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O.R. Application

Partial integration of frequency allocation within antenna positioning in GSM mobile networks

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Abstract

In this article we propose to partially integrate the *antenna positioning* (APP) and *frequency allocation problems* (FAP). The traditional wireless network design process examines these two major issues sequentially in order to avoid the very high complexity associated with the simultaneous resolution of the two problems. The proposed integration involves the introduction of interference protection guarantees within the APP. It is customary to define such guarantees in an intermediate step and to use them as input to FAP, in order to protect against interference in critical areas. The proposed approach consists of selecting these protections while solving the APP, allowing the optimization procedure to exploit the degrees of freedom that this would offer. Results on two real-life problem instances indicate a significant improvement in interference levels and resource utilization.

Keywords: OR in telecommunications; Tabu search; Antenna positioning; Frequency allocation; GSM

1. Introduction

In a GSM network a set of *transceivers* (TRX) transmit and receive calls through an antenna using radio frequencies. Each antenna provides service to a small area called a *cell*. The network design and upgrading process is concerned with decisions involving the deployment of equipment and radio resources so as to manage multiple design criteria. The *antenna positioning problem* (APP) and the *frequency assignment problem* (FAP) are two major components of the design process. Automatic tools for an efficient management of the network are increasingly critical for network operators due to competition and to the growth in the customer base.

In the FAP the aim is to allocate frequencies to TRXs in the network subject to constraints on the frequency set and to separation constraints which state that if TRXs i and j are assigned frequencies f and g, then $|f-g|>\delta_{ij}$. Several FAP formulations exist. They aim at minimizing the number or the range of frequencies used, or at minimizing total interference, the latter being the most common. A more extensive review of FAP models and solution approaches is provided by Aardal et al. (2001) and Touhami (2004).

Generally speaking, an APP model is built on a set of test points, M, which describe the service area. The radio propagation matrix describes the degradation of the signal strength from a set of the candidate antennas, N, to the test points. The purpose of the APP is to select a subset of antennas from N so as to provide service to the test points by providing a strong enough signal and ensuring call continuity as users move around. The APP is a multi-objective design problem involving coverage, capacity, costs and interference. A variety of models for the APP exist in the literature (see, e.g., Touhami (2004) and Touhami et al. (2006) for a review of these studies). In this paper, we focus on how to relate the APP and FAP in order to improve the overall network design.

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In addition to developing effective optimization algorithms for the APP and FAP, it is important to integrate the two problems. Traditionally, the design process is sequential: first the APP is solved assuming that all antennas operate on the same frequency (i.e., assuming there is no channel assignment) and thereafter the FAP is solved given the APP solution as input. A major drawback of the sequential decomposition with little or no integration is that the resulting solution is almost certain to be sub-optimal. In fact, the assignment of frequencies in minimum interference FAP aims at minimizing the area with high co-channel and adjacent-channel interference. This occurs when two transmitters use the same frequency or adjacent frequencies. At the same time, minimizing interference in the APP determines decisions such as which antennas to activate and their parameter settings, and hence determines the tradeoffs with respect to a number of design criteria (coverage, capacity, costs, interference). Thus by assuming a single frequency in the APP, the optimization attempts to reduce interference which may be eliminated by the FAP solution anyway. Therefore, the sequential decomposition approach is overprotective with respect to interference, at the expense of other design criteria. Within the sequential decomposition approach, some APP models do not include interference considerations. Even a good FAP algorithm may not be able to provide an acceptable level of interference for these cases.

In networks with Hierarchical Cell Structure (HCS) one would have a number of cells covering small areas with high traffic density and some larger cells covering several of the small cells. The top layer cells would capture all traffic that could not be served by the lower layer small cells. Such systems are increasingly used for high density networks. In such a context, cells in different layers will cover the same area and so the potential interference will be very high. If one is to use a co-channel interference scenario (as in sequential decomposition design), a good APP algorithm would try to avoid interference, and so by default the model has a negative bias against an HCS design. However, when the model provides a mechanism to protect cells in different layers the negative bias is eliminated and the HCS design may become an attractive alternative.

The next section reviews related work. In Section 3 we provide a complete description of the extended model for partial integration of the FAP and the APP, and in Section 4 we present the solution approach. Section 5 contains the details of the experimental study.

2. Literature review

The interest for APP seems to have arisen fairly late in comparison to FAP. Indeed, the first published article on this problem (Anderson and McGeehan (1994)) appeared in 1994 (see, e.g., Raisanen and Whitaker (2005)). After that date, however, it has been widely studied, notably in the context of three European Union-funded projects, namely, ARNO (Algorithms for Radio Network Optimization), STORMS (Software Tools for the Optimization of Resources in Mobile Systems), and MOMENTUM (Models and Simulation for Network Planning and Control of UMTS).

In Mühlenbein (1999), APP and FAP are treated as two sequential problems: "After the [Site Positioning] task has been completed, the next step is finding an assignment of frequencies to the antennas (the FAP problem)". Hurley (2002) hints at the sub-optimality of this approach: "If a radio network is poorly designed, then spectrum will be wasted and/or the quality of service will be degraded even if a good frequency assignment algorithm is used". Moreover, spectrum waste and service degradation may well occur even if the base stations are well configured, if this solution is obtained without consideration to FAP, the problem that comes next.

The idea of integrating several aspects of the network design problem is carried out in (Lissajoux et al., 1998), and particularly in (Reininger and Caminada, 2001), as part of the ARNO Project. In the latter, the authors partially relate APP and FAP by "optimizing location and parametrization of [the base stations] on one shot". They propose a model that considers three objectives, namely, minimum cost, maximum carried traffic and minimum "undesirable" overlap among cells, the latter being aimed at reducing total interference. As for the constraints, the aim at controlling the quality of downlink and uplink communications, facilitating handover, and ensuring that the cells are connected and have some desirable geometrical properties. The integration of locating and configuring base stations is carried further to UMTS networks by Amaldi et al. (2006). Specifically, the problem they consider is "to select the location and configuration of the base stations so as to minimize installation cots and to meet the traffic demand as well as possible". In (Jedidi et al., 2004), a trade-off is sought between minimum overlap and desirable cell shapes while the quality of radio coverage is controlled in the constraints. Zimmermann et al. (2003) also develop a multi-criteria model that involves minimum cost, minimum interference and optimum cell shapes. Jedidi et al. (2004) and Zimmermann et al. (2003) were also part of the ARNO Project.

