Using a New Heuristic Algorithm to Solve Channel Assignment Problems in Cellular Radio Networks

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Abstract—**In the channel assignment problem (CAP), frequencies are assigned to requested calls in a cellular mobile network subject to co-channel, adjacent channel and co-site constraints such that required bandwidth is minimized. In this paper, a new heuristic algorithm is proposed to solve the CAP and Node-color and Node-degree ordering of cells and Row-wise and Column-wise ordering of calls are used. The performance of proposed method is evaluated by solving 12 CAPs. Results show that this method can find more and better solutions with minimum required bandwidth in comparison with the other algorithms investigated in the paper. Other advantages of this method are convergence and simplicity.**

Keywords-component; Heuristic algorithms; channel assignment problem; cellular radio network;

I. Introduction

The appearance of cellular mobile communication systems and their rapid growth due to the portability and the availability of these systems provided an important alternative in the field of wireless mobile communications. The increasing demand of new services in this field, however, is in contrast to the capacity constraints inherent in the current communication systems. Hence, the use of techniques, which are capable of ensuring that the frequency spectrum assigned for use in mobile communications will be better utilized, is gaining an ever-increasing importance. This makes the task of channel assignment more and more crucial [1].

The Channel Assignment Problem (CAP) in this paper is based on a common model. The service area of the system is divided into a number of hexagonal cells. Every user is located in one cell. When a user requests a call in this system, a channel is assigned to that user to provide the communication service. This channel must satisfy the electromagnetic compatibility (EMC) constraints to avoid the radio interference between channels. Three types of EMC constraints have usually been considered in CAP.

1) The Co-channel Constraint (CCC): The same channel cannot be assigned to cells that have a certain distance from each other.

2) The Adjacent Channel Constraint (ACC): Adjacent channels cannot be assigned to adjacent cells simultaneously. In other words, any pair of channels in adjacent cells must have specified distance. Note that the distance indicates the difference in the frequency domain.

3) The Co-site Constraint (CSC): Any pair of channels in a cell must have a specified distance. This distance is usually larger than necessary distance for ACC.

In the simplest form of the CAP, the co-channel constraint only is considered, and the problem is known to be equivalent to a graph-coloring problem [2]. Since the graph-coloring problem is known to be Nondeterministic Polynomial-complete (NP-complete) [3], therefore, CAP is also NP-complete. Therefore, the calculation time and the computation complexity of searching for the optimum solution in the CAP grow exponentially with the problem size.

The rest of the paper is organized as follows. In section II, the CAP is formulated. In section III, new heuristic algorithm is introduced. In section IV, the quality of proposed method is assessed by using it to solve 12 CAPs and then compare the results with the two other algorithms in channel assignment. Finally, the conclusion is brought in section V.

II. Channel Assignment Problem Formulation

CAP in this paper follows the problem formulation by Gamst and Rave [4]. In 1982, Gamst and Rave [4] defined the general form of the CAP in an arbitrary inhomogeneous cellular radio network. In their definition, the EMC constraints in an n-cell network are described by a $n \times n$ symmetric matrix, which is called compatibility matrix C. Each non-diagonal element c_{ij} in C represents the minimum separation distance in the frequency domain between a frequency assigned to a call in cell #i and a frequency assigned to call in cell #j. The co-channel constraint is represented by c_{ij}=1, and the adjacent channel constraint is represented by c_{ij}=2. c_{ij}=0 indicates that calls in cell #i and cell #j are allowed to use the same frequency. Each diagonal element c_{ii} in C represents the minimum separation distance between any two frequencies assigned to calls in cell #i, which is called co-site constraint, where $c_{ii} \geq 1$ is always satisfied.

