A Simple Diversity Guided Particle Swarm Optimization

M. Pant, T. Radha, and V. P. Singh

Abstract— **In this paper we have proposed a new diversity guided Particle Swarm Optimizer (PSO), namely ATRE-PSO, which is a modification of attractive and repulsive PSO (ARPSO), suggested by Riget and Vesterstorm [1]. Depending on the diversity of the population the ATRE-PSO switches alternately between three phases of attraction, repulsion and a combination of attraction and repulsion, called the phase of positive conflict [2]. The performance of ATRE-PSO is compared with Basic PSO (BPSO) and ARPSO. The numerical results show that besides preserving the rapid convergence of the BPSO, ATRE-PSO also maintains a good diversity in the population. Under most of the test cases, simulations show that ATRE-PSO finds a better solution than BPSO as well as ARPSO.**

I. Introduction

PARTICLE swarm optimization technique is a population based stochastic search technique first suggested by Kennedy and Eberhart in 1995 [3]. Since then it has been used to solve a variety of optimization problems. Its performance has been compared with many popular stochastic search techniques like Genetic algorithms, Differential Evolution, Simulated Annealing etc. [4], [5], [6]. Although PSO has shown a very good performance in solving many test as well as real life optimization problems, it suffers from the problem of premature convergence like most of the stochastic search techniques, particularly in case of multimodal optimization problems.

The *curse* of premature convergence greatly affects the performance of algorithm and many times lead to a sub optimal solution. Aiming at this shortcoming of PSO algorithms, many variations have been developed to improve its performance. Some of these methods include fuzzy PSO [7], hybrid PSO [8], intelligent PSO [9], addition of a queen particle [10] etc.

In this paper we present a simple and effective PSO called ATRE-PSO which is a variation of ARPSO, a diversity guided PSO developed by Riget and Vesterstorm. Like ARPSO, ATRE-PSO uses diversity as a measure to guide the swarm population. In ARPSO if the diversity is above the certain threshold d_{high} then particles attract each other and if it is below d_{low}, then the particles repel each other until they meet the required high diversity d_{high}. In our modified version we have suggested an *in between* phase, called the phase of positive conflict which is a combination of attraction and repulsion. This phase is activated when the swarm diversity is lying between d_{low} and d_{high}. The remaining paper is organized as follows: in section II, we have briefly described the Basic Particle Optimization (BPSO), ARPSO and ATRE-PSO, in section III, the experimental setup, parameter settings and benchmark problems are reported. The experimental results are analyzed in section IV, finally the paper concludes with Section V.

II. Particle Swarm Optimization

A. Basic Particle Swarm Optimization (BPSO)

Particle Swarm Optimization (PSO) is a relatively newer addition to a class of population based search technique for solving numerical optimization problems. Its mechanism is inspired from the complex social behavior shown by the natural species like flock of birds, school of fish and even crowd of human beings. The particles or members of the swarm fly through a multidimensional search space looking for a potential solution. Each particle adjusts its position in the search space from time to time according to the flying experience of its own and of its neighbors (or colleagues).

For a D-dimensional search space the position of the ith particle is represented as $X_i = (x_{i1}, x_{i2}, \ldots, x_{iD})$. Each particle maintains a memory of its previous best position $P_i = (p_{i1}, p_{i2} \ldots p_{iD})$ and a velocity $V_i = (v_{i1}, v_{i2}, \ldots v_{iD})$ along each dimension. In each iteration, the P vector of the particle with best fitness in the local neighborhood, designated g, and the P vector of the current particle are combined to adjust the velocity along each dimension and a new position of the particle is determined using that velocity. The two basic equations which govern the working of PSO are that of velocity vector and position vector given by:

$$v_{id} = wv_{id} + c_1 r_1 (p_{id} - x_{id}) + c_2 r_2 (p_{gd} - x_{id}) \quad (1)$$

$$x_{id} = x_{id} + v_{id} \quad (2)$$

The first part of equation (1) represents the inertia of the previous velocity, the second part is the cognition part and it tells us about the personal thinking of the particle, the third part represents the cooperation among particles and is therefore named as the social component [11]. Acceleration constants c_1, c_2 [12] and inertia weight ω [13] are the predefined by the user and r_1, r_2 are the uniformly generated random numbers in the range of [0, 1].

