arrival processes and uniformly distributed holding times.

This paper is organized as follows. Section II discusses the demand model and spatial structure of cellular traffic. Section III presents a core IP (CIP_t) model, establishes its NP-hardness, and describes lower and upper bounding techniques. It also discusses the parameters and results of a DCA simulation that provides an estimate of the difference between the number of channels required under the two different co-channel interference models. Section IV presents the DCA heuristic and its results. Section V concludes the paper.

II. Traffic and Channel Demand Model

The influence of traffic inhomogeneity on the channel assignment problem is studied taking into consideration both spatial and temporal variations in the demand. Spatial variations in channel demand are addressed by classifying each cell in the network as a variable ($Type_v$) or constant ($Type_c$) demand rate

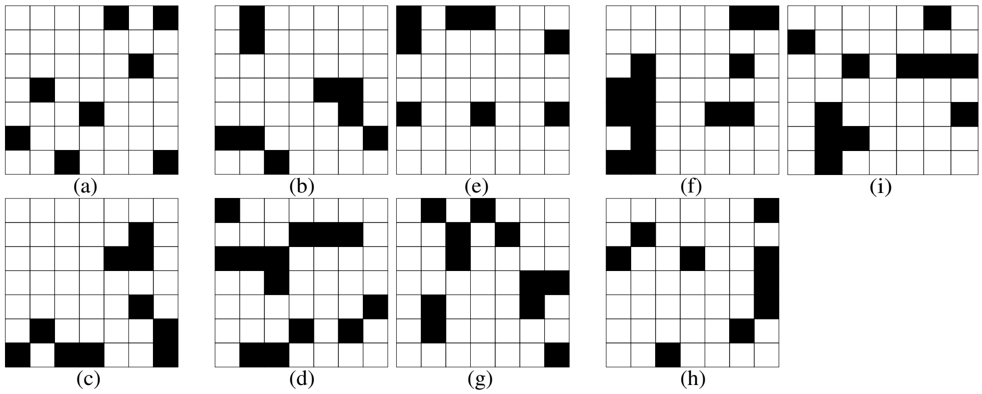

Figure 1: Stochastic spatial distribution of variable demand cells (shaded), with $Type_v$ cell parameter of 20% .

cell. The $Type_v$ cells are randomly distributed in space, driven by a two state on-off Markov arrival process and occupy the channels with uniformly distributed holding times. The demand in $Type_c$ cells is assumed to be deterministic in time and satisfied by D_c channels in each time unit.

The spatial assignment of cell type ($Type_v$ or $Type_c$) takes place at the initial time $t = 0$ and remains invariant in future time steps. The demand levels and channel assignments per cell are generated in successive discrete time intervals of unit duration. Each cell in the network is characterized by an aggregate demand function D_t, that specifies the number of channels required in the interval $(t, t + 1]$ to satisfy a zero-blocking condition. To maintain zero-blocking conditions, channel assignment algorithms designed in this work provide the required number of channels for every cell, while minimizing the total number of channels allocated.

The $Type_v$ cells are Bernoulli distributed in space. Each cell is characterized by probability p_v of being $Type_v$. The probability distribution of the number N_v of $Type_v$ cells in a network of c_{max} cells, where c_{max} is fixed, is given by the binomial distribution,

$$P[N_v = k] = \binom{c_{max}}{k} {p_v}^k (1 - p_v)^{c_{max}-k} \qquad (1)$$

for $k = 0, 1, \ldots c_{max}$ with expectation $E[N_v] = c_{max}\ p_v$ and variance $Var[N_v] = c_{max}\ p_v(1 - p_v)$. The manner in which the cells are traversed in assigning the cell types determines the average spacing between $Type_v$ cells. If S_k and S_{k+1} denote the spatial locations where the k^{th} and $(k+1)^{th}$ $Type_v$ cells occur, then the distribution of number of spaces between these cells is $P[S_k - S_{k+1} = m] = (1 - p_v)^{m-1}\ p_v, m = 1, 2, \ldots$, from which the expected spacing between $Type_v$ cells may be determined as $1/p_v$. Therefore, the choice of p_v allows control of both the fraction and average spacing of variable demand cells.

The channel demand rates in the off and on states of arrivals to $Type_v$ cells are D_{off} and D_{on} respectively. The channel holding time is $\tau_{min} \leq \tau \leq \tau_{max}$. This model enforces an inhomogeneous demand variation in time that can range from $(D_{off}\tau_{min} : D_{on}\tau_{max})$ number of channels per $Type_v$ cell.

MP formulations may not generally provide solutions as quickly as engineering models based on heuristics. However, the MP based algorithms can significantly reduce the required number of channels to satisfy a given spatio-temporal demand. Therefore, we first obtain solutions from IP-based algorithms in response to temporal variations that represent an upper bound envelope of the random demand function. For the traffic model considered, this corresponds to a linearly increasing aggregate demand function D_t in each $Type_v$ cell. We will assume in the examples provided that D_t increases linearly from 1 to 10 for $0 \leq t \leq 9$.

The IP formulation proposed is flexible enough to handle any other demand trend variations derived from random arrival processes and random channel holding times. The input to the algorithm at any time step is the function D_t for each cell in the network. The algorithm does not make any distinction based on how these demands are generated.

The cellular network considered is a 7×7 grid of square cells. Previous channel assignment papers [22, 23, 12, 4, 24] have considered networks of this size using hexagonal or square cellular regions. Furthermore, the ideas of the current paper can be applied to hexagonal cellular systems as well.

An ensemble of nine stochastic spatial distributions of $Type_v$ cells generated using the aforementioned Bernoulli statistics is depicted in Figure 1 for the parameter values $p_v = 0.2$ and $c_{max} = 49$. The shaded cells represent $Type_v$ cells. The unshaded cells are $Type_c$ cells with a demand of $D_c = 1$ remaining constant in time. The performance of the channel assignment models described in Section III and IV will be examined for these spatial configurations.

III. IP-Based Algorithm

This section presents an IP-based algorithm that simulates dynamic channel assignment (DCA) under space and time-varying channel demand. Unlike most channel assignment work we model geographic distance and the cumulative effect of interference across the cellular system. A cellular environment is modeled by associating with each transmitter a unique cell that represents its geographic transmission region of responsibility. Let $C = \{\vec{c_1}, \vec{c_2}, \ldots, \vec{c}_{c_{max}}\}$ denote a sequence of c_{max} cells, one for each transmitter. Demand at time t is denoted by $D_t(\vec{c_i})$. $D_t(\vec{c_i}) = t + 1$ for each $Type_v$ cell. The $Type_c$ cell demands are fixed at one channel in each time unit.

Section III.A describes a core IP model CIP_t that is constructed for a single time step t. Section III.B shows that solving CIP_t is at least as hard as solving an IP model whose co-channel interference constraints are based solely on reuse distance. For time period t, let R_t^* denote the optimal number of channels satisfying the demand constraints and co-channel interference constraints based solely on geographic reuse distance. Let C_t^* denote the optimal number of channels for CIP_t. Because IP is prohibitively expensive for large problems, to estimate the difference between C_t^* and R_t^*, a lower bound L_{r_t} on the number of channels required in the reuse distance case and an upper bound U_{c_t} on the number of channels required in the cumulative case are derived in Sections III.C and III.D, respectively. A lower bound L_{c_t} on the optimal number of channels in the cumulative interference case is provided in Section III.E that is at least as large as L_{r_t}. Section III.F discusses results of the IP-based DCA simulation.

