

Frequency Assignment Problem Using Discrete Particle Swarm Model

L. Benameur,

Laboratoire Conception & Systèmes
Faculté des Sciences,
Université Mohammed V, Rabat, Maroc
lamia_benameur@yahoo.fr

J. Alami, A. El Imrani

Laboratoire Conception & Systèmes
Faculté des Sciences,
Université Mohammed V, Rabat, Maroc
a_jihane79@yahoo.fr, elimrani@fsr.ac.ma

Abstract—The problem of the fixed-spectrum frequency assignment, where the objective is to minimize the cost due to the interference arising in a solution, is studied and solved in this paper using a discrete particle swarm optimization which is refined by a deterministic local search heuristic. Computational results, obtained for eight well-known benchmarks problem, confirm the effectiveness of discrete particle swarm optimization.

Keywords— Particle Swarm Optimization, frequency Assignment Problem, Interference Minimisation.

I. INTRODUCTION

The importance of the frequency assignment problem (FAP) for efficient use of the radio spectrum and minimization of interference is now well recognized. The majority of the early work on this problem concentrated on the minimum span problem [1]-[2]. In this variation of the FAP, constraints are specified, which, if satisfied, should lead to acceptable interference. It is then necessary to minimize the spectrum used, measured as the difference between the greatest and smallest assigned frequencies. It is now increasingly recognized that the fixed spectrum frequency assignment problem (FS-FAP) is of greater importance to network operators. In this variation of the FAP, the available spectrum to the operator is known in advance and is necessary to minimize some measure of interference. The constraints considered here are binary constraints, specifying the necessary frequency separation between given pairs of transmitters. There may be penalties (or weights) associated with the violation of each constraint. The objective can be to minimise the number of constraints violated or, as is increasingly done by operators, to minimise the sum of the weights associated with violated constraints.

The FAP is a generalization of the well known graph colouring problem [3], which is the problem of finding a colouring of a graph so that the number of used colours is minimized, subject to the constraints that each two adjacent vertices have two different colours: as such FAP is a NP- hard problem [4]-[5]. Such problems require the use of extremely time-consuming algorithms to obtain exact solutions. It is,

therefore, necessary to use more time-efficient algorithms that, however, cannot guarantee optimal solutions.

Different lower bounds on the optimal solution value for the frequency assignment problem have been proposed [6]-[7]. These techniques are useful both in assessing the quality of approximate solutions and in limiting the search for optimal assignments and are usually derived from graph-theoretic approaches, which adapt techniques originally developed for the colouring problem.

Exact algorithms have been proposed in [8]-[9]. Due to the NP-hardness of the problem, any exact optimization algorithm requires in the worst case an amount of time exponentially growing with the size of the instance. In order to obtain good solutions in a reasonable amount of time and due to the relevant actual importance of the FAP, much effort has been spent in studying heuristic algorithms. Different approaches have been used, including tabu search [10], simulated annealing [11], genetic algorithms [12], neural networks [4], dynamic programming [13], ant colony optimisation [14] and Cultural algorithm [15]. This work reports about the results obtained applying to FS-FAP discrete particle swarm optimization which includes a local search.

The paper is structured as follows. Section 2 provides background information on the FS-FAP problem. Section 3 presents an overview of discrete particle swarm optimization. Section 4 shows the computational results obtained for eight benchmarks problem instances.

II. THE FIXED-SPECTRUM FREQUENCY ASSIGNMENT PROBLEM(FS-FAP)

In this paper, the most common variation of FS-FAP, referred to the minimum interference frequency assignment problem, is considered. A system of n cells is represented by n vectors $X = \{x_1, x_2, \dots, x_n\}$. We assume that the channels are equally spaced in the frequency domain and are ordered from the low-frequency band to the high-frequency band with numbers $1, 2, \dots, m$. We use an $n \times n$ nonnegative symmetric matrix C , called a compatibility matrix, to represent the

electromagnetic compatibility constraints (EMC). The EMC is composed of three constraints: the co-site constraint (CSC): each pair of frequencies assigned to a cell should have a minimal distance between frequencies; the adjacent channel constraint (ACC): the adjacent frequencies in the frequency domain can not be assigned to adjacent cells simultaneously and the co-channel constraint (CCC): for a certain pair of cells, the same frequency can not be used simultaneously.

