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Frequency assignment, multiple interference and binary constraints

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Abstract The most accurate approaches to frequency assignment problems minimize a cost function based on signal-to-interference ratios at points where reception is required. The merits of this approach are counterbalanced by much greater requirements for computational resources than for the traditional approach using binary frequency separation constraints. This can make run times unrealistic for the largest problems. In this paper the merits of the signal-to-interference based cost function are confirmed, but it is shown that algorithms are faster and give better quality results if this cost function is combined with the binary constraint approach. Two types of algorithm are used to illustrate the combined approach, simulated annealing and a new ant colony system algorithm. The combined approach studied is applicable to all the main classes of frequency assignment problem.

Keywords Frequency assignment · Multiple interference · Binary constraints

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1 Introduction

The problem of assigning frequencies to the transmitters of a radio network is usually known as the *frequency assignment problem* (FAP) or *channel assignment problem* (CAP). The aim is to use the separation of assigned frequencies to minimize interference in the network, while at the same time using spectrum efficiently. The version of the problem considered here is the *fixed spectrum frequency assignment problem*. Here the spectrum available is defined initially and normally consists of a set of consecutive equally spaced channels, possibly with some gaps of one or more channels which are unavailable for some or all transmitters. Some measure of interference must then be minimized. This version of the FAP is important in a wide range of terrestrial and satellite based radio systems, both commercial and military.

There has been an enormous volume of literature devoted to the frequency assignment problem over the past 30 years; see [1, 2] for surveys. However, almost all of this literature is concerned with algorithms that use *binary constraints*. Binary constraints are of the form $|f_i - f_j| > k$ where f_i is the frequency assigned to transmitter i and $k+1$ is the smallest number of channels separation that will avoid interference between one of the transmitters i, j and a receiver of the other. The problem with formulations involving binary constraints is that no account is taken of interference from multiple sources. Multiple interference can be an important consideration in modern networks with high traffic levels involving all transmitters. Results indicating the extent to which frequency assignments generated using binary constraints are unsatisfactory are given in [3]. Further results can be found in Section 5.

Various approaches to the consideration of multiple interference in frequency assignment using nonbinary constraints have appeared in the literature, see for example [2, 4, 5]. However, for larger problems the number of nonbinary constraints required becomes unmanageable. A more satisfactory approach was proposed in [3], and independently in [6]. A set of *receiver points* at which a service is required is specified. For each receiver point the received signal strength from a serving transmitter and from all potentially interfering transmitters is available to the algorithm. The signal-to-interference ratio (SIR) at the reception points is then determined using off-tune rejection factors and compared with the required threshold. A cost function involves any deficits in the SIRs at reception points and this cost is zero if all receiver points have a satisfactory SIR. The reception points can be in the pattern of a regular grid or consist of points which are known a priori to be "worst case", for example at the boundaries of cells. The volume of data required is large but manageable.

The results presented here confirm that the approach described in [3, 6] is satisfactory, and more accurately minimizes interference than the standard binary constraint approach. However, experience with frequency assignment in a large satellite based system with finely tuned frequencies showed that the approach is generally successful, but run times can be exorbitant. Even with well designed data structures and efficient cost function updating the run times were many times longer than when using binary constraints, and were sometimes measured in weeks. In this paper the use of various combined approaches of binary constraint solution and SIR based cost function minimization will be explored. It will be shown that better solutions can be obtained, and obtained much more quickly, if the combined approach is used.

The advantages of the combined approach should be largely independent of the choice of algorithm. Thus two basic algorithms are considered, a standard simulated annealing algorithm and a new hybridised Ant Colony System algorithm referred to here as ANTS, which improves the algorithm in [7] and facilitates the optional use of binary constraints. Both the ANTS algorithm and the specific applications of the SA algorithm are new. However, the main innovation in the paper is in the way that SIR and constraint approaches are combined, and in the investigation of how this should best be done.

Benchmarks for multiple interference problems are inevitably more complex to define than binary constraint benchmarks. Two of the benchmarks here are existing benchmarks for terrestrial area coverage problems. The others are new reproducible problems designed to simulate typical satellite and terrestrial assignment problems. They should allow others to compare the results obtained here with different algorithms using a similar formulation.

2 The SIR model, multiple interference and constraints

2.1 The SIR model

The area in which a radio service is provided is taken to be a plane, and the Cartesian coordinates of the transmitters are given. The set of frequencies available for assignment to a transmitter is known as its *frequency domain*. The frequency domain need not necessarily be the same for all transmitters.

For a point r of the plane, the received signal strength (proportional to the power flux density) from a transmitter t is taken to be

$$S_{rt} = P_t L_{rt} \tag{1}$$

where P_t is the power of transmitter t, and L_{rt} is an attenuation factor that represents the signal loss on the path between t and r. L_{rt} should take account of transmitter antenna gain, receiver antenna gain and path loss. In practical applications the path loss is most accurately calculated using a terrain database. In the examples used in this work it will be assumed that the antenna gains are the same for all paths and, in fact, all constants of proportionality cancel in the SIR. For terrestrial problems the path loss can reasonably be represented by a formula such as $L_{rt} = d(r,t)^{-4}$, where $d(r,t)$ is the Euclidean distance between receiver point r and transmitter t. Other exponents between 2.5 and 4.3 have been used in the past to represent propagation loss in radio system planning. For a satellite problem the received signal strength from a beam (which depends on antenna characteristics) might reasonably be represented by $P_t \times (\frac{\sin cd}{cd})^2$ where d is the distance of the receiver from the beam centre and c is a constant chosen so that the received signal strength at the edge of the beam is half the signal strength at the centre [8, 9]. The interference at r comes from multiple sources and must take account of the frequency separations. Thus the interference at a given receiver r served by a transmitter t is given by

$$I_{rt} = \sum_{j \in T, j \neq t} P_j L_{rj} \theta_{j,\, t,\, |f_j - f_t|} \tag{2}$$

where T is the set of transmitters of the network and f_j and f_t are the frequencies (i.e. channel numbers in integers) assigned to transmitters j and t respectively. For homogeneous narrow band carriers the off-tune rejection factors $\theta_{j,\, t,\, |f_j - f_t|}$ depend only on the difference of the channels assigned to transmitters j and t. In such cases the notation can be simplified to $\theta_{|f_j - f_t|}$ and these values must be specified for all test problems. For wideband carriers (or combinations of wideband and narrowband carriers), accurate estimates of

$\theta_{j,\,t,\,|f_j - f_t|}$ can be obtained by integrating the (normalised) transmitter spectral function of j over the filter bandwidth of a receiver of t.

Using the definitions given above and selecting a set of test points, R, a frequency assignment A can be evaluated using the following formula, that assumes low values for a good assignment:

$$E(A, R) = \sum_{r \in R} \sum_{t \in W(r)} \left(\max \left\{ 0, \sigma - \frac{S_{rt}}{I_{rt}} \right\} \right)^a \tag{3}$$

where $W(r)$ is the set of serving transmitters for the point r and a is an even integer, taken as 2 here. σ is a threshold modelling the minimum accepted level of SIR. An alternative is to simply maximise the *coverage*, the percentage of served receivers with an adequate SIR. Given the possibility of small deficits being acceptable, Eq. (3) with $a = 2$ is preferred here as the result is still closely linked to coverage but large SIR deficits may be avoided. Larger (even) values of a would even more heavily penalise the existence of receivers with large SIR deficits, but would be less closely linked to coverage.

The choice of the set R of test points is a compromise between accuracy in estimating the adequacy of the SIR over the area served and algorithmic efficiency. Thus, whereas a fine grid is ideal, here the intersections of the edges of Voronoi polygons will be used. The Voronoi polygon surrounding a transmitter consists of the points which are closer to the transmitter than any other transmitter. Thus this set of reception points tends to be "worst case".

The model described here is essentially the same as those described in [3, 6]. Usually frequency assignment is considered to be the problem of satisfying a given set of frequency separation constraints given a limited set of channels. By contrast, the model described here can be considered as a "natural" model, in the sense that the real aim of frequency assignment is to provide a satisfactory SIR for as many receivers as possible given a limited set of channels.