III.A. Core IP Model

The CIP_t model's goal is to minimize the number of channels used while satisfying demand and co-channel interference constraints. There are f_{max} available channels. CIP_t consists of integer, binary variables, a minimization objective function, and a collection of linear constraints, as described below. In the square cellular system, cells are ordered by increasing row, followed by increasing column order.

The minimum carrier-to-interference (C/I) ratio threshold is represented by B. $S(\vec{c_i}, \vec{c_j})$ is the strength of the signal at $\vec{c_i}$ due to a transmitter at $\vec{c_j}$ and it satisfies Eqn. 2 below:

$$S(\vec{c_i}, \vec{c_j}) = \frac{1}{(|\vec{c_i} - \vec{c_j}|)^{\alpha}} \tag{2}$$

In Eqn. 2, α is a path-loss exponent and $|\vec{c_i} - \vec{c_j}|$ is the geographic (Euclidean) distance between $\vec{c_i}$ and $\vec{c_j}$. The ratio constraint is given by Eqn. 3:

$$\frac{S(\vec{c_i}, \vec{c_i})}{\sum_{j \neq i} S(\vec{c_i}, \vec{c_j})} \geq B \tag{3}$$

where the summation in the denominator represents the sum of interfering signal strengths.

$S(\vec{c_i}, \vec{c_j})$ may be derived from any channel model. In this work, a simple path loss model is specified based on $|\vec{c_i} - \vec{c_j}|$ and path loss exponent α as given in Eqn. 2. All transmitters transmit at the same power level. If cells are subject to the effects of fading and shadowing and the transmitters implement power control to counteract these effects, their transmission power level can change. In such a case each cell may be modeled by a function that describes the spread of transmission power in space. This adoption is outlined in Section V.

The formulation of CIP_t has two groups of integer binary decision variables. Assignment variables that show if a given frequency is assigned to a particular transmitter and usage variables that reflect whether or not a given frequency is used in the optimal solution are given in Eqns. 4 and 5 respectively for $1 \leq i \leq c_{max}, 1 \leq k \leq f_{max}$.

$$A_t(\vec{c_i}, f_k) = \begin{cases} 1 & if\ frequency\ f_k\ assigned\ to\ \vec{c_i} \\ 0 & otherwise \end{cases} \tag{4}$$

$$a_t(f_k) = \begin{cases} 1 & if\ \sum_{i=1}^{c_{max}} A_t(\vec{c_i}, f_k) \geq 1 \\ 0 & otherwise \end{cases} \tag{5}$$

The model CIP_t is stated as follows:

$$minimize \sum_{k=1}^{f_{max}} a_t(f_k) \tag{6}$$

$$subject\ to \sum_{k=1}^{f_{max}} A_t(\vec{c_i}, f_k) \geq D_t(\vec{c_i}) \quad 1 \leq i \leq c_{max}, \tag{7}$$

$$c_{max} a_t(f_k) \geq \sum_{i=1}^{c_{max}} A_t(\vec{c_i}, f_k) \quad 1 \leq k \leq f_{max}, \tag{8}$$

$$[1 - A_t(\vec{c_i}, f_k)] \sum_{j \neq i} S(\vec{c_i}, \vec{c_j}) + \frac{S(\vec{c_i}, \vec{c_i})}{B} A_t(\vec{c_i}, f_k) \geq \sum_{j \neq i} S(\vec{c_i}, \vec{c_j}) A_t(\vec{c_j}, f_k) \quad 1 \leq i \leq c_{max}, \quad 1 \leq k \leq f_{max} \tag{9}$$

Each cell has a demand constraint given by Eqn. 7 that ensures that demand for that cell is satisfied. The constraints of Eqn. 8 link the assignment variables to the usage variables so that channel assignments contribute to the objective function's value. If a channel is used in at least one cell, then a value of one is added to the objective function.

The C/I of Eqn. 3 is the basis for the linear constraints of Eqn. 9. There is one constraint of the form given in Eqn. 9 for each cell/frequency pair $(\vec{c_i}, f_k)$. The constraint ensures that channel f_k is only assigned to cell $\vec{c_i}$ when the cumulative co-channel interference due to other cells' usage of channel f_k is below the specified threshold. Enforcing Eqn. 3 whenever frequency f_k is assigned to cell $\vec{c_i}$ and also to another cell $\vec{c_j} \neq \vec{c_i}$ produces nonlinear constraints of the form in Eqn. 10 below. Eqn. 9 is used instead of Eqn. 10 because it is a linear expression that is logically equivalent to Eqn. 10.

$$S(\vec{c_i}, \vec{c_i}) A_t(\vec{c_i}, f_k) \geq B \sum_{j \neq i} [S(\vec{c_i}, \vec{c_j}) A_t(\vec{c_i}, f_k) A_t(\vec{c_j}, f_k)] \tag{10}$$

Note that the only part of this formulation that relates to the geometry of the cellular system is the calculation of Eqn. 2 which requires the Euclidean distance between two cells. Thus, it can be applied to a variety of cell geometries without modification.

Murphey *et al.* [9] survey IP formulations for channel assignment problems. Of the work examined there, the CALMA IP$-$2 formulation is the most similar to that presented here. The inclusion of *cumulative* co-channel interference is a point of departure in the current work.

III.B. Hardness

Here we show that solving the decision version[1] of CIP_t is NP-hard [25]. This uses the same binary variables and linear constraints as our minimization problem. However, Eqn. 6 is replaced by the question: "for a given $K \in Z^+$, does there exist a set of values for the binary variables such that $\sum_{k=1}^{f_{max}} a_t(f_k) \leq K$?" To establish NP-hardness, we use a reduction from Hale's NP-hard problem F*D-CCAP in [10]. F*D-CCAP is stated as follows for a set of transmitters C (i.e. cells) and a rational number r representing minimum reuse distance. Find an assignment of transmitters to channels that: 1) uses at most K channels, and 2) satisfies a reuse distance-based co-channel interference criterion such that, for $\vec{c_i}, \vec{c_j} \in C$, $i \neq j$, $A_t(\vec{c_i}, f_k) = A_t(\vec{c_j}, f_k) = 1$ only if $|\vec{c_i} - \vec{c_j}| > r$. To reduce an arbitrary instance of F*D-CCAP to an instance of our problem, we observe that there is a 1-1 mapping between variables of the two problems. Furthermore, the decision criterion of K channels is the same. To form our demand constraints we set $D_t(\vec{c_i}) = 1$ for each $\vec{c_i} \in C$.

