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On the Design Problem of Cellular Wireless Networks

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Abstract. In this paper, we deal with the problem of how to design cellular networks in a cost-effective way. We first propose an optimization model that deals with selecting the location of the base station controllers (BSCs) and mobile service switching centers (MSCs), selecting their types, designing the network topology and selecting the link types. In order to find a "good" solution, we propose a tabu search algorithm. Numerical results show that the tabu search algorithm produces solutions close to a proposed lower bound.

Keywords: cellular networks, topological design, BSC and MSC location, capacity planning, tabu search

1. Introduction

In a typical cellular network, the area of coverage is geographically divided into cells and the network topology is hierarchically organized in order to reduce costs. Each cell is equipped with a base transceiver station (BTS) that contains the radio transceivers providing the radio interface with mobile stations. One or more BTSs are connected to a base station controller (BSC) that provides a number of functions related to resource and mobility management as well as operation and maintenance for the overall radio network. One or more BSCs are connected to a mobile switching center (MSC) or switch that control call setup, call routing, while performing many other functions provided by a conventional communications switch. An MSC can be connected to other networks such as the public switched telephone network (PSTN), in order to provide a larger coverage.

Cellular network operators dedicate an important proportion of their budget to acquire, install and maintain the facilities (BTS, BSC, MSC, etc.) that carry traffic from cell sites to switches and other facilities. These facilities are often leased from local exchange carriers. The pressure to reduce costs adds new urgency to the search for optimized networks which can minimize the cost of required facilities while satisfying a set of predetermined constraints.

Typically, the design of cellular networks requires:

- the analysis of radio-wave propagation and/or the field topology to identify a set of possible base stations (BTS, BSC) locations;
- the selection of a least-cost subset of locations (network nodes) as hubs where the traffic is to be aggregated and switched;
- the assignment of each cell to a switch (MSC) while taking into account a certain number of constraints including capacity constraints, routing-diversity to assure reliability, handoffs frequency, and so on;
- the selection of the type of links between the nodes or network elements (BTS, BSC, MSC).

Many aspects of the overall design problem refer to a number of well-known operational research problems, such as graph partitioning [9,10] or p-fixed hubs location problem [13,14]. This kind of problems is $\mathcal{NP}$-hard, thus exact algorithms are practically inappropriate for moderate and large-size cellular networks. As a result, heuristic approaches have been largely used for solving these aspects of the design problem [1–3,5,10–12]. In particular, Cox and Sanchez [3] studied the whole design process and used a tabu search metaheuristic, with embedded knapsack and network flow subproblems to design a least-cost telecommunications networks to carry cell site traffic to wireless switches while meeting survivability, capacity, and technical compatibility constraints. In this context, each optimization problem is solved while accounting for its impacts on the other ones. Merchant and Sengupta [10] studied only the assignment problem. Their algorithm starts from an initial solution, which they attempt to improve through a series of greedy moves, while avoiding to be stranded in a local minimum. The moves used to escape a local minimum explore only a very limited set of options. These moves depend on the initial solution and does not necessarily lead to a good final solution. Other heuristics, strongly inspired by Merchant and Sengupta [10], were proposed by Bhattacharjee et al. [2]. These heuristics are based on the formation of cells clusters related to the same switch. The cell where the switch resides is the root of the cluster, then each cluster is extended by judiciously adding other cells. Several versions of the algorithm were proposed. In general, these algorithms improve the results of Merchant and Sengupta, but remains nevertheless ineffective for designing large-size cellular networks.

In this paper, we are interested in a global approach to the cellular network design problem. The proposed problem deals with jointly

- selecting the location of the BSCs and MSCs;
- selecting the BSC and MSC types;
- designing the network topology;
- selecting the link types.

This global problem has not been considered to date in the literature as well as the selection subproblem of the BSC and MSC types.

The paper is organized as follows. In Section 2, we present a model for the design problem of cellular networks. In Section 3, we present a tabu search algorithm. Numerical results are presented in Section 4. Conclusions and directions for further research are discussed in Section 5.

2. The design problem of cellular networks

In this section, we propose a model for the design of the network subsystem of cellular networks. This model is developed for the second generation of cellular networks using, for instance, GSM (Global System for Mobile communications), CDMA (Code Division Multiple Access) or TDMA (Time Division Multiple Access) systems [15]. However, it can also be used for the design of third generation networks using, for instance, WCDMA (Wideband CDMA) or CDMA2000 systems [15], if a tree-topology architecture is selected for the network subsystem.