The Final Report of the STORMS Project, carried between 1995 and 1998 (see Menolascino and Pizarroso, 1999), presents a suite of planning tools for UMTS. These include identification of cell sizes, radio coverage optimization, transmission power planning, dimensioning of radio communication, allocation of base station controllers, etc. Within this project, Cullen (1998) studies sequentially the selection of base stations and the dimensioning of radio resources.

Eisenblätter et al. (2003) was funded by the MOMENTUM Project. It assumes that the set of mobiles is partitioned into "snapshots" of approximately 540 mobiles, and integrates the following aspects of UMTS network design: site selection, antenna configuration, mobile assignment and power assignment. Eisenblätter et al. (2006) considers a similar problem in a larger context where snapshots are not considered. Finally, Whitaker and Hurley (2004) integrate site selection, infrastructure configuration and the estimation of a lower bound on the required number of sites.

Other models that simultaneously solve the APP and FAP have been proposed in the literature. In order to reduce problem complexity, these studies introduce several types of simplifications either to the APP or FAP aspects. For instance, Floriani and Mateus (1997) use a simplified definition of coverage and capacity, and consider frequency allocation through pattern reuse. This technique is based on the partitioning of the available set of frequencies into non-interfering groups. This grouping is not a function of interference but on the reuse distance, i.e., the distance reutilizing the same frequency group. This method may be effective under some very restrictive assumptions such as uniform traffic distribution, uniform signal propagation and the absence of restrictions on the frequency use (a frequency may not be allowed for some antennas due to some outside constraints). It has been argued that realistic networks rarely if ever obey such assumptions, making pattern reuse an easy but not so effective method for frequency allocation (see e.g., Bourjolly et al. (2002)).

The model proposed by Huang et al. (2000) focuses on minimizing the number of frequencies used and so the frequency allocation is integrated with the capacity aspects of the APP only. Mathar and Schmeink (2001) propose a model that maximizes the traffic captured in the network while ensuring that the interference at every point in the network is below a specified threshold value. For high traffic spots, finding a feasible frequency assignment may be difficult unless the threshold value is set high enough, which reduces the ability of the proposed model to manage interference (which was the initial objective behind the integration of the APP and FAP).

Mazzini et al. (2003) consider a network design problem that includes APP, FAP and the design of a fixed network. The latter problem refers to determining the topology, link type and capacity of the fixed network. The fixed network consists of a set of fixed switches that route calls through the network or to other networks. The model is designed to find feasible frequency assignments that exclude high interference between pairs of already specified antennas. This approach ignores the remaining interference which may be still high especially when considering the combined effect of antennas. In addition, the transmission power for each antenna in the candidate set is already determined (instead of being decided within the model). To cover the full range of transmission power, the candidate antenna set needs to be very large. Furthermore, the capacity planning and hence the frequency requirements are incorporated in the model, based on deterministic computations, ignoring the stochastic nature of traffic.

Despite the simplifying definitions and assumptions, models that simultaneously solve the APP and FAP are still very large and complex. The proposed models are often tested on small or simplified problem instances. There is a need to better integrate the design process. Second generation networks, such as GSM systems, still represent a very large portion of the technologies deployed, and operators wish to extend the life cycle of their investments as much as possible before moving to next generation networks. The effective management of 2G networks may help with this. Furthermore, techniques used to manage 2G networks may be adapted to work for the next generation systems.

In this paper we propose to partially relax the assumption of single frequency network which was used for an earlier model (see Touhami et al., 2006) by partially integrating FAP considerations without the high cost of frequency assignment. The single frequency model integrates several design aspects including precise characterization of costs, interference (by computing the combined effect of all interfering sources), coverage, capacity planning (by considering the stochastic nature of traffic through using the Erlang B function). The partial integration with the FAP involves the introduction of interference protection guarantees within the APP. This implies that the APP model will also select pairs of antennas that are guaranteed to use separate frequencies. This information increases the degrees of freedom of the APP optimization process. In order to satisfy these protection decisions, this information is passed to the FAP within the separation constraints. To solve this extended APP model, we add a fast deterministic antenna separation procedure to the tabu search algorithm proposed in a previous paper (see Touhami et al., 2006).

3. Problem formulation

3.1. Definitions and basic APP model

In this section we focus on presenting the extensions made to the single frequency APP model. Given a candidate set of antennas, the proposed model for the APP involves determining a set of antennas to activate and selecting their transmission power. We define:

M: the set of antennas in the candidate list;
K: the set of sites, corresponding to the antenna set in the candidate list;
N: the set of test point of interest;
a_n: the volume of traffic offered at test point n;
$x_i = 1$ if antenna i is active, 0 otherwise: these decision variables indicate if an antenna is active;
$w_k = 1$ if $x_i = 1$ for some $i \in k, 0$ otherwise: this is the indicator variable associated with every site;
q_{in}: the degradation in signal strength from antenna i received at test point n, expressed in dBm;
p_i: the transmission power of antenna, expressed in dBm. Thus, for any test point n, the received signal from antenna i is $p_{in} = x_i(p_i + q_{in})$;