B. ARPSO Algorithm

The ARPSO algorithm is diversity guided BPSO in which the behavior of the swarm is controlled as per the variation in diversity. The swarm population keeps shuttling between the phases of attraction and repulsion according to

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1-4244-1340-0/07/$25.00 ©2007 IEEE

the increase or decrease in diversity measure. In the attraction phase, the particles come towards each other following equation (1) as they do in BPSO. The movement of particles towards each other causes a gradual decrease in diversity of the population. When the diversity becomes lower then a certain specified value d_{low}, a repulsion phase obtained by inverting the velocity update formula is activated according to the following equation:

$$v_{id} = wv_{id} - c_1 r_1 (p_{id} - x_{id}) - c_2 r_2 (p_{gd} - x_{id}) \tag{3}$$

In this phase the particles are no longer attracted towards each other but move away or from each other. This generates a perturbation in the population and causes an increase in the diversity of the swarm population. The swarm particles stay in this phase until the diversity reaches a higher value d_{high}. As soon as the desired high diversity d_{high} is achieved, the swarm particles again come back to the attraction phase and the same process continues iteratively until the global optimum is obtained.

C. ATRE-PSO: A Variation of ARPSO

The ATRE-PSO proposed by us in this article is just a simple extension of ARPSO, where we have assumed a third phase which we call the *in between* phase or the phase of *positive conflict*. It is quite natural to think that (diversity <) d_{low} and (diversity >) d_{high} may not be the only two possibilities for deciding the movement of the swarm, but many times the diversity may lie in between the two threshold values. For this reason we decided to introduce a third phase which is activated when the diversity is greater than d_{low} but less then d_{high}. In ATRE-PSO, instead switching between just the attraction and the repulsion phase only, the swarm enters the third phase defined as

$$v_{id} = wv_{id} + c_1 r_1 (p_{id} - x_{id}) - c_2 r_2 (p_{gd} - x_{id}) \tag{4}$$

In this phase there is neither complete attraction nor a complete repulsion. The individual particle is attracted by its own previous best position p_{id} and is repelled by the best known particle position p_{gd} . In this way there is neither total attraction nor total repulsion but a balance between the two.

The swarm particles are guided by the following rule

$$v_{id} = \begin{cases} wv_{id} + c_1 r_1 (p_{id} - x_{id}) + c_2 r_2 (p_{gd} - x_{id}), div > d_{high} \\ wv_{id} + c_1 r_1 (p_{id} - x_{id}) - c_2 r_2 (p_{gd} - x_{id}), d_{low} < div < d_{high} \\ wv_{id} - c_1 r_1 (p_{id} - x_{id}) - c_2 r_2 (p_{gd} - x_{id}), div < d_{low} \end{cases}$$

The diversity (div) measure of the swarm is taken as [14]:

$$Diversity(S(t)) = \frac{1}{n_s} \sum_{i=1}^{n_s} \sqrt{\sum_{j=1}^{n_x} (x_{ij}(t) - \overline{x_j(t)})^2} \tag{5}$$

where S is the swarm, $n_s = |S|$ is the swarm size, n_x is the dimensionality of the problem, x_{ij} is the j'th value of the i'th particle and $\overline{x_j(t)}$ is the average of the j-th dimension over all particles, i.e.

$$\overline{x_j(t)} = \frac{\sum_{i=1}^{n_s} x_{ij}(t)}{n_s}$$

The idea behind the introduction of third phase is to improve the exploring and exploiting capabilities of ARPSO.