It should be noted that additional frequency separation constraints (other than those derived from an SIR computation as in the next section) are sometimes imposed. The most common examples are the frequency separation constraints $|f_i - f_j| \geq 2$ or $|f_i - f_j| \geq 3$ that must be satisfied for transmitters i and j at the same site in a mobile telephone problem. These constraints are easy to include in the algorithm. More complex cosite constraints concerned with intermodulation, spurious emissions and spurious responses can also be included if necessary [10]. In such cases the implementation is significantly more complex. It can then be seen that the combined approach is applicable to all the usual classes of frequency assignment problem studied, including mobile telephone networks, radio links problems, area coverage problems and satellite problems. The only real restriction is that of the SIR model itself; the model may become too large for the computer used.

2.2 Generating binary constraints

In order to obtain a set of binary constraints from the multiple interference model described in Section 2.1 the algorithm summarized in Fig. 1 is adopted. $\hat{\sigma}$ is a parameter representing a required SIR relative to interference from a single source. Generally $\hat{\sigma}$ must be chosen somewhat larger than σ if binary constraints are to give even a reasonable solution of the SIR model. Results in Section 5 will clarify how much larger $\hat{\sigma}$ should be than σ.

The procedure described in Fig. 1 calculates the lowest potential SIR at a receiver test point of either one of a pair of transmitters. One of the pair is the wanted transmitter for the test point and the other is an interferer. Starting from this situation, and not considering the other transmitters, the minimum separation required to have an SIR greater than or equal to the required SIR $\hat{\sigma}$ is determined and the necessary constraint for the binary model is stored.

Other binary constraints may be pre-specified for co-sited transmitters.

3 Algorithms

3.1 Simulated annealing

Simulated annealing [11, 12] is a stochastic computational technique derived from statistical mechanics for finding solutions to large optimisation problems that are close to minimum cost. The method is derived from thermodynamics, specifically from the way that liquids freeze or that metals cool and anneal. The aim of a simulated annealing algorithm here is to minimize a cost function $E = E(A, R)$ corresponding to an assignment A with a set R of reception test points. At each iteration a proposal is made to change the frequency assigned to one transmitter in the assignment A to a different frequency in that transmitter's frequency domain. The method is governed by a temperature $\mathcal{T}$ which reduces with time to a low value $\mathcal{T}_{\min}$ according to a cooling scheme given here by $\mathcal{T} := \beta\mathcal{T}$ where $0 < \beta < 1$.

Let ΔE denote the change in cost if the new proposal is accepted. The new proposal is accepted if $\Delta E \leq 0$ or else it is either accepted with probability $\exp(-\Delta E / k\mathcal{T})$ (with k a constant), or rejected. As $\mathcal{T} \to 0$ the algorithm converges on a solution which gives a low value of $E(A, R)$. The probabilistic option allows the algorithm to escape from local minima. The algorithm is run until $\mathcal{T} \leq \mathcal{T}_{\min}$ and reports the best solution A with the lowest $E(A, R)$ observed, denoted by A_{best} and E_{best} respectively.

Fig. 1 Binary constraint generation from the multiple interference model. The constraints will be of the form $|f_i - f_t| \geq C_{ti}$

INPUT:
T = set of transmitters;
R = set of receiver test points;
$W(r)$ = set of transmitters serving test point r;
$\{S_{ri}\}$ = received signal strengths at reception points r from each transmitter i;
$\hat{\sigma}$ = SIR threshold for a single source of interference;
$\{\theta_{i,t,\mathrm{Diff}}\}$ = set of off-tune rejection factors for the cases where $|f_t - f_i| = \mathrm{Diff}$;
$[M_{kj}]$, $[M_{kj}^{\mathrm{min}}]$, $[M_{kj}^{\mathrm{new}}]$ = matrices of SIR values;
OUTPUT:
$[C_{ti}],\ (i > t)$ = the upper triangle of the binary constraint matrix;

```
For j := 1 to |T|
    For k := 1 to |T|
        M_kj^min := ∞;
    end for
end for
For r := 1 to |R|
    For t ∈ W(r)
        For (i ∈ T) ∧ (i ≠ t)
            M_it := S_rt / S_ri;
            If (M_it ≤ M_it^min)
                M_it^min := M_it;
            end if
        end for
    end for
end for
For t := 1 to |T|
    For i := t + 1 to |T|
        M_ti^min := Min { M_ti^min ; M_it^min };
        Diff := 0;
        M_ti^new := M_ti^min;
        While ( M_ti^new < σ̂ )
            Diff := Diff + 1;
            M_ti^new := M_ti^min * θ_{i,t,Diff};
        end while
        C_ti := Diff;
    end for
end for
Return [C_ti], (i > t);
```

The simulated annealing algorithm is used in four ways in this paper:

1. *SA constraints.* To solve binary constraints, when $E(A, R) = E_{\mathrm{viol}}$ is just the number of constraints violated;

2. *SA SIR random start.* To minimize the SIR based cost $E(A, R) = E_{\mathrm{SIR}}$ given by (3), starting from a random assignment;

3. *SA SIR hybrid.* To minimize a weighted sum of the cost of (3) and the binary constraint cost. This will be referred to as a hybrid algorithm. For most of the results presented $E(A, R) = E_{\mathrm{SIR}} + E_{\mathrm{viol}}$ is used. Experience has not revealed any more effective choice of weights except in mobile telephone problems when certain "hard" cosite frequency separation constraints must be satisfied (see Section 5);

4. *SA SIR constraint start.* To minimize the SIR based cost $E(A, R) = E_{\mathrm{SIR}}$ given by (3), starting from an assignment obtained by solving binary constraints at a threshold $\hat{\sigma}$.

Thus 1 uses binary constraints only, 2 uses the cost of Eq. (3) only, while 3 and 4 use a combination of the two. A comparison of the results of 1 with 2, 3, 4 will confirm that binary constraints alone are inadequate. A comparison of the

Fig. 2 A simulated annealing algorithm

INPUT:
T = set of transmitters;
R = set of receiver test points;
$W(r)$ = set of transmitters serving test point r;
$\{S_{ri}\}$ = received signal strengths at reception points r from each transmitter i;
σ = SIR threshold;
$\{\theta_{i,t,\mathrm{Diff}}\}$ = set of off-tune rejection factors for $|f_t - f_i| = \mathrm{Diff}$;
$\mathcal{T}_0$ = initial temperature;
η = number of iterations at each temperature;
β = cooling constant;
OUTPUT:
A_{best}=Best assignment found;
E_{best}=Cost of A_{best};

Initialize temperature $\mathcal{T} := \mathcal{T}_0$;
Generate a random assignment A_{old} and calculate E_{old};
$A_{\mathrm{best}} := A_{\mathrm{old}}$;
$E_{\mathrm{best}} := E_{\mathrm{old}}$;
While ($\mathcal{T} > \mathcal{T}_{\min}$)
  For $i := 1$ to number η of updates at $\mathcal{T}$
    generate a new configuration A_{new} from A_{old};
    calculate new cost E_{new};
    calculate $\Delta E := E_{\mathrm{new}} - E_{\mathrm{old}}$;
    generate a uniform random number $U(0,1)$ in the range $0 \leq U(0,1) < 1$;
    If $U(0,1) < \exp(-\Delta E / k\mathcal{T})$
      $A_{\mathrm{old}} := A_{\mathrm{new}}$;
      $E_{\mathrm{old}} := E_{\mathrm{new}}$;
        If $E_{\mathrm{new}} < E_{\mathrm{best}}$
          $A_{\mathrm{best}} := A_{\mathrm{new}}$;
          $E_{\mathrm{best}} := E_{\mathrm{new}}$;
        end if
    end if
  end for
  $\mathcal{T} := \beta\mathcal{T}$ where $0 < \beta < 1$;
end while
Return $E_{\mathrm{best}}, A_{\mathrm{best}}$;

results of 2 with 3, 4 will confirm the benefit of combining the two approaches. A comparison of the results of 3 with 4 will show which of the two options is preferable.