The number of required frequencies for each cell in an n-cell network is described by an n-element vector, which is called demand vector D. Each element d_i in D represents the number of frequencies that must be assigned to calls in cell #i. If f_{ik} denotes the k^{th} frequency in cell #i that is assigned to a_{ik}, the k^{th} call in cell #i, and f_{jl} denotes the l^{th} frequency in cell #j

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that is assigned to a_{jl}, the l^{th} call in cell #j, then the EMC constraints are represented by:

$$\left|f_{ik} - f_{jl}\right| \geq c_{ij},\ i, j = 1, \ldots, n,\ k = 1, \ldots, d_i,\ l = 1, \ldots, d_j, \quad \text{if } i = j \Rightarrow k \neq l, \text{ and if } k = l \Rightarrow i \neq j. \tag{1}$$

Each f_{ik} is represented by a positive integer. The CAP is to assign a set of frequencies $\{f_{ik}\}$ to the set of calls $\{a_{ik}\}$ such that the bandwidth required by the system, i.e., max f_{ik}, is minimized, subject to EMC constraints.

In addition to the constraint matrix C and the demand vector D, another important parameter called lower bound (lb) is considered in the formulation of CAP. Parameter lb determines minimum value of the maximum f_{ik} for all i and k, so that no interference is caused (i.e. lb=min(max(f_{ik}))). This means if f_{ik}s can take values between 1 to lb, the values of f_{ik}s will not violate any constraints and a conflict-free channel assignment will be obtained. In fact, lb indicates the minimum required bandwidth for the CAP and if any smaller bandwidth is used, interference will be unavoidable and some constraints will be violated.

In this paper, Node-color (NC) and Node-degree (ND) ordering of cells and Row-wise (RW) and Column-wise (CW) ordering of calls are used. The degree of cell #i is defined as [5]:

$$m_i = \left(\sum_{j=1}^{n} d_j c_{ij}\right) - c_{ii}, \quad 1 \leq i \leq n.$$

In the Node-degree ordering, the cells are arranged in decreasing order of their degrees [5].

The Node-Color ordering of cells is obtained as follows [5]. The cell with the least degree is placed at the last place (n^{th}) in the list. Then, this cell is eliminated from the network and the degrees of the remaining cells are computed again. Then, the cell with the least degree is placed at the $(n\text{-}1)^{\text{th}}$ position in the list and eliminated from the network. This process is continued until only one cell is left, which is placed in the first position of the list.

Once the cells have been ordered, the calls are arranged in an ($n \times d_{max}$) matrix (A_c), where n is the number of cells and d_{max} is the maximum element of demand vector (d_{max}=max d_i) [5]. Each row of this matrix corresponds to the calls in a cell. Calls are arranged such that all the columns have nearly the same number of calls. This means calls in the 1st row start at the 1st column and calls in the 2nd row start at the $(d_1+1)^{\text{th}}$ column, if the 1st row has d_1 calls, and cyclically fill this row. Similarly, calls in the 3rd row start where the 2nd row ends and so on.

Once the calls have been arranged in a matrix, the Row-wise ordering of calls can be done as follows:

The calls in the 1st row are placed in the list then the calls in the 2nd row are placed after them and so on.

In the Column-wise ordering, the calls in the 1st column are placed in the list then the calls in the 2nd column are placed after them and so on.

By using two cell ordering methods and then two call ordering methods, four ordered list of calls are obtained. Since the ordered list of calls is obtained, frequencies f_{ik}s are assigned to calls a_{ik}s by the new heuristic algorithm.

III. New Heuristic Algorithm

According to CAP formulation, which is mentioned in section II, it is supposed to have an ordered list of calls ($a_1,\ldots,a_k,\ldots,a_N$). Here, a_{ij} (in section II) is replaced by a_k for simplification and N is the total number of calls in the network which can be obtained from the formula $\Sigma^n_{i=1}\, d_i$. First, one is assigned to all of the calls. Then, values of calls a_i ($i=2,\ldots,N$) are changed (if necessary) according to (1) and with respect to the value of the call a_1 subject to EMC constraints. For example, if a_1 is a call in k^{th} cell and a_i is a call in j^{th} cell, then condition $|a_1 - a_i| \geq c_{kj}$ must be satisfied. Otherwise, the value of a_i must be changed by using the following formula:

$$a_i = a_1 + c_{kj} \qquad i = 2, \ldots, N,\ \ k, j = 1, \ldots, n. \tag{2}$$

This is done for the other calls a_i ($i=2,\ldots,N$) respectively by keeping the values of calls $a_1,\ldots,a_i$ fixed and changing the values of calls a_j ($j=i+1,\ldots,N$) with respect to calls $a_1,\ldots,a_i$. This means the value of call a_j should satisfy the following formula:

