

PARTICLE SWARMS AND THE FREQUENCY
ASSIGNMENT PROBLEM

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Part I

Background

Chapter 1

Introduction

1.1 Test

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Chapter 2

Cellular Technology

2.1 Introduction

In the information age we currently live in almost every device has some sort of wireless technology it uses to provide a specific service. Radios for audio entertainment; Television remotes to change channels; Cellular phones for communication; Wireless access points to create wireless LAN's[2]. Wireless technology is now part of our everyday life.

The popularity and rapid adoption of wireless technology hasn't been kind to the management, planning and operation of wireless networks; It has actually worsen a problem known as the Frequency Assignment problem (FAP) which is present in all forms of wireless communication especially in GSM Cellular networks.

In this chapter we will start of by giving a brief history of the Frequency Assignment Problem. After the brief history overview we will give a description of the problem and explain some of the underlying concepts needed to understand the Frequency Assignment Problem. We will then move on to discuss the different variants present in the current domain. Further more we

will then discuss the two underlying approaches used in solving the Frequency Assignment Problem (FAP).

2.2 GSM Networks

The General System for Mobile Communications (GSM) is a system for multi-service cellular communication which is capable of providing voice as well as data services. Most cellular networks in operation are GSM based. The primary service that GSM caters for is voice communication, but other data services such as Short Message Service (SMS), Multimedia Message Service (MMS) and Internet connectivity services such as GPRS are becoming more important[7].

GSM is one of the most widely used radio communications technologies, which is why we need to look at the history behind it in order for us to understand the domain of radio communication better. We will now present a brief history of the GSM network specification.

2.2.1 A Brief History of GSM Networks

In the early 1981 a group known as the Groupe Speciale Mobile (GSM) was established to develop a Europe wide radio communication system using the reserved 900 MHz band¹.

At the start of the GSM specification in the early 1980's it was initially thought that the system would be analogue based, but this soon changed with the *Integrated Service Digital Network* (ISDN) specification nearing com-

¹In 1990 the United Kingdom requested that 1800MHz band be added to the scope of the GSM standard group. This variant of the GSM specification became known as the *Digital Cellular System 1800* (DCS1800) [19].

pletetion. As such the GSM specification started following much of the same design principles and access protocols that ISDN exhibited. After the completion of the ISDN specification and the advantages it brought to the field, it became unofficially clear that GSM would be based on digital transmission and that speech would be represented by a digital stream of 16 kbits/s [19].

Before the switch to digital transmission was finalized the GSM first wanted to evaluate the spectral efficiency of analogue and digital based transmission. Spectral efficiency plays an important part in wireless communication since the radio spectrum is a limited resource and whichever transmission technology is used, should maximise the utilization of the spectrum. Maximum utilisation is an important problem which we will discuss in detail in later sections of this chapter. The Spectral evaluation was conducted over a period of 3 years from 1984-1987. In 1987 a report was published about the 3 year evaluation and subsequently it became official that the GSM system would be digital based using *Time Division Multiple Access* (TDMA) [19, 18].

By the early 1990s GSM became an evolving standard and the first GSM based network was demonstrated in 1991². The following year a number of GSM networks were operating in Europe due to mobile terminals / equipment capable of operating on the networks becoming more widely available to the general public. In the same year an operator in Australia became the first non-European operator to implement a GSM based network[7].

The collective subscriber base of GSM networks surpassed the million subscriber mark in 1993. Due to this phenomenal growth in GSM network use, numerous extensions were made to the GSM specification. Some of the

²Near the end of 1991 the GSM group was renamed to *Speciale Mobile Group* (SMG) to eliminate confusion with the standard and the group.

extensions that were made are the following[7]:

- Half rate speech telephony
- Improved SMS
- Line Identification
- Call waiting
- Call holding

The specification with these extensions defined is known as the GSM Phase 2. As the world shifted towards more digital and data intensive services it became difficult to deliver these services over GSM networks. This difficulty was due to the restriction that data could only be transmitted at 9.6 Kbps. A move to eliminate this restriction was made with the specification of GSM Phase 2+.

The new specification defined new technologies such as General Packet Radio System (GPRS) and *Enhanced Data rate for GSM Evolution* (EDGE) which were designed with the primary goal of making more bandwidth available for data transmission. These new technologies have an inherent requirement that there be a higher signal to noise ratio present at transceivers. This requirement has an impact that effects radio interfaces and more importantly Frequency planning [7].

The actual signal to noise ratio at a receiver is dependent on a number of factors that include [2]:

- Frequency used at the transceiver
- Strength of the signal
- Weather conditions

- Shape of the surrounding environment
- Direction of the transmission

Even taking these factors into account the calculation of the signal to noise ratio at a transceiver is not trivial. For a more in depth discussion on the calculation the reader is directed at the survey by [2].

As the GSM standard matured as a cellular technology, industry experts already began specification of the next generation of cellular networks which would in time, replace the GSM cellular system. The specification of a new standard is considered to be a natural evolution of the technology. Each standard is designed with specific use cases in mind as to what its users might want to do as well as what is possible with the technology at the designers disposal. As time goes by, the technology improves and users habits and needs change, thus the standard must be improved upon to serve these new needs and incorporate new technology.

The *Universal Mobile Telecommunications System* (UMTS) can be considered the 3rd generation (3G) of cellular networks. UMTS was designed from the beginning to operate in parallel with the legacy GSM system. This decision was made to make the deployment of the system as hassle free as possible for the network operators. The first standard of the UMTS was issued in the beginning of 2000 and subsequently most modern networks are based on it or are migrating their networks to it.