2.1. Problem formulation

For the design problem of cellular networks, the following information is considered known:

- the location of the BTSs and their types;
- the traffic (in erlang) between the BTSs and from/to the public network;
- the location of potential sites to install the BSCs;
- the location of potential sites to install the MSCs;
- the different types of BSCs, their costs and capacities;
- the different types of MSCs, their costs and capacities;
- the installation cost of a given type of BSC at a given site (including the floor space, cables, racks, electric installations, labor, etc.);
- the installation cost of a given type of MSC at a given site (including the floor space, cables, racks, electric installations, labor, etc.);
- the costs of the different interface types plus the installation costs;
- the costs of the different link types plus the installation costs (including the cables or antennas for wireless links, the use of physical layer equipments, labor, etc.).

We also make the following assumptions about the organization of the network:

- each BTS is connected to exactly one BSC (with one or more BTS-BSC links);
- each BSC is connected to exactly one MSC (with one or more BSC-MSC links);
- each MSC is connected to the public network;
- the number of BTS-BSC links connected to a BSC cannot exceed the maximum number of BTS interfaces that can be put in that BSC;
- the sum of the capacities of the BTSs connected to a BSC cannot exceed its switch fabric capacity (in circuit);
- the number of BSC-MSC links connected to a BSC cannot exceed the maximum number of MSC interfaces that can be put in that BSC;
- the number of BSC-MSC links connected to an MSC cannot exceed the maximum number of BSC interfaces that can be put in that MSC;
- the sum of the rates of the interfaces installed in each MSC cannot exceed its switch fabric capacity (in circuit);
- at most one BSC can be installed at a BSC site;
- at most one MSC can be installed at an MSC site.

Solving the problem involves selecting the location of the BSCs and MSCs, selecting the BSC and MSC types, designing the network topology and selecting the link types. The objective of the problem is to minimize the network cost subject to all of the information and by taking into account the assumptions described above.

2.2. Notation

The following notation is used throughout the paper.

Sets: Let R be the set of BTS types (where α^r is the capacity (in circuit) of a BTS of type $r \in R$ and n^r is the number of BTS interfaces (or BTS-BSC links) necessary to connect it to the BSC) and I, the set of all BTSs, such that $I = \bigcup_{r \in R} I_r$ where I_r is the set of BTSs of type $r \in R$. Let J be the set of potential sites to install the BSCs, K the set of potential sites to install the MSCs and L, the set of BSC-MSC link types (where β^l is the capacity (in circuit) of the link of type $l \in L$). Let S be the set of BSC types (where m^s_{BTS} is the maximum number of BTS interfaces (or BTS-BSC links) that can be put in a BSC of type $s \in S$, m^s_{MSC} the maximum number of MSC interfaces (or BSC-MSC links) that can be put in a BSC of type $s \in S$ and η^s its switch fabric capacity (in circuit)) and T, the set of MSC types (where m^t_{BSC} is the maximum number of BSC interfaces that can be put in an MSC of type $t \in T$ and η^t its switch fabric capacity (in circuit)).

Decision Variables: Let v_{ij} be a 0–1 variable such that $v_{ij} = 1$ if and only if the BTS $i \in I$ is connected to a BSC installed at site $j \in J$, w_{jk} a 0–1 variable such that $w_{jk} = 1$ if and only if a BSC installed at site $j \in J$ is connected to an MSC installed at site $k \in K$ and x^l_{jk}, the number of links of type $l \in L$ installed between a BSC installed at site $j \in J$ is connected to an MSC installed at site $k \in K$. Let y^s_j be a 0–1 variable such that $y^s_j = 1$ if and only if a BSC of type $s \in S$ is installed at BSC site $j \in J$ and z^t_k, a 0–1 variable such that $z^t_k = 1$ if and only if an MSC of type $t \in T$ is installed at MSC site $k \in K$.

Traffic Variables: Let f_i be the traffic (in erlang) from BTS $i \in I$ to a BSC and f_j, the traffic (in erlang) from BSC $j \in J$ to an MSC.

Cost Parameters: Let a_{ij}^r be the link and interface costs (including the installation cost) for connecting BTS $i \in I(r)$ to a BSC installed at site $j \in J$ and b_{jk}^l, the link and interface costs (including the installation cost) for connecting a BSC installed at site $j \in J$ to an MSC installed at site $k \in K$ through a link and interfaces of type $l \in L$. Let c_j^s be the cost of a BSC of type $s \in S$ and installing it at site $j \in J$ and d_k^t, the cost of an MSC of type $t \in T$ and installing it at site $k \in K$.

Bandwidth Parameters: Let g^{oi} be the average number of communications per hour between from BTS $o \in I$ to BTS $i \in I$, g^{ip} the average number of communications per hour from BTS $i \in I$ to the public network and g^{pi}, the average number of communications per hour from the public network to BTS $i \in I$.