$$\left|a_i - a_j\right| \geq c_{kl} \tag{3}$$

In (3), it is supposed that call a_j is a call in cell k and call a_i is a call in cell l. If (3) is not satisfied, the value of call a_j must be changed as follows:

$$a_j = a_i + c_{kl}. \tag{4}$$

It is very important to note that the value of call a_j should be checked with the values of calls $a_1,\ldots,a_i$ so that frequency constraints are not violated. If the value of call a_j interferes with the values of other calls ($a_1,\ldots,a_i$), its value is increased by one and continue this until all of EMC constraints are satisfied. In this way, after doing this method once, solution (a_1=1,a_2=$A_2,\ldots,a_N$=A_N) is obtained where A_i is an integer value assigned to call a_i. Because this solution may not be the optimum solution, for finding other possible solutions or the possible optimum solutions, the above method is repeated with this difference that the value of call a_2 is equal to one and then the values of other calls are determined with respect to a_2 and so on. Finally, N solutions will be produced and we will choose one or more solutions that the maximum frequency assigned in them is smaller than the maximum frequency assigned in other solutions.

IV. Simulation Results

To test the new heuristic algorithm and compare its performance with the heuristic method proposed by Sivarajan et al [5] and the adaptive local-search algorithm proposed by Wang et al. [6], 12 CAPs are used that their compatibility matrix (C) and demand vector (D) are as follows:

$$C_1 = \begin{bmatrix} 5&4&0&0 \\ 4&5&0&1 \\ 0&0&5&2 \\ 0&1&2&5 \end{bmatrix},\ D_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 3 \end{bmatrix},\ C_2 = \begin{bmatrix} 2&2&4&4 \\ 2&2&2&4 \\ 4&2&2&1 \\ 4&4&1&2 \end{bmatrix},\ D_2 = \begin{bmatrix} 4 \\ 4 \\ 4 \\ 4 \end{bmatrix},\ C_3 = \begin{bmatrix} 4&3&4&3&4 \\ 3&4&3&2&2 \\ 4&3&4&2&4 \\ 3&2&2&4&2 \\ 4&2&4&2&4 \end{bmatrix},$$

$$D_4 = \begin{bmatrix} 2 \\ 4 \\ 4 \\ 1 \\ 4 \end{bmatrix},\ C_4 = \begin{bmatrix} 4&3&3&5&4 \\ 3&4&4&4&4 \\ 3&4&4&2&2 \\ 5&4&2&4&2 \\ 4&4&2&2&4 \end{bmatrix},\ D_4 = \begin{bmatrix} 5 \\ 5 \\ 5 \\ 5 \\ 5 \end{bmatrix},\ C_5 = \begin{bmatrix} 1&4&3&4&4&5 \\ 4&1&6&4&5&2 \\ 3&6&1&6&2&2 \\ 4&4&6&1&5&5 \\ 4&5&2&5&1&2 \\ 5&2&2&5&2&1 \end{bmatrix},\ D_5 = \begin{bmatrix} 5 \\ 3 \\ 5 \\ 4 \\ 3 \\ 5 \end{bmatrix},$$

$$C_6 = \begin{bmatrix} 5&1&6&4&3&4 \\ 1&5&3&3&2&4 \\ 6&3&5&5&4&4 \\ 4&3&5&5&2&3 \\ 3&2&4&2&5&5 \\ 4&4&4&3&5&5 \end{bmatrix},\ D_6 = \begin{bmatrix} 6 \\ 6 \\ 6 \\ 6 \\ 6 \\ 6 \end{bmatrix},\ C_7 = \begin{bmatrix} 7&3&6&1&6&7&7 \\ 3&7&6&4&4&2&5 \\ 6&6&7&3&7&5&3 \\ 1&4&3&7&5&3&4 \\ 6&4&7&5&7&7&5 \\ 7&2&5&3&7&7&2 \\ 7&5&3&4&5&2&7 \end{bmatrix},\ D_7 = \begin{bmatrix} 4 \\ 4 \\ 6 \\ 1 \\ 5 \\ 1 \\ 3 \end{bmatrix},$$