In the first case $k = 1$ in the algorithm. In the other cases ΔE can be very large and $k = 100000$ is used. Pseudo code for the algorithm is given in Fig. 2.

3.2 ANTS with binary search and local search

The *Ant Colony System*, originally proposed by Gambardella and Dorigo in [13] (see also Dorigo et al. [14]) is based on a computational paradigm inspired by the way real ant colonies function. A moving ant lays *pheromone trails* (Goss et al. [15]) on the ground. An ant encountering a previously laid trail can detect it and decide, with high probability, to follow it, thus reinforcing the trail with its own pheromone. A more detailed account of the analogy with real ant colonies can be found in [7], where an Ant Colony System algorithm for minimum span assignment was presented. Although this algorithm is easily modified for a fixed spectrum problem, it appears that the ants are unable to learn sufficiently quickly in very large problems. The Ant Colony System algorithm presented here (ANTS) allows an ant to more quickly learn a transmitter ordering, so that the most difficult transmitters to assign can be assigned first.

The main elements of the algorithm used here are *ants*, simple computational agents that individually and iteratively construct solutions for the problem. At each step, every ant k expands a current partial assignment by assigning to a currently unassigned transmitter a frequency selected from its frequency domain. The transmitter to assign is selected probabilistically, according to a probability distribution specified as follows. For ant k the probability p_j^k of assigning a frequency to a currently unassigned transmitter j is given by:

$$p_j^k = \frac{\tau_j}{\sum_{j \in U} \tau_j} \tag{4}$$

Fig. 3 Pseudo-code of the *ANTS* algorithm

INPUT:
T = set of transmitters;
R = set of receiver test points;
$W(r)$ = set of transmitters serving test point r;
$\{S_{ri}\}$ = received signal strengths at reception points r from each transmitter i;
σ = SIR threshold;
$\{\theta_{i,t,\mathrm{Diff}}\}$ = set of off-tune rejection factors for $|f_t - f_i| = \mathrm{Diff}$;
m = number of ants;
τ_{init},ρ, ω = algorithm constants;
OUTPUT:
a solution stored in *Sol*;

$E_{\mathrm{best}} := \infty$;
For each transmitter j
  $\tau_j := \tau_{\mathrm{init}}$;
end for
While (termination criteria not met)
  For $k := 1$ to m
    While (Ant k has not completed its solution)
      Choose the transmitter j to assign, with probability given by (4);
      Update the trail level τ_j by means of (5);
    end while
    Improve the current solution to its local optimum;
    E_{new} := Cost of the current solution;
    If ($E_{\mathrm{new}} < E_{\mathrm{best}}$)
      $E_{\mathrm{best}} := E_{\mathrm{new}}$;
      Sol := current solution;
    end if
  end for
  For each transmitter j
    Update the trail level τ_j by means of (7);
  end for
end while
Return *Sol*;

where U denotes the set of currently unassigned transmitters and τ_j is the "pheromone level" of transmitter j. τ_j measures how much interference was caused by transmitter j in past solutions. It represents *a posteriori* indication of the difficulty of the transmitter. In the beginning, when no information is available from past solutions, all τ_j are initialized to τ_{init}. Basically, transmitters which are judged to be difficult will have higher probabilities of being assigned early in the solution building process. Once a transmitter has been selected, it is assigned to the frequency which guarantees the smallest possible increase in the solution cost. When ant k assigns a frequency to transmitter j a local updating is performed on the pheromone matrix, according to the following *evaporation* rule:

$$\tau_j := \rho \times \tau_j \tag{5}$$

(where $0 < \rho < 1$ is a user defined value). An interesting aspect of the local updating is that while a transmitter is assigned to a frequency by an ant, assignment (5) makes the trail intensity diminish, making them less and less attractive, and favouring therefore the exploration of different solutions by the next ants of the colony.

After a solution is found by one ant some local search procedures are run to further improve the solution. The first is an optional binary constraint search. The other two searches use the cost function defined in (3). The optional nature of this fast binary search (which is the only part of the algorithm ANTS using binary constraints) allows the merit of combining the cost of Eq. (3) with binary constraints to be assessed in the context of the ANTS algorithm.

- *BTS*: This algorithm is run on the binary constraints model of the problem obtained as described in Section 2.2. In principle any algorithm for minimizing the number of constraint violations in a fixed spectrum frequency assignment problem could be used. Here tabu search [17, 18] is used; specifically the efficient algorithm for frequency assignment described in [19], with cost function E_{viol}.

- *LS1*: Repeatedly a transmitter is selected randomly and it is reassigned to a random frequency chosen from its frequency domain. If this exchange produces a reduction in the cost E_{SIR} the modified solution is accepted, otherwise the move is rejected and the old solution is restored.
- *LS2*: The transmitters are repeatedly scanned and each transmitter is selected for reassignment with a certain probability ω. The selected transmitters are deassigned and randomly ordered. Following this order, the transmitters are assigned to the frequency that produces the smallest value in the following formula, where T_C is the set of transmitters assigned in the solution under construction, and $I_{rt}(T_C)$ is the value returned by formula (2) when T is restricted to T_C:

$$\sum_{r\in R}\sum_{u\in W(r)\cap\{T_C\cup\{t\}\}}\frac{I_{ru}(T_C\cup\{t\})}{S_{ru}}-\sum_{r\in R}\sum_{u\in W(r)\cap T_C}\frac{I_{ru}(T_C)}{S_{ru}}. \quad (6)$$

Formula (6) measures the increase of interference due to the assignment of a frequency to transmitter t. The first term of (6) counts, for each reception point r and for each transmitter u serving r, the ratio between the interference on r due to the already assigned transmitters plus transmitter t and the signal strength from u to r. The second term calculates the same quantity before the assignment of transmitter t.
A solution is accepted only if there is an improvement, otherwise the old solution is retained.

Let T_{BTS}, T_{LS1} and T_{LS2} denote the time dedicated to local searches BTS, $LS1$ and $LS2$ respectively. For each problem T_{BTS} should be chosen to ensure that a good solution of the binary constraints is obtained. For the results presented here $T_{LS2} = 9 \times T_{LS1}$ as LS2 is the slowest of the three searches.

Once the m ants of the colony have completed their computation, the best known solution is used to globally modify the pheromone trail. In this way a "difficulty level" for transmitters is memorized in the pheromone trail matrix and future ants will exploit this information to generate new solutions which should be better. The pheromone matrix is updated as follows:

$$\tau_j := \tau_j \times \frac{I_j}{E_{\mathrm{best}}} \times |T| \quad (7)$$

where E_{best} is the total cost of the best solution found so far, and I_j is the cost involving transmitter j within this solution. Using (7) a transmitter which strongly contributes to the total cost will have the pheromone trail strongly reinforced, while a transmitter with a very marginal contribution to the cost will have the pheromone trail reduced.

Pseudo-code of the *ANTS* procedure is presented in Fig. 3.

4 Benchmark problems

In this section the six benchmark problems used are described. They include a variety of satellite and terrestrial problems. One has cosite constraints typical of a mobile telephone problem. The number of carriers in each satellite beam (or transmitters at a site in a terrestrial problem) must be specified. The values are typical of real problems, but were generated in an arbitrary way. To avoid the need to specify transmitter powers individually for many thousands of transmitters, they are assumed equal in each problem. As this constant transmitter power cancels in Eq. (3), the actual power need not be specified. The frequency domains used include consecutive channels, a single frequency domain with a gap, and multiple frequency domains. In the case of multiple frequency domains simple rules are used to associate domains with transmitters. This allows the compact presentation of reproducible benchmark problems which are similar to real problems. Problems using hexagonal cells are reproducible from the information given here. Transmitter co-ordinates for the problems with 95 and 458 transmitters can be obtained from the corresponding author.