UMTS is a large improvement of the GSM in two areas namely Data Transmission bandwidth and Frequency Planning due to UMTS utilising *DS-CDMA* (direct sequence code division multiple access) and *WCDMA* (Wide Band Code Division Multiple Access). The higher data transmission speed (2 Mb/s) can be attributed to UMTS using the DS-CDMA scheme. The scheme also allows more users to be served than previous generation of networks. A

direct consequence of WCDMA which sends information over a wide-band of 5 MHz is that no frequency planning problem comparable to that of GSM has to be solved[26, 7].

In this section we gave a brief overview of the history of the GSM Network specification. In the next section we will present an explanation of the topology of GSM network as well as look at the underlying problems present in a GSM networks.

2.3 Topology of a GSM Network

GSM networks consists of a variety of different subsystems to realise the goal of establishing a radio communication link between two parties. The hierarchy of systems and their respective connections to each other is illustrated in figure 1. We will now briefly explain each subsystem.

Mobile Station (MS)

A Mobile station (MS) as it is defined in the GSM spec refers to any mobile device that is capable of making and receiving calls on a GSM network. The MS is the main gateway for a user to gain access to the GSM network. The MS has two features which play an important role throughout the GSM Network, namely:

Subscriber Identification Module (SIM) — Usually inserted into a mobile devices. The SIM contains the *International Mobile Subscriber Identity* (IMSI) and is used throughout the network for Authentication as well as being a key part in providing encrypted transmissions.

International Mobile Equipment Identity(IMEI) — Used to identify mobile station equipment. Primarily used in the denial of service to

equipment that has been blacklisted³ and tries to gain access to the network.

The MS has the capability to change the transmission power it uses from its base value to a maximum value of 20 mW. The change in transmission power is automatically set to the lowest level by the Base Transceiver Station to ensure reliable communication after evaluating the signal strength as measured by the MS [18].

2.3.1 Base Station Subsystem (BSS)

According to the GSM Phase 2+ specification this system is viewed by the *Mobile Switching Centre* (MSC) through an Abis radio interface as the system responsible for communicating with Mobile Stations in a particular location area. The BSS usually consists of one *Base Station Controller* (BSC) with one or more *Base Transceiver Stations* (BTS) which it controls. The communication link between the MSC and BSC is called the A-interface and the communication link between the BSC and BTS is called the Abis interface. The definition of these communication interfaces is beyond the scope of this dissertation, the interested reader is directed to the book *GSM System Engineering* by Asha Mehrotra. A BTS has similar equipment to that of a Mobile Station. Both have transceivers, antennas and the necessary functions to perform radio communication.

In a GSM network the Service Area (SA) is subdivided into Location Areas (LA's) which are then further divided into smaller radio zones called cells [23]. A cell is served by only one BTS and is usually regarded to be in the center of a cell. Even though cells are modelled as being hexagons (See figure 3) the actual coverage area of a cell has no predefined regular shape.

³Equipment can be blacklisted for a variety of reasons e.g. theft

Futhermore a cell is divided into 1 to 3 service sectors and each sector is allocated an antenna/transceiver [18]. Depending on how many sectors are at a cell, the operating angle of the antennae needs to be adjusted accordingly to ensure 360 degree service. If there is only one sector an omni-directional antenna is used, otherwise the antennae operating angle are adjusted to $\frac{360^\circ}{n}$ where n is the amount of antennae [7].

Each sector operates one or more elementary transceivers called TRXs. The amount of TRXs per sector is determined by the expected peak traffic demand that the cell must be able to handle. Each TRX can handle 7 to 8 communication links or calls in parallel except the first TRX, which handles fewer calls than normal due to it being responsible for transmitting cell organisation and protocol information [7]. TRXs are able to handle 7-8 calls in parallel due to the use of *Frequency Division Multiplexing* (FDM) and *Time Division Multiplexing* (TDM) schemes. TRXs are assigned channels which enable them to provide conversion between digital traffic data on the network side and the radio communication between Mobile Stations and the GSM network. [16, 3]

2.3.2 Mobile Switching Centre (MSC)

The MSC is at the heart of cellular switching system and forms part of the *Network Switching Subsystem* (NSS). The MSC is responsible for the setting up, routing and supervision of calls between GSM subscribers. The MSC has interfaces on the one side to communicate with GSM subscribers (through the BSS) and on the other it has interfaces to communicate with external networks. The MSC interfaces with external networks to utilise their superior capability in data transmission as well as for the signalling and communication with other GSM entities [19].

The most basic functions that an MSC is responsible for in a network are the following [22]:

- Voice call initialization, routing, control and supervision between subscribers.
- Handover process between two cells.
- Location updating.
- MS authentication.
- SMS delivery.
- Charging and Accounting of services used by subscriber.
- Notification of other network elements.
- Administration input or output processing functions.

To achieve most of these functions the MSC has an integrated *Visitor Location Register (VLR)* database that stores call setup information for any MS that is currently registered for service with the MSC [19, 22].

The VLR retrieves this information from the *Home Location Register (HLR)* which contains all the registered GSM subscriber information for the network. This information enables the MSC to quickly retrieve the necessary information to setup a call between two entities [18, 23].