θ: the adjacent channel protection (set at -18 dBm in the test instances);
$p'_{in} = x_i(p_i + q_{in} + \theta)$: the equivalent adjacent-channels signal strength: when two signals are on adjacent channels, they interfere on each other but the level of interference is lower than if they were on the same one. The interference is equivalent to interference on the same channel but with a weaker signal because of the adjacent channel protection;
$p_{\min}$: the threshold on acceptable signal strength, expressed in dBm; for a communication to take place at a particular test point, we need $p_{in} > p_{\min}$;
Δ_i: the discrete set of possible values of p_i for antenna. For instance, for some antenna we may have $\Delta_i = \{\text{off}, 45, 46, 47, 48\}(\text{dBm})$;
$y_{in} = 1$, if $p_{in} = \max\{p_{jn}\}$ and $p_{in} > p_{\min}$, otherwise; this variable indicates if antenna i is the server of test point n by providing the strongest signal;
γ_n: the interference level at test point n, (this will be defined below);
$z_n = 1$, if $\gamma_{\text{n}} \leqslant \gamma^1, 0$, otherwise. This variable indicates if test point n is subject to high interference; γ^1 is the interference threshold below which CIR is considered too low.

Every APP solution provides an assignment of test points to antennas, as each point is assigned to the antenna with the strongest signal. The number of TRXs needed to provide the communication capacity is computed by an M/M/s queue with infinite sources and where blocked requests are lost (see Molina et al., 2000; Vasquez and Hao, 2001; Touhami et al., 2006). For technical reasons, this system also has a limit on the maximum number of TRXs/servers that can be installed. For the example used in this work, a maximum of 4 TRXs can be associated with every antenna. Table 1 shows the volume of traffic that can be served as a function of the number of TRXs installed for a 2% blocking probability, where traffic is expressed in Erlang or in the number of subscribers (0.022 Erlang per subscriber). If the offered traffic exceeds the capacity provided by the TRXs installed, then the excess traffic is not carried and hence is blocked.

3.2. Traffic overflow management

Consider the situation in which one antenna covers an area where the offered traffic exceeds the maximum capacity (number of TRXs) that could be installed in it, resulting in blocked traffic. Under certain conditions, the blocked traffic can be served by its second best server. We refer to this as *traffic overflow*. Hence, for a particular antenna we need to distinguish between direct offered traffic and overflow offered traffic. If we ignore this second type of traffic, then the number of TRXs installed may not be enough to satisfy the total demand, leading to more overflow traffic on some other antennas and more blocked traffic. In such case, one might observe a more severe traffic blocking than expected and hence a reduced quality of service. This concept may come to play an important role also in the context of HCS networks, where customers not served by lower level cells are redirected to higher level cells.

By definition, the signal from the best server is stronger than that from the second. So if both antennas use the same frequency, then the communication cannot be established on the second best server as the interference would be excessively high. Thus, for traffic overflow to take place, the first and second servers need to be on different frequencies (in the worst case, the best sever and the second server use adjacent frequencies). Also, for overflow to occur from antenna i to j, we need to have $p_{in} - p_{jn} < v$, where v is some fixed parameter. In the example we examine in this research, we have $v = 4$ dBm. This condition indicates that traffic overflow may occur on the cell boundaries only, which may correspond roughly to the handoff region. Although traffic overflow may occur on the third, or fourth best server (or further), we only consider the second best server in order not to increase the problem complexity too much. In addition, design engineers indicated to us that too much traffic overflow is not recommended. Traffic blocked beyond the second server is managed by a constraint that limits the fraction of blocked traffic. Using conversion Table 1, and assuming a maximum of 4 TRXs per antenna, we define the following:

$v_{in} = 1$ if $p_{in} = \max_{j:y_{jn}=0}\{p_{jn}\}, 0$ otherwise: these variables identify the second best server for each test point n;
$o_i = \sum_{n\in N} a_n y_{in}$: the total direct offered traffic at antenna;
$o_i\prime = o_i + \sum_{j\in M}\sum_{n\in N} a_n v_{jn}\delta^1_{ij} b_j / o_j$: the first term computes the direct offered traffic, while the second term compiles the fraction of blocked traffic served by other antennas that overflows on antenna as the second best server. δ^1_{ij} is a variable that ensures that overflow does not occur on the same frequency (details provided next).
$d_i = 1$ if $o_i \leqslant 132.1$; 2 if $132.1 < o_i \leqslant 405.5$; 3 if $405.5 < o_i \leqslant 709.3$; 4, otherwise: the number of TRXs to be installed at antenna; as a function of direct traffic only.
$b_i = 0$ if $o_i \leqslant 1027.3$; $o_i - 1027.3$ otherwise: the blocked traffic at antenna. This corresponds to the direct traffic

Table 1
Capacity and number of TRXs at 2% blocking, 0.022 Erlang per user

Number of TRXs	1	2	3	4	5	6	7
Traffic carried (Erlang)	2.93	9.00	15.76	22.82	30.05	37.45	44.90
Number of subscribers served	132.1	405.5	709.3	1027.3	1352.4	1685.4	2020.7

offered to antenna but that cannot be carried because not enough TRXs could be installed;

$c_i = o_i$ if $o_i \leqslant 1027.3, 1027.3$ otherwise: the carried direct traffic at antenna. This corresponds to the traffic served by antenna that could be processed by the installed TRXs;

$cap_i = 132.1$ if $d_i = 1$; 405.5 if $d_i = 2$; 709.3 if $d_i = 3$;1027.3 if $d_i = 4$: the capacity of antenna i, expressed as the total number of subscribers, offered in direct traffic, that can be served by the TRXs installed;

$e_i = 132.1 - o_i$ if $d_i = 1$; $405.5 - o_i$ if $d_i = 2$; $709.3 - o_i$ if $d_i = 3$;$1027.3 - o_i$ if $d_i = 4$; 0 otherwise: the excess capacity at antenna i. This corresponds to additional direct traffic that the currently installed number of TRXs could carry;

d'_i, b'_i, c'_i, cap'_i, e'_i: these variables are similar to, except they are a function of the combined direct and overflow traffic o'_i;

c^a_i: the cost of antenna;

c^s_k: the cost of site k;

c^t: the cost of a single TRX t.