Each diagonal element c_{ii} in C represents the CSC, and the rest of the elements, c_{ij} (where $i \neq j$), represent the ACC or CCC. A demand vector $R = \{r_1, r_2, \dots, r_n\}$ describes the channel requirements for each cell. Each element r_i in R represents the minimal number of channels to be assigned to cell x_i .

The FS-FAP is specified by the triple (X, R, C) , where X is a cell system, R is a requirement vector, and C is a compatibility matrix. Let $N = \{1, 2, \dots, m\}$ be a set of available channels, and let H_i be the subset of N assigned to x_i . The objective of FS-FAP is to find an assignment $H = \{H_1, H_2, \dots, H_n\}$, which satisfies the following conditions:

$$\begin{aligned} |H_i| &= r_i & \text{for } 1 \leq i \leq j \\ |h - h'| &\geq c_{ij}, & \text{for all } h \in H_i, h' \in H_j, \end{aligned} \quad (1)$$

$$\text{where } 1 \leq i \leq n, \text{ and } 1 \leq j \leq n, i \neq j, \quad (2)$$

$$|h - h'| \geq c_{ii}, \text{ for all } h, h' \in H_i, \text{ where } h \neq h', \quad (3)$$

Where $|H_i|$ denotes the number of channels in the set of H_i . We call such an assignment an admissible assignment.

The objective of the FS-FAP is to find an assignment that minimizes the total number of violations in an assignment. Formally we have:

$$\text{Min} \sum_{i=1}^n \sum_{a=1}^m \sum_{j=1}^n \sum_{b=1}^m p(i, a) \varepsilon(i, a, j, b) p(j, b) \quad (4)$$

Where

$$\varepsilon(i, a, j, b) = \begin{cases} 0 & \text{if the distance between channel } a \text{ and } b \text{ is greater or equal to } c_{ij} (|a - b| \geq c_{ij}), \\ 1 & \text{otherwise.} \end{cases}$$

And

$$p(i, a) = \begin{cases} 1 & \text{if the channel } a \text{ is assigned to the } i^{\text{th}} \text{ cell,} \\ 0 & \text{otherwise.} \end{cases}$$

$\varepsilon(i, a, j, b)$ is set to 1 if the assignment of channel a to cell x_i and channel b to cell x_j violates the EMC.

III. DISCRETE PARTICLE SWARM OPTIMIZATION ALGORITHM (DPSO)

In the standard PSO algorithm, each solution is called a "particle". All particles have their position, velocity, and fitness values. Particles fly through the n -dimensional space by learning from the historical information emerged from the swarm population. For this reason, particles are inclined to fly towards better search area over the course of evolution.

Let NP denote the swarm size represented as $X' = [X'_1, X'_2, \dots, X'_{NP}]$. Then each particle in the swarm population has the following attributes: a current position represented as $X'_i = [x'_{i1}, x'_{i2}, \dots, x'_{in}]$; a current velocity represented as $V'_i = [v'_{i1}, v'_{i2}, \dots, v'_{in}]$; a current personal best position represented as $P'_i = [p'_{i1}, p'_{i2}, \dots, p'_{in}]$; and a current global best position represented as $G' = [g'_1, g'_2, \dots, g'_n]$. Assuming that the function f is to be minimized, the current velocity of the j^{th} dimension of the i^{th} particle is updated as follows:

$$v'_{ij} = \omega' v'_{ij} + c_1 r_1 (p'_{ij} - x'_{ij}) + c_2 r_2 (g'_j - x'_{ij}), \quad (5)$$

where ω' is the inertia weight which is a parameter to control the impact of the previous velocities on the current velocity; c_1 and c_2 are acceleration coefficients and r_1 and r_2 are uniform random numbers between $[0, 1]$.

The current position of the j^{th} dimension of the i^{th} particle is updated using the previous position and current velocity of the particle as follows:

$$x'_{ij} = x'_{ij} + v'_{ij} \quad (6)$$

PSO was originally designed for working in multi-dimensional real spaces. The representation of a solution of the FS-FAP (and many other combinatorial optimization problems) consists of an ordered sequence of integer numbers.

Therefore, the PSO metaheuristic has to be adapted to work with this type of solution representation (i.e. the point of a particle is now a sequence of integer numbers). This is done by redefining the elements (point and velocity) and the operations (external multiplication of a coefficient by a velocity, sum of velocities and sum of a velocity plus a point) of Eq. (5) and (6). Moreover, it is also necessary to determinate how the initial population is set and the stopping criteria.

