



Bee Colony Optimization: Principles and Applications

Dušan Teodorović, Panta Lučić, Goran Marković, Mauro Dell' Orco

Abstract — The Bee Colony Optimization Metaheuristic (BCO) is proposed in the paper. The BCO represents the new metaheuristic capable to solve difficult combinatorial optimization problems. The artificial bee colony behaves partially alike, and partially differently from bee colonies in nature. In addition to proposing the BCO as a new metaheuristic, we also describe in the paper two BCO algorithms that we call the Bee System (BS) and the Fuzzy Bee System (FBS). In the case of FBS the agents (artificial bees) use approximate reasoning and rules of fuzzy logic in their communication and acting. In this way, the FBS is capable to solve deterministic combinatorial problems, as well as combinatorial problems characterized by uncertainty. The proposed approach is illustrated by three Case studies.

Keywords — Key words or phrases in alphabetical order, separated by commas.

I. INTRODUCTION

A great number of traditional engineering models and algorithms used to solve complex problems are based on control and centralization. Various natural systems (social insects colonies) lecture us that very simple individual organisms can create systems able to perform highly complex tasks by dynamically interacting with each other.

Bee swarm behavior in nature is, first and foremost, characterized by autonomy and distributed functioning and self-organizing. In the last couple of years, the researchers started studying the behavior of social insects in an attempt to use the Swarm Intelligence concept in order to develop various Artificial Systems.

The Bee Colony Optimization (BCO) Metaheuristic that represents the new direction in the field of Swarm Intelligence is introduced in this paper. The primary goal of this paper is to explore the possible applications of collective bee intelligence in solving combinatorial problems characterized by uncertainty. The paper is

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organized in the following way. The new computational paradigm - The Bee Colony Optimization is described in Section 2. Sections III, IV, and V are devoted to the description of Case Studies (Traveling Salesman Problem, Ride-Matching Problem, and Routing and Wavelength Assignment Problem). Conclusion is given in the Section VI.

II. THE BEE COLONY OPTIMIZATION: THE NEW COMPUTATIONAL PARADIGM

Social insects (bees, wasps, ants, termites) have lived on Earth for millions of years, building nests and more complex dwellings, organizing production and procuring food. The colonies of social insects are very flexible and can adapt well to the changing environment. This flexibility allows the colony of social insects to be robust and maintain its life in spite of considerable disturbances.

The dynamics of the social insect population is a result of the different actions and interactions of individual insects with each other, as well as with their environment. The interactions are executed via multitude of various chemical and/or physical signals. The final product of different actions and interactions represents social insect colony behavior. Interaction between individual insects in the colony of social insects has been well documented. The examples of such interactive behavior are bee dancing during the food procurement, ants' pheromone secretion, and performance of specific acts, which signal the other insects to start performing the same actions. These communication systems between individual insects contribute to the formation of the "collective intelligence" of the social insect colonies. The term "Swarm Intelligence", denoting this "collective intelligence" has come into use [1]-[3].

A. Bees in the Nature

Self-organization of bees is based on a few relatively simple rules of individual insect's behavior. In spite of the existence of a large number of different social insect species, and variation in their behavioral patterns, it is possible to describe individual insects' as capable of performing a variety of complex tasks [4]. The best example is the collection and processing of nectar, the practice of which is highly organized. Each bee decides to reach the nectar source by following a nestmate who has already discovered a patch of flowers. Each hive has a so-called dance floor area in which the bees that have discovered nectar sources dance, in that way trying to

convince their nestmates to follow them. If a bee decides to leave the hive to get nectar, she follows one of the bee dancers to one of the nectar areas. Upon arrival, the foraging bee takes a load of nectar and returns to the hive relinquishing the nectar to a food storer bee. After she relinquishes the food, the bee can (a) abandon the food source and become again uncommitted follower, (b) continue to forage at the food source without recruiting the nestmates, or (c) dance and thus recruit the nestmates before the return to the food source. The bee opts for one of the above alternatives with a certain probability. Within the dance area, the bee dancers "advertise" different food areas. The mechanisms by which the bee decides to follow a specific dancer are not well understood, but it is considered that "the recruitment among bees is always a function of the quality of the food source" [4]. It is also noted that not all bees start foraging simultaneously. The experiments confirmed, "new bees begin foraging at a rate proportional to the difference between the eventual total and the number presently foraging".