3.3. Performance criteria

The evaluation of any network configuration, S, is based on four dimensions: network coverage, network capacity, network interference, and network cost and profit. Any of these criteria can be incorporated in the objective function or in the problem constraint set. We use the following definitions to describe capacity and coverage-related performances of any given configuration S:

$A(S) = \sum_{i\in M} x_i$: the total number of active antennas in configuration S;

A^*: the maximum number of antennas that can be activated;

$U(S) = \sum_k w_k$: the total number of active sites in configuration S;

U^*: the maximum number of sites that can be opened;

$T(S) = \sum_{i\in M} d'_i$: the total number of installed TRXs in configuration S;

T^*: the maximum number of TRXs that can be installed;

$Cov(S) = \sum_{i\in M}\sum_{n\in N} y_{in}$: the total covered area in configuration S;

Cov^*: the minimum required level of coverage, expressed as a percentage of the total area;

$O(S) = \sum_{i\in M} o'_i$: the total network offered traffic in configuration S. This term measures the traffic coverage of configuration S and provides an estimate of the maximum revenue that can be generated by S;

O^*: the minimum required covered traffic volume, expressed as a percentage of the total offered traffic;

$C(S) = \sum_{i\in M} c'_i$: the total network carried traffic in configuration S. This term corresponds to the fraction of offered traffic that can be served by the installed TRXs;

C^*: the minimum required carried traffic, expressed as a percentage of the total offered traffic;

$B(S) = \sum_{i\in M} b'_i$: the total network blocked traffic in configuration S. Traffic is blocked when not enough TRXs could be installed at some antenna. This term measures lost revenue as well as the quality of service. Blocked communications may be frustrating to customers, and hence affect the perceived quality of service;

B^*: the maximum allowed blocking, expressed as a percentage of the traffic covered;

$Cap(S) = \sum_{i\in M} cap'_i$: the total network capacity in configuration S. This term describes the total number of customer that can be served by the proposed configuration. S;

$E(S) = \sum_{i\in M} e'_i$: the total network excess capacity in configuration S. This term describes the equipment efficiency of the proposed configuration and provides an estimate of the slack resources built in the proposed configuration (which could be seen as provision for future network development);

E^*: the maximum allowed excess capacity, expressed as a percentage of total network capacity.

Assuming that all signals are on the same frequency (co-channel interference case), the CIR level at any test point n is defined as the ratio of the serving signal to the combined effect of all interfering signals. This is given by

$$\gamma_n = \sum_{i\in M} p_{in} y_{in} - 10\log_{10} \sum_{j\in M} 10^{p_{jn}(1-y_{jn})/10} \tag{1}$$

The first term in Eq. (1) identifies the server signal of the test point under consideration, while the second term computes the combined effect of all interfering signals. Note that since power terms are expressed in dBm, they need to be transformed back to mW and added to determine their combined effect. We would like to have the CIR at all test points as high as possible. After solving the FAP, not all antennas in the summation in the second term of Eq. (1) will in fact interfere with the serving antenna of test point n. If we were to solve the APP and FAP simultaneously, we would need to include in this second term the variables that would indicate if the serving antenna is on an interfering frequency or not. This, however, would significantly add to the complexity of the problem. On the other hand, with a sequential resolution of the APP and FAP, while solving the APP, the assumption is that of a single frequency is assigned to all antennas. This means that all antennas are included in the second terms of Eq. (1). After solving the FAP, however, only a subset of antennas will in fact be assigned an interfering frequency. This means when evaluating the actual interferences in the network, i.e. after solving FAP, only a subset of antennas will be included in the second term of Eq. (1). This illustrates how the sequential decomposition approach is overprotective against interference. Furthermore, this has repercussions on the global network design aspects since the decisions made during the resolution of the APP attempt to find a balance between the cost, coverage, capacity and interference criteria.

3.4. Partial integration of APP and FAP

Our proposal for partial integration involves the introduction within the APP of guarantees that some pairs of antennas will use non-interfering frequencies. These indicate that a pair of antennas will not be assigned interfering frequency in an FAP solution. Such guarantees will allow any solution algorithm to the APP to drop some antennas from consideration in the second term of Eq. (1), and hence provide the algorithm with degrees of freedom when it comes to finding a compromise between the design objectives. These guarantees are satisfied during the resolution of the FAP by introducing separation constraints.

As stated earlier the aim of the FAP is to allocate frequencies to TRXs in the network subject to constraints on the frequency set and to separation constraints which state that if TRXs and are assigned frequencies f and g, then $|f-g| \geqslant \delta_{ij}$. Hence, $\delta_{ij} \geqslant 2$ indicates that cell i and j cannot use the same frequency or adjacent frequencies. If $\delta_{ij}=1$ then the worst acceptable case is adjacent-channel interference, whereas $\delta_{ij}=0$ indicates that no restrictions are imposed. In order to incorporate frequency-related information within the APP without solving the FAP, we propose to include the δ_{ij} as decision variables within APP. For each pair of antennas i and j, we define, and $\delta^2_{ij}=1$ if $\delta_{\mathrm{ij}} \geqslant 2$,0 otherwise.

If a solution to the APP indicates that $\delta^1_{ij}=1$, then in a feasible FAP solution, the worst that can happen is adjacent channel interference, if $\delta^2_{ij}=1$ then in a feasible FAP solution, antennas and will not interfere with each other, and if $\delta^1_{ij}=0$ and $\delta^2_{ij}=0$, then the pair of cells are not protected and no guarantees can be given with regard to interference. We need to define a modified measure of the CIR at every test point n to reflect the protection provided by separation guarantees. So we have

$$\gamma_n = \sum_{i\in M} p_{in}y_{in} - 10\log_{10} \sum_{\substack{j\in M\\ j\neq i*}} 10^{(p_{jn}(1-\delta^1_{i*j}-\delta^2_{i*j})+p'_{jn}\delta^1_{i*j})/10}, \tag{2}$$

where

$$i* = \arg\max_{i\in M \text{ and } x_i=1}\{p_{in}\}, \tag{3}$$

$$\delta^1_{ij}+\delta^2_{ij} \leqslant 1. \tag{4}$$

Note that if $\delta^1_{ij}=1$, indicating that as a worst case we would have adjacent-channel interference, we need to account for the adjacent channel protection. In such a situation, the interfering signal is no more p_{jn} but p'_{jn}. Constraint (4) indicates that only one type of separation constraint can be active.