$$C_8 = \begin{bmatrix} 4&4&1&1&3&1&3 \\ 4&4&5&2&1&5&5 \\ 1&5&4&1&1&2&5 \\ 1&2&1&4&1&3&5 \\ 3&1&1&1&4&5&5 \\ 1&5&2&3&5&4&2 \\ 3&5&5&5&5&2&4 \end{bmatrix},\ D_8 = \begin{bmatrix} 7 \\ 7 \\ 7 \\ 7 \\ 7 \\ 7 \\ 7 \end{bmatrix},\ C_9 = \begin{bmatrix} 5&2&4&6&7&3&3&7 \\ 2&5&3&7&7&3&3&1 \\ 4&3&5&2&5&2&7&2 \\ 6&7&2&5&2&8&4&3 \\ 7&7&5&2&5&3&4&4 \\ 3&3&2&8&3&5&5&2 \\ 3&3&7&4&4&5&5&1 \\ 7&1&2&3&4&2&1&5 \end{bmatrix},\ D_9 = \begin{bmatrix} 4 \\ 3 \\ 2 \\ 6 \\ 6 \\ 6 \\ 4 \\ 5 \end{bmatrix},$$

$$C_{12} = \begin{bmatrix} 13&3&12&14&3&3&12&11&10&12&10&13&5&7&4 \\ 3&13&1&2&13&6&8&14&4&13&10&13&8&15&6 \\ 12&1&13&8&4&8&7&10&10&14&4&3&9&15&1 \\ 14&2&8&13&13&1&8&4&11&5&14&3&8&14&14 \\ 3&13&4&13&13&6&7&8&3&3&8&11&7&6&5 \\ 3&6&8&1&6&13&13&10&4&15&10&4&11&10&3 \\ 12&8&7&8&7&13&13&1&5&3&2&13&4&1&11 \\ 11&14&10&4&8&10&1&13&9&5&6&1&13&13&5 \\ 10&4&10&11&3&4&5&9&13&9&3&9&6&10&13 \\ 12&13&14&5&3&15&3&5&9&13&9&15&12&3&13 \\ 10&10&4&14&8&10&2&6&3&9&13&4&10&10&12 \\ 13&13&3&3&11&4&13&1&9&15&4&13&6&7&8 \\ 5&8&9&8&7&11&4&13&6&12&10&6&13&10&3 \\ 7&15&15&14&6&10&1&13&10&3&10&7&10&13&1 \\ 4&6&1&14&5&3&11&5&13&13&12&8&3&1&13 \end{bmatrix},$$

$$D_{12} = \begin{bmatrix} 11 \\ 9 \\ 8 \\ 12 \\ 4 \\ 16 \\ 5 \\ 12 \\ 5 \\ 10 \\ 15 \\ 8 \\ 14 \\ 13 \\ 14 \end{bmatrix},\ C_{10} = \begin{bmatrix} 5&6&7&1&2&7&4&7 \\ 6&5&6&6&3&2&2&2 \\ 7&6&5&4&7&4&7&4 \\ 1&6&4&5&4&4&4&7 \\ 2&3&7&4&5&1&3&1 \\ 7&2&4&4&1&5&6&6 \\ 4&2&7&4&3&6&5&8 \\ 7&2&4&7&1&6&8&5 \end{bmatrix},\ D_{10} = \begin{bmatrix} 7 \\ 8 \\ 3 \\ 2 \\ 7 \\ 3 \\ 6 \\ 8 \end{bmatrix},$$