The basic principles of collective bee intelligence in solving combinatorial optimization problems were for a first time used in [5] and [6]. The authors introduced the Bee System (BS) and tested it in the case of Traveling Salesman Problem. The Bee Colony Optimization Metaheuristic (BCO) that has been proposed in this paper represents further improvement and generalization of the Bee System. The basic characteristics of the BCO Metaheuristic are described. Our artificial bee colony behaves partially alike, and partially differently from bee colonies in nature. The Fuzzy Bee System (FBS) capable to solve combinatorial optimization problems characterized by uncertainty is also introduced in the paper. Within FBS, the agents use approximate reasoning and rules of fuzzy logic in their communication and acting.

B. The Bee Colony Optimization Metaheuristic

Within the Bee Colony Optimization Metaheuristic (BCO), agents that we call - artificial bees collaborate in order to solve difficult combinatorial optimization problem. All artificial bees are located in the hive at the beginning of the search process. During the search process, artificial bees communicate directly. Each artificial bee makes a series of local moves, and in this way incrementally constructs a solution of the problem. Bees are adding solution components to the current partial solution until they create one or more feasible solutions. The search process is composed of *iterations*. The first iteration is finished when bees create for the first time one or more feasible solutions. The best discovered solution during the first iteration is saved, and then the second iteration begins. Within the second iteration, bees again incrementally construct solutions of the problem, etc. There are one or more partial solutions at the end of each iteration. The analyst-decision maker prescribes the total number of iterations.

When flying through the space our artificial bees perform *forward pass* or *backward pass*. During forward pass, bees create various partial solutions. They do this

via a combination of individual exploration and collective experience from the past.

After that, they perform backward pass, i.e. they return to the hive. In the hive, all bees participate in a *decision-making* process. We assume that every bee can obtain the information about solutions' quality generated by all other bees. In this way, bees *exchange* information about quality of the partial solutions created. Bees compare all generated partial solutions. Based on the quality of the partial solutions generated, every bee decides whether to abandon the created partial solution and become again uncommitted follower, continue to expand the same partial solution without recruiting the nestmates, or dance and thus recruit the nestmates before returning to the created partial solution. Depending on the quality of the partial solutions generated, every bee possesses certain level of *loyalty* to the path leading to the previously discovered partial solution. During the second forward pass, bees expand previously created partial solutions, and after that perform again the backward pass and return to the hive. In the hive bees again participate in a decision-making process, perform third forward pass, etc. The iteration ends when one or more feasible solutions are created.

Like Dynamic Programming, the BCO also solves combinatorial optimization problems in stages (Figure 1). Each of the defined stages involves one optimizing variable. Let us denote by $ST = \{st_1, st_2, \dots, st_m\}$ a finite set of pre-selected stages, where m is the number of stages. By B we denote the number of bees to participate in the search process, and by I the total number of iterations. The set of partial solutions at stage st_j is denoted by S_j ($j = 1, 2, \dots, m$).