Setting $\delta^2_{ij}=1$ for a pair of antennas i and j, would cancel out their mutual interference. Naturally, any algorithm would try to set all such variables to 1, canceling out all interferences. This is impossible for any practical size network (since a typical GSM operator would have only about 40 frequencies to serve hundreds of antennas). The same goes for the δ^1_{ij} variables. Therefore, we need to impose a constraint that will limit the number of protections that can be assigned to antennas. This constraint reflects the limited radio resources. To achieve this, we define ρ^1 and ρ^2 as the maximum allowed number of antenna pairs (i,j) such that $\delta^1_{ij}=1$ and $\delta^2_{ij}=1$, respectively. This requirement can then be translated to

$$\sum_{i,j\in M} x_i x_j \delta^1_{ij} \leqslant \rho^1 \quad \text{and} \quad \sum_{i,j\in M} x_i x_j \delta^2_{ij} \leqslant \rho^2 \tag{5}$$

Thus, for a given configuration S, two possible interference-related design criteria would be to maximize the average weighed CIR (weighed by the offered traffic), or minimize the traffic suffering from low CIR. Hence, we define

$$TQ(S)=\sum_{n\in N} a_n\gamma_n/O(S): \text{ the average weighed CIR across covered traffic in } N,$$

$$TR(S)=\sum_{n\in N} a_n z_n: \text{ the total traffic with CIR } <\gamma^1.$$

Operators only have a limited radio spectrum at their disposal. For realistic size networks, increasing the CIR would come at the expense of deploying more resources or reducing covered or carried traffic. In addition, at some CIR levels the communication quality is sufficiently acceptable and the benefits of a further CIR increase are imperceptible. The downside of maximizing average CIR is that the solution may have test points with unnecessarily high CIR levels while others suffer from significant interference levels. Therefore, we select minimizing the traffic with low CIR as the objective function. This also provides a link with FAP which employs the same type of criterion. After consulting with network design engineers, the threshold value, γ^1, is set at $\gamma^1 = 12$ dBm.

Operating costs and profit constitute the fourth dimension of evaluation for a network configuration. We define $Cost(S)=\sum_{k\in K} c^s_k w_k + \sum_{i\in N} c^a_i x_i + \sum_{i\in M} c^t d'_i$ as the total network operating costs. This corresponds to the summation of site, antenna and TRXs costs. We also define $Cost^*$ as the total available budget. Above we have listed a variety of design criteria that need to be managed simultaneously in order to obtain good APP solutions. This illustrates the need for an automatic optimization tool. For each criterion, we associate a target performance level. We can define Ω as the set of solutions achieving those target performance levels

$$\begin{aligned}\Omega = \{S:\ \text{such that}: Cov(s) \geqslant Cov_*|N|, O(S)\\ \geqslant O_*\sum_{n\in N} a_n,\ C(S)\geqslant C_*\sum_{n\in N} a_n,\ B(S)\\ \leqslant B^*O(S), E(S)\leqslant E^*Cap(S),\ Cost(S)\\ \leqslant Cost^*,\ A(S)\leqslant A^*,\ U(S)\leqslant U^*,\ T(S)\leqslant T^*\}.\end{aligned} \tag{6}$$

Note that the above definition of Ω does not include any interference-related design criterion. We opted to include such criteria in our model within the objective function. This is because by trying to minimize interference within the objective function, we link the APP to the FAP, which uses a similar interference reduction objective. Including interference consideration as a constraint requires determining an appropriate value for minimum average CIR or area with low CIR. This is not easy to achieve. Such considerations are more a consequence of the available resources and the business goals of the network operator. Any combination of the above criteria can be used as an objective function but because of these arguments, we use the objective function $F(S) = TR(S)$.

3.5. The extended APP model

We refer to the extended model as Ex-APP, and it is summarized as follows:

$$\text{(Ex-APP) Optimize } F(S) \tag{7}$$

Subject to

$$p_{in} = x_i(p_i + q_{in}) \quad \forall i \in M \text{ and } \mathrm{n} \in N, \tag{8}$$

$$0 \leqslant (p_{in} - p_{\min})y_{in} \quad \forall i \in M \text{ and } n \in N \tag{9}$$

$$P_{in} \geqslant p_{jn} - L(1 - x_i) - L(1 - y_{in}) \quad \forall i, j \in M \text{ and } n \in N, \tag{10}$$

$$p_{in} \geqslant p_{jn} - L(1 - x_i) - Ly_{in} - L(1 - v_{in}) \quad \forall i, j \in M \text{ and } n \in N \tag{11}$$

$$1 \geqslant \sum_{i \in M} y_{in} \quad \forall n \in N, \tag{12}$$

$$\gamma_n = \sum_{i \in M} p_{in} y_{in} - 10\log_{10} \sum_{\substack{j \in M \\ j \neq i*}} 10^{(p_{jn}(1-\delta^1_{i*j}-\delta^2_{i*j})+(p_{jn}+\theta)\delta^1_{i*j})/10} \quad \forall n \in N \text{ and } i* = \arg\max_{i \in M and x_i=1} \{p_{in}\}, \tag{13}$$

$$\delta^1_{ij} + \delta^2_{ij} \leqslant 1 \quad \forall i, j \in M, \tag{14}$$

$$\sum_{i,j \in M} x_i x_j \delta^1_{ij} \leqslant \rho^1 \tag{15}$$

$$\sum_{i,j \in M} x_i x_j \delta^2_{ij} \leqslant \rho^2 \tag{16}$$

$$Lz_n \geqslant (\gamma^1 - \gamma_n) \quad \forall n \in N \tag{17}$$

$$w_k \geqslant x_i \quad \forall i \in k \text{ and } \forall k \in K \tag{18}$$

$$S \in \Omega \tag{19}$$

$$p_i \in \Delta_i \quad \forall i \in M \tag{20}$$

$$x_i, y_{in}, z_n, w_k \quad \forall n \in N, \ \forall i \in M, \text{ and } \forall k \in K. \tag{21}$$