A requirement for being able to communicate with other network elements such as *Public Switching Telephone Networks (PSTN)* is the ability to multiplex and demultiplex signals to and from such network elements. This operation is a necessity, since the incoming or outgoing connection bitrate from the source entity might either be too low or too high for the receiving entity.

A typical scenario where this operation proves vital is when a mobile subscriber makes a call to a subscriber on a PSTN. The connection bit rate needs to be changed at the MSC from a wireless connection bitrate to a bitrate suitable for transmission over a PSTN.

2.3.3 Network Databases

The HLR, AUC (Authentication Center) and EIR (Equipment Identity Register) are the 3 'back-end' databases which stores and provides information for the rest of the GSM Network. We will now briefly discuss what the purpose of each database is and its core functions.

Home Location Register (HLR) The HLR is a database that permanently stores information pertaining to a given set of subscribers. The HLR needs to store a wide range of subscriber parameters because it is the reference source for anything GSM subscriber related in the network.

Subscriber parameters that are stored in the database include: Billing information, routing information, identification numbers, authentication parameters, subscribed services. The following information is also stored but the information is of a temporary nature and can change at anytime: Current VLR and MSC the subscriber is registered with; Whether the subscriber is roaming [18].

Authentication Center (AUC) The Authentication center is the entity in the GSM network that performs security functions and thus stores information that enables it to provide secure over the air communication. The information that is stored contains authentication information as well as keys that are used in ciphering of information.

During an authentication procedure no ciphering key is ever transmitted over the air, instead a challenge is issued to the mobile who needs to be authenticated. This challenge requires the mobile station to provide the correct *Signed Response*(SRES) with regard to the random number generated by the AUC. The random number and ciphering keys that are used change with each call that is made, thus an attacker would gain nothing by intercepting a key, since it will change with the next call [18].

Each mobile that is registered in the HLR database needs to be authenticated and each call that is instantiated needs to retrieve keys from the AUC to establish a secure communication link. The AUC is sometimes included with HLR to allow for fast communication between the two entities [18].

Equipment Identity Register (EIR) The EIR is a database that stores the IMEI numbers of all registered mobile equipment that has accessed the network. Only information about the mobile equipment is stored, nothing about the subscriber or call is stored in the database.

Typically there is only one EIR database per network and interfaces to the various HLR database contained in the network. The IMEI's are grouped into 3 categories: *White List*, *Black List* and the *Gray List*. The White list contains only the IMEI numbers of valid MS's; the Black List stores the IMEI numbers of equipment that has been reported stolen and the Gray List stores the IMEI numbers of equipment that has some fault (faulty software, wrong make of equipment).

2.3.4 GSM Network Management entities

In a GSM network most of the elements that form part of and make the network function are often distributed in a wide geographical area to provide

the best network coverage for the customer.

For a network to function properly and efficiently network engineers need to be kept up to date on the state of the network and be alerted if *any* problems occur. For this purpose there exists two systems in the GSM network architecture that allows for this functionality required by network engineers.

The one system is called the Operations and Management center which is responsible for centralized regional and local operational and maintenance activities. The other system called the Network Management System unlike the OMC provides global and centralized management for operations and maintenance of the network supported by the OMCs [18].

We will not discuss the OMC and NMC in a bit more detail where we'll define the most critical functions they perform.

Operational and Management Center The OMC is capable of communicating with GSM entities using two protocols namely SS7 and X.25. The SS7 protocol is usually used when the OMC is communicating within the GSM network over short and medium distances. The X.25 protocol is used for large external data transfers. All communication where the OMC is involved typically occurs over fixed line networks and/or leased lines. The OMC is usually used for day to day operation of a network [18].

The OMC has support for alarm handling. An alarm in a GSM network goes off whenever a predefined expected condition does occur. Engineers are able to define the severity of an alarm which defines who and what is further alerted when and if the alarm is escalated to a higher level [18].

To give one an idea of when and why an alarm goes off consider the following scenario: The MSC controls a set of BSS systems. Now for some reason a certain region experiences a power blackout. All the BSS affected

by the blackout switch over to reserve power if available. A first alarm is sounded to let the engineers / network know that the BSS is using reserve power. When the BSS reserve power is depleted the MSC sounds an alarm letting the network know that a specific BSS cannot be contacted.

Typically in the above scenario the severity of the first alarm will be of a medium priority. The second alarm is much more serious and its severity will be of a high priority.

The OMC is also capable of fault management in the GSM network. The OMC is able to activate, deactivate, remove and restore a service manually or automatically of network devices. Various tests can be run as well as diagnostic information can be retrieved on the network devices to detect any current or future defects [18].

Network Management Center The NMC is similar to the OMC but it is not restricted to only regional GSM entities as it is in charge of the all GSM entities in the network. The NMC provides traffic management for the global network and also monitors high priority alarms such as overloaded or failed network nodes. It is usually used in long term planning of a network, but it has the capability to perform certain OMC functions when an OMC is not staffed.

2.4 GSM Network problems

2.5 Summary

Chapter 3

The Frequency Assignment Problem

3.1 Introduction

The Frequency Assignment Problem (FAP)¹ is a generalisation of the graph-colouring problem and is subsequently an NP-hard problem. This is because one has a finite amount of frequencies which needs to be assigned to antennae/transceivers (TRX's) where the amount of transceivers to be assigned frequencies greatly out weigh the amount of available frequencies.