2.3. Network cost function

The cost function, representing the total cost of the network, is composed of the cost of the links and interfaces and the cost of the BSCs and MSCs.

The cost of the links and interfaces, noted $C_{L/I}(\mathbf{v}, \mathbf{x})$, is given by the following equation:

$$C_{L/I}(\mathbf{v}, \mathbf{x}) = \sum_{r\in R}\sum_{i\in I_r}\sum_{j\in J} a_{ij}^r v_{ij} + \sum_{j\in J}\sum_{k\in K}\sum_{l\in L} b_{jk}^l x_{jk}^l. \quad (1)$$

The cost of the BSCs and MSCs, noted $C_{B/M}(\mathbf{y}, \mathbf{z})$, is given below:

$$C_{B/M}(\mathbf{y}, \mathbf{z}) = \sum_{j\in J}\sum_{s\in S} c_j^s y_j^s + \sum_{k\in K}\sum_{t\in T} d_k^t z_k^t. \quad (2)$$

2.4. Model formulation

The model for the design problem of cellular networks, noted DPCN, can now be given.
DPCN:

$$\min C_{L/I}(\mathbf{v}, \mathbf{x}) + C_{B/M}(\mathbf{y}, \mathbf{z}) \quad (3)$$

subject to

BTS assignment constraints

$$\sum_{j\in J} v_{ij} = 1 \quad (i \in I) \quad (4)$$

BSC assignment constraints

$$\sum_{k\in K} w_{jk} = \sum_{s\in S} y_j^s \quad (j \in J) \quad (5)$$

BSC-type uniqueness constraints

$$\sum_{s\in S} y_j^s \le 1 \quad (j \in J) \quad (6)$$

MSC-type uniqueness constraints

$$\sum_{t\in T} z_k^t \le 1 \quad (k \in K) \quad (7)$$

BSC capacity constraints (BTS interface level)

$$\sum_{r\in R} n^r \sum_{i\in I_r} v_{ij} \le \sum_{s\in S} m_{\text{BTS}}^s y_j^s \quad (j \in J) \quad (8)$$

BSC capacity constraints (MSC interface level)

$$\sum_{l\in L}\sum_{k\in K} x_{jk}^l \le \sum_{s\in S} m_{\text{MSC}}^s y_j^s \quad (j \in J) \quad (9)$$

BSC capacity constraints (switch fabric capacity level)

$$\sum_{r\in R} \alpha^r \sum_{i\in I_r} v_{ij} \le \sum_{s\in S} \eta^s y_j^s \quad (j \in J) \quad (10)$$

MSC capacity constraints (BSC interface level)

$$\sum_{l\in L}\sum_{j\in J} x_{jk}^l \le \sum_{t\in T} m_{\text{BSC}}^t z_k^t \quad (k \in K) \quad (11)$$

MSC capacity constraints (switch fabric capacity level)

$$\sum_{l\in L} \beta^l \sum_{j\in J} x_{jk}^l \le \sum_{t\in T} \eta^t z_k^t \quad (k \in K) \quad (12)$$

BTS-BSC link capacity constraints

$$f_i \le \alpha^r \quad (i \in I_r,\ r \in R) \quad (13)$$

BSC-MSC link capacity constraints

$$f_j \le \sum_{k\in K}\sum_{l\in L} \beta^l x_{jk}^l \quad (j \in J) \quad (14)$$

Traffic flow conservation constraints

$$f_i = \sum_{o\in I}(g^{io} + g^{oi}) + g^{ip} + g^{pi} \quad (i \in I) \quad (15)$$

$$f_j = \sum_{i\in I} v_{ij}\left(\sum_{o\in I}(g^{io}+g^{oi})\right) + \sum_{i\in I} v_{ij}(g^{ip}+g^{pi}) \quad (j \in J) \quad (16)$$

Additional constraints

$$\sum_{l\in L} x_{jk}^l \le w_{jk} \max_{s\in S}\left\{m_{\text{MSC}}^s\right\} \quad (j \in J,\ k \in K) \quad (17)$$

$$w_{jk} \le \sum_{l\in L} x_{jk}^l \quad (j \in J,\ k \in K) \quad (18)$$

Integrality and nonnegativity constraints

$$\begin{aligned} &\mathbf{f} \in \mathbb{R}_+^{(|I|+|J|)}, \quad \mathbf{v} \in \mathsf{B}^{|I||J|}, \quad \mathbf{w} \in \mathsf{B}^{|J||K|}, \\ &\mathbf{x} \in \mathbb{N}^{|J||K||L|}, \quad \mathbf{y} \in \mathsf{B}^{|J||S|}, \quad \mathbf{z} \in \mathsf{B}^{|K||T|}. \end{aligned} \quad (19)$$