In these problems the off-tune rejection factors can be compactly defined by the formula given by Wang and Rappaport [20]:

$$\theta_{j,\,t,\,|f_j-f_t|} = \theta_{|f_j-f_t|} = \begin{cases} 1 & \text{if } f_j = f_t \\ 10^{\frac{-\alpha(1+\log_2|f(j)-f(t)|)}{10}} & \text{if } f_j \neq f_t \end{cases} \quad (8)$$

where α is an attenuation factor for adjacent channel interference, expressed in dB/octave. The value of α is specified for each benchmark problem and falls within the range of values used in the literature.

4.1 A satellite problem HEX358

In the problem HEX358 the region served is covered by 61 slightly overlapping circular satellite beams. These can be represented by regular hexagons as shown in Fig. 4. The use of hexagonal cells is an accurate representation of many satellite assignment problems.

The co-ordinates of the cell centres are given by $x = 1000 \times (j-1) + 500 \times i$, $y = 500\sqrt{3} \times i$, with $i, j \in \{1, 2, \ldots, 9\}$, $5 < i + j < 15$. They are generated sequentially in the order given by the sequence $(1, 5), (1, 6), \ldots, (1, 9), (2, 4), \ldots, (9, 5)$ for (i, j). The set of receiver test points R corresponds to the vertices of the regular hexagons. Each vertex may correspond to one, two or three receiver test points with different serving transmitters at the centre of an adjacent cell. Each beam has a *demand* value representing the number of carriers (transmitters) in

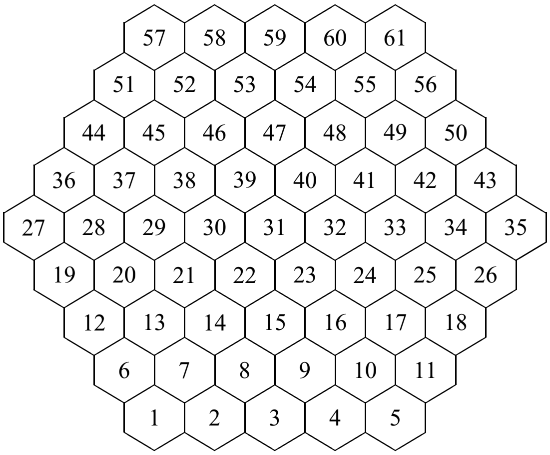

Fig. 4 The cellular geometry of HEX358, with 61 satellite beams

the beam. For HEX358 these are given by the vector:

$$(6, 8, 7, 7, 5, 4, 8, 8, 7, 5, 5, 5, 6, 4, 4, 4, 6, 8, 7, 7, 5, 4, 8,\\ 8, 7, 5, 5, 5, 6, 4, 4, 3, 3, 5, 5, 7, 6, 8, 7, 7, 5, 4, 8, 8, 7, 5,\\ 5, 5, 6, 4, 4, 4, 6, 8, 7, 7, 5, 4, 8, 8, 7),$$

where element i of the vector ($i \in \{1, 61\}$) is the demand for cell i. The received signal strengths P_i at the cell centres are considered equal and the received signal strength at distance d from the cell centre is given by $P_i \times (\frac{\sin cd}{cd})^2$, where $c = \frac{\sqrt{3}\times 1.391557378}{1000}$ so that the value at a receiver test point is half the value at the corresponding cell centre. The required SIR is 15 dB ($\sigma = 31.623$) and the value of α in Eq. (8) is 14. There are 3 frequency domains. Domain 0 is $\{0, 1, 2, \ldots 116\}$ with 117 frequencies. Domain 1 has 72 frequencies and is $\{45, 46, 47, \ldots 116\}$. Domain 2 also has 72 frequencies, and is $\{0, 1, 2, \ldots 71\}$. Number the carriers from 1 to 358, with the 6 carriers of cell 1 first, then the 8 carriers of cell 2, etc.. Then carrier i must be assigned a frequency from domain i mod 3.

4.2 A satellite problem HEX1794

In the problem HEX1794 the region served is covered by 310 slightly overlapping circular satellite beams, in a similar way to HEX358.

The co-ordinates of the 310 cell centres are given by $x = 1000 \times (j - 1) + 500 \times i$, $y = 500\sqrt{3} \times i$, with $i, j \in \{1, 2, \ldots, 20\}$, $10 < i + j < 32$. They are generated sequentially in the order given by the sequence $(1, 10), (1, 11), \ldots, (1, 20), (2, 9), \ldots, (20, 11)$ for (i, j). The set of receiver test points R corresponds to the vertices of the regular hexagons. For HEX1794 the demand values are given by the vector:

$$(6, 8, 7, 7, 5, 4, 8, 8, 7, 5, 5, 5, 6, 4, 4, 4, 6, 8, 7, 7, 5, 4, 8,\\ 8, 7, 5, 5, 5, 6, 4, 4, 3, 3, 5, 5, 7, 6, 8, 7, 7, 5, 4, 8, 8, 7, 5,\\ 5, 5, 6, 4, 4, 4, 6, 8, 7, 7, 5, 4, 8, 8, 7, 5, 5, 5, 6, 4, 4, 3, 3,\\ 5, 5, 8, 7, 7, 5, 4, 8, 8, 7, 5, 7, 6, 8, 7, 7, 5, 4, 8, 8, 7, 5, 5,\\ 5, 6, 4, 4, 4, 6, 8, 7, 7, 5, 4, 8, 8, 7, 5, 5, 5, 6, 4, 4, 3, 3, 5,\\ 5, 7, 6, 8, 7, 7, 5, 4, 8, 8, 7, 5, 5, 5, 6, 4, 4, 4, 6, 8, 7, 7, 5,\\ 4, 8, 8, 7, 5, 5, 5, 6, 4, 4, 3, 3, 5, 5, 8, 7, 7, 5, 4, 8, 8, 7, 5,\\ 7, 6, 8, 7, 7, 5, 4, 8, 8, 7, 5, 5, 5, 6, 4, 4, 4, 6, 8, 7, 7, 5, 4,\\ 8, 8, 7, 5, 5, 5, 6, 4, 4, 3, 3, 5, 5, 7, 6, 8, 7, 7, 5, 4, 8, 8, 7,\\ 5, 5, 5, 6, 4, 4, 4, 6, 8, 7, 7, 5, 4, 8, 8, 7, 5, 5, 5, 6, 4, 4, 3,\\ 3, 5, 5, 8, 7, 7, 5, 4, 8, 8, 7, 5, 7, 6, 8, 7, 7, 5, 4, 8, 8, 7, 5,\\ 5, 5, 6, 4, 4, 4, 6, 8, 7, 7, 5, 4, 8, 8, 7, 5, 5, 5, 6, 4, 4, 3, 3,\\ 5, 5, 7, 6, 8, 7, 7, 5, 4, 8, 8, 7, 5, 5, 5, 6, 4, 4, 4, 6, 8, 7, 7,\\ 5, 4, 8, 8, 7, 5, 5, 5, 6, 4, 4).$$

The received signal strengths P_i at the cell centres are again considered equal and the received signal strength at distance d from the cell centre is the same as for HEX358. The required SIR is again 15 dB and the value of α in Eq. (8) is 14. There are 3 frequency domains. Domain 0 is $\{0, 1, 2, \ldots 256\}$ with 257 frequencies. Domain 1 has 212 frequencies and is $\{45, 46, 47, \ldots 256\}$. Domain 2 also has 212 frequencies, and is $\{0, 1, 2, \ldots 211\}$. Number the carriers from 1 to 1794, with the 6 carriers of cell 1 first, then the 8 carriers of cell 2 etc.. If $i \le 1000$ then carrier i must be assigned a frequency from domain i mod 3. If $i > 1000$ then carrier i must be assigned a frequency from domain 0.

4.3 A terrestrial area coverage problem with 95 transmitters

This problem is a non-cellular area coverage problem (so all demands are 1) and has been used in [7, 16]. The transmitter locations are of uneven density appropriate to an urban area surrounded by countryside. There are 541 reception points defined, which are the corner points of the Voronoi polygons defined by the transmitters. The powers of the transmitters are assumed all to equal a constant P and the received signal strength at distance d from a transmitter is taken to be $P \times d^{-4}$. Here the required SIR is 14 dB ($\sigma = 25.11886$) and the value of α in Eq. (8) is 15. There is a single frequency domain consisting of 24 consecutive channels.