III. Experimental Settings

In order to make a fair comparison of BPSO, ARPSO and ATRE-PSO, we fixed the same seed for random number generation so that the initial swarm population is same for all the three algorithms. The number of particles in the swarm (swarm size) was 30. A linearly decreasing inertia weight is used which starts at 0.9 and ends at 0.4, with the user defined parameters c_1=2.0 and c_2=2.0. The diversity controlling parameters d_{low} and d_{high} were taken as $5.0*10^{-6}$ and 0.25 respectively. For each algorithm, the maximum number of iterations allowed was set to 10,000. A total of 30 runs for each experimental setting were conducted and the average fitness of the best solutions throughout the run was recorded.

For evaluating the modified ATRE-PSO, we used a test suite of 19 standard benchmark functions. The test suite consists of a diverse set of problems of different dimensions including unimodal and multimodal functions and also a noisy test function. The dimensions of the problems vary from 2 to 20. In Table 1, the benchmark problems have been listed.

IV. Experimental Results

The results of the benchmark problems f_1 - f_{19} are shown in Table II in terms of mean best fitness, diversity and standard deviation. Fig. 1 shows the mean best fitness curves and the diversity curves for selected benchmark problems. Functions starting from f1 to f8 are of dimensions 20. For f1, ATRE-PSO gave a better result than ARPSO and BPSO. f2 is a very simple sphere function and all the algorithms converged to the global minimum. However, ATRE-PSO took slightly longer time to converge. On f3 and f4 ATRE-PSO gave a better performance in comparison to ARPSO and BPSO. Function f5 is a noisy function and as expected BPSO was not as efficient as other two, ATRE-PSO gave a better fitness function value than ARPSO. For f6 also ATRE-PSO was better than ARPSO and BPSO. For some reason BPSO outperformed the other algorithms in f7, but for f8 again it was ATRE-PSO which was better than ARPSO and BPSO. Functions f9 onwards are of two variables problems and all the algorithms gave more or less similar results. But there also in some cases ATRE-PSO gave a better fitness function value. Although f19 is known to be a notorious function because of the presence of several local and global minima, surprisingly all the algorithms converged to the exact global minimum

The slow convergence of ATRE-PSO in some cases is quite natural and expected because of the presence of an additional phase. However, a noticeable thing about it is the fact that it maintains a high level of diversity without disturbing the convergence.

TABLE I
NUMERICAL BENCHMARK FUNCTIONS.

REMARKS: 1. FUNCTIONS SINE AND COSINE TAKE ARGUMENTS IN RADIANS.
2. THE FUNCTION u USED IN f_8 AND THE MATRIX a USED IN f_9 ARE ALL DEFINED IN THE APPENDIX.