It is inevitable that a network will have interference and we can thus only minimise the amount of interference that might occur on the network - an optimisation problem. Using exact algorithms to find a solution is not practical since the time to find a solution will be polynomial. Generally Metaheuristic algorithms are used to find optimal solutions to NP-hard problems[16]. We will discuss Metaheuristic algorithms which are generally used to find

¹Also known as Automatic Frequency Planning (AFP) or Channel Assignment Problem (CAP)[16]

solutions for NP-hard problems in Chapter 4.

A contributing factor to the difficulty of the FAP is due to the scarcity of usable frequencies in the radio spectrum, which forces network operators to reuse their allocated/licensed frequencies in their respective networks. The scarcity of the usable frequencies in the spectrum can be attributed to the overuse of certain bands as well as large scale reuse of frequencies in networks. This has put strain on the spectrum and has complicated the management of networks significantly because interference is more likely to occur.

Frequency assignment is the last step in a long process of network setup. Before frequencies are assigned base stations need to be placed and need to be configured, which include azimuth and tilt of the antenna for optimal network coverage. After the base stations are configured they need to be allocated a certain amount of transmitters to achieve a target network capacity [6].

Frequency assignment is only a means to achieve the targeted network coverage as well as network capacity.

In this section we gave a brief introduction as to what the Frequency Assignment Problem is and why it occurs as well as why it is a problem. In the next section we will describe the types of Frequency Assignment in use today and we will also discuss some of the variants of FAP and also formally define which of the variants we will concentrate on.

3.2 Frequency Assignment Types

In this section we will discuss the different methods that exist to allocate frequencies to cells in a cellular network. We will also state which method we will use through out this paper.

Within the FAP domain there exists different types of the FAP which

have emerged over the years as the domain and requirements have changed. We will discuss these FAP variants in section 3.4.

There are a variety of FAP in the domain (which will be discussed in later sections of this chapter) but most of these problems can be classified into two categories based on the assignment scheme they use:

- (a) *Fixed Frequency/Channel Assignment* (FFA/FCA) is the process of permanently assigning frequencies to cells (cellular towers). The frequencies assigned are fixed and cannot be changed on the fly while the network is active, since the frequencies assigned to the cell form part of a delicate frequency plan designed to keep interference to minimum.
- (b) *Dynamic Frequency/Channel Assignment* (DFA/DCA) is the process of allocating channels to cells as they require it to meet the current traffic demand imposed on them by clients.

Each cell can be assigned multiple frequencies based on the amount of transmitters or TRX's it has. The amount of TRX's in a cell depends on the expected amount of traffic the particular cell must handle.

Most of the research in the FAP has concentrated on the FFA. The reason for this is because FFA is a static technique, which allows it to come up with a better solution since it has more time for calculation. FFA is also easier to implement in practise and allows the network operators to cater for the worst case scenario - heavy traffic load on the network.

The DFA is at the moment a very hard problem because the network frequency plan is constantly changing, which means as the traffic on the network increases the longer the DFA focused algorithm will take to allocate a frequency. This increase in processing time is because the algorithm has to take into account more constraints with a lower available frequency pool.

DFA must do this process within seconds since a cell needs to serve clients.

Most researchers have concentrated on solving the FFA using heuristic approaches like neural networks, local search techniques and more recently meta heuristic approaches which include genetic algorithms, simulated annealing, ant colony optimisation and particle swarm optimisation.

In this section we have given a description of the Frequency Assignment Problem and introduced some concepts which we will use throughout the dissertation. In the next section we will present the Mathematics that govern the Frequency Assignment Problem.

3.3 Interference

In this section our discussion will focus on Interference. We will describe what interference is and why it is important for cellular networks as well as give an overview of when interference occurs.

Interference occurs when frequencies assigned to connections differ by a small margin. The amount of interference on a connection defines the *Quality of Service* (QoS). One can naturally make the deduction that the more frequencies differ used on connections in a area, the better quality of service one will experience in that area.

Cellular networks use the amount of interference on their networks as qualitative measure for their QoS. A network with high interference would experience a lot of dropped connections/calls, which occurs when the interference is too high to sustain a connection or call for communication, consequently their QoS degrades as interference increases.

In the literature a variety of methods are given to calculate the amount of interference in a network. These methods range from theoretical approaches

to precise measurements. Regardless of what method is used the end result which they all produce is called an *Interference Matrix*[16].

An Interference Matrix consists of a number of cell pairs (i,j) , where i is the cell receiving interference and j the cell whose allocated channel is providing the interference. Each cell pair in the matrix has two corresponding values that indicate the level of interference if the *Electromagnetic constraints* are violated [7, 2, 16]. These values are usually normalized to be between 0.0 and 1.0 [6].

Primarily Interference occurs when the Electromagnetic constraints are violated, which are defined as:

Co-Channel — When a cell i and a cell j operate on the same frequency or channel interference will occur [7, 9, 2, 16, 25]. This is called co-channel interference. This constraint is the most important constraint that must not be violated to ensure proper performance and reliability of a modern cellular network[9].

Adjacent Channel — When a cell i and a cell j operate on adjacent channels, their allocated frequencies differ by one i.e. cell i operates on channel f then if cell j operates on either channel $f - 1$ or $f + 1$ then adjacent channel interference will occur [7, 9, 2, 16, 25]

Co-Site — If cell i and cell j are located at the same site, then their allocated frequency ranges must differ by a certain distance in the frequency domain. This distance is known as the reuse distance [28, 10].