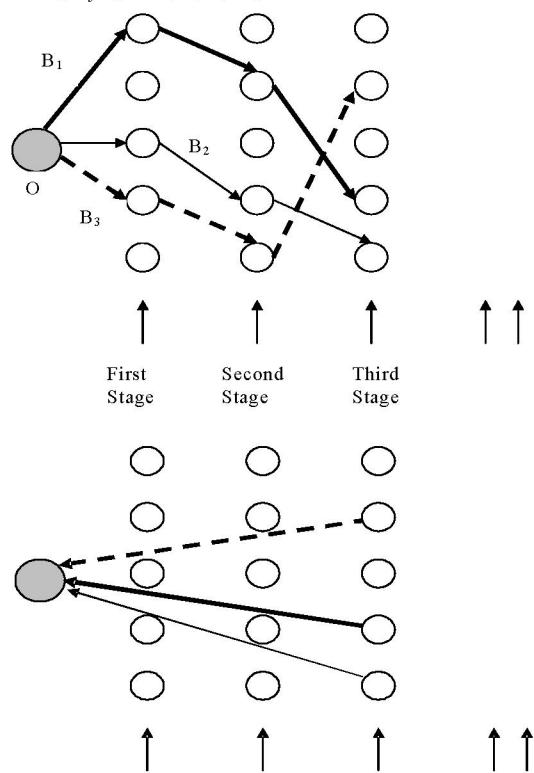


Fig. 1. First forward pass and the first backward pass

The following is pseudo-code of the Bee Colony Optimization:

Bee Colony Optimization

(1) *Initialization.* Determine the number of bees B , and the number of iterations I . Select the set of stages $ST = \{st_1, st_2, \dots, st_m\}$. Find any feasible solution x of the problem. This solution is the *initial best solution*.

(2) Set $i := 1$. Until $i = I$, repeat the following steps:

(3) Set $j = 1$. Until $j = m$, repeat the following steps:

Forward pass: Allow bees to fly from the hive and to choose B partial solutions from the set of partial solutions S_j at stage st_j .

Backward pass: Send all bees back to the hive. Allow bees to exchange information about quality of the partial solutions created and to decide whether to abandon the created partial solution and become again uncommitted follower, continue to expand the same partial solution without recruiting the nestmates, or dance and thus recruit the nestmates before returning to the created partial solution. Set, $j := j + 1$.

(4) If the best solution x_i obtained during the i -th iteration is better than the best-known solution, update the best known solution ($x := x_i$).

(5) Set, $i := i + 1$.

Alternatively, forward and backward passes could be performed until some other stopping condition is satisfied. The possible stopping conditions could be, for example, the maximum total number of forward/backward passes, or the maximum total number of forward/backward passes between two objective function value improvements.

During the forward pass bees will visit certain number of nodes, create partial solution, and after that return to the hive (node O). In the hive, bees will participate in a decision making process. Bees compare all generated partial solutions. Based on the quality of the partial solutions generated, every bee will decide whether to abandon the generated path and become again uncommitted follower, continue to fly along discovered path without recruiting the nestmates, or dance and thus recruit the nestmates before returning to the discovered path. Depending on the quality of the partial solutions generated, every bee possesses certain level of loyalty to the path previously discovered. For example, bee B_1 , B_2 , and B_3 participated in the decision-making process. After comparing all generated partial solutions, bee B_1 decided to abandon already generated path, and to join bee B_2 .

The bees B_1 , and B_2 fly together along the path generated by the bee B_2 . When they reach the end of the path, they are free to make individual decision about next node to be visited. The bee B_3 will continue to fly along discovered path without recruiting the nestmates (Figure 2). In this way, bees are again performing forward pass.

During the second forward pass, bees will visit few more nodes, expand previously created partial solutions, and after that perform again the backward pass and return to the hive (node O). In the hive, bees will again participate in a decision making process, make a decision,

perform third forward pass, etc. The iteration ends when the bees have visited all nodes.

Within the proposed BCO Metaheuristic, various heuristic algorithms describing bees' behavior and/or "reasoning" could be developed and tested. In other words, various BCO algorithms could be developed. These algorithms should describe the ways in which bees decide to abandon the created partial solution, to continue to expand the same partial solution without recruiting the nestmates, or to dance and thus recruit the nestmates before returning to the created partial solution.

