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A Study of Global Optimization Using Particle Swarms

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Abstract. A number of recently proposed variants of the particle swarm optimization algorithm (PSOA) are applied to an extended Dixon-Szegö bound constrained test set in global optimization. Of the variants considered, it is shown that constriction as proposed by Clerc, and dynamic inertia and maximum velocity reduction as proposed by Fourie and Groenwold, represent the main contenders from a cost efficiency point of view. A parameter sensitivity analysis is then performed for these two variants in the interests of finding a reliable general purpose 'off-the-shelf' PSOA for global optimization. In doing so, it is shown that inclusion of dynamic inertia renders the PSOA relatively insensitive to the values of the cognitive and social scaling factors.

Key words. constriction factor, dynamic inertia, global optimization, numerical investigation, particle swarm.

1. Introduction

The particle swarm optimization algorithm (PSOA), first proposed by Kennedy and Eberhart [1, 2], models the optimal exploration of a problem space by a population of agents or particles; the success histories of the agents influences both their own search patterns and those of their peers. The search is focused toward promising regions by biasing each particle's velocity vector toward both the particle's own 'remembered' best position and the 'communicated' best ever swarm location. The relative weights of these two positions are scaled by two factors, aptly called the cognitive and social scaling parameters [3]. Incidentally, these two components are the among the main governing parameters of swarm behavior (and algorithm efficiency), and have previously been the topic of extensive studies [4–6].

A newcomer among optimization algorithms, the derivative-free PSOA has recently received a lot of attention, with some conferences devoted solely to this topic. The reasons for the interest in the PSOA are numerous, but include the following: The algorithm can easily be parallelized on massive parallel processing machines, since the individual searches of the simulated particles are independent

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of each other, and communication between particles is only required once all particles have evolved to the same pseudo time state.

Furthermore, the PSOA is simpler, both in formulation and computer implementation, than the genetic algorithm (GA). In addition, the PSOA seems to outperform the GA for a number of difficult programming classes, notably the unconstrained global optimization problem [7].

Previously, the PSOA has been applied to analytical test functions, mostly univariate or bivariate without constraints, by Kennedy [8] and Shi and Eberhart [9]. Kennedy also applied the algorithm to multimodal problem generators. Previously, Kennedy [4] used the PSOA as an optimization paradigm that simulates the ability of human societies to process knowledge.

The PSOA is suited to the training of neural networks and has been applied to this class of optimization problem by a number of workers, e.g. Eberhart and Hu [10] and van den Bergh and Engelbrecht [11]. Recent contributions by, amongst others, Carlisle and Dozier [12] and Eberhart and Shi [13], include modifications which allow the PSOA to track a changing extremum over time in a dynamic environment. Lately, the PSOA was successfully applied to optimal structural design by Fourie and Groenwold [14–16].

Notwithstanding it's recent popularity, the PSOA has a number of drawbacks, one of which is the presence of problem dependent parameters. Previously, a number of workers have attempted to find 'universal' values for the PSOA parameters, the most recent being Carlisle and Dozier in their paper aptly called 'An off-the-shelf PSO' [17].

A further drawback of the original algorithm proposed by Kennedy and Eberhart lies therein that the algorithm is known to quickly converge to the approximate region of the global minimum. However, the algorithm does not maintain this efficiency when entering the stage where a refined local search is required to pinpoint the minimum exactly. This has led to a number of variations on the original PSOA being proposed to overcome this shortcoming. Some of the most notable of these formulations are the introduction of an inertia term by Shi and Eberhart [3], and more recently, the so-called constriction factor by Clerc [18] in his 'swarm and queen' approach.

Constriction seems superior to the introduction of inertia [19]. In the latter approach, the inertia term is either kept constant or decreased linearly as the search progresses, with the linear decrease in inertia more efficient than a constant inertia term. Recently, Fourie and Groenwold [15] dynamically reduced the inertia of the swarm based on the instantaneous success of the search. This variant allows the inertia method an efficiency and reliability on par with constriction. More importantly, the algorithm becomes relatively insensitive to the values of the cognitive and social scaling factors, a desirable attribute in finding a general 'off the shelf' PSOA for global optimization.

In this paper we evaluate some recent variants of the PSOA, while we also attempt to propose optimal values for the parameters of the most successful variants of the algorithm for unconstrained global optimization. Our paper is structured as follows: In Section 2 we present the global optimization problem. This is followed by an outline of the original PSOA in Section 3. The variants of the PSOA under consideration are then detailed in Section 4. In Section 5 the PSOA and it's variant are applied to an extended Dixon-Szegö test set, to assess the efficiency and reliability of the different variants. In Section 6 we perform a parameter sensitivity study for the most successful variants of the PSOA, namely constriction, and dynamic inertia and maximum velocity reduction. Finally, we propose settings for a general 'off-the-shelf' PSOA for global optimization in Section 7, while conclusions are drawn in Section 8.