The objective function (3) of DPCN, as mentioned before, is composed of three terms, representing the total network cost. The BTS assignment constraints (4) require each BTS to be connected to exactly one BSC and the BSC assignment constraints (5) impose each BSC to be connected to exactly one MSC. The BSC-type uniqueness constraints (6) require that at most one BSC type be installed at site $j \in J$ and MSC-type uniqueness constraints (7) impose that at most one MSC type be installed at site $k \in K$. Constraints (8) require the total number of BTS-BSC links connected at site $j \in J$ be less than or equal to the maximum number of BTS interfaces that can be put in the BSC type installed at that site and constraints (9) impose the total number of BSC-MSC links connected at site $j \in J$ be less than or equal to the maximum number of MSC interfaces that can be put in the BSC type installed at that site. Constraints (10) require the sum of the BTS rates connected to the BSC type installed at site $j \in J$ be less than or equal to its switch fabric capacity and constraints (11) impose the maximum number of BSC-MSC links connected to site $k \in K$ be less or equal to the maximum number of BSC interfaces that can be put in the MSC type installed at that site. Constraints (12) require the sum of the rates of the BSC-MSC links connected to the MSC type installed at site $k \in K$ be less than or equal to its switch fabric capacity. Constraints (13) and (14) are respectively the BTS-BSC and the BSC-MSC link capacity constraints, constraints (15) and (16) are traffic flow conservation constraints and constraints (17) and (18) impose, for all $j \in J$ and $k \in K$, $w_{jk} = 1$ if and only if $\sum_{l \in L} x^l_{jk} \geq 1$. Constraints (20) are integrality and nonnegativity constraints.

Note that DPCN can not be decomposed exactly into two or more subproblems. As a result, the only way to find an optimal solution is to consider the problem as a whole. Moreover, DPCN is $\mathcal{NP}$-hard (transformation from the uncapacity facility location problem [4]). That is the reason why, in the rest of this paper, we concentrate our efforts on the development of efficient heuristics.

3. The tabu algorithm

In this section, we propose a tabu search (TS) algorithm for DPCN, called TS-DPCN. The basic principle of the tabu search is to define a set of possible solutions and, starting from the current solution, to find a better one in its neighborhood. In order for the algorithm to move away from a local minimum, the search allows moves resulting in a degradation of the objective function value and the solutions obtained recently are considered tabu to prevent the algorithm from examining a local minimum more than once. For an introduction to tabu search, see Glover and Laguna [6].

The following notation is used in the presentation of TS-DPCN. Let e_j be the state of the site $j \in J$ such that $e_j = 0$ if there is no BSC installed at site j and $e_j = s$ (for $s \in S$) if a BSC of type s is installed at site j. Similarly, let e_k be the state of the site $k \in K$ such that $e_k = 0$ if there is no MSC installed at site k and $e_k = t$ (for $t \in T$) if an MSC of type t is installed at site k.

Let $\mathbf{y}(\mathbf{e})$ and $\mathbf{z}(\mathbf{e})$ be the vectors $\mathbf{y}$ and $\mathbf{z}$ when the state vector of all sites $\mathbf{e} = \{e_j\}_{j \in J} \bigcup \{e_k\}_{k \in K}$ is fixed.

In the next subsection, we propose a decomposition approach to solve DPCN($\mathbf{e}$), i.e., the model DPCN when the decision vectors $\mathbf{y}$ and $\mathbf{z}$ are set respectively to vectors $\mathbf{y}(\mathbf{e})$ and $\mathbf{z}(\mathbf{e})$.

3.1. Solving DPCN($\mathbf{e}$)

When vectors $\mathbf{y}$ and $\mathbf{z}$ are fixed, DPCN can be decomposed into two subproblems. The first subproblem, noted $\overline{\text{DPCN}}(\mathbf{y}, \mathbf{z})$, is given below.

$\overline{\text{DPCN}}(\mathbf{y}, \mathbf{z})$:

$$\min_{\mathbf{v}} \sum_{r \in R} \sum_{i \in I_r} \sum_{j \in J} a^r_{ij} v_{ij} \tag{20}$$

subject to (4), (8), (10) and

$$\mathbf{v} \in \mathsf{B}^{|I||J|}. \tag{21}$$

The purpose of this subproblem is to connect the BTSs to the BSCs while respecting assignment, BSC degree and capacity constraints. Since this subproblem is $\mathcal{NP}$-hard (transformation from the knapsack problem [4]) and we may have to solve it several thousands instances of it during the tabu search procedure, we propose a heuristic to find solutions quickly. This heuristic, called HFS (Heuristic for the First Subproblem), is presented below.