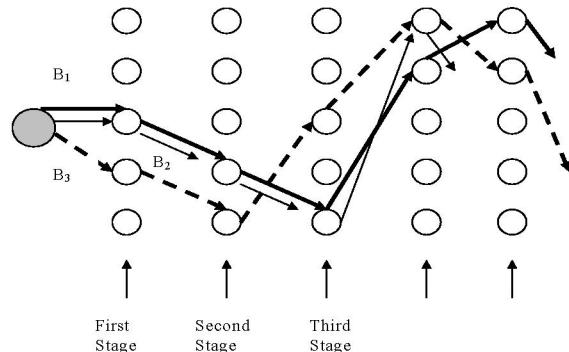


Fig. 2. Second forward pass

In addition to proposing the BCO as a new metaheuristic, we also describe in the paper two BCO algorithms that we call the Bee System (BS) and the Fuzzy Bee System (FBS). The BS proposed in [5] is described in more details within the Case Study of the Traveling Salesman Problem. In the case of FBS [7], the agents (artificial bees) use approximate reasoning and rules of fuzzy logic in their communication and acting. In this way, the FBS is capable to solve deterministic combinatorial problems, as well as combinatorial problems characterized by *uncertainty*. The FBS is described in details within the Case Study of Ride-Matching problem.

III. CASE STUDY #1: SOLVING THE TRAVELING SALESMAN PROBLEM BY THE BEE COLONY OPTIMIZATION

The proposed Bee System was tested on a large number of numerical examples. The benchmark problems were taken from the following Internet address: <http://www.iwr.uni-heidelberg.de/iwr/comopt/software/TSPLIB95/tsp/>.

The following problems were considered: Eil51.tsp, Berlin52.tsp, St70.tsp, Pr76.tsp, Kroa100.tsp and a280.tsp. All tests were run on an IBM compatible PC with PIII processor (533MHz). The results obtained are given in Table 1.

We can see from the Table 1 that the proposed BCO produced results of a very high quality. The BCO was able to obtain the objective function values that are very close to the optimal values of the objective function. The times required to find the best solutions by the BCO are very low. In other words, the BCO was able to produce "very good" solutions in a "reasonable amount" of computer time.

TABLE 1: THE RESULTS OBTAINED BY THE BEE COLONY OPTIMIZATION

| Problem Name (Number of nodes) | Optimal Value (O) | The best value obtained by the Bee System (B) | $\frac{(B-O)}{(O)}\%$ | CPU (sec) |
|--------------------------------|-------------------|---|-----------------------|-----------|
| Eil51 (51) | 429.983 | 431.121 | 0.26% | 44 |
| Berlin52 (52) | 7544.36 | 7544.366 | 0% | 18 |
| St70 (70) | 678.597 | 678.621 | 0.0035% | 238 |
| Pr76 (76) | 108159 | 108790 | 0.58% | 127 |
| Kroa100 (100) | 21285.4 | 21441.5 | 0.73% | 58 |
| A280 (280) | 2586.77 | 2740.63 | 5.95% | 1855 |

IV. CASE STUDY #2: SOLVING THE RIDE-MATCHING PROBLEM BY THE FUZZY BEE SYSTEM

Urban road networks in many countries are severely congested, resulting in increased travel times, increased number of stops, unexpected delays, greater travel costs, inconvenience to drivers and passengers, increased air pollution and noise level, and increased number of traffic accidents.

Expanding traffic network capacities by building more roads is extremely costly as well as environmentally damaging. More efficient usage of the existing supply is vital in order to sustain the growing travel demand. Ridesharing is one of the widely spread Travel Demand Management (TDM) techniques that assumes the participation of two or more persons that all together share vehicle when traveling from few origins to few destinations. All drivers that participate in ride-sharing offer to the operator the following information regarding trips planned for the next week: (a) Vehicle capacity (2, 3, or 4 persons); (b) Days in the week when person is ready to participate in ride-sharing; (c) Trip origin for every day in a week; (d) Trip destination for every day in a week; (e) Desired departure and/or arrival time for every day in a week.

The ride-matching problem considered in [7] could be defined in the following way: Make routing and scheduling of the vehicles and passengers for the whole week in the "best possible way". The following are potential objective functions: (a) Minimize the total distance traveled by all participants; (b) Minimize the total delay; (c) Make relatively equals vehicle utilization. We deal with the deterministic combinatorial optimization problem in the case when the desired departure and/or arrival times are fixed (For example "I want to be picked-up exactly at 8:00 a.m."). On the other hand, in many real-life situations the desired departure and/or arrival times are fuzzy (I want to be picked-up about 8:00 a.m.). In this case, the ride-matching problem should be treated as a combinatorial optimization problem characterized by uncertainty. We solve the problem described by the Fuzzy Bee System.

