



## Formulation and Computationally Efficient Algorithms for an Interference-Oriented Version of the Frequency Assignment Problem

S. KOTROTSOS, G. KOTSAKIS, P. DEMESTICHAS, E. TZIFA, V. DEMESTICHA and  
M. ANAGNOSTOU

National Technical University of Athens, Computer Science Division, 9 Heroon Polytechniou Str.,  
Zographou 15773 Athens, Greece  
E-mail: pdemest@cc.ece.ntua.gr

**Abstract.** The frequency assignment problem will maintain its importance for several years, since future versions of legacy cellular systems, e.g., those of GSM, will continue to exist. This paper elaborates on an interference-oriented version of the frequency assignment problem. The objective function is associated with the interference levels that are imposed by the frequency allocation, while the constraints are related to the allocation of the frequencies required in each cell and the prevention of some unacceptable interference situations. The problem is formally stated, mathematically formulated and solved by means of computationally efficient heuristics. Finally, results are obtained and concluding remarks are made.

**Keywords:** GSM, FDMA/TDMA, co-channel and adjacent channel interference, frequency assignment.

### 1. Introduction

An important research, development and standardisation area in the recent years is the specification of future versions of legacy cellular communications systems, e.g., those of the Global System for Mobile communications (GSM) [1–5]. Moreover, it is widely accepted that third generation cellular and broadband radio access systems [6–10] will, at least in their first phases, co-exist with the future versions of legacy cellular systems, and especially, those of GSM (e.g., [3–5]), which is expected to have a significant role as a dominating standard until 2010. In summary, the future situation in the cellular and radio access market may be as the one depicted in Figure 1.

The success of any cellular or radio access system is coupled with the efficient utilisation of the scarcely available radio spectrum. In principle, legacy cellular systems (e.g., GSM) split the available spectrum into traffic and signalling channels according to a Frequency Division Multiple Access/Time Division Multiple Access (FDMA/TDMA) scheme. Moreover, they use Fixed Carrier Allocation (FCA) schemes [11, 12], even though more dynamic schemes, e.g., based on Dynamic Carrier Allocation (DCA) or Hybrid Carrier Allocation (HCA) [13, 14], have been proposed [15]. In the FCA case the allocation of carriers to cells is determined in the design phases of the system (Figure 2), through the solution of the appropriate instance of the *frequency assignment (planning)* problem [11, 12], which is known to be very complex. The frequency allocation is occasionally re-engineered (re-designed) due to the accumulated experience, or in order to handle anticipated situations (e.g., an expected increase in the traffic demand in some area of the network).