A. Point of the particle

A point consists of a solution which represents frequency assignments. Each population member is made up of Rtot integer parameters, where Rtot is the total number of frequency request in the cellular system. For example, a point could be the sequence (1,6,10,2,8,3).

B. Velocity of the particle

The expression $(X_2 - X_1)$, where X_2 and X_1 are two points, represents the difference between two points and the velocity needed to go from X_1 to X_2 . This velocity is an ordered list of transformations (called movements) that must be applied sequentially to the particle so that its current point, X_1 changes to the other one X_2 . A movement is a pair of values (α/j) . For each position u in the sequence (point) X_1 , the algorithm determines whether the unit that is in position u of sequence X_1 is the same unit that is in position u of sequence X_2 . If the units are different, α is the unit in position u of X_2 and j is equal to position u . Thus, this movement denotes that to go from the sequence X_1 to the sequence X_2 , the unit in position j must be exchanged for the unit α .

For example, let $X_2 = (A_1, C_1, B_2, C_2, A_2, C_4, B_1, C_3)$ and $X_1 = (A_1, B_1, C_2, C_1, B_2, C_4, A_2, C_3)$. The sub-indices of the units are fictitious identifiers used to distinguish between the units of the same product. Thus, the velocity $(X_2 - X_1)$, is formed by the list of movements

$$\begin{aligned} X_1 &= (A_1, B_1, C_2, C_1, B_2, C_4, A_2, C_3), \\ (C_1/2) &\rightarrow (A_1, C_1, C_2, B_1, B_2, C_4, A_2, C_3) \\ (B_2/3) &\rightarrow (A_1, C_1, B_2, B_1, C_2, C_4, A_2, C_3) \\ (C_2/4) &\rightarrow (A_1, C_1, B_2, C_2, B_1, C_4, A_2, C_3) \\ (A_2/5) &\rightarrow (A_1, C_1, B_2, C_2, A_2, C_4, B_1, C_3) \\ (B_1/7) &\rightarrow (A_1, C_1, B_2, C_2, A_2, C_4, B_1, C_3) = X_2 \end{aligned}$$

C. External Multiplication of a Coefficient by a Velocity

The values of the coefficients ω , c_1 and c_2 in Eq. (5) are between 0 and 1. When a coefficient is multiplied by a velocity, it indicates the probability of each movement to be applied. For example, if we multiply the coefficient 0.6 by the velocity $[(C_1/2), (B_2/3), (C_2/4), (A_2/5), (B_1/7)]$, five random numbers between 0 and 1 are generated for comparison with the value 0.6. If the random number is lower than 0.6, the movement is applied. Therefore, if the values of the random numbers are 0.8, 0.3, 0.7, 0.4 and 0.2, movements $(B_2/3)$, $(A_2/5)$ and $(B_1/7)$ are applied, whereas movements $(C_1/2)$ and $(C_2/4)$

are not. The resulting velocity of the multiplication is therefore $[(B_2/3), (A_2/5), (B_1/7)]$, which, as previously stated, represents a list of movements to be applied to a point.

D. Sum of Velocities

The sum of two velocities is simply the concatenation of their own list of movements.

E. Sum of Velocity and point

The sum of a velocity plus a point gives the result of sequentially applying each movement of the velocity to the point.

F. Initial Population

The initial population is generated by setting a void velocity and a random point for each particle. As has been previously mentioned, each point consists of a solution represented by a sequence of integer numbers. A random solution is generated as follows: for each position in the sequence, a product to be sequenced is chosen at random. The probability of each product is equal to the number of units of this product that remain to be sequenced divided by the total number of units that remain to be sequenced.

G. Stopping Criteria

The PSO algorithm stops after it has run for a preset time.

H. Introducing a Deterministic Local Search Procedure into PSO

At each generation, a deterministic local search procedure is performed to the n individual. The basic idea is that each solution is taken to its local minimum by the application of a deterministic local optimization heuristic [15]. This heuristic consists in choosing, for each channel call of each cell that violates the electromagnetic constraints, a channel, if exists, that validates the EMC. The new optimized solutions are considered as the final solutions produced in the current generation.

IV. SIMULATION RESULTS AND DISCUSSION

To test the performance of discrete particle swarm algorithm, eight well-known benchmarks [16], are used in this paper. For these benchmarks problem, the number of available channels allows us to obtain channel assignment solutions without interference.