2. Problem Formulation

Consider the unconstrained (or bounds constrained) mathematical programming problem represented by the following: Given a real valued objective function $f(\boldsymbol{x})$ defined on the set $\boldsymbol{x} \in D$ in $\mathbb{R}^n$, find the point $\boldsymbol{x}^*$ and the corresponding function value f^* such that

$$f^* = f(\boldsymbol{x}^*) = \min\{f(\boldsymbol{x}) | \boldsymbol{x} \in D\}, \tag{1}$$

if $\boldsymbol{x}^*$ exists and is unique. Alternatively, find a low approximation $\tilde{f}$ to f^*.

If the objective function and/or the feasible domain D are non-convex, then there may be many local minima which are not optimal. Hence, from a *mathematical* point of view, problem (1) is essentially insolvable, due to a lack of mathematical conditions characterizing the global optimum, as opposed to a strictly convex continuous function, which is characterized by the Karush-Kuhn-Tucker conditions at the minimum.

The problem of globally optimizing a real valued function is inherently intractable (unless hard restrictions are imposed on the objective function) in that no practically useful characterization of the global optimum is available. Indeed the problem of determining an accurate estimate of the global optimum is mathematically ill-posed in the sense that very similar objective functions may have global optima very distant from each other [20]. Nevertheless, the need in practice to find a relative low local minimum has resulted in considerable research over the last decades to develop algorithms that attempt to find such a low minimum. A comprehensive survey of global optimization up to 1990 is presented by Törn and Zilinskas [21].

3. Particle Swarm Optimization

The basic PSOA is constructed as follows: Consider a swarm of p particles or birds. For particle i, Kennedy and Eberhart [1, 2] originally proposed that the position $\boldsymbol{x}^i$ is updated in the following manner:

$$\boldsymbol{x}^i_{k+1} = \boldsymbol{x}^i_k + \boldsymbol{v}^i_{k+1}, \tag{2}$$

with the velocity $\boldsymbol{v}^i$ calculated as follows:

$$\boldsymbol{v}_{k+1}^i = \boldsymbol{v}_k^i + c_1 r_1 (\boldsymbol{p}_k^i - \boldsymbol{x}_k^i) + c_2 r_2 (\boldsymbol{p}_k^g - \boldsymbol{x}_k^i). \tag{3}$$

Here, subscript k indicates an (unit) pseudo-time increment. $\boldsymbol{p}_k^i$ represents the best ever position of particle i at time k, with $\boldsymbol{p}_k^g$ representing the global best position in the swarm at time k. r_1 and r_2 represent uniform random numbers between 0 and 1. Kennedy and Eberhart proposed that the cognitive and social scaling parameters c_1 and c_2 are selected such that $c_1 = c_2 = 2$, in order to allow a mean of 1 (when multiplied by the random numbers r_1 and r_2). The result of using these proposed values is that the particles overshoot the target half the time.

4. Variations on Kennedy and Eberhart's Original PSOA

In this section a number of variations on the original PSOA proposed by Kennedy and Eberhart are presented. In doing so, we do not aim to be exhaustive. Instead, we list the most significant and commonly used variants.

4.1. INTRODUCTION OF CONSTANT INERTIA WEIGHT

This variant, due to Shi and Eberhart [3], constitutes the first significant variation on the original particle swarm algorithm. An inertia term w is introduced into the original velocity rule (3) as follows:

$$\boldsymbol{v}_{k+1}^i = w\boldsymbol{v}_k^i + c_1 r_1 \left(\boldsymbol{p}_k^i - \boldsymbol{x}_k^i\right) + c_2 r_2 \left(\boldsymbol{p}_k^g - \boldsymbol{x}_k^i\right). \tag{4}$$

The scalar w performs a scaling operation on the velocity $\boldsymbol{v}_k$, analogous to introducing 'momentum' to the particle. Higher values for w results in relatively straight particle trajectories, with significant 'overshooting' or 'overflying' at the target, resulting in a good global search characteristic. Lower values for w result in erratic particle trajectories with a reduction in overshoot, both desirable properties for a refined localized search.

The most serious drawback of the introduction of constant inertia is the problem dependency of w. In a typical implementation, an intermediate value for w is selected, resulting in a search that is unoptimal during both the 'global' and 'local' phases of the search.

4.2. LINEAR INERTIA REDUCTION

Linear inertia reduction, also proposed by Shi and Eberhart [3, 9], is a variation on the introduction of constant inertia as discussed in Section 4.1 above. This variation attempts to eliminate some of the drawbacks of constant inertia, and entails the linear scaling of the inertia parameter w during the search, usually between 0.8 and 0.4, in a specified number of function evaluations. This ensures that the PSOA gradually transitions from an algorithm suitable for a global search to an algorithm suitable for refining an optimum in a local search. The optimum rate for reducing w is still problem dependent, and constitutes the main drawback of this variation.

4.3. LIMITATION OF MAXIMUM VELOCITY

In this variation, Shi and Eberhart [9, 19] limit the velocity of each particle to a specified maximum velocity $\boldsymbol{v}^{\max}$. This represents an attempt to reduce excessively large step sizes in the position rule (2). The maximum velocity is calculated as a specified fraction γ of the distance between the bounds of the search domain:

$$\boldsymbol{v}^{\max} = \gamma(\boldsymbol{x}_{\mathrm{UB}} - \boldsymbol{x}_{\mathrm{LB}}) \tag{5}$$

where x_{UB} and x_{LB} respectively represent the upper and lower bounds of the domain D. This once again prevents excessively large steps during the initial phases of a search. Previously, Carlisle and Dozier [17] and Eberhart and Shi [19] showed that this variation increases reliability and reduces cost.