A. The Fuzzy Bee System

Bees face many decision-making problems while searching for the best solution. The following are bees' choice dilemmas: (a) What is the next solution component to be added to the partial solution?; (b) Should the partial

solution be abandon or not?; (c) Should the same partial solution be expanded without recruiting the nestmates?

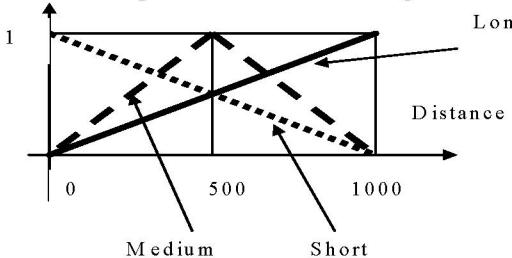


Fig. 3. Fuzzy sets describing distance

The majority of the choice models are based on random utility modeling concepts. These approaches are highly rational. They are based on assumptions that decision-makers possess perfect information processing capabilities and always behave in a rational way (trying to maximize utilities). In order to offer alternative modeling approach, researchers started to use less normative theories. The basic concepts of Fuzzy Sets Theory [8], linguistic variables, approximate reasoning, and computing with words have more understanding for uncertainty, imprecision, and linguistically expressed observations. Following these ideas, we start in our choice model from the assumption that the quantities perceived by bees are "fuzzy" [8]. Artificial bees use approximate reasoning and rules of fuzzy logic in their communication and acting. During the j -th stage bees fly from the hive and choose B partial solutions from the set of partial solutions S_i at stage s_j (forward pass). When adding the solution component to the current partial solution during the forward pass, specific bee perceives specific solution component as "less attractive", "attractive", or "very attractive". We also assume that an artificial bee can perceive a specific attributes as "short", "medium" or "long" (Figure 3), "cheap", "medium", or "expensive", etc.

B. Calculating the solution component attractiveness and choice of the next solution component to be added to the partial solution

The approximate reasoning algorithm for calculating the solution component attractiveness consists of the rules of the following type:

If the attributes of the solution component are VERY GOOD

Then the considered solution component is VERY ATTRACTIVE

The main advantage of using the approximate reasoning algorithm for calculating the solution component attractiveness is that it is possible to calculate solution component attractiveness even if some of the input data were only approximately known. Let us denote by f_i the attractiveness value of solution component i . The probability p_i for solution component i to be added to the partial solution is equal to the ratio of f_i to the sum of all considered solution component attractiveness values:

$$p_i = \frac{f_i}{\sum_j f_j} \quad (1)$$

In order to choose next solution component to be added to the partial solution, artificial bees use a proportional selection known as the “roulette wheel selection.” (The sections of roulette are in proportion to probabilities p_i). In addition to the “roulette wheel selection,” several other ways of selection could be used.

C. Bee's partial solutions comparison mechanism

In order to describe bee's partial solutions comparison mechanism, we introduce the concept of *partial solution badness*. We define partial solution badness in the following way:

$$L_k = \frac{L^{(k)} - L_{\min}}{L_{\max} - L_{\min}} \quad (2)$$

where:

L_k - badness of the partial solution discovered by the k -th bee

$L^{(k)}$ - the objective function values of the partial solution discovered by the k -th bee

L_{\min} -the objective function value of the best-discovered partial solution from the beginning of the search process

L_{\max} - the objective function value of the worst discovered partial solution from the beginning of the search process

The approximate reasoning algorithm to determine the partial solution badness consists of the rules of the following type:

If the discovered partial solution is BAD

Then loyalty is LOW

Bees use *approximate reasoning*, and compare their discovered partial solutions with the best, and the worst discovered partial solution from the *beginning* of the search process. In this way, “historical facts” discovered by the *all members* of the bee colony have significant influence on the future search directions.