The remaining task is to map the reuse distance-based co-channel interference condition to our co-channel interference constraints. We do this by redefining the function $S(\vec{c_i}, \vec{c_j})$ as follows:

$$S(\vec{c_i}, \vec{c_j}) = \left\{ \begin{array}{cc} 1 & if\ i \neq j\ and\ |\vec{c_i} - \vec{c_j}| > r \\ \infty & if\ i \neq j\ and\ |\vec{c_i} - \vec{c_j}| \leq r \\ c_{max} - 1 & if\ i = j \end{array} \right\} \tag{11}$$

We also specify that $B = 1$. With these definitions, if $|\vec{c_i} - \vec{c_j}| \leq r$ for any $i \neq j$, then if $A_t(\vec{c_j}, f_k) = 1$ for some f_k, the right-hand side of Eqn. 10 will be $= \infty$, so the constraint will only be satisfied if $A_t(\vec{c_i}, f_k) = 0$. Furthermore, if $|\vec{c_i} - \vec{c_j}| > r$ for every $i \neq j$ for which $A_t(\vec{c_j}, f_k) = 1$ for some f_k, then the right-hand side of Eqn. 10 will be $\leq c_{max} - 1$, so the constraint can be satisfied when $A_t(\vec{c_i}, f_k) = 1$ since that gives the left-hand side the value $c_{max} - 1$. Thus, Eqn. 10 is satisfied if and only if the F*D-CCAP reuse condition is satisfied. This completes the reduction and establishes NP-hardness. Thus, our co-channel interference constraints are at least as powerful as ones based only on reuse distance.

III.C. Lower Bound on R_t^*

For time period t, recall that R_t^* is the optimal number of channels satisfying the demand constraints of Eqn. 7 and co-channel interference constraints based solely on the geographic reuse distance r. That is, for $\vec{c_i}, \vec{c_j} \in C$, $i \neq j$, $A_t(\vec{c_i}, f_k) = A_t(\vec{c_j}, f_k) = 1$ only if $|\vec{c_i} - \vec{c_j}| > r$. From the ratio threshold B and path-loss exponent α of the channel model, r can be calculated (see Section III.F).

The lower bound L_{r_t} on R_t^*, given by Eqn. 12 below, comes from a straightforward application of the definition of a *cluster*. A cluster is defined in [26] as "a maximal mutually interfering group of cells," where maximality is a function of the reuse distance r. Anderson [3], Jordan *et al.* [26] and others have observed that since each cell in the cluster must be allocated a different channel, the total number of calls across all cells in a cluster provides a lower bound on the number of channels required for that cluster as well as for the entire cell system. In a square grid of square cells

[1] Alternatively, NP-completeness of an integer-coefficient version of our problem can be established using an integer-coefficient feasibility version of our problem.

a cluster can be defined as the largest square group of cells such that each pair of cells in the group is separated by distance $\leq r$. Such a cluster is a square group of cells whose side length is $\rho = 1 + \lfloor r/\sqrt{2} \rfloor$. The lower bound is based only on reuse distance, so $R_t^* \geq L_{r_t}$.

$$L_{r_t} = \max\{\sum_{\vec{c_i} \in h} D_t(\vec{c_i}) | h \; is \; a \; cluster\} \tag{12}$$

III.D. Locality-Based C_t^* Upper Bound

Recall that C_t^* denotes the optimal number of channels for CIP_t. $C_t^* \geq R_t^*$ because geographic separation less than or equal to r violates Eqn. 3 and therefore Eqns. 9 and 10. An upper bound U_{c_t} on the minimum number of channels C_t^* required by the CIP_t of Section III.A is obtained here along with channel assignments A_t corresponding to that bound. The upper bound algorithm satisfies the CIP_t demand and co-channel interference constraints, so:

$$U_{c_t} \geq C_t^* \geq R_t^* \geq L_{r_t} \tag{13}$$

Consequently, any bound on $U_{c_t} - L_{r_t}$ also bounds the difference $C_t^* - R_t^*$ and therefore provides an estimate of the effect of using cumulative co-channel interference instead of just reuse distance. (See Section III.F for an empirical estimate on this bound.)

One way to obtain U_{c_t} is to run an IP solver on CIP_t, limit the execution time, and use the best feasible solution found within the time limit (if any). Given the NP-completeness of IP binary, integer-coefficient feasibility [25], it is not surprising that this fails to consistently produce good results, even for a 7×7 sized grid. In order to generate good feasible solutions we therefore use an alternate approach involving highly constrained versions of CIP_t.

In this method, a geographic locality $Y_t^g(\vec{c_i})$ of size g is specified around a cell $\vec{c_i}$. Geographic localities are used to identify the collection of cells Z_t^g that are close to a cell whose demand changes from time step $t-1$ to time step t. Let g_{max} be the minimum value of g for which $Z_t^g = C$. Thus, g_{max} is the maximum relevant locality size for a given cell geometry and spatial distribution of $Type_v$ cells[2]. A sequence of g_{max} IP models $I_t^0, \ldots, I_t^{g_{max}}$ is solved, where $I_t^{g_{max}} = CIP_t$, and the model yielding the minimum number of channels produces U_{c_t}. Each model I_t^g is CIP_t augmented with additional constraints built using Z_t^g. Let $Q_t^g(\vec{c_i})$ be the set of additional assignment constraints associated with cell $\vec{c_i}$. According to $Q_t^g(\vec{c_i})$, each cell $\vec{c_i}$ that is not sufficiently close to a cell whose demand changes from time step $t-1$ to time step t retains its time step $t-1$ assignments in time step t. This is achieved using equality constraints of the form $A_t(\vec{c_i}, f_k) = A_{t-1}(\vec{c_i}, f_k)$.

Because I_t^g is CIP_t augmented with additional constraints, the optimal number of channels for I_t^g is $\geq C_t^*$ and therefore $U_{c_t} \geq C_t^*$. Note that limiting the execution time in solving an individual I_t^g also produces an upper bound on C_t^*. Because I_t^g may be highly constrained for small localities, the amount of execution time required to obtain good feasible solutions for these models may be significantly less than the time required for large localities.

The locality policy helps to reduce the number of assignment changes across time steps. The amount of control is proportional to locality size. Reassignments are limited the most for small localities and the least for g_{max} since $I_t^{g_{max}} = CIP_t$.

UPPER BOUND ALGORITHM:

$U_{c_t} \leftarrow f_{max}$
for $g \leftarrow 0$ to g_{max}
  for each cell $i = 1$ to c_{max}
    $Y_t^g(\vec{c_i}) \leftarrow \{\vec{c_j} \mid g\sqrt(2) \geq |\vec{c_i} - \vec{c_j}|\}$
  $Z_t^g \leftarrow \bigcup_{i | D_t(\vec{c_i}) \neq D_{t-1}(\vec{c_i})} Y_t^g(\vec{c_i})$
  for each cell $i = 1$ to c_{max}
    $Q_t^g(\vec{c_i}) \leftarrow \begin{cases} \{A_t(\vec{c_i}, f_k) = A_{t-1}(\vec{c_i}, f_k)\} & if \; \vec{c_i} \notin Z_t^g \\ \emptyset & otherwise \end{cases}$
  $I_t^g \leftarrow CIP_t \cup [\bigcup_{\vec{c_i}} (Q_t^g(\vec{c_i}))]$
  $N_t^g \leftarrow$ number of channels used by I_t^g
  if $N_t^g < U_{c_t}$
    then $U_{c_t} \leftarrow N_t^g$
      $A_t \leftarrow$ channel assignments for I_t^g
return $\{U_{c_t}, A_t\}$

III.E. Locality-Based C_t^* Lower Bound

For a time step t, a lower bound L_{c_t} on C_t^* is calculated as follows, where $CIP_t(h)$ denotes CIP_t applied to cell group h:

$$L_{c_t} = \max\{optimal \; number \; of \; channels \; for \; CIP_t(h) | h \; is \; an \; (\rho+1) \times (\rho+1) \; cell \; group\} \tag{14}$$

Because the co-channel interference constraints of CIP_t are stronger than those based solely on reuse distance and a $(\rho+1) \times (\rho+1)$ sized square contains cluster-sized $\rho \times \rho$ cell groups, $L_{c_t} \geq L_{r_t}$. Furthermore, since each cell group h is a subset of the larger cell grid and CIP_t is applied to h, $C_t^* \geq L_{c_t}$.