Heuristic HFS

Step 1: Order the elements in R such that $\alpha^1 > \alpha^2 > \cdots > \alpha^{|R|}$.

Step 2: For $r := 1$ to $|R|$ do

Solve the following problem, called $\overline{\text{DPCN}}_r(\mathbf{v}, \mathbf{y}, \mathbf{z})$, that consists of connecting the BTSs in I_r to the BSCs considering that the BTSs in $I_1 \cup \ldots \cup I_{r-1}$ are already connected to BSCs.

$\overline{\text{DPCN}}_r(\mathbf{v}, \mathbf{y}, \mathbf{z})$:

$$\min_{\{v_{ij} : i \in I_r, j \in J\}} \sum_{i \in I_r} \sum_{j \in J} a^r_{ij} v_{ij} \tag{22}$$

subject to

$$\sum_{j \in J} v_{ij} = 1 \quad (i \in I_r) \tag{23}$$

$$\sum_{i \in I_r} v_{ij} \leq \left\lfloor \frac{1}{n^r} \left(\sum_{s \in S} m^s_{\text{BTS}} y^s_j - \sum_{o=1}^{r-1} \sum_{i \in I_o} n^o v_{ij} \right) \right\rfloor \quad (j \in J) \tag{24}$$

$$\sum_{i \in I_r} v_{ij} \leq \left\lfloor \frac{1}{\alpha^r} \left(\sum_{s \in S} \eta^s y^s_j - \sum_{o=1}^{r-1} \sum_{i \in I_o} \alpha^o v_{ij} \right) \right\rfloor \quad (j \in J) \tag{25}$$

$$v_{ij} \in \mathbb{R}_+ \quad (i \in I_r; j \in N). \tag{26}$$

This subproblem is an instance of the linear assignment problem; to solve it, we use the shortest augmenting path algorithm LAPJV of Jonker and Volgenant [8].

Step 3: Return the vector $\mathbf{v}$ and the cost of the subproblem using (20).

The second subproblem, noted $\overline{\overline{\text{DPCN}}}(\mathbf{v}, \mathbf{y}, \mathbf{z})$, is given below.

$\overline{\overline{\text{DPCN}}}(\mathbf{v}, \mathbf{y}, \mathbf{z})$:

$$\min_{\mathbf{f}, \mathbf{w}, \mathbf{x}} \sum_{j \in J} \sum_{k \in K} \sum_{l \in L} b^l_{jk} x^l_{jk} \tag{27}$$

subject to (5), (9), (11)–(18) and

$$\mathbf{f} \in \mathbb{R}_+^{(|I|+|J|)}, \quad \mathbf{w} \in \mathsf{B}^{|J||K|}, \quad \mathbf{x} \in \mathbb{N}^{|J||K||L|}. \tag{28}$$

The purpose of this subproblem is to connect the BSCs to the MSCs while respecting assignment constraints, BSC degree constraints, MSC degree and capacity constraints, BSC-MSC link capacity constraints and traffic flow conservation constraints. Since this subproblem is $\mathcal{NP}$-hard (transformation from the knapsack problem [4]), we propose a heuristic to find solutions quickly. This heuristic, called HSS (Heuristic for the Second Subproblem), is presented below.

Heuristic HSS

Step 1: For all $j \in J$ do

1.1. Calculate Traffic(j) defined as the traffic (in erlang) passing through BSC j that should be switched by an MSC.

$$\text{Traffic}(j) := \sum_{i\in I} v_{ij}\left(\sum_{o\in I}(g^{io}+g^{oi})\right)+\sum_{i\in I} v_{ij}(g^{ip}+g^{pi}).$$

1.2. Calculate MinLinks(j) defined as the minimum number of links needed from BSC j to an MSC.

$$\text{MinLinks}(j) := \left\lceil \frac{\text{Traffic}(j)}{\max_{l\in L}\{\beta^l\}} \right\rceil.$$

1.3 If MinLinks$(j) \le \sum_{s\in S} m^s_{\text{BTS}} y^s_j$ go to Step 1.4. Otherwise, stop, the subproblem is not feasible.

1.4 Determine by enumeration the number of links of each type NumLinks(j, l) for all $l \in L$, such that the total number of links is MinLinks(j) and the total capacity, noted TotalLinkCapacity(j) is minimum but greater or equal to Traffic(j).

Step 2: Let $\kappa^1, \kappa^2, \ldots, \kappa^{|P|}$ be the different values of TotalLinkCapacity(j) for all $j \in J$ such that $\kappa^1 > \kappa^2 > \cdots > \kappa^{|P|}$ and set $J_p = \{j \in J : \text{TotalLinkCapacity}(j) = \kappa^p\}$ for $p = 1, \ldots, |P|$. Moreover, let σ^p be the minimum number of links to obtain a capacity of κ^p, for $p = 1, \ldots, |P|$.