4.4. CONSTRICTION FACTOR

A notable recent variation on the original velocity rule (3) is the introduction of the constriction factor proposed by Clerc [18], in his swarm and queen approach, as further explored by Eberhart and Shi [19]. This method introduces a constriction factor K into velocity rule (3), which has the effect of reducing the velocity of the particles as the search progresses, thereby contracting the overall swarm diameter. This in turn results in a progressively smaller domain being searched.

The value of the constriction factor K is calculated as a function of the cognitive and social parameters c_1 and c_2:

$$\boldsymbol{v}^i_{k+1} = K * \left[\boldsymbol{v}^i_k + c_1 r_1 \left(\boldsymbol{p}^i_k - \boldsymbol{x}^i_k\right) + c_2 r_2 \left(\boldsymbol{p}^g_k - \boldsymbol{x}^i_k\right)\right], \tag{6}$$

$$K = \frac{2}{|2 - \varphi - \sqrt{\varphi^2 - 4\varphi}|} \quad \text{where} \quad \varphi = c_1 + c_2, \quad \varphi > 4. \tag{7}$$

In their search for an ‘off-the-shelf’ PSOA, Carlisle and Dozier [17] show that cognitive and social values of $c_1 = 2.8$ and $c_2 = 1.3$ yield good results for their test set.

4.5. DYNAMIC INERTIA AND MAXIMUM VELOCITY REDUCTION

This variation, proposed by Fourie and Groenwold [15], aims to reduce the sensitivity to problem dependent parameters associated with previous implementations of inertia [9, 19]. In this approach, a simultaneous dynamic reduction in inertia and maximum velocity is implemented to decrease the swarm domain in a controlled fashion. The approach is outlined as follows: Firstly, the initial inertia w_0 is prescribed, while the initial maximum velocity vector $\boldsymbol{v}^{\max}$ is again calculated as a fraction of the domain using (5). The swarm domain is then effectively reduced by decreasing the inertia and maximum velocity by fractions α and β respectively, if no improvement in the swarm fitness values $\boldsymbol{p}_k^g$ and $\boldsymbol{p}_k^i$ occur after a to be specified number of iterations h:

$$\text{if } \; f(\boldsymbol{p}_k^g) \geqslant f(\boldsymbol{p}_{k-h}^g), \text{ then } \; w_{k+1} = \alpha w_k, \boldsymbol{v}_k^{\max} = \beta \boldsymbol{v}_k^{\max}, \tag{8}$$

with $0 < \alpha, \beta < 1$, prescribed. Rather than reducing the inertia and maximum velocity in a linear fashion, dynamic inertia reduction allows the adjustment of the algorithm parameters according to the success history of the swarm. For reasons of clarity, we will denote h the 'dynamic delay period' in this paper.

5. Numerical Results with the Different PSOA Variants

In this section, numerical results are presented for an extended version of the original Dixon-Szegö test set [22] (Table 1). The acronyms used to denote the PSOA variants are tabulated in Table 2. In our numerical experiments, each problem in the extended Dixon-Szegö test set is analyzed 50 times. We then record the average optimal function values f_{ave} and the standard deviation $\bar{\sigma}$ thereof, the number of function evaluations N_{fe}, and the reliability R_S, defined as the 'success ratio', i.e. the number of runs out of 50 that converged to the *a priori* known optimum. In addition, we also report the best (lowest) function value f_{best} found.

For the sake of brevity, we only present tabulated results for the G1 problem. For the other problems in the test set, the results are summarized in graphical form in Figure 1. However, detailed tabulated results are available from the authors on request.

Since it is not the objective to test the performance of different stopping criteria, but rather the PSO algorithm itself, a simple *a priori* stopping criteria is used, in which the algorithm is stopped when the result is within a prescribed tolerance ε_a of the best known solution. (ε_a is also given in Table 1).

Table 1. The extended Dixon-Szegö test set

No.	Name	n	ε_a
1	Griewank G1	2	0.001
2	Griewank G2	10	0.1
3	Goldstein-Price	2	0.001
4	Six-hump camelback	2	0.001
5	Shubert, Levi No. 4	2	0.001
6	Rastrigin	2	0.001
7	Branin	2	0.001
8	Hartman 3	3	0.001
9	Hartman 6	6	0.001
10	Shekel 5	5	0.001
11	Shekel 7	7	0.001
12	Shekel 10	10	0.001

Table 2. Acronyms used to denote algorithm variants

Acronym	Name
PSO-S	Standard PSOA
PSO-CI	PSOA with constant inertia
PSO-CIV	PSOA with constant inertia and maximum velocity limitation
PSO-LI	PSOA with linear inertia reduction
PSO-LIV	PSOA with linear inertia and maximum velocity limitation
PSO-C	PSOA with constriction
PSO-DIV	PSOA with dynamic inertia and velocity reduction

In implementing the respective variants, an asynchronous implementation for updating the swarm best value $\boldsymbol{p}_k^g$ and particle best value $\boldsymbol{p}_k^i$ is used. Numerical studies by Carlisle and Dozier [17] indicate that the asynchronous method is in general superior to the the synchronous method, in which the swarm achievements are only updated once a time increment is completed. Furthermore, a global neighborhood [6, 23] is used throughout.