[2]It is well-defined if C contains at least one $Type_v$ cell. It is equal to 0 if every cell is a $Type_v$ cell.

Note that this use of geographic locality is quite different than in Section III.D. In Section III.D the entire grid is used but neighborhoods of $Type_v$ cells produce constraints on assignments. Here no constraints on assignments are added to CIP_t, but only a portion of the grid is considered.

III.F. Results

This section discusses the effectiveness of the new DCA strategy of Section III.D with respect to two goals. The first is to assess in Section III.F.1 the impact of cumulative interference constraints versus those based solely on reuse distance by empirically bounding an estimate on $C_t^* - R_t^*$ using $U_{c_t} - L_{r_t}$. The second goal is to lay the foundation for a fast, centralized DCA heuristic. We characterize U_{c_t} in Section III.F.2 in order to gain sufficient insight to create the new heuristic of Section IV.

A time-step simulation is used. $D_t(\vec{c_i}) = t + 1$ for each $Type_v$ cell. The simulation is executed for 10 time steps, starting at $t = 0$. The C/I threshold B given in Eqn. 3 is obtained so as to yield the largest channel reuse distance smaller than one that yields $\rho = 2$. A greedy sequential assignment algorithm is applied iteratively over a range of B values so as to find the interval that satisfies the required reuse distance. The algorithm is described in the web link [27] that accompanies this paper. We note that the thresholds are specific to the channel interference model assumed. For the simple path-loss model considered here with $\alpha = 3.5$,[3] a threshold value of $B = B_2 = 27,234$ satisfies the reuse constraints for $\rho = 2$. An additional set of results for the case $\rho = 3$ using $B_3 = 125,000$ is presented in [27].

Our DCA strategy is evaluated on the spatial configurations of Figure 1. For these configurations the percentage of $Type_v$ cells is set to 20%, i.e. $p_v = 0.2$. For each step t in the simulation, each IP model is solved using the Mixed Integer Programming solver component of the CPLEX 7.0^{TM} [4] optimization software package. An upper bound on CPLEX computation time is imposed as follows, where computation time is in seconds on a 450 MHz SPARC Ultra[5]. For each IP model I_t^g in the algorithm of Section III.D the limit is 10 seconds plus a factor proportional to the difference between the upper and lower bounds on C_t^*. This consistently produces good feasible solutions in our experiments.

Some of the centralized channel assignment literature considers published benchmarks for evaluating algorithms. These benchmarks are for problems that also constrain frequency separation distance [23, 22, 28, 29]. In [30] interference constraints are modeled using an interference graph. Although the constraints model reuse distance they do not consider the cumulative effect of interference across the cellular system when assigning a channel to a cell as done in this work. The published benchmarks are therefore not directly applicable to this work. The web link accompanying this paper [27] includes instance data so that future work can be compared with ours.

III.F.1. Empirical Bound on $C_t^* - R_t^*$

Based on Eqn. 13, this section provides an empirical estimate of $C_t^* - R_t^*$ by bounding $U_{c_t} - L_{r_t}$. Figure 2 shows U_{c_t} and L_{r_t} for representative spatial distributions of Figure 1 and threshold B_2 [6]. For linearly increasing demand, the number of channels required for L_{r_t} can be expressed in the form $D_v v(t + 1) + D_c c$, where v and c are the numbers of $Type_v$ and $Type_c$ cells, respectively, in a maximal-demand cluster and D_v is the demand rate of change per unit time for a $Type_v$ cell. Since we assume $D_v = 1$ and $D_c = 1$, $L_{r_t} = vt + (v + c)$. Linear regression on U_{c_t} yields a line whose slope relative to v differs by only 3% when averaged across the nine spatial distributions. The slope of the regression line for U_{c_t} varies the most from v for distribution (b). In that case $U_{c_t} = L_{r_t}$ for $t = 6, 8, 9$, so the IP solutions for those time steps are provably optimal.

Because the slope of U_{c_t} is approximately equal to the slope of L_{r_t}, the size of the $U_{c_t} - L_{r_t}$ gap appears to be independent of t and comes from the gap $U_{c_0} - L_{r_0}$. To assess this gap, we first evaluate L_{r_0}. Section III.C establishes that the size of a cluster is ρ^2. At $t = 0$ every cell in a cluster requires one channel and every cluster is a maximal-demand cluster, so $L_{r_0} = v + c = \rho^2$.

To analyze U_{c_0}, we first remark that $U_{c_0} = 10$ for all 9 spatial distributions for $\rho = 2$. Thus, $U_{c_0} \approx (\rho + 1)^2$. U_{c_0} can be improved as follows. Let CIP' be a modified version of CIP_0 that uses the cumulative interference constraints of CIP_0. In CIP' each cell's demand is one channel, but the demand con-

[3]Hac and Mo [12] also use $\alpha = 3.5$. Reuse distance of 2 is used by Battiti *et al.* in [24].

[4]CPLEX 7.0 is a trademark of ILOG Corporation.

[5]SPARC Ultra is a trademark of Sun Microsystems Corporation.

[6]Figure 2 also shows results for the lower bound L_{c_t} and our new, fast heuristic denoted H_t. These results are discussed in Sections III.F.2 and IV.B, respectively. Figures for the complete set of nine spatial distributions are at the web link accompanying this paper [27].