D. Calculating the number of bees changing the path

Every partial solution (partial path) that is being advertised in the dance area has two main attributes: (a) the objective function value, and (b) the number of bees that are advertising the partial solution (partial path). The number of bees that are advertising the partial solution is a good indicator of a bees' collective knowledge. It shows how bee colony perceives specific partial solutions.

The approximate reasoning algorithm to determine the advertised partial solution attractiveness consists of the rules of the following type:

If the length of the advertised path is SHORT and the number of bees advertising the path is SMALL

Then the advertised partial solution attractiveness is MEDIUM

Path attractiveness calculated in this way can take values from the interval $[0,1]$. The higher the calculated value, the more attractive is advertised path. Bees are less or more loyal to “old” paths. At the same time, advertised

paths are less, or more attractive to bees. Let us note paths p_i and p_j . We denote by n_{ij} the number of bees that will abandon path p_i , and join nestmates who will fly along path p_j .

The approximate reasoning algorithm to calculate the number of shifting bees consists of the rules of the following type:

If bees' loyalty to path p_i is LOW and path p_j 's attractiveness is HIGH

Then the number of shifting bees from path p_i to path p_j is HIGH

In this way, the number of bees flying along specific path is changed before beginning of the new forward pass. Using collective knowledge and sharing information among themselves, bees concentrate on more promising search paths, and slowly abandon less promising paths. ways of selection could be used.

E. Numerical experiment

The authors in [7] tested the proposed model in the case of ridesharing demand from Trani, a small city in the southeastern Italy, to Bari, the region capital of Puglia

They collected the data regarding 97 travelers demanding for ridesharing, and assumed, for sake of simplicity, that the capacity is 4 passengers for all their cars. In our case, the algorithm chooses $24*4 = 96$ out of 97 travelers to build up the “best” path. The authors used a hive of 15 bees, leaving at once. Bees have generated only six “foraging paths”. The other generated paths were abandoned eventually.

Changes of the best discovered objective function values are shown in Figure 4.

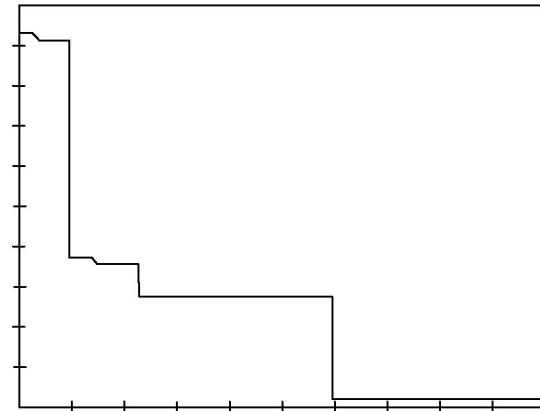


Fig. 4. Changes of the best-discovered objective function values

V. CASE STUDY #3: ROUTING AND WAVELENGTH ASSIGNMENT (RWA) IN ALL-OPTICAL NETWORKS

Every pair of nodes in optical networks is characterized by a number of requested connections. The total number of established connections in the network depends on the routing and wavelength assignment procedure. Routing and wavelength assignment (RWA) problem in all-optical networks could be defined in the following way: Assign a path through the network and a wavelength on that path for each considered connection between a pair of nodes in

such a way to maximize the total number of established connections in the network.

In [9] the authors proposed the BCO heuristic algorithm tailored for the RWA problem. They called the proposed algorithm the BCO-RWA algorithm. Bees decide to choose a physical route in optical network in a random manner. Logit model is one of the most successful and widely accepted discrete choice model. Inspired by the Logit model, the authors in [9] assumed that the probability p_r^{sd} of choosing route r in the case of origin-destination pair (s,d) equals:

$$p_r^{sd} = \begin{cases} \frac{e^{\nu_r^{sd}}}{\sum_{i=1}^{|R^{sd}|} e^{\nu_i^{sd}}}, & \forall r \in R^{sd} \text{ i } W_r > 0 \\ 0, & \forall r \in R^{sd} \text{ i } W_r = 0 \end{cases} \quad (3)$$

where $|R^{sd}|$ is the total number of available routes between pair of nodes (s,d) . The route r is available if there is at least one available wavelength on all links that belong to the route r .