The maximum number of function evaluations allowed is set at 30000. Hence, if the *a priori* stopping condition is not satisfied within this number, the search is deemed to have failed to converge. In each case, a swarm consisting of 20 particles is used, with the cognitive and social parameters c_1 and c_2 both set to 2, for all the variants but constriction. For constriction, the cognitive and social parameters c_1 and c_2 are set to 2.8 and 1.3 respectively, as recommended by Carlisle and Dozier [17]. For the dynamic inertia and maximum velocity variant, $\alpha = \beta = 0.99$ and $h = 10$ are used. $\gamma = 1.0$ is used for all problems where the maximum velocity limitation is applied.

In the interest of obtaining a robust and versatile algorithm no attempt is made to optimize the algorithm parameters for individual problems. The results are briefly discussed in the following subsections and are summarized in Figure 1.

5.1. STANDARD PSOA

The standard PSOA converged for only 3 out of the 12 test problems, and then only with a very poor reliability. Hence we refrain from a detailed discussion of this variant.

5.2. CONSTANT INERTIA WEIGHT AND MAXIMUM VELOCITY LIMITATION

Upon the addition of a constant inertia weight ($w = 0.6$) the numerical results are largely improved as compared to the results with the standard version of the PSOA. Even so, the cost associated with the higher dimensional Shekel problems are extremely high as compared to the other variants tested.

Additional enforcement of a maximum velocity limit marginally reduces the cost for the complete test set, with the exception of the Shekel S5 problem.