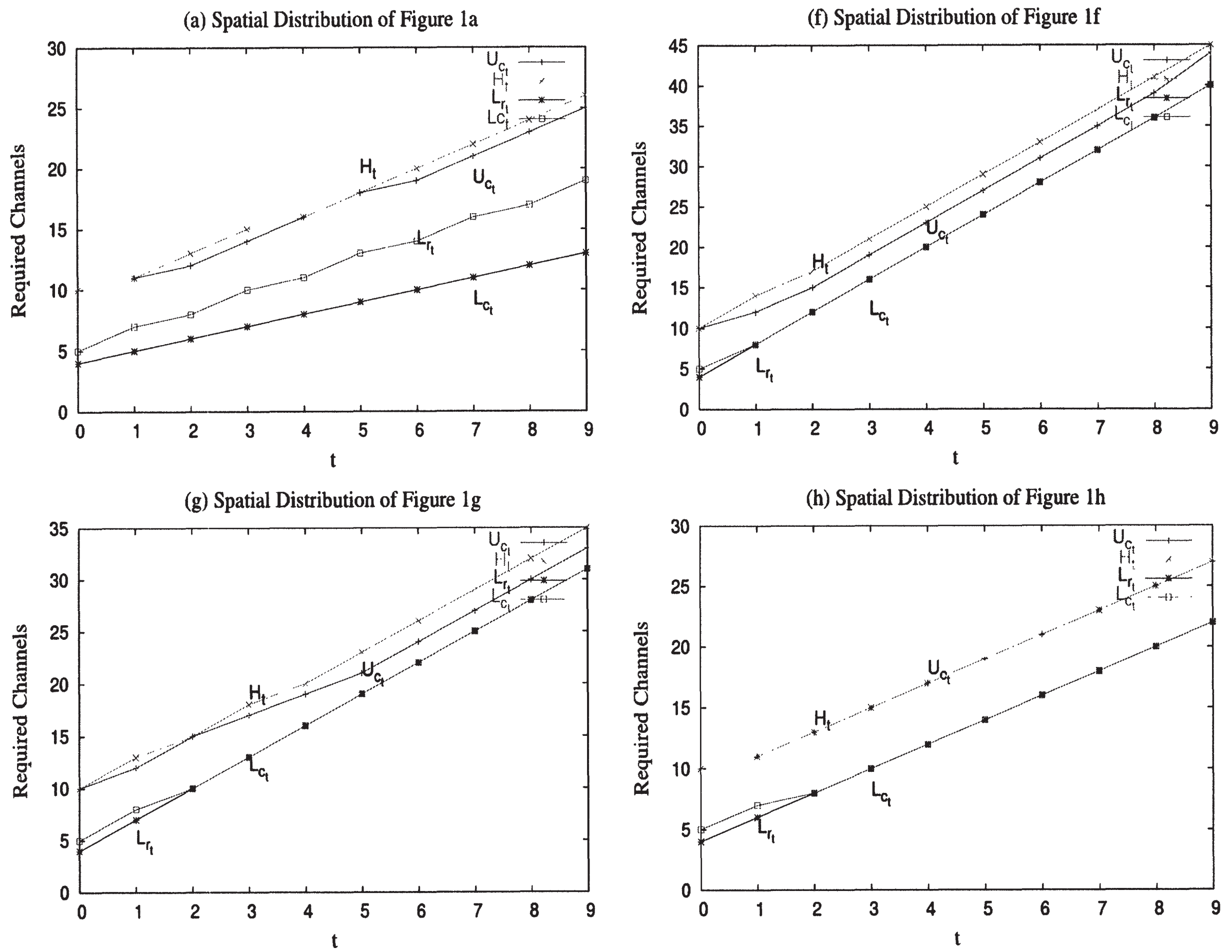

Figure 2: Minimum number of channels for representative spatial distributions of Figure 1 using threshold B_2 for cluster size $\rho = 2$.

straints are soft constraints. CIP' uses only 1 channel and maximizes the number of reuses of that channel to attempt to satisfy demand across the entire grid. For the 7×7 grid and interference threshold B_2, the solution of CIP' yields a maximum number of channel reuses $= 9 = (\rho+1)^2$. One way this can be achieved is to assign a channel to cells with row/column: (1,1), (1,4), (1,7), (4,1), (4,4), (4,7), (7,1), (7,4), and (7,7). This reuse pattern can be used to create assignments for the entire grid that are feasible with respect to the interference constraints. The first step is to assign a different channel to each cell in the $(\rho+1) \times (\rho+1)$ cell group whose upper left corner is at cell (1,1). The next step is to reuse each of those channels according to the above pattern. Note that, under this assignment, each cell receives exactly one channel. Thus, demand is satisfied using only 9 channels. This improves U_0 from 10 to 9, making it exactly $= (\rho+1)^2$ for B_2.

We conclude that, empirically, $U_{c_0} - L_{r_0} = (\rho+1)^2 - \rho^2$. Thus, in our experiments the gap $C_t^* - R_t^*$ that reflects the additional impact of cumulative interference constraints versus those based only on reuse distance is $\leq (\rho+1)^2 - \rho^2$ and is characterized using only ρ which depends only on the reuse distance r. This impact is the number of extra cells added to a cluster to form a square cell group whose side is one larger than that of a cluster.

III.F.2. Empirical Bounds on C_t^*

This section summarizes results on lower and upper bounds on C_t^* to lay the groundwork for the new heuristic of Section IV.

LOWER BOUND ON C_t^*: Section III.E establishes L_{c_t} as a lower bound on C_t^*. The values of L_{c_t} for threshold value B_2 are shown in Figure 2 for representative configurations of Figure 1. As expected, $L_{c_t} \geq L_{r_t}$. For most of the spatial configurations, the difference between L_{c_t} and L_{r_t} is small. In fact, for each configuration except (a), there is no difference beyond the second time step. For B_3 there is no difference for any time steps. Thus, the observations about the empirical bound on $U_{c_t} - L_{r_t}$ essentially hold for $U_{c_t} - L_{c_t}$ as well. Furthermore, the small difference between L_{r_t} and L_{c_t} suggests that, for small geographic localities, sufficient channel reuse occurs so that the impact of the cumulative interference constraints beyond those based solely on reuse distance is small. More specifically, for a given $(\rho + 1)^2$ cell group h, let $L_{r_t}(h)$ be the result of Eqn. 12 applied just to the part of the cellular grid defined by h. At least $L_{r_t}(h)$ channels must be used in solving $CIP_t(h)$. Most of the spatial distributions contain a cluster for which $L_{r_t}(h)$ is sufficiently large enough to allow these $L_{r_t}(h)$ channels to be reused in the additional $(\rho + 1)^2 - \rho^2$ cells, satisfying their additional demand without violating the cumulative interference constraints.

UPPER BOUND on C_t^*: Section III.F.1 improves U_{c0} to $(\rho + 1)^2$ for B_2. This, combined with the observation that the slope of the U_{c_t} curve is approximately v, allows us to conclude that $U_{c_t} \approx (\rho + 1)^2 + vt$. In order to design a fast, new heuristic whose channel usage is similar to the computationally expensive IP-based approach, we studied the channel reuse patterns for U_{c_t}. For B_2 the number of reuses per channel, averaged across the 10 time periods and 9 spatial configurations, is 4.5. A heuristic that comes close to matching the 4.5 channel reuse rate is likely to perform as well as the IP-based strategy. The maximum number of reuses per channel is optimally computed to be 9 for the 7×7 grid and B_2 by CIP' (see Section III.F.1). The 9-channel reuse pattern given there spreads out channel reuses uniformly across the grid. This reuse pattern and similar ones suggest that the IP solver, while maximizing channel reuse, uses a strategy that roughly resembles maximizing the minimum geographic distance between assignments of the same channel. This observation is used to design the heuristic of Section IV.

Here we compare U_{c_t} results across locality sizes where the upper bound g_{max} on locality size is as defined in Section III.D. For the spatial distributions of Figure 1, values of g_{max} range from 2 to 5. For B_2 and averaging across time steps and spatial distributions, locality 0 requires the smallest number of channels roughly 93% of the time. Recall that for locality size 0 only assignments for $Type_v$ cells can be changed. These results indicate that allowing assignments for $Type_c$ cells to change rarely results in a fast improvement in the IP solver's locality size 0 solution. Using locality size 0 therefore has the dual benefits of providing the best results on number of channels within the execution time limit as well as limiting reassignments across time periods.

IV. DCA Heuristic

Even though the upper bound algorithm of Section III uses highly constrained forms of the CIP_t model, due to its locality policy, its long execution time makes its use prohibitive. Section IV.A describes a centralized DCA heuristic that uses the cumulative interference constraints of Eqn. 3. Experience with the constrained CIP_t model under linearly increasing demand led to the design of the new heuristic. The effectiveness of the heuristic is analyzed in Section IV.B for both linearly increasing demand and that modeled by a Markov process in time. The heuristic uses significantly less computation time than the algorithm of Section III yet the number of channels it uses is comparable to that algorithm.