$$C_{11} = \begin{bmatrix} 2&1&1&0&1&0&1&1&1&1&0&1&1&1&1&0&0&0&0&0&0&0&0&0&0 \\ 1&2&1&0&1&0&1&1&0&1&0&1&1&1&1&0&0&0&0&0&0&0&0&0&0 \\ 1&1&2&1&1&1&1&1&1&1&1&1&1&1&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&1&2&0&0&1&1&1&1&1&1&1&0&0&0&0&0&0&0&0&0&1&1&1 \\ 1&1&1&0&2&0&0&0&0&1&1&1&1&1&1&1&0&0&0&0&0&0&0&0&0 \\ 0&0&1&0&0&2&1&1&1&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 1&1&1&1&0&1&2&1&1&1&1&1&1&0&0&0&0&0&0&0&0&0&0&0&0 \\ 1&1&1&1&0&1&1&2&1&1&1&1&1&0&0&0&0&0&0&0&0&0&0&1&0 \\ 1&0&1&1&0&1&1&1&2&1&1&1&0&0&0&0&0&0&0&0&0&0&0&1&1 \\ 1&1&1&1&1&1&1&1&1&2&1&1&1&1&1&1&0&0&0&0&0&1&0&1&0 \\ 0&0&1&1&1&0&1&1&1&1&2&0&1&1&1&1&0&1&1&1&1&1&1&1&1 \\ 1&1&1&1&1&0&1&1&1&1&0&2&1&1&0&0&0&0&0&0&0&0&0&0&0 \\ 1&1&1&1&1&0&1&1&0&1&1&1&2&1&1&1&1&1&1&1&0&0&0&0&0 \\ 1&1&1&0&1&0&0&0&0&1&1&1&1&2&1&1&1&1&1&1&0&0&0&0&0 \\ 1&1&0&0&1&0&0&0&0&1&1&0&1&1&2&1&1&1&1&1&1&1&0&0&0 \\ 0&0&0&0&1&0&0&0&0&1&1&0&1&1&1&2&1&1&1&1&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&1&1&1&1&2&1&1&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&1&0&1&1&1&1&1&2&1&1&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&1&0&1&1&1&1&1&1&2&1&1&1&1&0&0 \\ 0&0&0&0&0&0&0&0&0&0&1&0&1&1&1&1&0&1&1&2&1&1&1&0&0 \\ 0&0&0&0&0&0&0&0&0&0&1&0&0&0&1&0&0&0&1&1&2&1&1&0&0 \\ 0&0&0&0&0&0&0&0&0&1&1&0&0&0&1&0&0&0&1&1&1&2&1&1&1 \\ 0&0&0&1&0&0&0&0&0&0&1&0&0&0&0&0&0&0&1&1&1&1&2&1&1 \\ 0&0&0&1&0&0&0&1&1&1&1&0&0&0&0&0&0&0&0&0&0&1&1&2&1 \\ 0&0&0&1&0&0&0&0&1&0&1&0&0&0&0&0&0&0&0&0&0&1&1&1&2 \end{bmatrix},\ D_{11} = \begin{bmatrix} 10 \\ 11 \\ 9 \\ 5 \\ 9 \\ 4 \\ 5 \\ 7 \\ 4 \\ 8 \\ 8 \\ 9 \\ 10 \\ 7 \\ 7 \\ 6 \\ 4 \\ 5 \\ 5 \\ 7 \\ 6 \\ 4 \\ 5 \\ 7 \\ 5 \end{bmatrix}.$$

Problem 1 is given in most references ([5], [6], [7], [8], etc) as a simple example and includes a 4-cell network with 6 channels. Problems 2-10 and 12 are generated randomly. Problem 11 is a practical CAP taken from [8], [9], in Helsinki, Finland that is composed from a 25-cell network with 167 channels. The new heuristic algorithm is implemented in C. This program was run on a PC with a Pentium 4 (3.2 GHz) CPU.

Table I shows the results produced by the new heuristic method for the 12 stated CAPs. The 1st column of this table shows problem number. Constraint matrix (C) and demand vector (D) for each problem are given in the 2nd and 3rd columns, respectively. The 4th column shows the maximum frequencies obtained by ND ordering of cells and RW ordering of calls. In the 5th column, the maximum frequencies obtained by ND ordering of cells and CW ordering of calls are shown. The maximum frequencies obtained by NC ordering of cells and RW ordering of calls are shown in the 6th column. We have shown the maximum frequencies obtained by using NC ordering of cells and CW ordering of calls in the 7th column.