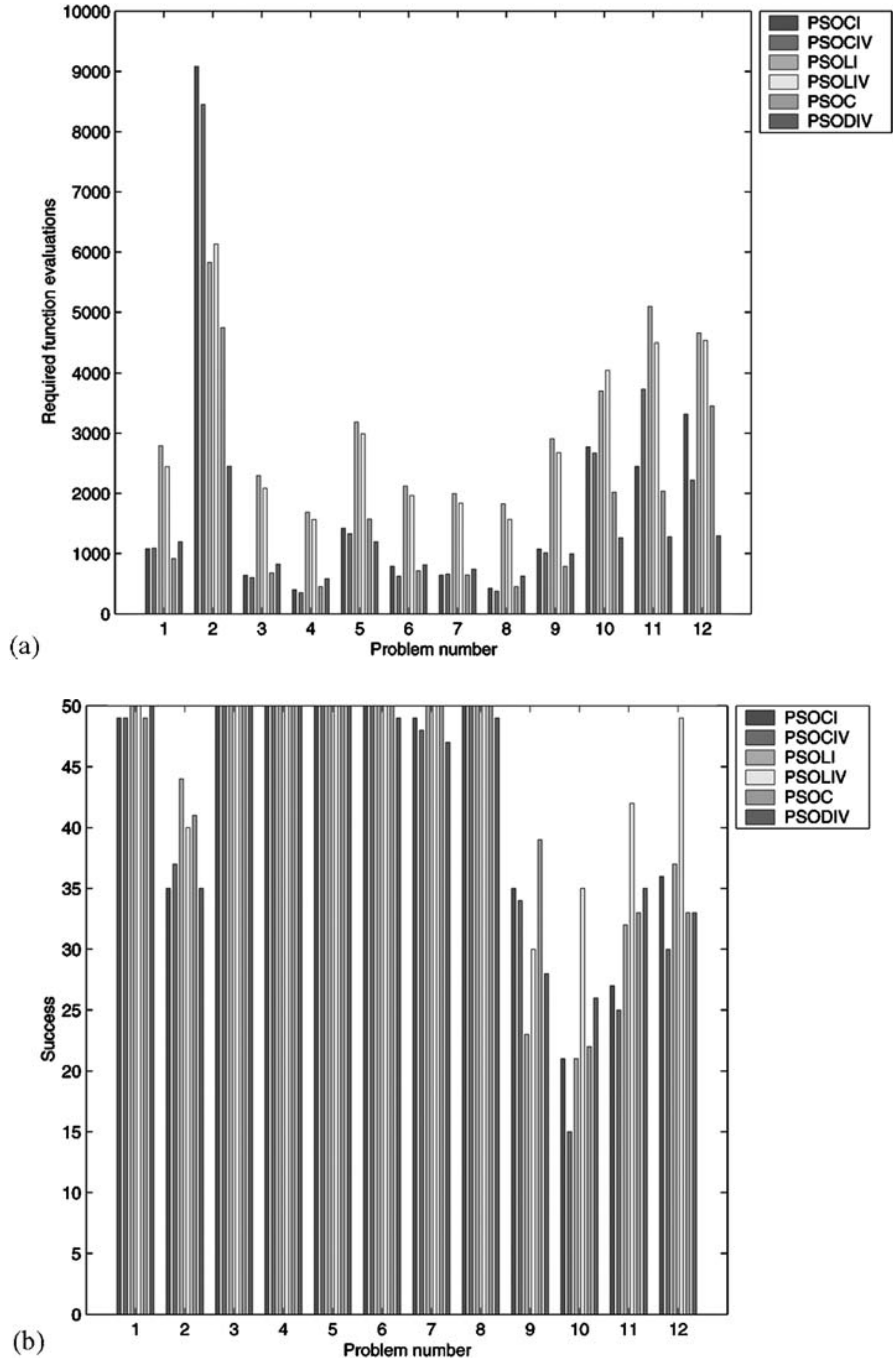


Figure 1. Summary of cost and reliability of the different variants studied.

Table 3. Griewank 1

	Optimum solution	PSO algorithm					
		PSO-CI	PSO-CIV	PSO-LI	PSO-LIV	PSO-C	PSO-DIV
ε_s		0.001	0.001	0.001	0.001	0.001	0.001
f_{ave}		0.00053	0.00054	0.00049	0.00053	0.00054	0.00050
$\bar{\sigma}$		0.00025	0.00031	0.00030	0.00027	0.00032	0.00032
N_{fe} (ave.)		1081	1089	2789	2443	918	1197
Reliability		49/50	49/50	50/50	50/50	49/50	50/50
f_{best}	0.00000	0.00007	0.00001	0.00002	0.00000	0.00003	0.00000
x_1	0.00000	−0.00223	0.00099	0.00626	0.00091	0.00387	0.00023
x_2	0.00000	0.01617	−0.00643	−0.00361	0.00119	0.00944	−0.00066
N_{fe}		842	2093	4171	2512	831	932

5.3. LINEARLY DECREASING INERTIA AND MAXIMUM VELOCITY LIMITATION

In the experiments with this variant, the inertia weight w is scaled linearly between 0.8 and 0.4 during the first 4000 function evaluations of the search. This variation is more costly than the constant inertia variation for all but the Griewank G2 problem. This probably indicates that the optimum rate of inertia reduction is problem dependent. The algorithm yields improved reliability on all of the Shekel group of problems as compared to the constant inertia PSOA variation.

When a limitation on maximum velocity is also introduced, both the average cost and the reliability are in general improved.

5.4. CONSTRICTION FACTOR

Numerical experimentation and work done by others [17] indicate that limitation of the maximum velocity does not contribute to an increased efficiency for this variant of the PSOA if bounds constraints are enforced.

The numerical results are impressive, with the cost notably reduced as opposed to the variants with inertia. However, for some of the more difficult higher dimensional problems, the algorithm reliability is decreased.

5.5. DYNAMIC INERTIA REDUCTION AND MAXIMUM VELOCITY LIMITATION

In terms of reliability, this variant is marginally outperformed by constriction for most of the lower dimensionality problems. For the more difficult higher dimensionality problems, the average cost decreases as compared to constriction, with reliability similar to constriction.

5.6. DISCUSSION

It is noted that the coordinates of some of the minima found deviates from the listed (known) global minima. For the very difficult Griewank 2 test function, this reflects the inability of the PSOA to find the global minimum of this function reliably (there are a few thousand local minima present in the region of interest). For the Schubert problem, all the minima found represent global minima (there are 760 local minima present, of which 18 are global).

6. Parameter Sensitivity Study

While we note that linear inertia reduction yields good reliability, the results presented in Section 5 indicate that constriction and the dynamic inertia/velocity reduction variants are the main contenders when both reliability and cost are considered. Choosing between these two contenders seems difficult, and should probably be judged in future for problems with higher dimensionality than considered herein.

In the interests of presenting a reliable general purpose PSOA, we now perform a parameter sensitivity study for the two main contenders.

6.1. COGNITIVE/SOCIAL RATIO

Previously, Kennedy asserted that the sum of the cognitive and social values c_1 and c_2 should approximately equal 4.0 [5]. For constriction, Carlisle and Dozier [17] have shown that it is advantageous to adjust the cognitive/social ratio to favor cognitive learning (an individualistic swarm). They report that values of 2.8 and 1.3 respectively for the cognitive and social components yield the best performance for the test set they consider. The set considered by Carlisle and Dozier is smaller, and includes unimodal and multimodal functions.

In the following subsections, we investigate whether this is true for the extended Dixon-Szegö test set under consideration, and establish the influence on dynamic inertia reduction. Numerical results presented in Figures 2 and 3 are obtained by varying the cognitive value c_1 between 0 and 4.1, with the social value calculated in each case as $c_2 = 4.1 - c_1$, as suggested by Carlisle and Dozier [17].

6.1.1. *Constriction*

Figure 2(a) indicates that the optimum cognitive value for the extended Dixon-Szegö test set tends to be in the region between 1.5 and 3, indicating that the recommended setting of 2.8 by Carlisle and Dozier is also appropriate for the problems in the extended Dixon-Szegö test set when considering cost. Figure 2(b) shows the optimum value for reliability to reveal a greater problem dependency, with graphs of the Hartman and Shekel family of problems peaking at c_1 values in the region of 3.5. The Griewank G2 problem however shows a sharp decrease in reliability for values of c_1 above 3. Again, a value of 2.8 would probably be a realistic compromise to ensure reasonable reliability.