L is a large number. Eq. (8) are used to compute the received signals from all sources at each pixel. Constraints (9) ensure that a test point is covered only if the received signal exceeds the prescribed threshold. Constraints (10) and (11) identify the best server of the pixel and its second best server. Inequality (12) ensures that each test point has at most one best server and Eq. (13) are used to compute their CIR (best signal relative to the combined effect of all interfering signals). Note that the best server is used to compute direct traffic offered as well as in the CIR computations. The second best server is used to compute traffic overflow. The direct and overflow traffic are then combined to compute antenna capacity and blocking rate. Constraints (14)–(16) are used to manage the imposed antenna separation as presented above. Inequalities (17) are used to identify test point with low CIR and inequalities (18) are used to count the number of sites with at least one active antenna. The minimum acceptable network performance is specified in constraints (19). These are followed by the definition of the domains of the model variables.

4. Proposed heuristic algorithm

The non-linear integer programming model provided in the previous section highlights the complexity of the problem. Heuristic optimization can provide the flexibility needed to manage the variety of design criteria while handling non-linearity and allow for model enhancements. We propose to extend the tabu search algorithm proposed for the single frequency APP model (referred to as SF-APP, see Touhami et al., 2006 for details). This algorithm iterates between two phases: constraint management phase and system stabilization phase. During each phase the solution neighborhood is defined by different types of neighborhood operators: in the former phase, the operators favor solutions that would restore feasibility while in the latter phase the operators are more focused on managing the objective function. Due to the problem complexity, the solution evaluation time is long and so we opted to use targeted neighborhood operators in order to speed up the search. A penalty term, associated with each of the constraints used in defining Ω, is incorporated within the objective function.

A deterministic procedure is used for the assignment of separation guarantees. This procedure is called when evaluating APP solutions. By keeping the separation assignment outside the tabu search algorithm, we avoid increasing the size of the search space. In addition, by using a construction heuristic for the assignment of separations, the added complexity of the Ex-APP relative to SF-APP is limited. This also allows us to gain insights into the effect of introducing antenna separation since we maintain everything else unchanged.

The separation allocation procedure determines the values for the variables δ^1_{ij} and δ^2_{ij}, given the set of active antennas and their assigned power levels. This procedure is called while solving Ex-APP and can be applied to any other APP solution. In fact we use this procedure to allocate separations for the SF-APP solution as well as for the solution that was provided to us for the network instances tested in this paper.

The proposed separation procedure is a heuristic that attempts to allocate separations where they would be most profitable. Consider a test point n, its best server i^* and any other active antenna j, so we have $p_{i^*n} > p_{jn} > p_{\min}$. We define the two variables:

$$\varepsilon_n^c(j) = (p_{i*n} - p_{jn})/a_n \quad \text{and} \quad \varepsilon_n^a(j) = (p_{i*n} - p_{jn} - \theta)/a_n.$$

Both variables measure the relative impact caused by the interfering signal on the best signal, weighed by the inverse of the volume of offered traffic at n. The variable $\varepsilon_n^c(j)$ considers the case where the best and interfering signal are on the same frequency, whereas $\varepsilon_n^a(j)$ handles the case of signals on adjacent frequencies.

These quantities are indicative of the need to impose a separation between antennas i^* and j: the closer the two signals are from each other, the more we need to separate them; and the heavier the traffic at a test point, the more advantageous it is to separate them. Therefore, we would like to see that the test points for which $\varepsilon_n^c(j)$ and $\varepsilon_n^a(j)$ are small to benefit in priority from the available number of possible separations. For each pair of active antennas i and j, we define

$$Sep^c(i,j) = \sum_{n/i=\arg\max_j\{p_{jn}\}} \varepsilon_n^c(j) \quad \text{and} \quad Sep^a(i,j) = \sum_{n/i=\arg\max_j\{p_{jn}\}} \varepsilon_n^a(j).$$

These two variables provide a surrogate measure of the need to separate each pair of antennas: the smaller the values, the higher the need to separate the antenna pair (if an antenna is not active then the two variables are set to a large number). We can summarize the separation allocation procedure as follows:

4.1. Procedure separate (S)

1. For all test points n, for all antennas j, compute $\varepsilon_n^c(j)$ and $\varepsilon_n^a(j)$.
2. For all antennas i and$j$: compute $Sep^c(i,j)$ and $Sep^a(i,j)$.
3. Order all $Sep^c(i,j)$ in increasing order in list Co, and order all $Sep^a(i,j)$) in increasing order in list Ad.
4. Starting from the antenna pair (i,j) with the smallest $Sep^c(i,j)$ value in list Co, and going up, assign $\delta_{ij}^2 = 1$. Stop before condition (16) is violated.
5. Starting from the antenna pair (i,j) with the smallest $Sep^a(i,j)$ value in list Ad, and going up, assign $\delta_{ij}^2 = 1$ (if $\delta_{ij}^2 = 1$). Stop before condition (15) is violated.