IV.A. Approach

The heuristic takes as input the spatial demand distribution at a time instance. Considering each available channel sequentially, it attempts to maximize the reuse of a channel by controlling the change in cumulative interference experienced by cells that have already been assigned this channel. Each potential cell in which this channel can be assigned is associated with a cost factor that is given by the sum of the squared difference between the cumulative interference before and after the channel is assigned to the new cell. Rather than selecting the position that maximizes or minimizes the interference change, the position that corresponds to the median change in interference is selected. Under this constraint, the channel is reused until the co-channel interference constraint of Eqn. 3 is violated at all remaining cells requiring a channel. This choice of placing the channel in a position that is intermediate to the maximum and minimum changes in the residual interference is motivated by examination of channel reuse patterns for the IP model. As observed in Section III.F.2, it resembles maximizing the minimum distance between assignments of the same channel. The heuristic attempts to quickly achieve this effect using a greedy strategy. Choosing the position that maximizes the additional interference would pack channel reuses close together. This would have the positive effect of minimally fragmenting the remainder of the available positions and therefore preserve flexibility for future assignments.

However, it would have the negative effect of creating so much additional cumulative interference that the number of future reassignments would be severely limited. Choosing the position that minimizes the additional interference would spread out channel reuses. This would have the positive effect of minimizing the additional interference so that the number of future reassignments could be larger. However, it would have the negative effect of fragmenting the remainder of the available positions and therefore reducing flexibility for future assignments[7]. Choosing the median change in interference is a compromise between these two extreme policies. This is expected to adequately treat both competing objectives. Other factors that influence the heuristic include assigning the first use of a channel to the cell with the maximum demand in its $(\rho+1) \times (\rho+1)$ neighborhood.

The worst case asymptotic running time of the heuristic is in $O(t_{max}|D_t|c_{max}^3)$, where t_{max} is the number of time steps and D_t is the demand for time step t. The c_{max}^3 term comes from the *cost* calculation.

<u>DCA HEURISTIC</u>:

for each time step t
 if demand across cells is nonuniform
  then Initialize $C = \{\vec{c_1}, \vec{c_2}, \ldots, \vec{c}_{c_{max}}\}$:
   Find demand in $(\rho+1) \times (\rho+1)$ cell groups.
   Normalize demand in each group by group size.
   Order groups by demand density.
   Order individual cells for assignment starting
    from cells in highest density regions.
  else Initialize C in column order.
 $f \leftarrow$ first channel
 while demand D_t is not yet satisfied
  if f has not been assigned to any location
   then $L_{nf} \leftarrow$ all cells with unsatisfied demand.
    Assign f to cell with maximum number of
     neighbors with unsatisfied demand
     (in $(\rho+1) \times (\rho+1)$ cell group)
   else $L_{af} \leftarrow$ locations where f has been assigned
    $L_{nf} \leftarrow$ locations where f can be assigned
     without violating Eqn. 3 at cells in L_{af}
    if L_{nf} is empty
     then $f \leftarrow$ next channel
     else for each cell $\vec{c_i} \in L_{af}$
       $I_e(\vec{c_i}) \leftarrow$ existing interference level
        due to assignments of f
      for each cell $\vec{c_j} \in L_{nf}$
       for each cell $\vec{c_i} \in L_{af}$
        $I_n(\vec{c_i}) \leftarrow$ interference at $\vec{c_i}$
         if f assigned to $\vec{c_j}$
        $cost(\vec{c_j}) \leftarrow \sum_{\vec{c_i} \in L_{af}} |I_e(\vec{c_i}) - I_n(\vec{c_i})|^2$
       Sort *cost* in ascending cost order.
       Assign f to cell with median cost value.

[7]Hac and Mo [12] make similar observations in a distributed DCA setting about policies that always choose the channel with largest or smallest interference level.

IV.B. Results

IV.B.1. Linearly Increasing Demand

The DCA heuristic is first compared to the IP strategy by considering the linearly increasing demand function of $Type_v$ cells. The results of the DCA heuristic for representative spatial distributions of Figure 1 are shown in Figure 2 under the label H_t and may be compared to the IP solution denoted by U_{c_t}. The number of channels found required by the heuristic is seen to be comparable to that generated by the IP algorithm. Typically, the heuristic matches the IP solution to within one channel, and deviates by at most two channels in cases (e,f,g). For case (c), the heuristic improves on the IP performance by requiring one less channel. The performance may also be compared with respect to the average channel reuse afforded by the two approaches. The heuristic reuses each channel on average 4.64 times, in comparison with the IP which reuses each channel an average of 4.5 times. The policy of reusing a channel at a location that creates a median change in residual interference was compared to two other policies where the location is selected so as to minimize or maximize the change in residual interference. The latter policy is often referred to as maximum packing. The median policy was typically the better choice, particularly in cases where the $Type_v$ cells were spread across the entire spatial grid. Although the minimum policy was often comparable to the median solution, this approach can lead to divergent solutions in cases such as (d,f), where large $Type_v$ clusters reduce the probability of finding a location that is furthest from a group of interfering cells. The maximum packing policy is in all cases the worst performing policy since the cumulative interference quickly constrains the channel reuse factor. The heuristic's execution time is observed to increase linearly as a function of demand, which is consistent with the analysis of Section IV.A. The heuristic is fast, with typical running time of .002 seconds per unit of demand on a 600 MHz Compaq Alpha server.

IV.B.2. Markovian Demand Model

The application of the DCA heuristic based algorithm is examined next in the context of blocking probabilities generated for randomly varying demand in time. $Type_c$ cells generate a constant demand of $D_c = 1$