4.4 A terrestrial area coverage problem with 458 transmitters

This problem is a non-cellular area coverage problem (so all demands are 1) and has also been used in [7, 16]. The transmitter locations are again of uneven density appropriate to an urban area surrounded by countryside. There are 2675 reception points defined, which are the corner points of the Voronoi polygons defined by the transmitters. There are 3 frequency domains. Domain 1 has 18 frequencies and is $\{0, 1, 2 \ldots 17\}$. Domain 2 also has 18 frequencies, and is $\{5, 6, 7, \ldots 22\}$. Domain 3 also has 18 frequencies, and is $\{10, 11, 12, \ldots 27\}$. Then carriers $1, 2, 3 \ldots, 100$ and $301, 302 \ldots, 400$ must be assigned a frequency from domain 1, carriers $101, 102, 103 \ldots, 200$ and $401, 402 \ldots, 458$ must be assigned a frequency from domain 2 and carriers $201, 202, 203 \ldots, 300$ must be assigned a frequency from domain 3. Other characteristics are the same as for the 95 transmitter problem.

4.5 A terrestrial mobile telephone problem HEX1225

In the terrestrial problem HEX1225 the region served is covered by 310 hexagonal cells, with an identical geometry to HEX1794 except that the cell centres correspond to terrestrial transmitter locations. The demand values in this problem represent the number of transmitters at the location and are given by the vector:

(2, 5, 4, 5, 5, 4, 3, 2, 3, 5, 5, 5, 2, 4, 4, 4, 2, 3, 5, 4, 5, 4, 3,
3, 3, 5, 5, 5, 2, 4, 4, 3, 3, 5, 5, 2, 4, 3, 5, 3, 5, 4, 2, 4, 3, 5,
5, 5, 3, 4, 4, 4, 3, 5, 5, 3, 5, 4, 4, 3, 4, 5, 5, 5, 2, 4, 4, 3, 3,
5, 5, 3, 2, 5, 5, 4, 5, 4, 3, 5, 2, 3, 4, 5, 5, 5, 4, 4, 3, 3, 5, 5,
5, 3, 4, 4, 4, 5, 4, 5, 5, 5, 4, 4, 3, 2, 5, 5, 5, 3, 4, 4, 3, 3, 5,
5, 4, 3, 3, 4, 5, 5, 4, 5, 4, 4, 5, 5, 5, 3, 4, 4, 4, 5, 4, 3, 4, 4,
4, 4, 5, 2, 5, 5, 5, 2, 4, 4, 3, 3, 5, 5, 4, 5, 4, 5, 4, 3, 3, 4, 5,
5, 3, 4, 4, 3, 5, 4, 3, 4, 4, 5, 5, 5, 2, 4, 4, 4, 5, 3, 2, 2, 5, 4,
5, 3, 4, 5, 5, 5, 3, 4, 4, 3, 3, 5, 5, 4, 4, 3, 4, 4, 5, 4, 3, 3, 4,
5, 5, 5, 2, 4, 4, 4, 4, 3, 3, 3, 5, 4, 3, 2, 3, 5, 5, 5, 2, 4, 4, 3,
3, 5, 5, 3, 4, 4, 5, 4, 3, 3, 4, 5, 5, 4, 4, 3, 3, 5, 4, 4, 3, 2, 5,
5, 5, 2, 4, 4, 4, 2, 3, 3, 4, 5, 4, 4, 4, 3, 5, 5, 5, 2, 4, 4, 3, 3,
5, 5, 4, 5, 3, 4, 4, 3, 4, 2, 3, 4, 5, 5, 5, 3, 4, 4, 4, 3, 5, 4, 4,
5, 4, 3, 3, 4, 5, 5, 5, 2, 4, 4).

The frequencies assigned to each pair i, j of transmitters at the same location satisfy a *cosite constraint* $|f_i - f_j| \geq 3$ which is *hard* (i.e. must be satisfied). Again the set of receiver test points R corresponds to the vertices of the regular hexagons and each vertex may correspond to one, two or three receiver test points with different serving transmitters. The transmitter powers P_i are considered equal and the received signal strength at distance d from the cell centre is given by $P_i \times d^{-4}$. The required SIR is 12 dB ($\sigma = 15.8489$) and the value of α in Eq. (8) is 15. The frequency domain consists of 35 consecutive channels.

4.6 A terrestrial cellular problem HEX3710

The terrestrial problem HEX3710 has 3710 transmitters and is the largest problem studied here. It is similar to HEX1225 but differs in that it has a more extensive cellular structure, and all demands are 1 so there are no hard constraints. Specifically, the transmitter locations are given by $x = 1000 \times (j - 1) + 500 \times i,\ y = 500\sqrt{3} \times i$, with $i, j \in \{1, 2, \ldots, 20\}$, $35 < i + j < 107$. There is a single frequency domain $\{0,1,2,3,4,5,8,9,10\}$ with a single gap and 9 frequencies for all transmitters. Other characteristics are the same as for HEX1225.

5 Results

Results will now be given for the six benchmark problems. The software was compiled using Microsoft Visual C++ 6.0 and run on a 2.67 GHz Xeon processor with 4G Bytes of memory. Run times are reported in hours rather than number of iterations to allow a fair comparison of SA and ANTS, given the very different meaning of "iterations" for the two algorithms. The main sets of results given in Tables 1–6 demonstrate the advantages of using binary constraints in combination with the SIR based cost function in terms of solution quality. For simulated annealing $\beta = 0.999$ for all results and $\mathcal{T}_0$ is given in the column of the tables headed "Start temperature". The value of $\mathcal{T}_{\min}$ is adjusted for the target run time (which is always below 24 hours). For these results $\eta = 2000$ for HEX358, the 95 transmitter problem and the 458 transmitter problem, $\eta = 700$ for HEX1794, HEX 1225 and $\eta = 300$ for HEX3710. For the ANTS algorithm $\tau_{\text{init}} = 1.0$, $\rho = 0.9$, $\omega = 0.2$ and the number m of ants is 3. $T_{LS1} + T_{LS2}$ is 900s for HEX358, the 95 transmitter problem and the 458 transmitter problem, 9000s for the HEX1794 and 24000s for HEX3710. For the problem HEX1225 the cosite frequency separation constraints are hard and must be satisfied. Thus the second option for simulated annealing (minimize (3) only) might not give a valid solution. The correct comparison to illustrate the benefit of constraints computed as in Section 2.2 can still be made with our implementation. It involves the application of the following methods to HEX1225:

Table 1 Results for the HEX358 problem with a SIR threshold of $\sigma = 31.623$ (15 dB). $\alpha = 14$ and up to 117 frequencies are available, but some frequencies are blocked for certain transmitters

Method	Binary constraints used?	SIR used?	Threshold $\hat{\sigma}$ when generating constraints (in dB).	Start temperature	Constraint violations	Cost	Run time (hours)
SA constraints	Y	N	15	1.0	0	84797.09	0.007
SA constraints	Y	N	16	1.0	0	23833.22	0.01
SA constraints	Y	N	17	1.0	0	23833.22	0.01
SA constraints	Y	N	18	1.0	0	2723.63	0.11
SA constraints	Y	N	19	1.0	88	742426.88	0.94
SA SIR random start	N	Y	–	1.0	–	4009.22	12.57
SA SIR hybrid	Y	Y	18	1.0	–	3109.29	12.64
SA SIR constraint start	Y	Y	15	0.001	–	6490.46	2.84
SA SIR constraint start	Y	Y	16	0.001	–	6014.93	6.84
SA SIR constraint start	Y	Y	17	0.001	–	4129.34	9.34
SA SIR constraint start	Y	Y	18	0.0001	–	**905.56**	8.43
SA SIR constraint start	Y	Y	19	0.0001	–	29487.70	8.56
ANTS no constraints	N	Y	–	–	–	22718.05	8.79
ANTS with constraints	Y	Y	15	–	–	13105.03	14.33
ANTS with constraints	Y	Y	16	–	–	8066.51	8.55
ANTS with constraints	Y	Y	17	–	–	6636.33	18.59
ANTS with constraints	Y	Y	18	–	–	**844.23**	16.26
ANTS with constraints	Y	Y	19	–	–	31726.02	15.66