Table 1 summarizes the characteristics of these eight problems.

A. Simulation Results

In this section, results obtained for some of the benchmarks problem mentioned below are presented. Note that the parameters of the algorithm used in this study are the population size and the maximum number of generations.

1) Results of problem #1

This problem is simple to solve. The DPSO is applied using a population of 10 individuals evolving during 10 generations. Table 2 depicts channels assignment for each

cell. From this table, these assignments validate all the electromagnetic compatibility constraints. Note that several solutions are obtained.

TABLE I. SPECIFICATIONS OF THE EIGHT BENCHMARK PROBLEMS USED

Problem #	No. of Cells	No. of Channels	Compati- bility Matrix (C)	Demand Vector (D)
1	4	11	C1	D1
2	25	73	C2	D2
3	21	381	C3	D3
4	21	533	C4	D3
5	21	533	C5	D3
6	21	221	C3	D4
7	21	309	C4	D4
8	21	309	C5	D4

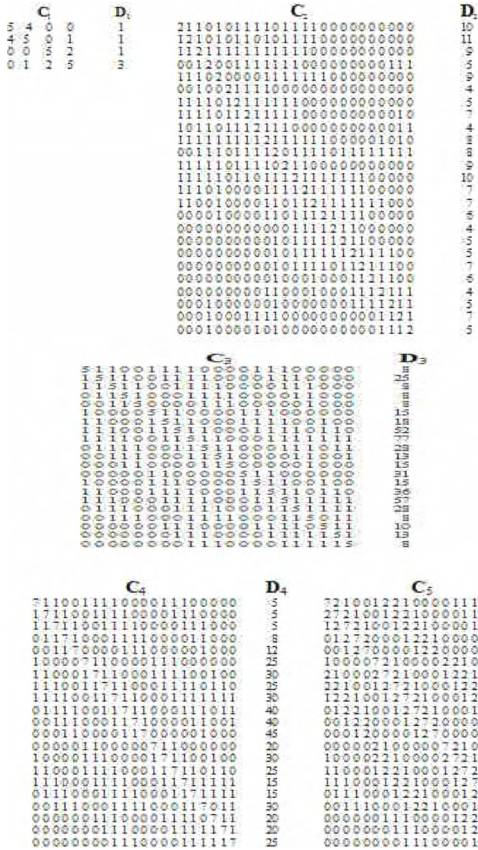


TABLE II. CHANNEL ASSIGNMENT SOLUTION FOR PROBLEM #1

Cells #	Channels Assignment
1	6
2	1
3	8
4	10

2) Results of problem #2
Table 5 displays the solution obtained using 100 individuals which are evolved during 80 generations.

Table 3 presents the channel assignment solution for problem #2. For this problem, the number of cells is 25 and the total number of frequency request in the cellular system is 167, while the available spectrum is 73.

The DPSO is performed using a population of 10 individuals which evolve for 10 generations.

As reported in table 3, the obtained solution includes channels that do not violate the electromagnetic compatibility constraints and use the total available spectrum [0...72].

TABLE III. CHANNEL ASSIGNMENT SOLUTION FOR PROBLEM #2

Cells #	Channels Assignment										
1	8	0	59	2	4	6	10	12	14	46	
2	1	3	5	11	13	16	18	20	9	22	24
3	17	19	21	23	25	27	37	29	31		
4	1	3	10	12	14						
5	28	30	35	39	33	41	43	45	47		
6	0	2	4	6							
7	28	30	33	35	38						
8	39	47	41	43	45	49	52				
9	9	11	5	18							
10	50	53	48	58	64	55	66	61			
11	2	4	20	24	26	36	46	51			
12	32	26	36	51	56	60	15	62	65		
13	57	63	54	67	69	71	40	44	7	34	
14	49	52	42	68	70	72	38				
15	17	19	21	23	25	27	31				
16	0	3	5	10	12	14					
17	1	4	6	11							
18	13	16	18	22	28						
19	35	29	9	37	41						
20	1	11	30	6	32	43	39				
21	0	5	10	12	18	14					
22	28	33	38	3							
23	19	21	25	27	15						
24	23	29	13	0	16	34	32				
25	6	17	8	22	35						

3) Results of problem #3

This problem is more complicated than the previous one in term of the total number of channel request in the cellular system which is equal to 481 demands. In addition, the co-site constraints (CSC), given by the diagonal elements of the compatibility matrix, are set to 5.