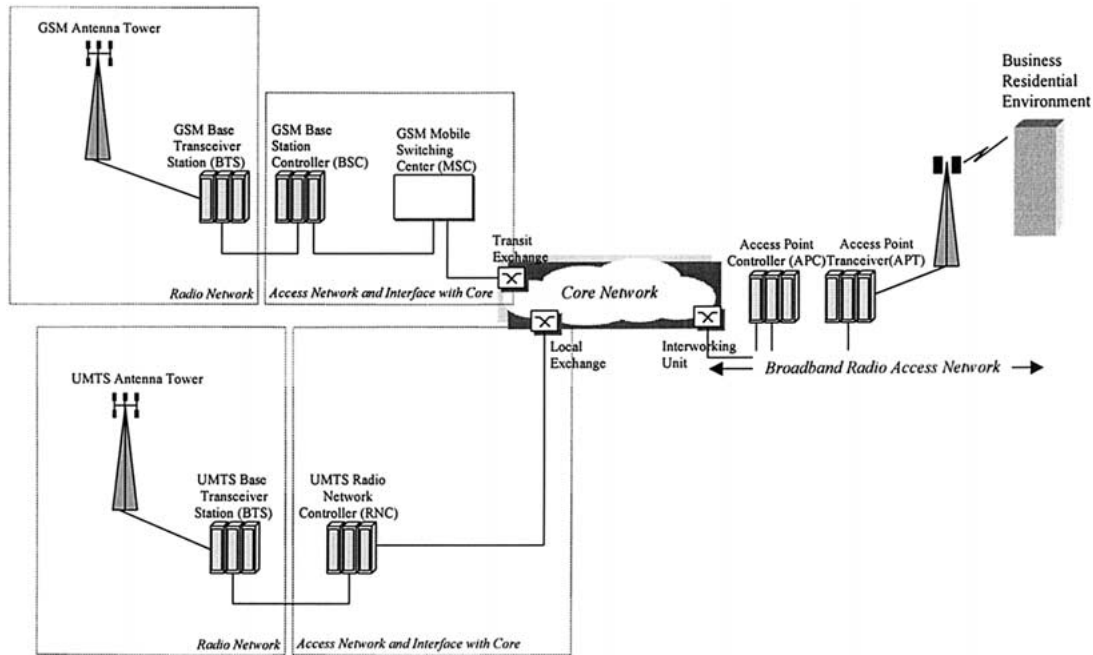


Figure 1. A view of the radio access and cellular network world of the future.

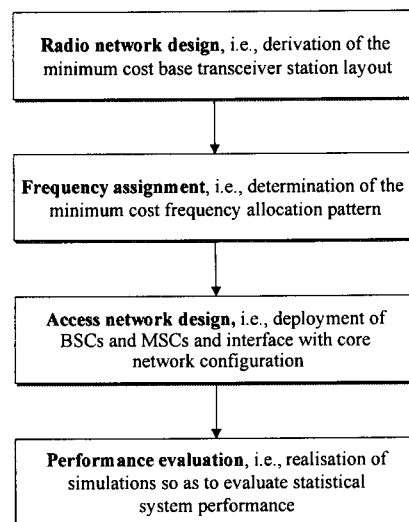


Figure 2. Phases in the design of a legacy (non-CDMA-based) cellular system.

The fact that GSM (or more specifically, its future versions, e.g., [3–5]) will continue to play a significant role in the years to come, entails that the frequency assignment (planning) problem will maintain its importance at least in the near future. Pertinent software tools may be seen as essential means for solving a complex problem that enables the efficient exploitation of the available radio spectrum. The solution of a version of the frequency assignment (planning) problem is the general context of this paper.

In general, the frequency assignment (planning) problem may be described as follows (Figure 3): “Given the cell layout of the system, the set of available frequencies, the load