A fourth constraint, known as the Handover constraint, is also applicable in Cellular networks. This constraint imposes a separation in frequencies when one cell hands over a call to another cell. If this constraint is violated a mobile

subscriber will experience a dropped call since the handover between cells fails. The above constraints only account for factors that are in our control.

Interference also occurs due to technical limitations, natural phenomena and other external factors like other systems. Thus another constraint is imposed on the frequency that is allocated to cell. This constraint is known as the *separation* constraint which imposes a minimum separation between frequencies assigned to a cell [7, 25]. To avoid clashes with other operator frequencies each cell may also have a set of locally forbidden frequencies which are not allowed to be used under any circumstance.

In this section we described what interference is and what the consequences are of too much interference in a network. We also laid out under which circumstances interference can occur in a wireless network. In the next section we will give a brief overview of in which industries the Frequency Assignment Problem is applicable.

3.4 FAP in the industry

In this section we will list some of the industries where the FAP is encountered. We provide a brief overview how the problem differs compared to other industries. We will also give some references to literature where the FAP and the particular domain are discussed.

3.4.1 Satellite communication

The FAP in the Satellite communication domain occurs with respect to the ground terminals that transmit and receive signals via a satellite. One would assume that the problem includes the satellite, but the problem is only concerned with the frequencies that the ground terminals use. It is interesting

to note that the ground terminals can be a base station or a handheld device (e.g GPS or Satellite phone).

In Satellite communication, a signal is transmitted to one or more satellites via an uplink from a ground terminal. The signal is received by the recipient satellites and relayed to the interested ground terminals who receive the signals via a downlink.

The frequencies used by the ground terminals for uplink and downlink communication are separated by a large distance in the frequency domain - the typical distance is much larger than the bandwidth. When frequencies are assigned to transmitters, downlink transmitters are ignored and only uplink transmitters are considered [2].

A radical difference with regard to the use of frequencies compared to the standard FAP in Cellular Networks is that, frequencies are only allowed to be used once. This is specific to the satellite domain to avoid interference[2].

There isn't a lot of research on the FAP in Satellite Communication. One of the few recent papers on FAP in this domain is a paper by Lui et al.[15] where the authors employ a Chaotic Neural Network to minimize the interference in a satellite communication system with which they achieve very good results which in most benchmarks finds the global optimum.

Another paper concentrating on this domain is a paper by Houssin et al.[12] where the allocation of frequencies assigned in the satellite system is optimised using Space Division Multiple Access (SDMA). SDMA was developed for use in 3G networks and forms part of the Multiple Access family of techniques (CDMA, TDMA) that are in use in wireless networks today.

It is interesting to note that the authors concentrate on optimising the amount of users served by the system and not interference incurred by the allocated frequencies.

3.4.2 Wireless mesh networks and WLANs

Wireless mesh networks and WLANs² are the most recent applications where the FAP is encountered.

Multiple WLANs are increasingly being used to provide backbone support for large fixed line networks, enterprise networks, campuses and metropolitan areas. To be able to provide backbone support for these networks, a primary design goal when designing and deploying these networks is capacity. A limiting factor for WLAN capacity is interference which affects multihop hop settings. Thus the overall network interference needs to be minimized to increase the capacity of the network [27].

Most wireless networks operate on the IEEE 802.11 a/b/g standard. A IEEE 802.11n standard is available but hasn't been finalized yet, even though one can already find wireless hardware operating on this standard. According to the IEEE 802.11g standard only 13 Frequencies are available for use and in some geographical areas a further limiting constraint is imposed which only allows a certain subset of frequencies to be used [2].

Typical approaches allocating frequencies include using DCA and FCA³. DCA isn't very popular since the dynamic switching of channels lowers the response time on commodity hardware since there is a delay in milliseconds when switching channels. Typical packet transmission times are in microseconds. To guarantee uptime and high responsiveness, FCA is the preferred approach [27].

The FAP in Wireless Mesh Networks and WLANs differ to the standard problem in that it introduces an extra constraint. Channels assigned to links

²Both applications use the same standard and encounter similar problems in their respective domains.

³Discussed in section 3.2 on page 26

on a node cannot be more than the available interfaces on that particular node. This constraint is known as the *interface constraint* [27]. Another aspect to consider is the placement of access points (AP) in the network, which is similar to the problem cellular networks face with regard to base station placement [2].

There is a wealth of literature on Wireless Mesh Networks. The paper by Subramanian et al.[27] formulate a lower bound using semi-definite techniques and linear programming. Using these lower bounds with their discussed algorithms they get very promising results on a simulation benchmark (ns2). Their discussed solution imposes no specific hardware or topology changes in the wireless network.

Another interesting paper by Chen et al.[4] the authors follow a different route than traditional proposed algorithms with regard to interference. The authors present algorithms that focus on the interference as perceived by the user and not the AP. They also use site specific knowledge provided by Blueprints, Google Earth and Google Maps in their algorithms to predict potential path loss when allocating a frequency to a AP.

3.4.3 Military field communication

In a Military context the FAP is a very difficult problem to be solved due to its dynamic nature. During deployment connections need to be established rapidly between nodes with not gauruntee that the nodes would stay static at locations. Usually nodes are military field phones can be any transceiver device.

Due to the nature of the problem the DCA scheme is used to allocate frequencies to nodes. The Military FAP differs due to the property that any of the nodes are mobile and can move at any moment to a new location,

potentially interfering with another connection. Two frequencies need to be assigned to each connection that is established, one for each direction of communication. These allocated frequencies must also differ by a certain distance in the frequency domain to prohibit alternating directions of communication interfering.