In table II like table I, the results obtained by the heuristic method [5] and by using frequency exhaustive strategy [5], NC and ND ordering of cells and RW and CW ordering of calls are shown. Also, the 8th column of table II shows the bandwidth obtained by the adaptive local-search algorithm [6].

Comparing the corresponding rows and columns of both tables, we can see the required bandwidth obtained by new method in all problems is smaller or equal to the necessary bandwidth of the heuristic method [5] and adaptive local-search algorithm [6].

Table III shows the number of solutions obtained by the new heuristic method and by using different ordering of cells and calls in each problem. The heuristic method [5] can find only one solution for each ordering of cells and calls. As shown in table III, the new heuristic algorithm can find more solutions than the heuristic method [5].

V. CONCLUSION

In this paper, a new heuristic algorithm is proposed to solve CAPs in cellular radio networks and Node-color and Node-degree ordering of cells and Row-wise and Column-wise ordering of calls are used. The performance of the new algorithm is compared with another heuristic method and adaptive local-search algorithm used in CAPs and is showed through simulations that new method can find more and better solutions with the least necessary bandwidth that does not violate any EMC constraints. Other advantages of this method are convergence and simplicity. Tables IV-XV contain solutions of problems 1-12, respectively.

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TABLE I. RESULTS OBTAINED BY THE NEW METHOD FOR 12 CAPS WITH NC & ND ORDERING OF CELLS AND RW & CW ORDERING OF CALLS.

Problem number	C	D	ND & RW	ND & CW	NC & RW	NC & CW
1	C_1	D_1	11	11	11	11
2	C_2	D_2	25	37	25	37
3	C_3	D_3	38	41	38	41
4	C_4	D_4	67	67	69	68
5	C_5	D_5	35	76	40	89
6	C_6	D_6	114	105	115	110
7	C_7	D_7	105	105	105	115
8	C_8	D_8	85	103	86	101
9	C_9	D_9	99	111	96	112
10	C_{10}	D_{10}	133	161	138	161
11	C_{11}	D_{11}	73	73	73	73
12	C_{12}	D_{12}	809	896	806	904

TABLE II. RESULTS OBTAINED BY HEURISTIC METHOD [5] AND ADAPTIVE LOCAL-SEARCH ALGORITHM [6] FOR 12 CAPS.

Problem number	C	D	Sivarajan et al. [5]				Wang et al. [6]
			ND & RW	ND & CW	NC & RW	NC & CW	
1	C_1	D_1	11	11	11	11	11
2	C_2	D_2	27	39	25	37	27
3	C_3	D_3	39	42	40	43	38
4	C_4	D_4	76	68	75	69	67
5	C_5	D_5	35	76	43	89	35
6	C_6	D_6	115	105	116	111	105
7	C_7	D_7	105	107	105	119	105
8	C_8	D_8	87	136	107	101	87
9	C_9	D_9	100	124	96	122	98
10	C_{10}	D_{10}	136	173	139	177	138
11	C_{11}	D_{11}	77	73	73	73	73
12	C_{12}	D_{12}	809	908	895	999	806

TABLE III. THE NUMBER OF SOLLUTIONS OBTANIED BY THE NEW HEURISTIC ALORITHM BY USING DIFFERENT ORDERING OF CELLS AND CALLS.

Problem number	ND & RW	ND & CW	NC & RW	NC & CW
1	4	4	4	4
2	1	4	1	4
3	5	5	3	5
4	1	5	3	5
5	1	4	1	3
6	10	18	11	6
7	8	4	11	1
8	1	7	2	7
9	1	1	1	2
10	1	1	1	1
11	37	102	45	137
12	1	1	1	1

TABLE IV. SOLUTION OF PROBLEM 1

Cell Number			
1	2	3	4
6	2	3	1
			6
			11

TABLE V. SOLUTION OF PROBLEM 2

Cell Number			
1	2	3	4
19	11	2	1
21	13	4	3
23	15	6	5
25	17	8	7

TABLE VI. SOLUTION OF PROBLEM 3

Cell Number				
1	*2*	*3*	*4*	*5*
34	19	1	3	17
38	23	5		21
	27	9		25
	31	13		29

TABLE VII. SOLUTION OF PROBLEM 4

Cell Number				
1	*2*	*3*	*4*	*5*
32	51	13	23	3
36	55	17	27	7
40	59	21	1	11
44	63	25	5	15
48	67	29	9	19