Step 3: For $p := 1$ to $|P|$ do

Solve the following problem, called $\overline{\overline{\text{DPCN}}}_p(\mathbf{x}, \mathbf{v}, \mathbf{y}, \mathbf{z})$, that consists of connecting the BSCs in J_p to the MSCs considering that the BSCs in $J_1 \cup \cdots \cup J_{p-1}$ are already connected to MSCs.

$\overline{\overline{\text{DPCN}}}_p(\mathbf{x}, \mathbf{v}, \mathbf{y}, \mathbf{z})$:

$$\min_{\{w_{jk}: j\in J_p,\, k\in K\}} \sum_{j\in J_p}\sum_{k\in K}\left(\sum_{l\in L}\text{NumLinks}(j,l)b^l_{jk}\right)w_{jk} \tag{29}$$

subject to

$$\sum_{k\in K} w_{jk} = 1 \quad (j \in J_p) \tag{30}$$

$$\sum_{j\in J_p} w_{jk} \le \left\lfloor \frac{1}{\sigma^p}\left(\sum_{t\in T} m^t_{\text{BSC}} z^t_j - \sum_{j\in\cup_{o=1}^{p-1} J_o} \text{MinLinks}(j) w_{jk}\right)\right\rfloor \quad (k \in K) \tag{31}$$

$$\sum_{j\in J_p} w_{jk} \le \left\lfloor \frac{1}{\kappa^p}\left(\sum_{t\in T} \eta^t z^t_j - \sum_{j\in\cup_{o=1}^{p-1} J_o} \text{TotalLinkCapacity}(j) w_{jk}\right)\right\rfloor \quad (k \in K) \tag{32}$$

$$w_{jk} \in \mathbb{R}_+ \quad (j \in J_p,\ k \in K). \tag{33}$$

This subproblem is an instance of the linear assignment problem; to solve it, we use the shortest augmenting path algorithm LAPJV of Jonker and Volgenant [8].

Step 4: For all $j \in J$, $k \in K$ and $l \in L$ set $x^l_{jk} :=$ NumLinks$(j, l)w_{jk}$.

Step 5: Return the vector $\mathbf{x}$ and the cost of the subproblem using (27).

3.2. The tabu algorithm

Each move of tabu search consists of modifying the state of a given site in the current solution. At each iteration of the search, we determine the best move (among the $|J||S|+|K||T|$ moves) while taking into account the tabus as well as the aspiration criterion. The chosen site is declared tabu for a number of iterations that is randomly determined according to a uniform discrete distribution on the interval $[L, U]$.

The aspiration criterion states that if the use of tabu site allows us to discover a solution better than any other found so far, we may remove the tabu from this site.

We now proceed to a detailed description of TS-DPCN.

Algorithm TS-DPCN

Step 1: (Initial solution)

Find an initial solution by solving DPCN($\mathbf{e}$) where vector $\mathbf{e}$ is determined as follows. Set $e_j := \max\{\eta^s : s \in S\}$, for all $j \in J$ and set $e_k := \max\{\eta^t : t \in T\}$, for all $k \in K$.

Repeat Steps 2 to 3 for MaxIter iterations

Step 2: (Exploring the neighborhood)

2.1. Determine the best move while taking into account the tabus and the aspiration criterion. For each move $\mathbf{e} \to \mathbf{e}'$, which modifies the state of a given site in the current solution, we solve DPCN($\mathbf{e}'$) as described in Section 3.1. The cost of a solution is given by the objective function (3).

2.2. Determine the number of iterations according to a uniform distribution on the interval $[L, U]$ for which the chosen site is tabu.

Step 3: (Best solution update)
If the cost of the current solution is less than the cost of the best solution found so far, update this best solution.

4. Numerical results

In this section we present a study of the performance of the proposed heuristics. The algorithms TS-DPCN and IH-DPCN were implemented in the C language on a SunFire 4800 workstation (900 MHz and 8 GB of RAM). For the tests, three BTS types, three BSC types and three MSC types are used. Their features are presented respectively in Tables 1–3. Moreover, DS-1 links are used to connect the BTSs to the BSCs; DS-1 and DS-3 links are used to connect the BSCs to the MSCs. The interfaces costs are presented in Table 4 and the link costs in Tables 5 and 6.

Table 1
Features of the BTS types.

	Type A	Type B	Type C
Capacity (circuits)	96	288	576
Number of BTS DS-1 interfaces	1	3	6

Table 2
Costs of the BSC types (including the installation costs).