A lot of literature can be found on Military field communication. This is due to two organizations CELAR⁴ and EUCLID⁵ making data available to various research groups and allowing them to develop algorithms for frequency assignment.

A comprehensive study by Dupont et al.[5] on the 36 instances of real life data obtained from CELAR. The Authors state that the CELAR data actually has 3 subproblems which occurs in separate stages. In the first stage a Constrained Satisfaction Problem is encountered when assigning initial frequencies. The second problem occurs when new links are established and frequencies need to be assigned, this is known as the second stage. The last and final stage occurs when a new link cannot be assigned a frequency and thus a repair is needed. For each stage the authors developed algorithms to try and optimally solve it to produce an overall optimal solution.

For the instances made available by EUCLID the study by Aardal et al.[1] provides results from various groups who worked on the instances provided, which is known as the CALMA project. Various optimization and approximations algorithms were implemented by the research groups and new lower bounds were also found. The authors present each algorithm implemented by the underlying research group. Results are shown for Minimum Interference problems as well as Minimum Span problems.

⁴Centre Electronique de L'Armement

⁵European Cooperation on the Long Term Defense

3.4.4 Television and Radio Broadcasting

The FAP encountered in broadcasting very much resembles the problem domain found in Cellular networks. The only notable difference is the required distance allocated frequencies must differ in the frequency domain is larger in broadcasting than in cellular networks [2].

Since the problem resembles the problem found in Cellular networks, there are few articles that specifically discuss frequency assignment in broadcasting as a main topic. A paper that does specifically discuss FAP in broadcasting is presented by Idoumghar and Schott [14]. The authors present a distributed hybrid genetic algorithm and a cooperative distributed tabu search algorithm. They compare these algorithms with their sequential counterparts of their algorithms and with a ANTS algorithm. The benchmark instances they used were provided by the TDF-C2R Broadcasting and Wireless research center.

3.4.5 Cellular Communication

Cellular communication⁶ can be considered the main driving force with regard to research in the Frequency Assignment domain. As new standards are developed and used in 3G networks, in general a frequency assignment problem still needs to be solved since these techniques do not eliminate interference entirely, but they do make it *less* likely to occur.

One such technique that is mostly used in GSM networks is called Frequency Hopping, which as the name implies, the transmitters "hops" onto different frequencies according to a predefined sequence of frequencies. The frequency can change per packet if the underlying hardware can handle it

⁶An overview of a Cellular Communication technology called GSM is presented in Chapter 2.

otherwise it switches per connection [2, 20, 7].

The FAP in the Cellular domain is the most researched topic and is considered the default domain of the problem. As such, most of the literature concentrates on this domain and one can find a lot of articles in the literature presenting viable algorithms that produce real world solutions [7].

Because the problem is NP-Hard most presented algorithms are either of the metaheuristic type or more recently of the swarm intelligence type. Both of these algorithmic types are discussed in Chapter 3 and 4 respectively.

Besides optimizing algorithms, there is also a wealth of literature on upper and lower bounds for the FAP. Using lower bounds in FAP orientated algorithms can produce very favourable results as demonstrated in the paper presented by Montemanni and Smith [21]. Using a lower bound in conjunction with their algorithm they achieved a new optimum in a variety of benchmarks, most notably in the COST 259 benchmark.

Other papers in the literature contribute by providing different modeling techniques such as the paper by Borndrfer et al. [3]. The interested reader who wants to know more about the problem domain, general modeling techniques used as well as the most common algorithms used in directed to the study by Aardall et al.[2].

In this section we discussed the different industries in which the Frequency Assignment Problem is encountered and also gave a brief description on how the problems are different with regard to what constraint it imposes on the frequencies for the particular industry. We also gave some brief references to some literature where the FAP is discussed with regard to the particular domain. In the next section we will give a brief description of the different types of Frequency Assignment Problems as well as give a small discussion on the literature found for the individual problems.

3.5 Frequency Assignment Problem types

I We will start of giving a brief overview on one of the first and oldest problems in the domain and we will end of discussing the problem we will base our implementation on.n this section we will give a brief explanation on some of the various problems that exist for the FAP. We will start of giving a brief overview on one of the first and oldest problems in the domain and we will end of discussing the problem we will base our implementation on.

3.5.1 Minimum Order FAP

The Minimum Order FAP (MO-FAP) was the first FAP that emerged in the 70's. The MO-FAP is concerned with assigning frequencies to transmitters while interference is minimized as well as minimizing the amount of different frequencies that are used.

In MO-FAP frequency re-use is prioritised and the usage of a frequency has a certain cost associated with it. The reason for this is because when the wireless network industry started out, operators were billed according to the amount of different frequencies they used. In the beginning frequencies weren't cheap since they were sold per unit [2, 20].

Over the years as the law governing the wireless spectrum changed and new technology as well standards emerged, thus MO-FAP has lost its relevancy. Companies aren't billed according to the different frequencies they use, but they purchase licenses from a regulatory body. This license usually stipulates what frequency band the network is allowed to use.

In Some instances a certain band of frequencies is put up for auction by a regulatory body, to which interested parties can bid to own the specified spectrum. Due to the shift in how frequencies are allocated to network, nei-

ther the regulatory bodies nor the network operators care about the amount of different frequencies are used. Thus MO-FAP has lost its relevancy in the modern wireless industry.