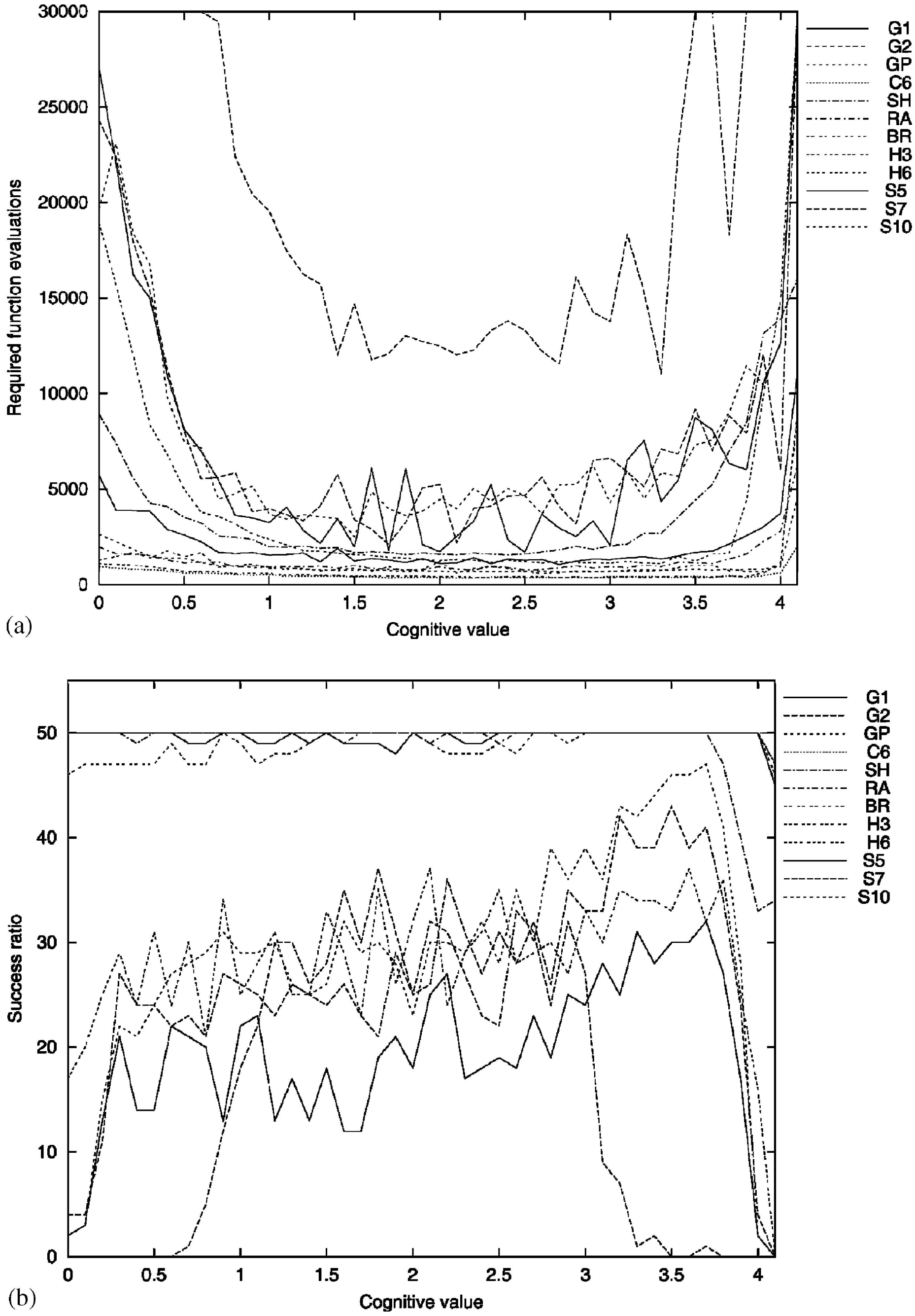


Figure 2. Constriction: Cost and reliability as a function of the cognitive parameter c_1.