Table 2 Results for the HEX1794 problem with a SIR threshold of $\sigma = 31.623$ (15 dB). $\alpha = 14$ and up to 257 frequencies are available, but some frequencies are blocked for certain transmitters

Method	Binary constraints used?	SIR used?	Threshold $\hat{\sigma}$ when generating constraints (in dB).	Start temperature	Constraint violations	Cost	Run time (hours)
SA constraints	Y	N	15	1.0	0	287445.97	0.01
SA constraints	Y	N	16	1.0	0	161603.89	0.01
SA constraints	Y	N	17	1.0	0	161603.89	0.01
SA constraints	Y	N	18	1.0	0	97248.54	0.02
SA constraints	Y	N	19	1.0	0	19345.59	1.63
SA constraints	Y	N	20	1.0	0	15724.39	1.55
SA constraints	Y	N	21	1.0	202	1577874.5	22.70
SA SIR random start	N	Y	–	1.0	–	2982.14	18.33
SA SIR hybrid	Y	Y	19	1.0	–	4361.56	15.91
SA SIR hybrid	Y	Y	20	1.0	–	5557.26	22.80
SA SIR constraint start	Y	Y	15	0.001	–	2437.00	22.83
SA SIR constraint start	Y	Y	16	0.001	–	2454.00	16.51
SA SIR constraint start	Y	Y	17	0.001	–	2454.00	13.44
SA SIR constraint start	Y	Y	18	0.001	–	2781.71	23.06
SA SIR constraint start	Y	Y	19	0.0001	–	1080.20	12.68
SA SIR constraint start	Y	Y	20	0.0001	–	**358.93**	18.17
ANTS no constraints	N	Y	–	–	–	9289.06	15.70
ANTS with constraints	Y	Y	15	–	–	10112.85	15.86
ANTS with constraints	Y	Y	16	–	–	9474.40	16.02
ANTS with constraints	Y	Y	17	–	–	9972.81	13.31
ANTS with constraints	Y	Y	18	–	–	11479.71	13.26
ANTS with constraints	Y	Y	19	–	–	4535.09	15.59
ANTS with constraints	Y	Y	20	–	–	**2673.25**	23.64

Table 3 Results for the 95 transmitter problem with a SIR threshold of $\sigma = 25.11886$ (14 dB). $\alpha = 15$ and 24 consecutive frequencies are available

Method	Binary constraints used?	SIR used?	Threshold $\hat{\sigma}$ when generating constraints (in dB).	Start temperature	Constraint violations	Cost	Run time (hours)
SA constraints	Y	N	14	1.0	0	9163.62	0.0003
SA constraints	Y	N	15	1.0	0	3202.80	0.0003
SA constraints	Y	N	16	1.0	0	1401.17	0.003
SA constraints	Y	N	17	1.0	4	14885.94	0.05
SA constraints	Y	N	18	1.0	9	30873.51	0.042
SA constraints	Y	N	19	1.0	12	44875.44	0.096
SA SIR random start	N	Y	–	1.0	–	147.35	1.14
SA SIR hybrid	Y	Y	16	1.0	–	**83.58**	0.17
SA SIR hybrid	Y	Y	17	1.0	–	204.66	1.23
SA SIR constraint start	Y	Y	14	0.001	–	133.08	0.21
SA SIR constraint start	Y	Y	15	0.001	–	189.55	0.28
SA SIR constraint start	Y	Y	16	0.001	–	**100.11**	0.41
SA SIR constraint start	Y	Y	17	0.001	–	255.68	0.46
SA SIR constraint start	Y	Y	18	0.001	–	161.20	0.43
SA SIR constraint start	Y	Y	19	0.001	–	348.65	0.44
ANTS no constraints	N	Y	–	–	–	113.53	0.50
ANTS with constraints	Y	Y	14	–	–	127.27	2.25
ANTS with constraints	Y	Y	15	–	–	189.99	0.50
ANTS with constraints	Y	Y	16	–	–	**79.61**	0.60
ANTS with constraints	Y	Y	17	–	–	183.07	1.50
ANTS with constraints	Y	Y	18	–	–	132.15	0.60
ANTS with constraints	Y	Y	19	–	–	113.04	3.30

Table 4 Results for the 458 transmitter problem with a SIR threshold of $\sigma = 25.11886$ (14 dB). $\alpha = 15$ and up to 28 frequencies are available, but some frequencies are blocked for certain transmitters

Method	Binary constraints used?	SIR used?	Threshold $\hat{\sigma}$ when generating constraints (in dB).	Start temperature	Constraint violations	Cost	Run time (hours)
SA constraints	Y	N	15	1.0	0	14983.24	0.02
SA constraints	Y	N	16	1.0	0	9761.59	0.02
SA constraints	Y	N	17	1.0	0	4885.82	0.02
SA constraints	Y	N	18	1.0	0	3664.27	1.00
SA constraints	Y	N	19	1.0	10	50452.50	0.10
SA constraints	Y	N	20	1.0	33	140974.77	0.10
SA SIR random start	N	Y	–	1.0	–	235.00	17.86
SA SIR hybrid	Y	Y	17	1.0	–	75.01	11.10
SA SIR constraint start	Y	Y	15	0.001	–	96.27	20.02
SA SIR constraint start	Y	Y	16	0.001	–	86.43	18.02
SA SIR constraint start	Y	Y	17	0.001	–	112.71	16.29
SA SIR constraint start	Y	Y	18	0.001	–	63.07	16.40
SA SIR constraint start	Y	Y	19	0.0001	–	542.17	15.08
SA SIR constraint start	Y	Y	20	0.001	–	**57.24**	17.03
ANTS no constraints	N	Y	–	–	–	3183.17	20.03
ANTS with constraints	Y	Y	15	–	–	3065.93	20.50
ANTS with constraints	Y	Y	16	–	–	2202.20	20.02
ANTS with constraints	Y	Y	17	–	–	1786.12	20.30
ANTS with constraints	Y	Y	18	–	–	1022.45	21.39
ANTS with constraints	Y	Y	19	–	–	**540.29**	23.70
ANTS with constraints	Y	Y	20	–	–	582.35	23.03

Table 5 Results for the HEX1225 problem with a SIR threshold of $\sigma = 15.8489$ (12 dB). $\alpha = 15$ and 35 consecutive frequencies are available

Method	Binary constraints used?	SIR used?	Threshold $\hat{\sigma}$ when generating constraints (in dB).	Start temperature	Constraint violations	Cost	Run time (hours)
SA constraints	Y	N	12 + hard	1.0	0	41625.18	0.10
SA constraints	Y	N	13 + hard	1.0	0	9834.04	0.12
SA constraints	Y	N	14 + hard	1.0	0	14358.32	0.79
SA constraints	Y	N	15 + hard	1.0	63	166095.75	0.78
SA constraints	Y	N	16 + hard	1.0	41	110594.66	21.82
SA constraints	Y	N	17 + hard	1.0	178	451722.94	24.00
SA SIR hybrid	Y	Y	hard only	1.0	–	104.08	7.40
SA SIR hybrid	Y	Y	14 + hard	1.0	–	172.45	12.13
SA SIR constraint start	Y	Y	12 + hard	1.0	–	130.22	12.53
SA SIR constraint start	Y	Y	13 + hard	1.0	–	91.32	20.10
SA SIR constraint start	Y	Y	14 + hard	1.0	–	**47.84**	11.80
SA SIR constraint start	Y	Y	15 + hard	1.0	–	66.62	12.10
SA SIR constraint start	Y	Y	16 + hard	1.0	–	225.14	12.25
SA SIR constraint start	Y	Y	17 + hard	1.0	–	197.68	9.62