TABLE VIII. SOLUTION OF PROBLEM 5

Cell Number					
1	*2*	*3*	*4*	*5*	*6*
14	8	21	1	27	31
15	9	22	2	28	32
16	10	23	3	29	33
17		24	4		34
18		25			35

TABLE IX. SOLUTION OF PROBLEM 6

Cell Number					
1	*2*	*3*	*4*	*5*	*6*
54	55	1	83	85	25
59	60	6	88	90	30
64	65	11	93	95	35
69	70	16	98	100	40
74	75	21	103	105	45
79	80	114	108	110	50

TABLE X. SOLUTION OF PROBLEM 7

Cell Number						
1	*2*	*3*	*4*	*5*	*6*	*7*
77	80	36	60	1	105	39
84	87	43		8		46
91	94	50		15		53
98	101	57		22		
		64		29		
		71				

TABLE XI. SOLUTION OF PROBLEM 8

Cell Number						
1	*2*	*3*	*4*	*5*	*6*	*7*
4	61	2	3	1	30	32
8	65	6	7	62	34	36
12	69	10	11	66	38	40
16	73	14	15	70	42	44
20	77	18	19	74	46	48
24	81	22	23	78	50	52
28	85	26	27	82	54	56

TABLE XII. SOLUTION OF PROBLEM 9

Cell Number							
1	*2*	*3*	*4*	*5*	*6*	*7*	*8*
68	66	3	34	29	1	63	13
73	71	8	39	36	6	86	18
78	76		44	41	11	91	23
83			49	46	16	96	90
			54	51	21		95
			59	56	26		

TABLE XIII. SOLUTION OF PROBLEM 10

Cell Number							
1	*2*	*3*	*4*	*5*	*6*	*7*	*8*
1	37	123	2	13	109	35	68
6	42	128	7	18	114	40	73
11	47	133		23	119	45	78
16	52			28		50	83
21	57			65		55	88
26	62			79			93
31	70			84			98
	75						103

TABLE XIV. SOLUTION OF PROBLEM 11

Cell Number																								
1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	*21*	*22*	*23*	*24*	*25*
5	36	21	32	39	5	39	18	1	2	5	57	1	18	21	32	5	43	2	12	32	1	22	21	2
7	38	23	34	41	7	41	20	3	4	7	59	3	20	23	34	7	45	4	14	34	3	24	23	4
9	40	25	36	43	9	43	22	42	6	9	61	58	22	25	36	9	47	6	16	36	18	26	25	6
11	42	27	38	45	11	45	24	44	8	11	63	60	24	27	38	11	49	8	35	38	20	28	27	8
13	44	29	40	47		47	26		10	13	65	62	26	29	40		51	10	37	40		30	29	10
15	46	31		49			28		12	15	67	64	28	31	42				39	42			31	
17	48	33		51			30		14	17	69	66	30	33					41				33	
19	50	35		53					16	19	71	68												
32	52	37		55							73	70												
34	54											72												
	56																							

TABLE XV. SOLUTION OF PROBLEM 12

Cell Number														
1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*
331	512	750	461	7	495	59	157	646	4	121	46	313	1	258
344	525	715	503	20	506	72	170	475	17	176	164	326	14	134
357	538	728	516	33	519	85	183	488	30	189	180	339	27	147
370	551	741	529	683	532	98	196	657	43	202	193	352	40	318
383	564	754	542		545	111	209	670	56	215	206	365	53	335
396	577	767	555		558	696	222		69	228	219	378	66	348
409	590	780	568		571		235		82	241	232	391	79	361
422	603	793	581		584		248		95	254	245	404	92	374
435	616		594		597		261		108	267		417	105	387
448			607		610		274		701	280		430	118	400
806			620		623		287			293		443	131	413
			633		636		300			306		456	144	426
					649					711		469		439
					662					724		482		452
					675					737				
					688									