For this problem, the population size used is set to 40 and the maximum of generations is 30. Table 4 depicts the obtained solution which does not violate the electromagnetic compatibility matrix.

4) Results of problem #8

For this system of 21 cells, the available spectrum is [1...309], the total number of channels request is 470 and the co-site constraints is 7.

V. CONCLUSION

The fixed spectrum frequency assignment problem is studied in this paper. The objective of this study is to minimize the total interference of an assignment plan.

Due to the NP-hardness of this problem, any exact optimization algorithm requires in the worst case an amount of time exponentially growing with the size of the instance. For this reason, the heuristic methods are more suitable for this case. These heuristics attempt to identify an acceptable solution in a reasonable amount of time.

For that reason, a hybrid discrete particle swarm optimization algorithm is used to solve the frequency assignment problem. This algorithm implements a deterministic local search procedure within discrete particle swarm paradigm. The hybridization of a local search procedure, based on enumerative method, and discrete particle swarm algorithm allows exploiting both determinist feature of the local search in finding the exact solution and stochastic feature of particle swarm algorithm to reduce the time of computation required to convergence.

In conclusion, for cellular systems with high number of cells, this algorithm can be efficiently applied to find the exact solution in an acceptable time of computation.

REFERENCES

- [1] W. K. Hale, "New spectrum management tools", Proc. IEEE Int. Symp. Electromag. Compatibility, pp. 47- 53, (1981).
- [2] S. Hurley, D. H. Smith, and S. U. Thiel, "FASoft: A system for discrete channel frequency assignment", Radio Science, vol. 32, no. 5, pp. 1921-1939. (1997).
- [3] W. K. Hale, "Frequency assignment: theory and applications", Proc. IEEE (68), 1497-1514. (1980).
- [4] D. Kunz, "Channel assignment for cellular radio using neural network", IEEE Transactions on Vehicular Technology, Vol. 40, pp. 188-193. (1991).
- [5] R. Mathar and J. Mattfeldt, "Channel assignment in cellular radio networks", IEEE Transactions on Vehicular Technology, Vol. 42, 647-656. (1993).
- [6] D. W. Tcha, Y. J. Chung and T. J. Choi, "A New Lower Bound for the Frequency Assignment Problem", IEEE Trans. on Networking 5(1):34-39. (1997).
- [7] R. Montemanni, D.H Smith., S.M. Allen, "An improved algorithm to determine lower bounds for the fixed spectrum frequency assignment problem", European Journal of Operational Research (156)736 -751, (2004).
- [8] K. I. Aardal, A. Hipolito, S. Van Hoesel and B. Jansen, "A Branch-and-Cut Algorithm for the Frequency Assignment Problem", Technical Report Annex T-2.2.1 A, CALMA project, T.U. Eindhoven and T.U. Delft, (1995).
- [9] V. Maniezzo, and R. Montemanni, "An exact algorithm for the radio link frequency assignment problem", Report CSR 99-02, Computer Science, Univ. of Bologna, (1999).
- [10] R. Montemanni, N. J. Moon and D. H. Smith, "An improved Tabu search algorithm for the fixed spectrum frequency assignment problem", IEEE Transactions on Vehicular Technology, vol. 52, No.3, (2003).
- [11] M. Duque-Anton, D. Kunz, B. Ruber, "Channel assignment for cellular radio using simulated annealing", IEEE Trans. Vehicular Technol. (42)14 - 21, (1993).
- [12] W. Crompton, S. Hurley, N. M. Stephens, "A parallel genetic algorithm for frequency assignment problems", in Proceedings of the IMACS =IEEE Conference on Signal Processing, Robotics and Neural Networks, Lille, France, pp 81 -84. (1994).
- [13] S.A.G. Shirazi, H. Amindavar, "Fixed channel assignment using new dynamic programming approach in cellular radio networks", Computers and Electrical Engineering (31)303 -333. (2005).
- [14] J. Alami, A. El Imrani, "A Hybrid Ants Colony Optimization for Fixed- Spectrum Frequency Assignment Problem", IA. (2006).
- [15] Z. Li. Stan, "Markov Random Field, Modeling in computer vision", Spring Verlag, 1995.
- [16] R-H. Cheng, C. W. Yu and T-K. Wu., "A Novel Approach to the Fixed Channel Assignment Problem", Journal of Information Science And Engineering 21, 39-58 (2005).