Function	Dim	Ranges	Mini. Value
$f_1(x)=\sum_{i=1}^{n}(x_i^2-10\cos(2\pi x_i)+10)$	20	[-5.12,5.12]	$f_1(\overline{0})=0$
$f_2(x)=\sum_{i=1}^{n}x_i^2$	20	[-5.12,5.12]	$f_2(\overline{0})=0$
$f_3(x)=\frac{1}{4000}\sum_{i=0}^{n-1}x_i^2-\prod_{i=0}^{n-1}\cos(\frac{x_i}{\sqrt{i+1}})+1$	20	[-600,600]	$f_3(\overline{0})=0$
$f_4(x)=\sum_{i=0}^{n-1}100(x_{i+1}-x_i^2)^2+(x_i-1)^2$	20	[(-30,30]	$f_4(\overline{1})=0$
$f_5(x)=(\sum_{i=0}^{n-1}(i+1)x_i^4)+rand[0,1]$	20	[-1.28,1.28]	$f_5(\overline{0})=0$
$f_6(x)=-\sum_{i=1}^{n}x_i\sin(\sqrt{\lvert x_i\rvert})$	20	[-500,500]	$f_6(\overline{420.97})$ = -418.9829 x n
$f_7(x)=-20\exp(-0.2\sqrt{\frac{1}{n}\sum_{i=1}^{n}x_i^2})-\exp(\frac{1}{n}\sum_{i=1}^{n}\cos(2\pi x_i))+20+e$	20	[-32,32]	$f_7(\overline{0})=0$
$f_8(x)=(0.1)\{(\sin(3\pi x))^2+\sum_{i=0}^{n-2}[(x_i-1)^2(1+(\sin(3\pi x_{i+1}))^2)]$ $(x_n-1)(1+(\sin(2\pi x_n))^2)\}+\sum_{i=0}^{n-1}u(x_i,5,100,4)$	20	[-50,50]	$f_8(\overline{1,1,\ldots,-4.76})$ = -1.1428
$f_9(x)=(\frac{1}{500}+\sum_{j=0}^{24}(j+1+\sum_{i=0}^{1}(x_i-a_{ij})^6)^{-1})^{-1}$	2	[-65.54,65.54]	$f_9(\overline{-31.95})$ = 0.998
$f_{10}(x)=4x_0^2+2.1x_0^4+\frac{1}{3}x_0^6+x_0x_1-4x_1^2+4x_1^4$	2	[-5,5]	$f_{10}(\overline{0.09,-0.71})$ = -1.0314
$f_{11}(x)=(x_1-\frac{5.1}{4\pi^2}x_0^2+\frac{5}{\pi}x_0-6)^2+10(1-\frac{1}{8\pi})\cos(x_0)+10$	2	[-5,15]	$f_{11}(\overline{9.42,2.47})$ = 0.397886
$f_{12}(x)=\{1+(x_0+x_1+1)2(19-14x_0+3x_0^2-14x_1+6x_0x_1+3x_1^2)\}$ $\{30+(2x_0-3x_1)^2(18-32x_0+12x_0^2+48x_1-36x_0x_1+27x_1^2)\}$	2	[-2,2]	$f_{12}(\overline{0,-1})=3$
$f_{13}(x)=0.5+\frac{(\sin\sqrt{x^2+y^2})^2-0.5}{(1+0.001(x^2+y^2))^2}$	2	[-100,100]	$f_{13}(\overline{0})=0$
$f_{14}(x)=(x_2+x_1^2-11)^2+(x_1+x_2^2-7)^2+x_1$	2	[-5,5]	$f_{14}(\overline{-3.788,-3.286})$ = -3.783962
$f_{15}(x)=-\cos(x_1)\cos(x_2)\exp(-((x_1-\pi)^2+(x_2-\pi)^2))$	2	[-100,100]	$f_{15}(\overline{3.1416,3.1416})$ = -1
$f_{16}(x)=\sum_{i=1}^{n}x_i^2+\sum_{i=1}^{n}(0.5ix_i)^2+\sum_{i=1}^{n}(0.5ix_i)^4$	2	[-10,10]	$f_{16}(\overline{0})=0$
$f_{17}(x)=x_1^2+2x_2^2-0.3\cos(3\pi x_1)-0.4\cos(4\pi x_2)+0.7$	2	[-100,100]	$f_{17}(\overline{0})=0$
$f_{18}(x)=-\sum_{i=1}^{n}\sin(x_i)(\sin(i\frac{x_i^2}{\pi}))^{2m}$, m = 10.	2	[-π,π]	$f_{18}(\overline{2.25,1.57})$ = -1.801
$f_{19}(x)=\sum_{j=1}^{5}j\cos((j+1)x_1+j)\sum_{j=1}^{5}j\cos((j+1)x_2+j)$	2	[-10,10]	$f_{19}(\overline{x})$ = -186.7309

TABLE II

RESULTS FOR ALL ALGORITHMS ON BENCHMARK PROBLEMS OF DIMENSIONALITY 20 OR LESS (MEAN OF 30 RUNS, DIVERSITY AND STANDARD DEVIATIONS (STDDEV)).