	Type A	Type B	Type C
Switch fabric capacity (circuits)	5000	10000	15000
Maxumum number of BTS interfaces	15	30	60
Maxumum number of MSC interfaces	15	30	60
Cost ($)	50000	90000	120000

Table 3
Costs of the MSC types (including the installation costs).

	Type A	Type B	Type C
Switch fabric capacity (circuits)	100000	200000	300000
Maximum number of BSC interfaces	50	100	150
Cost ($)	200000	350000	500000

Table 4
Costs of the interfaces types (including the installation costs).

Interface type	Capacity (circuits)	Cost ($)
DS-1	96	500
DS-3	2688	2500

Table 5
Costs of the BTS-BSC links (including the installation costs).

BTS type	Number of DS-1s	Capacity (circuits)	Cost ($/km)
A	1	96	2000
B	3	288	3000
C	6	576	4000

Table 6
Costs of the BSC-MSC links (including the installation costs).

Link type	Capacity (circuits)	Cost ($/km)
DS-1	96	2000
DS-3	2688	4000

For the tests, the MaxIter parameter of the TS-DPCN heuristic is set to 500 and the interval $[L, U]$ to [5,10] because the best solutions was found within 500 iterations and with this interval.

All the heuristic results were compared to a lower bound (denoted LB) obtained as follows. The bound LB was obtained by using the CPLEX Mixed Integer Solver [7] to solve DPCN without integrality constraints on **v**, **w** and **x** variables. The default settings of CPLEX are used and the branch-and-bound node limit was set to 50 000 and the upper limit on the size of the tree to 500 MB.

For the evaluation of the heuristic performance, 60 test problems are generated as follows. $|I|$ points corresponding to BTSs' locations, $|J|$ points corresponding to the candidate BSC sites and $|K|$ points corresponding to the candidate MSC sites were generated in a square region of length of 100 km following a uniform distribution. The type of each BTS is selected randomly among the three BTS types considered (see Table 1). Finally, the demand between each pair of BTSs and between the BTSs and the public network is generated randomly in the interval [0, 0.2] erlang, following a uniform distribution.

The results are presented in Table 7. In this table, columns 1 to 3 present respectively the number of BTSs, the number of potential BSC sites and the number of MSC sites. Column 4 presents the value of the lower bound or the best bound found by the branch-and-bound algorithm if the node limit (NL) or the tree limit (TL) is reached and, column 5 presents the CPU execution time to find it. Columns 6 to 8 present the solution found by algorithm TS-DPCN, the CPU execution time to find it and the GAP that indicates the percentage gap between the heuristic solution and the lower bound (with respect to the value of the lower bound).

The following observations can be made from Table 7. First, TS-DPCN finds solutions close to optimality for the problems considered (within 5.79%, on average, from the lower bound) and the mean CPU time is 1052.8 seconds. As a result, the proposed algorithm is recommended for designing cellular networks. Moreover, the following observations can be made:

1. TS-DPCN produces solutions close to the lower bound for test networks containing 50 and 100 BTSs;
2. for test networks with more than 100 BTSs, the gaps are larger;
3. for a fixed number of BTSs, the gap does not change so much according to the number of BSC sites and MSC sites.

The second observation can be explained as follows. For all solutions found by TS-DPCN containing more than 100 BTSs, DS-3 links are used between BSCs and MSCs. However, since

Table 7
Computational results.