Table 6 Results for the 3710 transmitter problem with a SIR threshold of $\sigma = 25.11886$ (14 dB). $\alpha = 15$ and 9 frequencies are available

Method	Binary constraints used?	SIR used?	Threshold $\hat{\sigma}$ when generating constraints (in dB).	Start temperature	Constraint violations	Cost	Run time (hours)
SA constraints	Y	N	12	1.0	0	119743.80	0.06
SA constraints	Y	N	13	1.0	0	19797.30	0.10
SA constraints	Y	N	14	1.0	0	19797.30	0.11
SA constraints	Y	N	15	1.0	71	36869.10	0.65
SA constraints	Y	N	16	1.0	71	36868.87	0.68
SA constraints	Y	N	17	1.0	966	548752.88	0.75
SA SIR random start	N	Y	–	1.0	–	22.73	23.32
SA SIR hybrid	Y	Y	14	1.0	–	247.98	23.78
SA SIR hybrid	Y	Y	15	1.0	–	1339.86	24.00
SA SIR constraint start	Y	Y	13	0.0001	–	3.74	16.84
SA SIR constraint start	Y	Y	14	0.0001	–	5.11	17.20
SA SIR constraint start	Y	Y	15	0.0001	–	4.00	16.59
SA SIR constraint start	Y	Y	16	0.0001	–	**2.40**	16.61
SA SIR constraint start	Y	Y	17	0.0001	–	4.22	16.63
ANTS no constraints	N	Y	–	–	–	459.18	20.22
ANTS with constraints	Y	Y	12	–	–	653.60	13.60
ANTS with constraints	Y	Y	13	–	–	361.09	20.29
ANTS with constraints	Y	Y	14	–	–	336.88	13.55
ANTS with constraints	Y	Y	15	–	–	184.87	20.22
ANTS with constraints	Y	Y	16	–	–	**4.01**	7.04
ANTS with constraints	Y	Y	17	–	–	964.17	13.81

1. Simulated annealing solution of both hard cosite frequency separation constraints and constraints computed as in Section 2.2, with $E_{\text{viol(hard)}}$ and $E_{\text{viol(2.2)}}$ denoting the respective numbers of violations.
2. Hybrid simulated annealing algorithm with hard cosite frequency separation constraints only from a random start. The cost is $E_{\text{SIR}} + 1000E_{\text{viol(hard)}}$.
3. Hybrid simulated annealing algorithm with both hard cosite frequency separation constraints and constraints computed as in Section 2.2, from a random start. The cost is $E_{\text{SIR}} + 1000E_{\text{viol(hard)}} + E_{\text{viol(2.2)}}$.
4. Hybrid simulated annealing algorithm with hard cosite frequency separation constraints only from a constraint solution start as in the first case above. The cost is $E_{\text{SIR}} + 1000E_{\text{viol(hard)}}$.

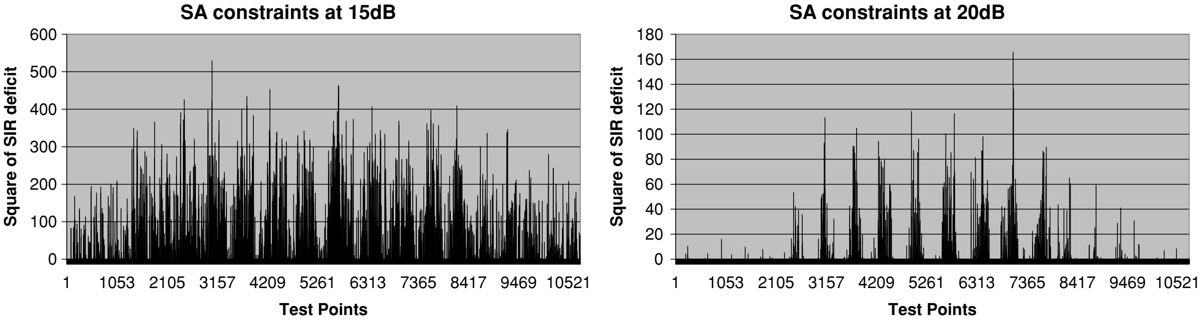

Fig. 5 Cost distribution for constraint solutions to HEX1794

In the case of the hybrid algorithm the hard binary constraints are given a greatly increased weighting of 1000 and the other costs are given a weighting of 1. This differs from all other applications of the hybrid algorithm, and ensures that the hard constraints are satisfied once the cost is below 1000.

5.1 Discussion

It should be noted from Tables 1–6 that the SA constraints solutions are obtained very quickly, so runs for several values of $\hat{\sigma}$ can be performed. Longer run times only rarely improve the solutions with residual constraint violations. The costs obtained by solving constraints are very poor in comparison with the use of an SIR based cost function. This gives more extensive confirmation of the advantage of SIR based cost functions claimed in [3, 6].

In general the largest value of $\hat{\sigma}$ that leads to a zero constraint solution leads to the lowest cost of any of the constraint solutions (except for HEX1225 where it gives the second lowest cost). It is also true that these values of $\hat{\sigma}$ are the best ones to use in conjunction with SA SIR or ANTS. The only exceptions are the 458 transmitter problem and HEX3710, when this choice of $\hat{\sigma}$ still gives excellent solutions. The best starting temperature for SA SIR methods can be determined by some fast initial experiments (with the SA temperature reducing quickly).

The meaning of the cost values can be interpreted in Figs. 5–7 where the cost contribution of each reception point is shown. It therefore represents, for each reception point, the square of the SIR deficit. The problem illustrated is HEX1794 and the assignments are (i) the constraint solution at 15 dB, (ii) the constraint solution at 20 dB (iii) SA from a random start (iv) SA from the 20 dB constraint solution (v) the ANTS solution without using binary constraints (vi) the ANTS solution using binary constraints at 20 dB. Noting the very different vertical scales, the improved quality of the lowest cost solution is apparent. In the assignment obtained by SA SIR from a 20 dB constraint start, each frequency is reused several times. A fragment of the assignment of cost 358.93 showing the frequency reuse for the first 10 frequencies is shown in Table 7. Here the first row shows the six transmitters assigned the lowest numbered channel (frequency), the second row shows the six transmitters assigned the second lowest numbered channel etc.. Considering all frequencies in this example, each frequency is reused between 5 and 9 times.

The relative merit of the different methods is summarized by the ranking in Table 8. For five problems the SA constraint solutions are the worst obtained and are very much worse than the SA SIR solutions in terms of the SIR over the network. For the exceptional case of HEX358 the SA constraints solutions are again much worse than the best results obtained. It can also be noted from Table 8 that the best and second best methods always use both binary constraints and the SIR based cost function. This supports the main claim of this paper. SIR solutions from a constraint start are better than SIR solutions from a random start if the correct constraints are chosen. They