Fun.	BPSO			ATREPSO			ARPSO		
	Mean	Diversity	Stddev	Mean	Diversity	Stddev	Mean	Diversity	Stddev
f_1	22.339158	0.000115	15.932042	19.425979	7.353246	14.349046	22.305996	0.000115	15.95881
f_2	1.167749 e-45	2.426825 e-23	5.222331 e-46	4.000289 e-17	8.51205	0.000246	8.551321 e-19	0.005389	8.761187 e-19
f_3	0.031646	0.000710	0.025322	0.025158	574.74139	0.02814	0.031155	0.000710	0.025406
f_4	22.191725	2.551408	1.615544 e+04	19.49082	1.586547	3.964335 e+04	22.191721	1.586547	3.965460 e+04
f_5	8.981602	0.340871	9.001534	8.046617	2.809409	8.862385	8.144808	0.340871	8.861699
f_6	-6178.5599	0.072325	4.893329 e+02	-6183.6776	199.95052	469.611104	-6154.7202	0.072325	4.801525 e+02
f_7	3.483903 e-18	3.651635 e-18	8.359535 e-19	0.018493	42.596802	0.014747	8.195287 e-10	0.001593	5.136632 e-10
f_8	-1.150072	0.000935	0.001972	-1.149317	55.844364	0.003303	-1.150072	0.000935	0.001972
f_9	1.000000	0.033500	1.152356 e-07	1.000000	21.500517	1.152356 e-07	1.000000	0.033500	1.192093 e-07
f_{10}	-1.031364	0.142718	2.304713 e-07	-1.031364	2.138688	2.304713 e-07	-1.031364	0.142718	2.384186 e-07
f_{11}	0.399886	8.239762 e-05	6.963319 e-15	0.397886	1.550852	8.34465 e-08	0.398886	8.239762 e-05	8.940697 e-08
f_{12}	3.000000	1.17719 e-05	0.000000	3.000000	1.35496	4.352908 e-08	3.000000	1.17719 e-05	0.000000
f_{13}	0.000000	4.273782 e-09	0.000000	0.000000	29.148478	0.000000	0.000000	0.001322	0.000000
f_{14}	-3.331488	2.747822 e-05	1.24329	-3.751458	3.214462	0.174460	-3.331488	2.747822 e-05	1.243290
f_{15}	-1.000000	8.519092 e-05	0.000000	-1.000000	31.124617	0.000000	-1.000000	8.519092 e-05	0.000000
f_{16}	0.000000	2.495595 e-25	0.000000	0.000000	2.636646	0.000000	0.000000	0.611920	0.000000
f_{17}	-5.551115 e-17	3.292352 e-11	0.000000	-5.751115 e-17	25.291439	0.000000	-5.551115 e-17	0.004130	0.000000
f_{18}	-1.774591	2.606517 e-05	0.143838	-1.801301	0.816007	1.771024 e-06	-1.774591	2.606517 e-05	0.143838
f_{19}	-186.730941	0.362224	1.424154 e-05	-186.730941	5.410105	1.424154 e-05	-186.730942	0.070456	1.525879 e-10