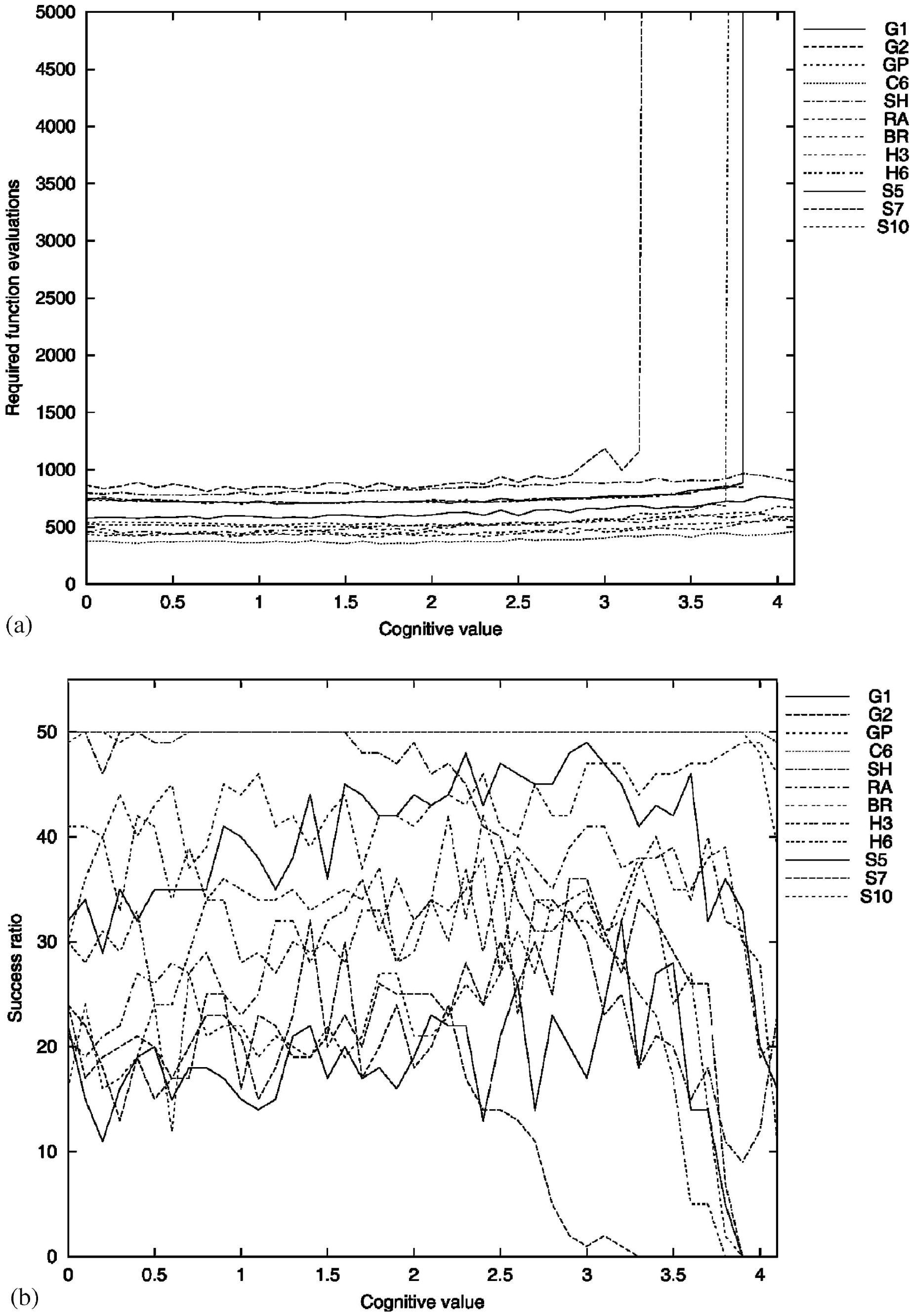


Figure 3. Dynamic inertia reduction: Cost and reliability as a function of the cognitive parameter c_1.

6.1.2. *Dynamic Inertia Reduction and Maximum Velocity Limitation*

Results with dynamic inertia reduction and maximum velocity limitation (Figure 3) indicates that this variant of the PSOA is relatively insensitive to the cognitive/social ratio. The cost remains low throughout the range of variation of the c_1 parameter, with a sharp increase to 30000 function evaluations at values above 3.8, indicating that none of the iterations converged. The reliability is also relatively insensitive to the cognitive parameter c_1 for the majority of the problems, with a drop in reliability at values above 3. The insensitivity to low values of cognitive learning indicates the successfulness of the purely 'social' swarm (e.g. see [4]). A reasonable value for this variant is 2.0, which was initially suggested by Kennedy and Eberhart [1] for the 'standard' PSOA.

6.2. SWARM POPULATION SIZE

The effect of swarm population size on constriction has been extensively studied by Carlisle and Dozier [17], Eberhart and Shi [19], and Shi and Eberhart [24].

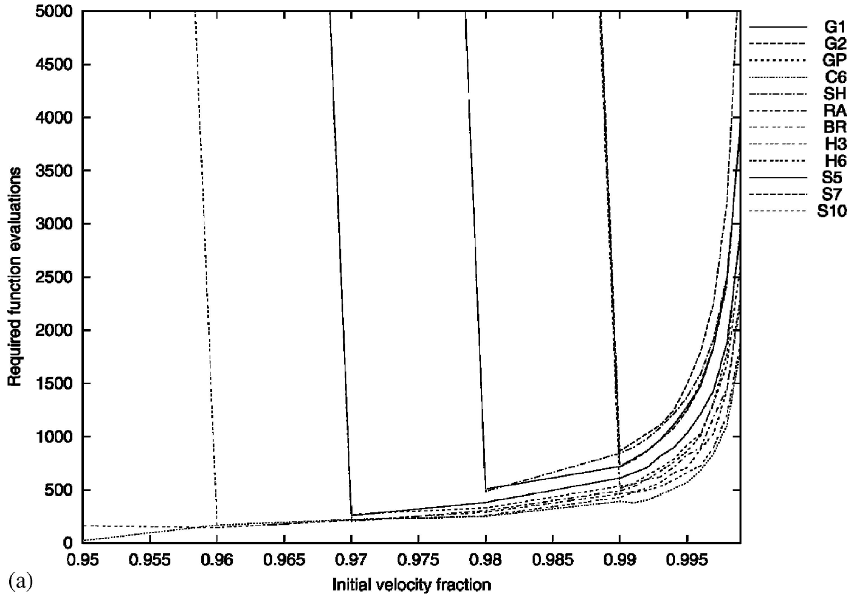
For constriction, our findings closely supports the findings of Carlisle and Dozier, who maintain that, while an increase in population tends to lessen the required swarm *iterations*, the accompanying *cost* (N_{fe}) increases (not shown). Although populations of as little as 5 particles find the optimum at low cost, the sharp decrease in reliability with small population sizes dictate a lower bound when reliability is considered. A swarm population of 20–30 seems a reasonable compromise between cost and reliability.