The complete evaluation procedure of a solution S can be summarized as follows:

4.2. Procedure evaluate (S)

1. For all test points n: determine the best and second best servers: i_n^1 and i_n^2, and compute γ_n with $\delta_{ij}^1 = \delta_{ij}^2 = 0$.
2. For all active antennas j: compile all variables.
3. Determine separation variables δ_{ij}^1 and δ_{ij}^2 using *Procedure separate (S)*.
4. For all test points n, recompute γ_n by including the impact of δ_{ij}^1 and δ_{ij}^2.
5. For all active antennas j: compile all variables o_j^i, d_j', b_j', c_j', cap_j', and e_j'.
6. Compute $F(S)$.

Note that while solving SF-APP, there is no allocation of separations, hence Steps 3 through 6 are skipped. The same principle of generating the antenna separation using a construction heuristic can also use to extend to the model to manage GSM networks with power management or to 3G networks (which also use power management). This would require redefining the CIR measurements.

5. Experimental results

The proposed model and solution approach for the APP are tested on a realistic data set for a medium size city. The data is provided by AirTel, Montreal. This study examines a network at two development points. The first point corresponds to the green field scenario and the second corresponds to a network upgrade scenario, where a major increase in network capacity is implemented. In both cases, we compare the configurations deployed by the operator to those produced by the proposed algorithm. The main characteristics of the network are:

- Total area: 33.742 km × 35.182 km with a grid resolution of 100 m.
- Number of relevant test points: 63345.
- Green field scenario: 13,743 traffic test points with 36,479 subscribers.
- Network upgrade scenario: 18586traffic test points with 124,942 subscribers.
- Average traffic per subscriber: 0.022 Erlang.
- Minimum required signal strength: −92 dBm.
- The adjacent channel protection: $\theta = -18$ dBm.

The candidate antenna set contains 334 antennas on 107 sites that have been considered by the operator as feasible. The antenna power level set is limited to $|\Delta_i| = 7$ (including the off state). Even though the equipment has a wider operating set, this limitation would provide margin for manual fine tuning of the proposed solutions.

For the purpose of comparison, we apply the SF-APP model and the Ex-APP to test instances. The target performance levels used to define Ω are determined so that feasible solutions are at least as good as the operator's solution. In addition, in order to be able to compare solutions, the maximum allowed number of separations is set equal to the number of separations used by the network operator. Note that this operator uses sophisticated planning tools such as DOCAFTM and AtollTM (see Bourjolly et al., 2002) and pays special attention to the optimization of its network.

All computations are carried out on Pentium III PC, at 500 MHz and with 576 Mb of RAM. The algorithm is coded in Visual C++. For each scenario, we evaluate the configurations implemented by the network operator and compare them to the optimized solutions of SF-APP and Ex-APP. Solutions are obtained after running the tabu search algorithm for three hours in the green field scenario and five hours for the network upgrade scenario. For the frequency allocation, we use a tabu search algorithm proposed by Touhami (2004) and the results are reported for one and two hours for the two scenarios, respectively.

The results for the two scenarios are summarized in Tables 2 and 3. The first column of each of the two tables refers to the operator's antenna positioning solution and the operator's separation matrix. The second column refers to the operator's solution but with the separation matrix built using the heuristic assignment procedure which we propose in this paper (labeled Opt-Separation). The last two columns describe the two solutions generated by the SF-APP and Ex-APP models. For the last column, the separation procedure is integrated within the optimization, while for the other columns, the separation is applied after the APP solution is obtained (this is labeled Post-APP). For each of these cases, the solution is described by the amount of the resources utilized and the performance level achieved. The traffic overflow that results from separations is indicated by the label "*overflow*" in each evaluation criteria.

The two tables show the percentage of traffic with high interference (i.e., CIR < 12 dBm) both without and with the effect of separation. These rows are followed by the value of the objective function after solving the FAP on each of the APP solutions. The FAP and APP model interference use different methods: in the former case, pair-wise antenna interference is considered, while in the latter case, the combined interference is used. Because of this, the last row shows the final evaluation: the percentage of traffic with low CIR after incorporating the effect of the frequency allocation. In fact, modeling interference under the FAP considers only the pairwise effect: In the FAP, the aim is

Table 2
Optimization results for green field scenario

APP solution source Separation matrix source	Operator Operator Post-APP	Operator Opt-Separation Post-APP	SF-APP Opt-Separation Post-APP	Ex-APP Opt-Separation In APP
%Coverage	45.26	45.26	50.48	61.32
%Traffic coverage	81.93	81.93	84.43	85.06
# Antennas	91	91	47	40
# Sites	34	34	30	31
# TRX direct (+overflow)	194 (194)	194 (194)	147 (147)	137 (138)
% Excess capacity direct (+overflow)	30.4 (30.2)	30.4 (30.1)	16.2 (15.9)	11.9 (12.2)
% Blocked traffic direct (+overflow)	3.5 (3.3)	3.5 (3.0)	2.9 (2.5)	3.5 (3.0)
% Carried traffic direct (+overflow)	79.1 (79.2)	79.1 (79.4)	81.9 (82.3)	82.0 (82.5)
# Separations	637	637	637	636
Average weighted CIR (+separations)	67.3 (616.6)	67.3(640.4)	195.1 (791.7)	171.2 (1435.2)
%Traffic <12 dB (+separations)	63.0 (46.9)	63.0 (41.9)	52.8 (16.9)	59.5 (7.5)
FAP: Cochannel Interference	0	0	0.294	0.834
FAP: Adjacent-channel Interference	0	0	0	0
%Traffic CIR < 12 dB (after FAP)	0.432	0.495	0.668	1.528