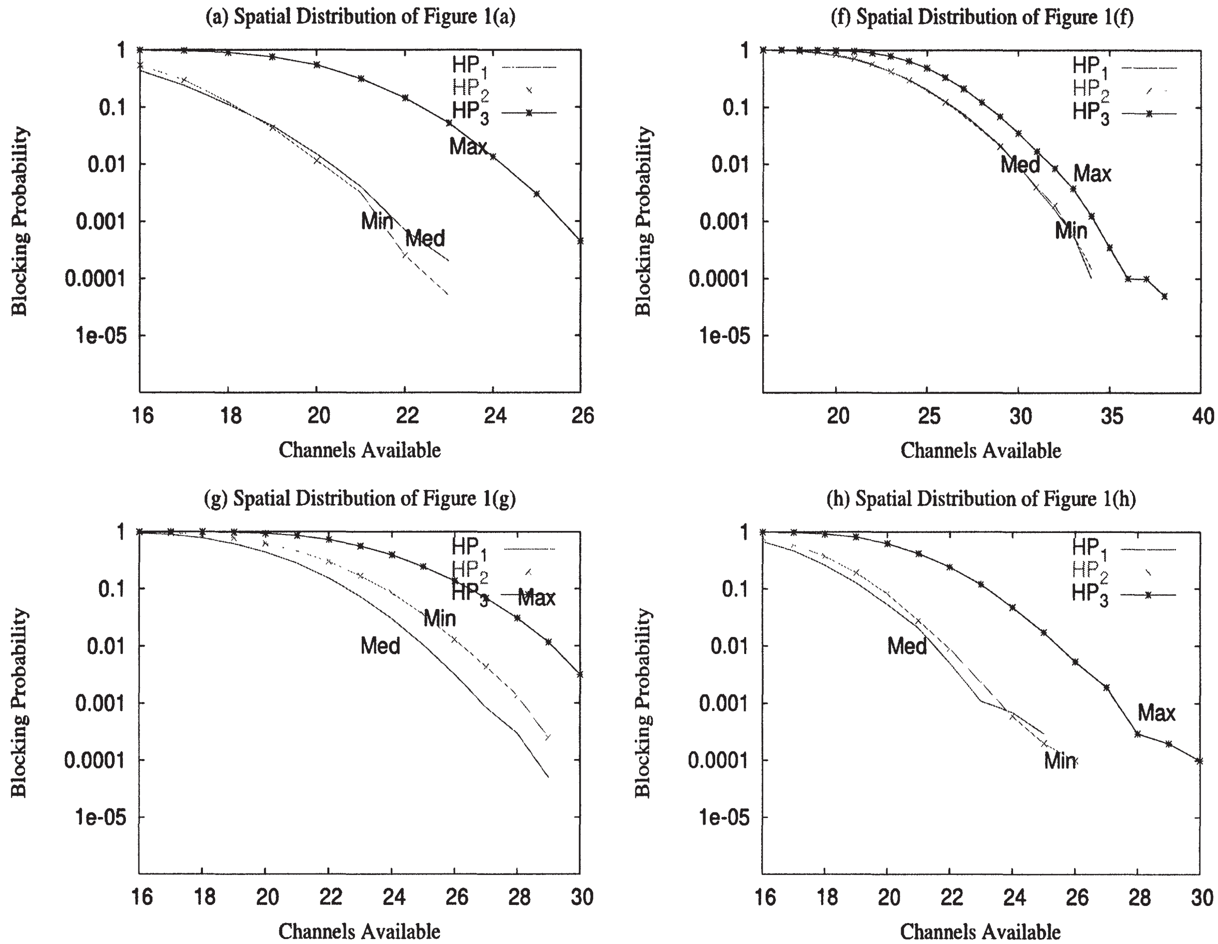

Figure 3: Blocking probabilities using DCA heuristic for on-off arrival processes and uniformly distributed holding times. (median, minimum and maximum packing policies compared)

channel in each time unit. $Type_v$ cells generate demands using a two state (on-off) discrete-time Markov chain model with uniformly distributed holding times. In the on and off states the cell demand rates are D_{on} and D_{off} respectively, with $D_{on} > D_{off}$. The channel holding times in the on and off states are characterized by independent identically distributed random variables with an average of $\hat{\tau}$ time units.

Denote the parameters of the Markov chain transition matrix as,

$$P = \begin{bmatrix} p_{00} & p_{01} \\ p_{10} & p_{11} \end{bmatrix} \tag{15}$$

with states 0 and 1 representing the off and on states respectively.

The expected demand per $Type_v$ cell is obtained using the sum of the average on and off periods of the Markov chain as a reference cycle duration. Let $\hat{T}_{on}$ and $\hat{T}_{off}$ denote the expected durations of the arrival process in the on and off periods respectively. Note that $\hat{T}_{on} = \frac{1}{p_{10}}$ and $\hat{T}_{off} = \frac{1}{p_{01}}$. The average holding times may in general be different in the on and off periods and are denoted as $\hat{\tau}_{on}$ and $\hat{\tau}_{off}$ respectively.

The demand variation in a particular state, say the on state during $\hat{T}_{on}$, may be defined in terms of three regimes: (i) the channel acquire period in which a $Type_v$ cell demands D_{on} channels with a holding time of $\hat{\tau}_{on}$. This period lasts for $\hat{\tau}_{on}$ time units for distributions characterized by finite holding times; (ii) the peak demand period where the demand saturates at $D_{on}\hat{\tau}_{on}$ until the average duration of the on state terminates; (iii) the channel release period where demand decreases at a rate of D_v per unit time, lasting for time period $\hat{T}_{on} + \hat{\tau}_{on}$. This demand change during $\hat{T}_{on}$ can be summarized as:

$$\begin{aligned} d_{on}[t] &= D_{on}t, \quad 1 \le t \le \hat{\tau}_{on} \\ &= D_{on}\hat{\tau}_{on}, \quad \hat{\tau}_{on} + 1 \le t \le \hat{T}_{on} \\ &= D_{on}\hat{\tau}_{on} - D_v\left[t - \hat{T}_{on}\right], \end{aligned}$$

$$\hat{T}_{on} + 1 \leq t < \hat{T}_{on} + \hat{\tau}_{on} \quad (16)$$

A similar equation can be defined for the demand changes $d_{off}[t]$ during the average period of the off-state, $\hat{T}_{off}$. The average demand during an on-off cycle of the Markov chain is then given by

$$\hat{D} = \frac{\sum_{t:(\hat{\tau}_{on}+\hat{T}_{on})} d_{on}[t] + \sum_{t:(\hat{\tau}_{off}+\hat{T}_{off})} d_{off}[t]}{\hat{T}_{on} + \hat{T}_{off}} \quad (17)$$

Eqn. 16 when integrated over the duration $\hat{\tau}_{on} + \hat{T}_{on}$, yields the aggregate demand generated during the on-state activity, $\hat{D}_{on} = D_{on}\ \hat{\tau}_{on}\ \hat{T}_{on}$. Similarly, $\hat{D}_{off} = D_{off}\ \hat{\tau}_{off}\ \hat{T}_{off}$.

The average channel demand per $Type_v$ cell may therefore be obtained by relation,

$$\hat{D} = \frac{(D_{off}\ \hat{\tau}_{off}\ \hat{T}_{off}) + (D_{on}\ \hat{\tau}_{on}\ \hat{T}_{on})}{\hat{T}_{on} + \hat{T}_{off}} \quad (18)$$

The blocking performance is examined for the case where $D_{on} = 2$, $D_{off} = 1$ and the channel holding times are discrete equiprobable values ranging from 1 to 5 unit time durations. The average holding times in the on and off states are $\hat{\tau}_{off} = \hat{\tau}_{on} = 3$. These parameters bound the per $Type_v$ cell demand variation between values of 1 and 10 respectively in each time interval. Selecting the Markov parameters as, $p_{00} = 0.55$, $p_{01} = 0.45$, $p_{10} = 0.2$, $p_{11} = 0.8$, the average channel demand rate per $Type_v$ cell is given by Eqn. 18 as 5.1 channels.

The blocking probabilities were simulated by providing a fixed number of channels F and applying the DCA heuristic to perform the assignment in each time step of the Markov process evolution. The blocking probabilities plotted in Figure 3 for representative ensembles given in Figure 1 are obtained by counting the number of blocked calls in simulation lengths of $30,000$ time steps. The results in each panel demonstrate the effect of choosing assignments based on the three different cost metrics in the DCA heuristic discussed above. Probabilities denoted as HP_1 depict the case where channels were assigned to positions that resulted in a median change in interference, HP_2 denotes the case when the change in interference was minimized (channels placed furthest from existing assignments) and HP_3 show the results when channel reuse was subjected to the maximum packing condition. It is seen that the maximum packing constraint demonstrates the worst performance and the median interference change proves to be the better policy in almost all cases. We note that the maximum packing constraint has been applied as a candidate in several DCA algorithms [4, 24] for evaluating how a good dynamic channel assignment algorithm will perform. Our results demonstrate that when cumulative interference is a constraint, the maximum packing policy is not a good indicator of DCA performance.