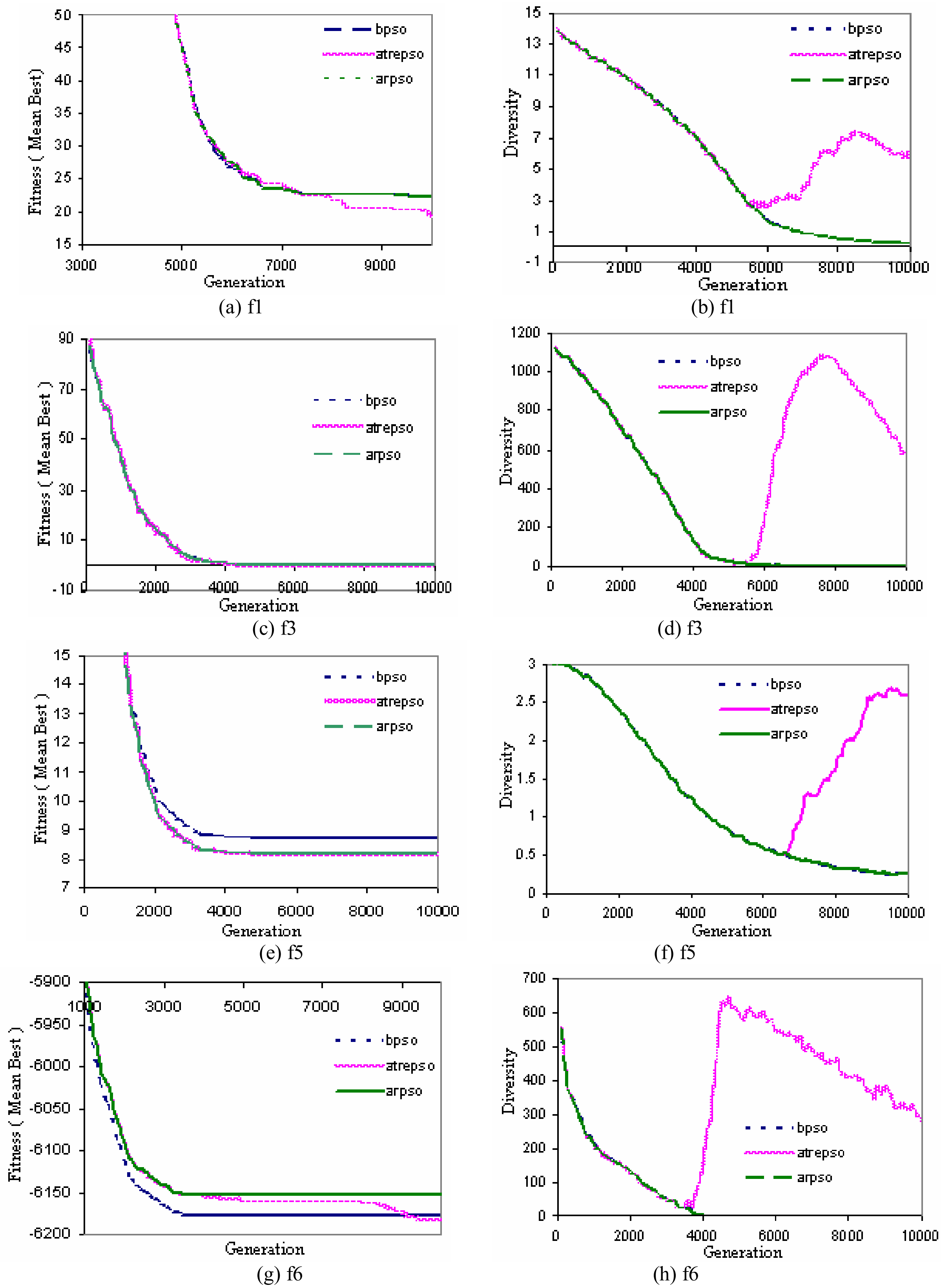

Fig. 1. Mean best fitness and Diversity curves for selected benchmark problems. All results are means of 30 runs

V. CONCLUSION

It is a well known fact that maintaining a high diversity while preserving fast convergence are two contradicting features. As far as the diversity is concerned ATRE-PSO did a very good job of maintaining a high level of diversity. In terms of average fitness function value also, ATRE-PSO gave a better performance than BPSO and ARPSO in many test problems, but since the experiments have been done on a rather narrow platform, making any definite or concrete conclusion is not justified. We may say that ATRE-PSO is only a variant of BPSO or ARPSO (or any diversity guided PSO). On philosophical terms we may say that it represents the complexity of human nature which is most of the times in a state (or phase) of interpersonal conflict (which we have called the phase of positive conflict). A lot of work, both theoretically and experimentally, has to be done to make any strong judgment about the ATRE-PSO.

APPENDIX

f_8 : $u(x, a, b, c) = b\,(x-a)^c$ if $x > a$,
$u(x, a, b, c) = b\,(-x-a)^c$ if $x < -a$,
$u(x, a, b, c) = 0$ if $-a \le x \le a$.

$$f_9 : \quad a = \begin{pmatrix} -32,-16,0,16,32,...,-32,-16,0,16,32 \\ -32,...,-16,...,0,...,16,...,32,... \end{pmatrix}$$

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