TABLE IV. CHANNEL ASSIGNMENT SOLUTION FOR PROBLEM #3

Cells#	Channels Assignment																							
1	128	2	7	12	17	22	27	32																
2	188	3	8	13	18	23	28	38	43	48	53	58	63	68	73	78	83	88	93	98	103	108	113	118
3	123	4	9	14	19	24	29	34																
4	62	2	7	12	17	22	27	32																
5	41	1	6	11	16	21	26	31																
6	80	1	6	11	16	21	26	31	36	41	46	51	56	61	66									
7	123	4	9	14	19	24	29	34	39	44	49	54	59	64	69	74	79	84						
8	285	5	10	15	20	42	47	52	57	62	67	72	77	82	87	92	97	102	107	112	117	122	127	132
	137	142	147	152	157	162	167	172	177	182	187	192	197	202	207	212	217	222	227	232	237	242	247	252
	257	262	267	272																				
9	1	6	11	16	21	26	31	36	41	46	51	56	61	66	71	76	81	86	91	96	101	106	111	116
	121	126	131	136	141	146	151	156	161	166	171	176	181	186	191	196	201	206	211	216	221	226	231	236
	241	246	251	256	261	266	271	276	281	286	291	296	301	306	311	316	321	326	331	336	341	346	351	356
	361	366	371	376	381																			
10	37	44	49	54	59	64	69	74	79	84	89	94	99	104	109	114	119	124	129	134	139	144	149	154
	159	164	169	174																				
11	68	3	8	13	18	23	28	33	38	43	48	53	58											
12	80	4	9	14	19	24	29	34	39	45	50	55	60	65	70									
13	157	2	7	12	17	22	27	32	37	42	47	52	57	62	67	72	77	82	87	92	97	102	107	112

[illegible]

TABLE V. CHANNEL ASSIGNMENT SOLUTION FOR PROBLEM #8

Cells#	Channels Assignment																											
1	48	6	13	20	27																							
2	1	8	15	24	31																							
3	70	77	42	49	56																							
4	23	30	90	97	104	118	2	9																				
5	174	181	188	32	111	18	4	11	25	62	69	76																
6	201	157	194	166	173	183	1	9	16	23	30	37	44	51	58	65	72	79	86	93	100	107	118	125	132			
7	139	146	153	160	239	251	3	11	18	25	32	39	46	53	60	67	74	81	88	95	102	109	116	181	188			
	196	203	210	217	224																							
8	29	230	22	57	36	43	50	64	71	78	85	92	99	112	119	129	162	169	176	237	287	244	253	260	267			
9	173	180	187	264	271	257	250	5	12	19	26	33	40	47	54	61	68	75	82	89	96	103	110	117	124			
	131	138	145	152	159																							
10	37	289	7	14	296	303	44	51	58	65	72	79	86	93	100	114	121	128	135	142	149	156	163	170	177			
	184	191	198	205	212	268	275	282																				
11	270	277	284	291	20	298	305	27	39	46	53	60	67	74	81	88	95	102	109	116	123	130	137	144	151			
	158	165	172	179	186	242	249	256																				
12	1	8	15	22	29	36	43	50	57	64	71	78	85	92	99	106	113	120	127	134	141	148	155	162	169			
	176	183	190	197	204	260	267	274	281	288	295	302	309															
13	122	129	136	143	150	103	4	12	19	26	33	40	47	54	61	68	75	82	89	96								
14	226	191	198	205	212	219	7	14	21	28	35	42	49	56	63	70	77	84	91	98	105	113	120	127				
	134	141	148	155	163	170																						
15	249	123	137	144	151	158	165	172	179	186	193	200	207	214	221	228	235	242	256	263	270	277	284	291	298			
16	274	246	126	133	140	182	2	9	16	190	197	204	211	218	225													
17	175	278	285	292	194	201	21	28	63	208	215	222	106	229	236													
18	294	287	189	132	139	125	196	3	10	17	24	203	34	41	48	55	210	217	224	231	238	245	252	259	266			
	273	280	146	153	160																							
19	114	121	128	135	142	149	6	13	20	27	34	41	48	55	62	72	79	86	93	100								
20	109	116	130	161	147	154	4	11	18	25	32	39	46	53	60	67	74	81	88	95								
21	164	171	178	185	192	199	1	8	15	30	43	50	57	69	76	83	90	97	104	111	118	127	134	141	150			