To compare the blocking probabilities across the nine different ensembles, the unique spatial features of the $Type_v$ cells must be taken into account. This can be achieved by considering the IP solutions U_{c_t} or heuristic solutions H_t obtained, for that time instant when all $Type_v$ cells assume the same demand value of $\hat{D}$. For the case analyzed here, $\hat{D} = 5.1$ and $16 \leq H_t[n] \leq 25$ across the nine spatial ensembles, $n : [a - i]$. Denoting $H_t[n]$ under the constraint of fixed t as F_{avg}, the effective system load may be represented as $\gamma = \frac{F_{avg}}{F}$. The solution F_{avg} is unique to each spatial distribution and it effectively captures both spatial and average temporal dynamics of the network demand.

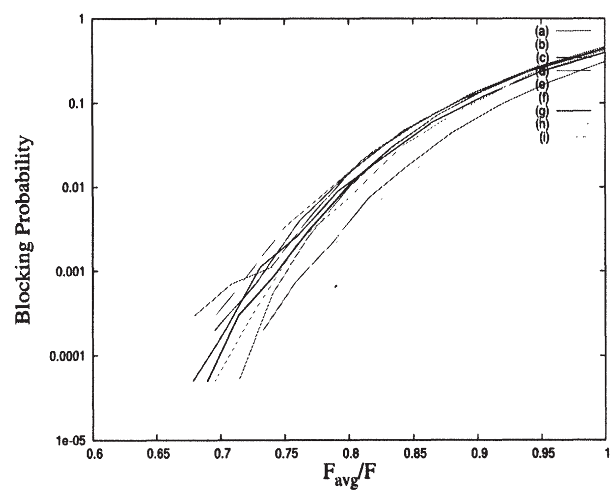

Figure 4: Blocking probabilities of all nine spatial ensembles as a function of $\gamma = F_{avg}/F$ for threshold B_2 and policy HP_1.

Figure 4 depicts the blocking probabilities for policy HP_1 of all nine ensembles as a function of γ. The relative invariance in the structure of the individual blocking probabilities when represented with respect to γ indicates that F_{avg} captures the necessary differences among the spatial distributions and can be applied as a suitable renormalization factor. These results provide an estimate of the maximum operating load that will maintain blocking probabilities below a specified threshold. This estimate is expected to be valid for the class of spatio-temporal distributions specified by the set of average parameters $p_v c_{max}$ and $\hat{D}$, capturing the spatial and temporal features respectively.

IV.B.3. Uniform Traffic Comparison

To evaluate the benefits of dynamic channel assignment using the new heuristic, the number of channels required under fixed channel assignment considering peak demand conditions were obtained. For the case where all of the cells are assumed to be of $Type_c$, demanding the maximum of 10 channels per cell, the DCA heuristic was found to require 92 channels to satisfy the network demand. We note that 9 channels are sufficient to ensure zero blocking when all $c_{max} = 49$ cells require one channel (see the modified IP formulation CIP' in Section III.F.1). The proposed heuristic only requires 2 additional channels in this case. If, however, one additionally considers that the peak demand arises from only the 20% fraction of $Type_v$ cells as in Figure 1, under DCA solutions U_{c_t}, then the savings in the number of channels utilized ranges from $51 - 72\%$.

V. Conclusions

This paper presents schemes for centralized DCA under spatio-temporal demand variation. Cumulative co-channel interference constrains reuses of the same channel. In this context the paper makes two main contributions. First, it establishes that obtaining the optimal number of channels using cumulative co-channel interference constraints is at least as hard as finding the optimal number of channels using interference constraints based only on reuse distance. Then, to quantify this, an empirical bound is obtained on the difference between the optimal number of channels required using cumulative co-channel interference constraints versus interference constraints based only on reuse distance. The bound derives from an upper bound on the optimal number of channels in the cumulative case and a lower bound on the optimal number of channels in the reuse distance case. For linearly increasing demand, the difference is approximately equal to the number cells in a cluster whose size is based only on reuse distance and which is independent of the time parameter. A geographic locality-based technique is used to generate an upper bound on the minimum number of channels in the cumulative interference case. It uses an IP model that enforces the demand and cumulative co-channel interference constraints. The geographic locality technique has the advantage of limiting channel reassignments across simulation time steps. Reassignments are limited the most for the smallest locality size and constant demand cells.

The second contribution is a new, fast centralized DCA heuristic. The heuristic's design is based on analysis of channel reuse patterns used by the IP solver in the geographic locality-based technique. The average number of reuses per channel for the 7×7 square grid and interference threshold of $27,234$ is 4.5 for the IP solver. To achieve this, the IP solver appears to follow a policy that maximizes the minimum distance between channel reuses while attempting to maximize the number of channel reuses. The new heuristic tries to achieve this effect by selecting a channel that does not induce the maximum or minimum change in interference but, rather, produces a median level of change. This policy's results are comparable to those of the IP solver for linearly increasing demand. The heuristic achieves average channel reuses of 4.64, exceeding the IP solver's rate by approximately 3%. The heuristic's execution time is fast and increases linearly as a function of demand.

The application of the DCA heuristic for estimating blocking probabilities of on-off Markov arrival processes with uniformly distributed holding times is demonstrated. Knowledge of the channels required for zero blocking under average demand conditions allows us to represent the blocking probabilities of the different spatial ensembles by a single function.

Simulation experiments with a variety of spatial traffic demand distributions show that the savings obtained with the new heuristic by considering non-uniformity of traffic in space range from $51 - 72\%$ with respect to uniform traffic.

The proposed DCA strategies can also be considered when power levels vary across transmitters. This can be accommodated by specifying a path-loss exponent α_i for each transmitter i. The path-loss exponent α appears in Eqn. 2 and is used to calculate signal strength $S(\vec{c_i}, \vec{c_j})$. If α_i is used in Eqn. 2 instead of α, the result is signal strength at cell $\vec{c_j}$ due to transmission from the base station at cell $\vec{c_i}$. $S(\vec{c_i}, \vec{c_j})$ is a constant for the purposes of the core IP model and our algorithms. The structure of the model and algorithms do not need to be changed to accommodate different $S(\vec{c_i}, \vec{c_j})$ values. The minimum interference threshold satisfying a particular reuse distance can still be determined as described in Section III.F. This reuse distance would be the minimum one across all the transmitters. Since this represents the minimum geographic separation between uses of the same channel, it can still be used to define the size of a cluster. The new heuristic of Section IV uses reuse distance to determine the size of a cell group for assessing local demand density. This would not need to be changed if using α_i. Examining the effects of different α_i values on the empirical bounds presented here is a subject for future work.

The DCA schemes in this paper assume centralized control over channel assignment and power levels of transmitters do not change over time. The new heuristic is appropriate for situations in which adaptive power control is not used and minimizing the number of channels used is important enough to justify the overhead of centralized control. The new DCA results presented here can also be used as benchmarks for distributed DCA heuristics in the following two ways. First, they can serve as empirical lower bounds on the number of channels required by distributed DCA heuristics that do not use adaptive power control. Second, distributed DCA heuristics that use power control could use the number of channels achieved here as a goal. Consequently, one direction for future work is the design of a distributed DCA heuristic that comes close to assigning the same number of channels as this paper's centralized heuristic.

A web site accompanies this paper [27]. It elaborates on the results described in this paper and contains information to assist other researchers in comparing their work with ours. Web resources include demand files, a description of the greedy sequential algorithm used to generate interference threshold values, figures associated with our second threshold value, B_3, and an annotated bibliography of relevant DCA research.

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