3.5.2 Minimum Span FAP

The Minimum Span FAP (MS-FAP) is a problem that is very relevant today, especially when network operators want to deploy a new network in a region. The MS-FAP is concerned with keeping the interference below a certain level during assignment as well as minimizing the span. The interference threshold used, is specified by the network designer as the minimum allowable interference on the network.

The span is defined as an interval on the frequency domain. This interval is calculated by taking the difference of the maximum and minimum frequencies used during assignment. With the span value, network operators are able to request certain frequency bands and know their network will be able to operate at suitable interference levels [2, 20, 24].

The MS-FAP and MO-FAP are two very similar problems, the only difference is that MO-FAP focuses on minimizing different frequencies and MS-FAP focuses on minimizing the interval of frequencies used during assignment [2]. The Philadelphia benchmark is usually used to gauge how good the algorithm performs.

3.5.3 Minimum Interference FAP

The Minimum Interference FAP (MI-FAP) or Fixed Spectrum FAP (FS-FAP) is typically encountered after the network operator has obtained a frequency band from a regulatory body. Other problems use matrices to forbid certain frequencies from being with certain transmitter [2, 7, 20, 8].

Unlike previous discussed problems, in MI-FAP any available frequency in the allocated band may be used even though it produces interference. The other problems are concerned with the frequencies used, even though they might be violating some constraints that incur a huge amount of interference. The interference value doesn't play a large role in their respective objective functions. In MI-FAP the objective is to minimize the total amount of interference on the network. It is important to note that this amount of interference might not necessarily be zero [2, 7, 20, 8].

The MI-FAP is the most encountered problem currently in cellular networks, since there are more operating networks than new networks being designed in the cellular industry today. This particular problem forms the focus for this dissertation.

Since MI-FAP is very close to real world instance problems, authors tend to use real world instances or benchmarks that resemble real world instances to test the quality and efficiency of their algorithms [2, 7, 20, 8]. We'll benchmark the quality and efficiency of our solution with the COST 259 benchmark which is discussed in section 3.7.

In this section we laid out the different types of Frequency Assignment Problems there are in the literature. We also gave a brief discussion on some of the literature found on the individual problems. Finally we formally stated on which one of the frequency assignment problems we will be concentrating on, namely the Fixed Spectrum Minimum Interference Frequency Assignment Problem (MI-FAP).

In the next section we will give a Mathematical definition for the Fixed Spectrum MI-FAP which will form the bases for the objective/cost function that we are going to minimize to find an optimal frequency plan.

3.6 Fixed Spectrum MI-FAP Mathematical Formulation

In this section we will give a Mathematical definition of the Frequency Assignment Problem which will form the core of what our algorithm discussed in this dissertation will optimize. We'll start of by denoting the symbols we will use and then we will give the Mathematical definition of the cost function we will minimize.

The Frequency Assignment Problem can be represented as a graph colouring problem hence it is known to be NP-Complete. Before we can formally define the Frequency Assignment Problem we first need to introduce some symbol definitions.

$$G = (V, E) \tag{3.1}$$

$$V = \{v_0, v_1, \dots, v_i\} | i \in \mathbb{N} \tag{3.2}$$

$$E = \{v_0v_1, v_0v_2, \dots, v_iv_j\} | v \in V, \forall i, j \in \mathbb{N}, i \neq j \tag{3.3}$$

$$D = \{d_{01}, d_{02}, \dots, d_{ij}\} | \forall \{i, j\} \in E, \exists d_{ij} \in \mathbb{N}^+ \tag{3.4}$$

$$P = \{\{p_{00}, \bar{p}_{01}\}, \{p_{10}, \bar{p}_{11}\}, \dots, \{p_{i0}, \bar{p}_{i1}\}\} | \forall \{i, j\} \in E, \exists p_{ij} \in \mathbb{N}^+ \tag{3.5}$$

$$F = \{0, 1, 2, 3, \dots, k\} | \forall k \in \mathbb{N}, \forall v \in V \exists f \in F \tag{3.6}$$

$$d_{ij} < |f(i) - f(j)|, \forall i, j \in \mathbb{N}, i \neq j \tag{3.7}$$

Let G (see 3.1) be a weighted undirected graph, where V (see 3.2) is a set of vertices. Each $v \in V(G)$ represents a transmitter in the frequency assignment problem.

E (see 3.3) is a set of edges. An edge consists of two vertices v_i and v_j that are joined because there exists a constraint on the frequencies that can be assigned between the two vertices or transmitters. Each edge has two

associated labels d_{ij} and p_{ij} [3, 21].

The label d_{ij} that is part of the set D (see 3.4) denotes the maximum separation that is required to exists between frequencies assigned to two transmitters v_i and v_j . Using $f(i)$ to denote the frequency assigned to i , we can determine using equation 3.7 if the interference involving the transmitters v_i and v_j is acceptable[3, 21]

The other label, p_{ij} , forms part of the set P (see 3.5) which is referred to as the Interference Matrix⁷. Each label p_{ij} contains two values which represents interference⁸:

- p_{i0}^- represents the value for co-channel interference [3, 21].
- $p_{i1}^=$ represents the value for adjacent channel interference[3, 21].

Lastly we have the set F (see 3.6) that denotes a set of consecutive frequencies for every transmitter in V [3, 21].