For dynamic inertia reduction, very similar results to those of constriction are obtained (not shown). A swarm size of 20 seems sufficient as a threshold value to prevent reduced reliability at the low end of the graph, while retaining reasonable cost.

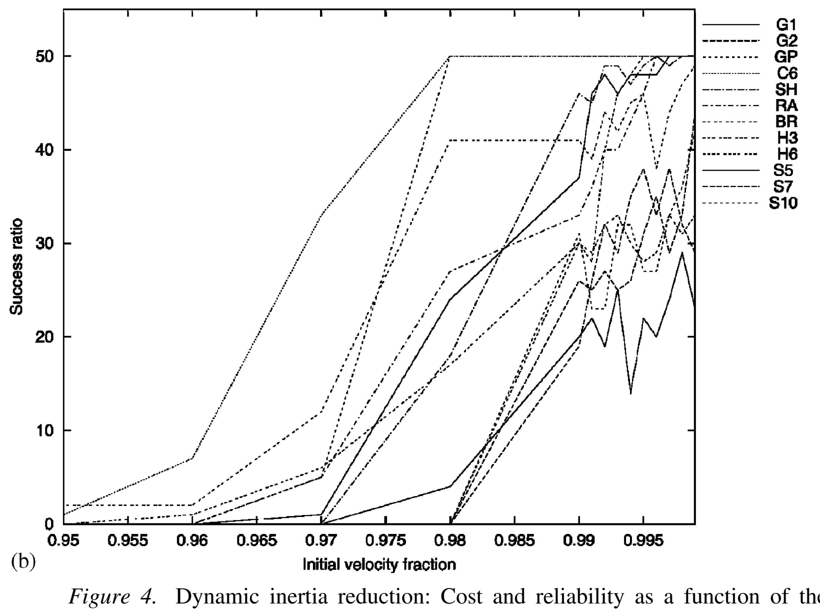
6.3. DYNAMIC DELAY PERIOD AND REDUCTION PARAMETERS

Both cost and reliability are quite insensitive to the value of the dynamic delay period h (not shown). The only exception to this is the Griewank G2 problem, which reveals a reduction in the reliability for values of h above 10.

The effect of the reduction parameters α and β in (8) are studied in Figure 4. For the sake of simplicity, we select $0.95 \leqslant \alpha = \beta \leqslant 1$. The study reveals a rapid increase in cost for $\alpha = \beta > 0.99$ (Figure 4(a)), since the algorithm approximates the constant inertia variant as α, β approach 1. For values of $\alpha = \beta < 0.99$, the reliability decreases sharply (Figure 4(b)), suggesting a optimal value of 0.99 for the extended Dixon-Szegö test set.



(a)



(b)

Figure 4. Dynamic inertia reduction: Cost and reliability as a function of the reduction parameters α and β.

6.4. INITIAL VELOCITY FRACTION

Dynamic inertia reduction is rather insensitive to the value of the initial velocity fraction γ (not shown), although for some problems, the reliability decreases sharply below $\gamma = 0.3$. A practical setting would probably be $\gamma = 0.5$.

7. Recommendations

We propose that either the constriction or the dynamic inertia reduction variants of the PSOA are used in global optimization. For constriction, we support the previously proposed values of $c_1 = 2.8$ and $c_2 = 1.3$. For dynamic inertia reduction, we propose $c_1 = c_2 = 2.0$, $h = 10$, and $\alpha = \beta = 0.99$. As far as swarm population size is concerned, a population size of roughly 20 seems optimal for both constriction and dynamic inertia reduction.

8. Conclusions

We have applied the PSOA and some of its variants to an extended Dixon-Szegö test set in global optimization. We show that constriction and dynamic inertia reduction are the main contenders when considering both reliability and cost.

For problems of low dimensionality, dynamic inertia reduction is marginally outperformed by constriction. For problems of higher dimensionality, dynamic inertia reduction seems slightly superior.

Dynamic inertia reduction is shown to be less sensitive to parameter variations than constriction, for which the optimum choice of cognitive c_1 and social c_2 scaling parameters tends to be problem dependent.

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