In the hive every bee makes the decision about abandoning the created partial solution or expanding it in the next forward pass. The authors in [9] assumed that every bee can obtain the information about partial solution quality created by every other bee. They calculated the probability that the bee b will at beginning of the $u+1$ forward pass use the same partial tour that is defined in forward pass u in the following way:

$$p_b = e^{-\frac{C_{max} - C_b}{u}} \quad (4)$$

where:

C_b – the total number of established lightpaths from the beginning of the search process by the b -th bee

C_{max} – the maximal number of established lightpaths from the beginning of the search process by any bee

u – ordinary number of forward pass, $u=1,2,\dots,U$

TABLE 2: THE RESULTS COMPARISON

| Total number of requested light-paths | Number of wave-lengths | Number of established lightpaths | | CPU time [s] | | Relative error [%] |
|---------------------------------------|------------------------|----------------------------------|---------|--------------|---------|--------------------|
| | | ILP | BCO-RWA | ILP | BCO-RWA | |
| 28 | 1 | 14 | 14 | 4 | 4.33 | 0 |
| | 2 | 23 | 23 | 94 | 4.58 | 0 |
| | 3 | 27 | 27 | 251 | 4.68 | 0 |
| | 4 | 28 | 28 | 313 | 4.66 | 0 |
| 31 | 1 | 15 | 14 | 4 | 4.73 | 6.67 |
| | 2 | 25 | 25 | 83 | 5.00 | 0 |
| | 3 | 30 | 30 | 235 | 5.19 | 0 |
| | 4 | 31 | 31 | 1410 | 5.21 | 0 |
| 34 | 1 | 15 | 14 | 4 | 5.19 | 6.67 |
| | 2 | 27 | 26 | 148 | 5.50 | 3.70 |
| | 3 | 33 | 33 | 216 | 5.64 | 0 |
| | 4 | 34 | 34 | 906 | 5.64 | 0 |
| 36 | 1 | 16 | 15 | 23 | 5.64 | 6.25 |
| | 2 | 27 | 26 | 325 | 6.09 | 3.70 |
| | 3 | 34 | 34 | 788 | 6.11 | 0 |
| | 4 | 36 | 36 | 1484 | 6.13 | 0 |
| 38 | 1 | 17 | 16 | 16 | 5.67 | 5.88 |
| | 2 | 28 | 27 | 247 | 6.09 | 3.57 |
| | 3 | 35 | 35 | 261 | 6.23 | 0 |
| | 4 | 38 | 38 | 1773 | 6.33 | 0 |
| 40 | 1 | 17 | 16 | 31 | 6.00 | 5.88 |
| | 2 | 28 | 27 | 491 | 6.28 | 3.57 |
| | 3 | 35 | 35 | 429 | 6.61 | 0 |
| | 4 | 40 | 40 | 1346 | 6.67 | 0 |

The authors in [9] calculated the probability p_P that the P -th advertised partial solution will be chosen by any of the uncommitted follower using the following relation:

$$p_P = \frac{e^{C_p}}{\sum_{p=1}^P C_p} \quad (5)$$

where C_P is the total number of the established lightpaths in the case of the P -th advertised partial solution

The BCO-RWA algorithm was tested on a few numerical examples. The authors in [9] formulated corresponding Integer Linear Program (ILP) and discovered optimal solutions for the considered examples. In the next step, they compared the BCO-RWA results with the optimal solution. The comparison for the considered network is shown in the Table 2.

We can see from the Table 2 that the proposed BCO-RWA algorithm has been able to produce optimal, or a near-optimal solutions in a reasonable amount of computer time.

VI. CONCLUSION

The successful applications of the Bee Colony Optimization to difficult combinatorial optimization problems are very encouraging. It is of a great importance to investigate in future research both advantages and disadvantages of autonomy, distributed functioning and self-organizing in relation to traditional engineering methods relying on control and centralization.

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