Formally the Fixed Spectrum Frequency Assignment Problem (FS-FAP) can now be defined as a 5-tuple $FS-FAP = \{V, E, D, P, F\}$ with a required mapping of $f : V \rightarrow F$ [21]. The objective of the FS-FAP is to find an assignment of frequencies to transmitters that minimizes the sum of total interference (see 3.9).

$$c(p_i) = \begin{cases} p_{i0}^- & , \text{if } |f(i) - f(j)| = 0 \\ p_{i1}^= & , \text{if } |f(i) - f(j)| \leq d_{ij} \\ 0 & , \text{if } |f(i) - f(j)| > d_{ij} \end{cases} \quad (3.8)$$

$$TotalInterference = \sum_{i=0}^{\mathbb{P}} c(p_i), p \in P \quad (3.9)$$

⁷Discussed in section 3.3 page 28

⁸Interference values can be zero in some cases

In this section we Mathematically defined the Frequency Assignment Problem using the symbols we defined. In the next section we will give a brief discussion on the different Frequency Assignment Benchmark Problems that exist and also define the benchmark we will be using in our implementation.

3.7 FAP Benchmarks

In this sections will discuss some of the most used benchmarks in the FAP domain. We will start of with the first benchmark that was introduced in the 70s and end of with a disuccsion on the benchmark we will be using to test our implementation.

3.7.1 Philedelphia Benchmark

The Philedelphia benchmarks are derived from an instance that was introduced in 1973 by Anderson. Each instance is a hexagonal grid of cells that overlaps the area of interest. At the center of each cellthere is a transmitter. Past approaches used these hexagonal systems to model modern cellular networks [2, 17].

In this benchmark interference is measured by a co-channel reuse distance. This distance stipulates that the difference of the frequencies assigned to two cells must greater of equal to a certain value d . A frequency cannot be assigned to a cell if it violates this minimum distance [2, 17].

These benchmarks are typically used to test algorithms developed for MS-FAP, since there is no concept of cost or penalty for interference incurred by violating constraints.

3.7.2 CELAR

In 1994 EUCLID introduced a project called CALMA which was a combined effort by various European governments that were part of EUCLID to investigate algorithms for Military applications. The project was granted to six research groups. Within the project 36 instances were made available by CELAR for Radio Link Frequency Assignment [2, 5].

All the CELAR instances have the constraint that the difference between frequencies assigned to interfering radio links must be greater than a certain predefined distance in the frequency domain. This is a soft constraint and may be violated. Another constraint in the CELAR instances is that each pair of parallel links must differ by an exact predefined distance. This constraint is a hard constraint and may not be violated [5].

These instances were initially not available to the general public as it was contained to be within the CALMA project. In 2001 the CELAR launched an the International ROADEF challenge, where certain instances from the CALMA project were made available for the research teams taking part in the challenge. The instances made available had been modified to take polarizations and controlled relaxations of certain EMC constraints [11].

3.7.3 COST 256

The COST (COoperation europeene dans le domaine de la recherche Scientifique et Technique) 259 is a set of real world GSM instances made available by the European Union. The instances are publicly available and can be downloaded for free at <http://fap.zib.de/> (FAP Web 2007). The website also contains the most recent results obtained by researchers using these instances [2, 7].

The instances are fairly difficult due to the large amount of transmitters (900 - 4000) that need to be assigned frequencies, with a relatively small amount spectrum of frequencies. The main important characteristic of this benchmark is that it resembles real world GSM network data, which is why we the authors have selected this as the primary benchmark we will be concentrating on [2, 13].

More specifically we will concentrate on a small subset of the instances that are available, namely Siemens1, Siemens2, Siemens3 and Siemens4. In the paper by Montemanni and Smith [21] the same subset of problems is used and to date their algorithm has produced some of the best results.

3.8 Summary

Chapter 4

Metaheuristics Algorithms

4.1 Introduction

Metaheuristics is a sub domain of the artificial intelligence domain. It evolved out of a need for more efficient search techniques with regard to hard problems.

Metaheuristics forms part of a collective body of algorithms that use heuristics to search a particular domain's problem space, for the most optimal solution adhering to certain hard and soft constraints. Some of the most important Algorithms that form part of this collective body is:

- Tabu Search
- Simulated Annealing
- Genetic Algorithm

The above mentioned algorithms aren't the only algorithms to form part of this sub-domain, but they are the algorithms that have received the most attention in the literature and generally produce good results.

In this chapter our main focus will be to discuss each of the above listed algorithms. We will start of by briefly discussing the characteristics of metaheuristic algorithms after which we will discuss each of the above algorithms in detail. We will also provide a literature study for each algorithm inorder for us to see how an algorithm needs to be changed and optimised for particular problem domain.

4.2 Characteristics of Metaheuristics

NP-Complete problems have been proven to not be solveble in polinomial by local search methods such as 1,2,3. Metaheuristic algorihtms on the other hand are much more efficient in search the problem space and produce much better results in a short amount of time.

Metaheuristic Algorihtms do not search a problem space statically by testing every possible permutation in the problem space and evaluating it. Instead these algorithms make use of local and global search methods that explore the search space probilistically; moving in the problem space to a region with a high probability of finding high qaulity candidate solutions.

4.3 Tabu Search

4.3.1 Basic Algorithm

4.3.2 Literature study

4.4 Simulated Annealing

4.4.1 Basic Algorithm

4.4.2 Literature study

4.5 Genetic Algorithm

4.5.1 Basic Algorithm

4.5.2 Literature study

4.6 Summary

Chapter 5

Swarm Intelligence

5.1 Introduction

Part II

Implementation

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