			LB		TS-DPCN		
$\|I\|$	$\|J\|$	$\|K\|$	OBJ [k$]	CPU [sec]	OBJ [k$]	CPU [sec]	GAP [%]
50	10	10	4051.5	1.0	4083.5	91.3	0.79
100	10	10	6865.3	1.3	7215.8	200.9	5.11
50	20	10	3316.6	4.0	3321.1	126.3	0.14
100	20	10	6244.2	8.8	6599.4	219.9	5.69
150	20	10	8571.8	8.1	9346.3	370.3	9.04
200	20	10	10608.2	14.1	11843.7	492.4	11.65
50	30	10	3518.3	17.0	3525.2	175.9	0.20
100	30	10	6114.5	137.8	6217.3	381.2	1.68
150	30	10	7725.0	125.3	8294.5	593.0	7.37
200	30	10	9281.2	53.8	10216.8	709.2	10.08
250	30	10	11365.9	552.5	12415.5	902.3	9.23
300	30	10	13360.1	105.4	15024.8	1113.7	12.46
50	40	10	3267.9	55.5	3304.2	253.9	1.11
100	40	10	5399.7	605.6	5607.3	572.9	3.85
150	40	10	8234.0	512.1	8817.5	885.4	7.09
200	40	10	9111.7	213.3	10030.0	1158.1	10.08
250	40	10	10901.4	1688.8	12064.4	1402.0	10.67
300	40	10	12947.3	1082.5	13975.0	1736.8	7.94
350	40	10	TL (16350.8)	2442.5	18491.7	1579.8	–
400	40	10	TL (16244.7)	2307.2	18217.9	1929.9	–
50	10	20	4148.0	1.3	4218.0	159.0	1.69
100	10	20	6573.1	1.8	6944.9	320.9	5.66
50	20	20	3557.4	21.4	3583.2	182.2	0.72
100	20	20	6934.9	25.5	7150.9	417.2	3.11
150	20	20	8208.2	36.5	8913.1	773.8	8.59
200	20	20	11106.3	49.4	12231.6	861.9	10.13
50	30	20	3583.8	56.8	3591.7	244.7	0.22
100	30	20	5606.1	168.9	5804.1	613.9	3.53
150	30	20	7551.4	845.0	8092.9	758.5	7.17
200	30	20	10304.1	164.5	11070.6	1030.0	7.44
250	30	20	NL (11983.6)	1749.4	13103.5	1184.1	–
300	30	20	12666.1	369.6	13749.5	1563.3	8.55
50	40	20	3318.8	191.4	3366.1	362.7	1.43
100	40	20	5249.7	433.5	5377.7	731.0	2.44
150	40	20	7360.4	1295.5	8037.9	1086.1	9.20
200	40	20	9598.3	3564.2	10467.4	1488.5	9.05
250	40	20	10795.8	2014.6	11885.9	1996.1	10.10
300	40	20	NL (12999.1)	5496.9	14236.4	2296.6	–
350	40	20	NL (13883.8)	4567.4	15290.7	2526.5	–
400	40	20	NL (15992.4)	7394.4	17644.6	2815.7	–
50	10	30	3886.2	5.0	3942.8	248.8	1.46
100	10	30	6594.2	1.8	6917.6	515.7	4.90
50	20	30	3516.1	20.6	3588.0	238.9	2.04
100	20	30	5739.6	36.8	5949.5	682.4	3.66
150	20	30	8732.8	75.2	9335.9	739.5	6.91
200	20	30	9814.4	41.6	10605.8	1203.4	8.06
50	30	30	3467.2	251.8	3525.2	331.4	1.67
100	30	30	5862.2	885.6	6079.2	714.4	3.70
150	30	30	7743.3	1310.2	8214.7	1125.1	6.09
200	30	30	10237.3	64.3	11255.0	1146.4	9.94
250	30	30	11051.7	357.1	11902.5	1740.5	7.70
300	30	30	NL (13840.9)	3460.6	15079.8	2281.0	–
50	40	30	3232.8	424.1	3298.5	495.9	2.03
100	40	30	NL (5167.3)	2406.6	5279.5	739.9	–
150	40	30	NL (7408.6)	5076.9	7894.1	1519.4	–
200	40	30	NL (9347.7)	5243.4	10391.4	1955.9	–
250	40	30	10828.0	2196.8	11992.0	2393.9	10.75
300	40	30	NL (12155.7)	6447.6	13206.7	2970.8	–
350	40	30	NL (17423.8)	9432.5	19257.0	2472.3	–
400	40	30	NL (16214.9)	9905.6	17820.2	3344.7	–

the integrality constraints on the **x** variables are not considered to find the lower bound, solutions far from feasibility are found. We have tried to improve the value of the lower bound by considering these constraints but we did not succeed to solve it using CPLEX.

5. Concluding remarks

In this paper, we have studied the problem of designing cellular networks and proposed an optimization model for selecting the location of the network nodes (BSC and MSC) and their types, designing the network topology and selecting the link types. After defining the basic components or network elements of such networks and making some realistic assumptions about their organization, we have presented a new mathematical formulation of this design problem. A heuristic for generating initial solutions and a tabu search algorithm for improving this initial solution have been also proposed.

All the results were compared to a proposed lower bound obtained by relaxing a subset of the original constraints in the model. These results have shown that the tabu-based heuristic produces solutions close to the lower bound for test networks containing 50 and 100 BTSs; for test networks with more BTSs, the gaps are larger. For a fixed number of BTSs, the gap does not change so much according to the number of BSC sites and MSC sites. Finally, whatever the number of BTS from 50 to 400, the gap is always less than 12%. As a result, the heuristic proposed is recommended for designing cellular networks.

It would be interesting to improve the lower bound formulation in order to evaluate effectively the proposed tabu search heuristic. Another research direction is the adaptation of our model and algorithms to the design of third-generation systems, by taking into account traffic forecast, network node dimensioning, interface design, transmission network, placement of network nodes and overall network topology.

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