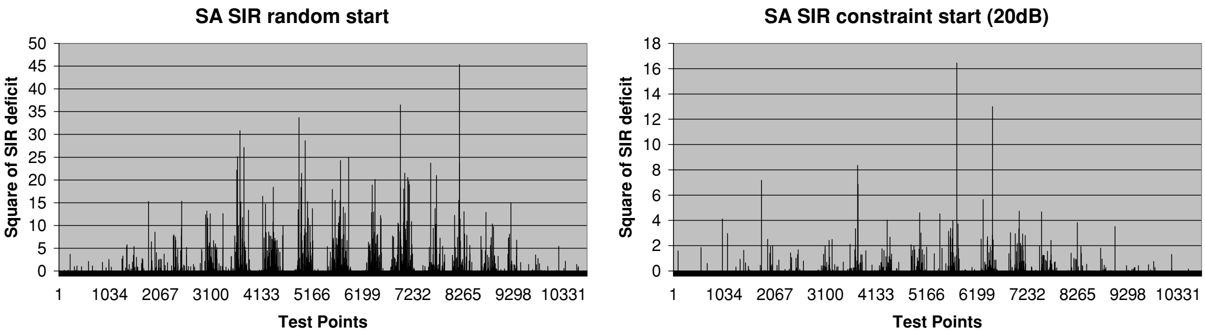

Fig. 6 Cost distribution for SIR solutions to HEX1794

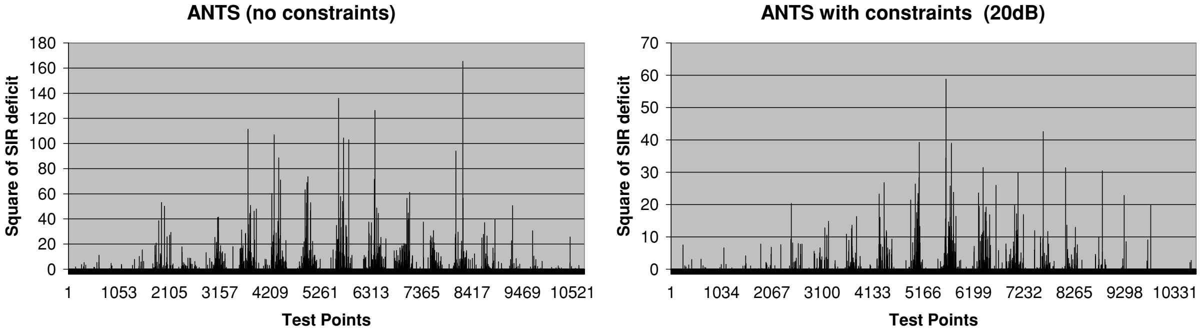

Fig. 7 Cost distribution for ANTS solutions to HEX1794

are also better than the hybrid method except for the rather small 95 transmitter problem where the best hybrid result is a little better. ANTS found the best assignments only for the two smaller problems, but again always gave its best results when the binary constraint search BTS was included.

Simulated annealing results for many further variations of some of the problems presented here can be found in [8]. They lead to the same general conclusions.

6 Problem complexity and run times

It has been known for many years (see for example [21]) that constraint based frequency assignment problems include graph colouring problems as a special case and so are NP-complete. Capone and Trubian [6] prove that the formulation described in Section 2.1 is also NP-complete by reducing the three-dimensional matching problem (known to be NP-complete) to this problem. This situation has never prevented the successful application of meta-heuristic and other algorithms to frequency assignment problems, although as noted previously run times are much longer with the SIR approach.

In Fig. 8 the reduction in $\log_{10}(\text{cost})$ with time is illustrated for the 95 transmitter problem. The behaviour for simulated annealing is typical of that usually observed for frequency assignment problems. The solution from a constraint solution starts with lower cost and converges more quickly than for the random start. The step-like behaviour of the ANTS algorithm arises because new complete iterations do not necessarily produce lower cost than the previous best. The run time recorded is generally the time taken to obtain the given cost, but the algorithm was left to run for longer to check that no significantly better assignment could be obtained with a mildly increased run time. One of the advantages of meta-heuristic algorithms for frequency assignment is that reasonably good assignments are available from an early stage in the run.

Some faster runs were performed to demonstrate that reasonable solutions can be obtained fairly quickly when the combination of binary constraints and the cost function (3) is used. For each problem the constraint start SA approach and the best ANTS approach was pursued. The results are presented in Table 9. The column marked "Iterations" contains η for SA and $T_{LS1} + T_{LS2}$ for ANTS. Run times are considerably reduced, but the quality of the assignments obtained is still remarkably better than the quality of the traditional constraint solutions.

Table 7 Frequency reuse for the first 10 frequencies of the cost 358.93 assignment of HEX1794

Frequency	Transmitters							
0	275	318	881	959	1037	1570	1605	
1	90	495	639	1166	1501	1520	1718	
2	309	353	668	839	1339	1387	1662	
3	411	873	923	968	1458	1579		
4	116	470	722	866	1025	1562	1604	
5	95	635	707	1205	1251	1709	1754	
6	441	512	674	1084	1237	1568		
7	137	234	659	848	1125	1499	1632	1673
8	212	252	461	911	1060	1202	1517	1784
9	32	501	641	891	1133	1272	1316	

Table 8 Ranking of different method types according to the best cost obtained

Method type	Binary constraints used?	SIR used?	HEX 358	HEX 1794	95 trans.	458 trans.	HEX 1225	HEX 3710
SA constraints	Y	N	3	6	6	6	4	6
SA random start	N	Y	5	3	5	3	–	3
SA SIR hybrid	Y	Y	4	4	2	2	2,3	4
SA SIR constraint start	Y	Y	2	1	3	1	1	1
ANTS no constraints	N	Y	6	5	4	5	–	5
ANTS with constraints	Y	Y	1	2	1	4	–	2

Table 9 Results of faster runs for all six benchmark problems

Problem	Method	Threshold when generating constraints (in dB).	Start temperature	Iteration	Cost	Run time (hours)
HEX358	SA SIR constraint start	18	0.0001	2000	1088.97	0.07
HEX358	ANTS with constraints	18	–	2500	1167.41	1.40
HEX1794	SA SIR constraint start	20	0.0001	70	1147.87	2.12
HEX1794	ANTS with constraints	20	–	4400	3665.01	3.87
95	SA SIR constraint start	16	0.001	2000	103.60	0.25
95	ANTS with constraints	16	–	900	168.67	0.50
458	SA SIR constraint start	18	0.0004	700	99.63	1.73
458	ANTS with constraints	19	–	900	822.68	1.20
HEX1225	SA SIR hybrid (constraint start)	hard, 14 + hard	0.00000019	500	389.49	0.54
HEX3710	SA SIR constraint start	16	0.0001	60	36.89	3.22
HEX3710	ANTS with constraints	16	–	5000	17.09	3.22

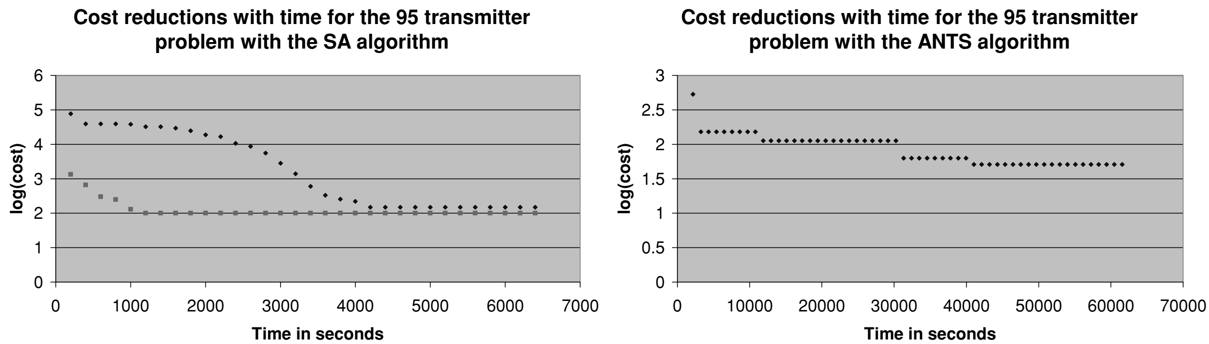

Fig. 8 Logarithmic reduction of cost with time for the 95 transmitter problem. For the simulated annealing algorithm the upper data series represent a random start and the lower series represents a start from a 16 dB constraint solution

7 Conclusion

The results demonstrate the merit of combining the binary constraint approach with the SIR approach for two quite different algorithms. No lower bounds are available to evaluate the SIR model used for fixed spectrum problems. However, even when constraint solutions meet a known lower bound they give a much worse distribution of SIR over the network than the SIR results presented here.

For simulated annealing the best approach is to start from a good binary constraint solution and improve it with the simulated annealing SIR algorithm. ANTS sometimes gives the best results, but its performance seems more variable than simulated annealing. The approach presented mitigates the extra computational resources required for SIR based methods. Even a very short improvement phase of a constraint solution by simulated annealing from a low starting temperature (or hillclimbing) will give very significant improvements in assignment quality.

Many alternative frequency assignment algorithms in the literature could be modified in similar ways to combine binary constraints and the SIR cost function. The reproducible benchmarks presented here will allow these algorithms to be evaluated in the SIR context.

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