Table 3
Optimization results for network upgrade scenario

APP solution source Separation matrix source	Operator Operator Post-APP	Operator Opt-Separation Post-APP	SF-APP Opt-Separation Post-APP	Ex-APP Opt-Separation In APP
% Coverage	96.55	96.55	82.9	88.5
% Traffic coverage	99.51	99.51	98.1	98.4
# Antennas	183	183	149	135
# Sites	66	66	65	66
#TRX direct (+overflow)	485 (487)	485 (487)	454 (463)	440 (455)
% Excess capacity direct (+overflow)	16.0 (15.2)	16.0 (14.7)	11.5 (11.8)	12.2 (11.3)
% Blocked traffic direct (+overflow)	23.4 (22.3)	23.4 (21.1)	20.9 (19.3)	23.1 (21.2)
% Carried traffic direct (+overflow)	76.3 (77.3)	76.3 (78.5)	77.6 (79.1)	75.7 (77.5)
# Separations	2061	2061	2061	2061
Average weighted CIR (+separation)	43.4 (85.2)	43.4 (312.4)	211.7 (343.4)	161.6 (364.4)
%Traffic CIR < 12 dB (+separation)	82.8 (70.3)	82.8 (63.5)	80.3 (55.9)	80.7 (46.9)
FAP: Cochannel Interference	1.437	1.232	5.375	4.930
FAP: Adjacent-channel Interference	0	0	0	0
%Traffic CIR < 12 dB (after FAP)	4.273	3.952	8.146	6.04

to assign frequencies to transmitters so as to minimize the traffic (or area) with high interference levels. The inputs are the co-channel and adjacent-channel interference matrices that indicate, for each antenna pair, the percentage traffic (or area) with low CIR, if they use the same or adjacent frequencies. This implies that transmitters are considered in pairs under the FAP, while in the APP all transmitters are considered simultaneously. The pairwise measure is not accurate. First, a single antenna may not cause interference when considered in isolation but when combined with other interferers, the interference may become significant. Second, in the FAP, the objective function adds the fractions of pairwise interfered traffic. This leads to the case where affected traffic is counted more than once. On the other hand, SF-APP and Ex-APP consider the combined effect of all interfering signals at the level of the test points, producing an accurate measure of the interference.

In both scenarios, the proposed models and algorithm generate solutions that utilize fewer resources than the operator's solution, while achieving or improving the target performances and significantly reducing system interference. In the network upgrade scenario, the area coverage requirements are set lower than the operator's solution as a large portion of the service area does not carry traffic. The reduction in interference is reflected by both an increase in the average test point CIR and the decrease in the fraction of traffic with critically low CIR (below 12 dB). The Ex-APP solutions outperform the SF-APP and the operator's solutions in terms of reducing the number of resources deployed and minimizing the fraction of traffic with low CIR when the impact of separations is incorporated. We also note that SF-APP outperforms Ex-APP with respect to the fraction of traffic with low CIR without the impact of separations. This is mainly due to the difference in the objective functions: SF-APP ignores the impact of separations while Ex-APP explicitly considers their impact during the optimization process. The results show huge reduction in interference for Ex-APP for the green field scenario. As for the network upgrade scenario, the gain is not as large. This is mainly due to the relatively small size of the candidate antenna set.

The separation procedure proposed in this paper appears to improve upon the separation matrix used by the operator, not only with respect to interference but also when incorporating the effect of traffic overflow. The proposed separation procedure is able to better exploit the available resources as expressed by the reduction in blocked traffic.

One may also observe in Tables 2 and 3 that a better FAP solution is obtained for the APP solutions used by the operator than the APP solutions produced by SF-APP and Ex-APP for both scenarios. Because the APP solutions for these latter models use fewer antennas, the area served by each antenna needed to be increased in order to satisfy the performance requirements, by readjusting the power levels for some antennas. By doing so, the overlap between cells may increase, causing higher interference levels. In fact, the addition of new antennas to an existing network is used both to increase network capacity and to combat interference.

The evaluation of the combined impact of the APP and FAP solutions indicates that the APP models proposed in this paper generate solutions with fewer resources but with a limited increase in interference. This outcome is expected since maintaining the same level of network coverage while reducing the number of active antennas requires increasing the area covered by the remaining antennas. This in turn increases the interactions between active antennas, and hence increases the total interference. For the green field scenario, this increase is very small and is largely compensated by the additional traffic carried by the network and the gains in costs and overall quality of service. In the network upgrade scenario, the interference levels achieved by the APP models are a little higher than under the operator's solution, which costs much more. In addition for this case, the candidate antenna set is relatively small and so further reductions in the interfered traffic could be expected.

For this test problem, we observed that the constraints on blocked traffic and on carried traffic are the most sensitive. Thus relaxing the limit of four TRXs per cell may be worth considering for the operator. Also, it may be worth noting that the algorithm visits a number of different solutions with similar objective function values. However, relaxing the constraints on the number of antennas, sites and TRXs and replacing them by a budget constraint (through the use of some arbitrary cost figures), we observe that the number of these quasi-equivalent solutions decreases. This implies that careful consideration is required while determining the target performances.

6. Conclusion

In this paper we present a proposal for partial integration of frequency allocation considerations within the APP. The results indicate that incorporating explicit consideration of the frequency allocation problem leads to improved solutions. Antenna separation is often used to protect the handoff procedure. By extending its use to managing interference in general and by including them within the APP, one can obtain solutions that use fewer resources while achieving the same design objectives and better quality of communication. Applied to two real-life problem instances, the proposed heuristic algorithm combined with the separation generation procedure was able to improve upon the solutions generated by the operator, highlighting the appropriateness of (at least partial) integration to managing this complex problem for realistic networks. Our approach only partially integrates APP and FAP and thus remains suboptimal. Further enhancements can be brought for example by using a fast FAP algorithm instead of the separation constraints to evaluate interference.

Future work involves allowing the models to handle power management is another issue being investigated.

Power management allows transmitters in the network to dynamically adjust the transmission power in order to manage interference as function of variations in traffic distribution. This could also be useful to extend the proposed models to UMTS (Universal Mobile Telecommunications System), which is based on wideband CDMA. The design of such systems is very dependent on the selection of the appropriate antenna configurations as they involve transmission on a single frequency. The proposed models could be modified to handle such systems by changing the interference calculation formulae to account for the use of power management.

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