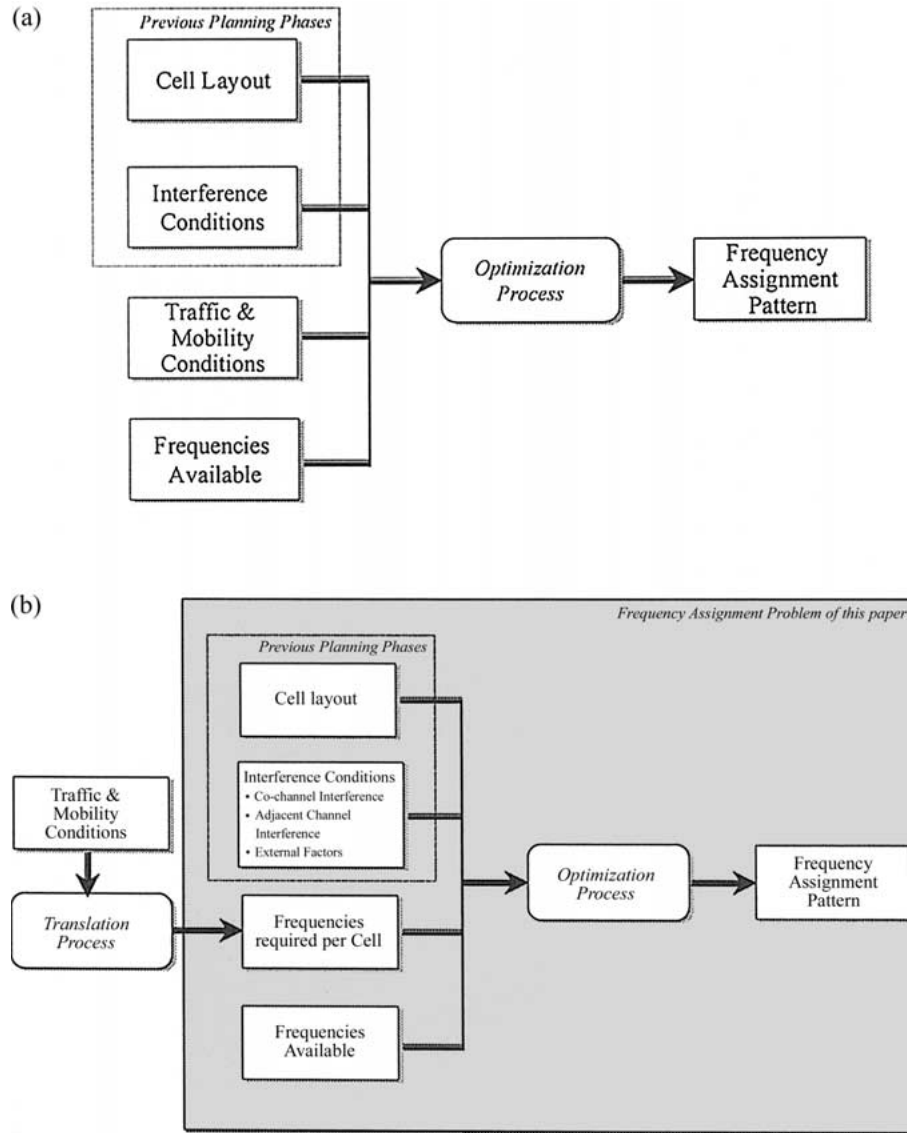


Figure 3. (a) General description of the frequency assignment problem; (b) Description of the frequency assignment problem addressed in this paper.

expected in each cell (aggregating the service preferences and the mobility characteristics of the users), and interference conditions in the network, find an allocation of frequencies to cells that optimises an objective function and satisfies the problem constraints". Various objective functions and constraints have been considered in the literature. References [13, 14] are well-known surveys on (static and dynamic) radio resource management schemes, while references [16, 17] include classifications of frequency assignment (planning) problems. A general classification scheme separates the versions of the problem into the *interference oriented* and the *traffic oriented* categories. Problems of the first category optimise the interference levels of the system, under constraints that derive from the traffic that should be carried in each cell and some interference conditions that should be avoided. Problems of the second (and perhaps

largest) category allocate frequencies to cells so as to optimise a quality criterion associated with the traffic that is carried by the system, subject to interference constraints.

The version of the problem that is addressed in this paper falls into the interference-oriented category. This choice is motivated by the anticipated augmented complexity of the interference conditions of the environment. Our constraints will be targeted to the following: First, the allocation of the frequencies required in each cell, so as to adequately cope with the expected traffic. Second, the prevention of some unacceptable interference situations. We believe that these aspects yield a realistic version of the frequency assignment problem, which will maintain its importance in the future cellular and broadband radio access markets. The identified problem will be formally described and mathematically formulated as a combinatorial optimisation problem [18, 19]. The computational effort associated with the optimal solution of the problem will lead us to the design of two computationally efficient heuristic algorithms that fall into the *simulated annealing* (SA) [20, 21] and the *genetic algorithm* (GA) [22–25] categories. These are well known techniques for determining near-optimal solutions to hard combinatorial optimisation problems (e.g., references [26, 27] are recent sample application areas).

The literature is rich in terms of solution methods for various versions of the frequency assignment problem. Important existing approaches are based on graph theoretic methods [30–34], and well-established methods for solving difficult combinatorial optimisation problems, e.g., neural networks [35] and simulated annealing [36, 37]. In general, most of these papers fall into the traffic-oriented category. Moreover, commercial frequency planning tools are being developed, e.g., [38, 39]. Nevertheless, in the corresponding documentation it is often stated that even today often frequency planning is done manually and that there is room for new problem areas and automatic solutions. The contribution of this paper is in the following areas. First, the elaboration on a problem version that is realistic (in the sense that it comprises features that are important from the operators' perspective) and extensible (as shown in subsequent sections). The specific problem version, even though important in the commercial domain (i.e., operators are interested for pertinent tools), has not often been addressed in the literature. The second contribution of this paper is the provision of the corresponding formal statement and novel optimal formulation; both can serve to other researchers and developers as a basis for progressing the work. The last contribution of this paper the design, implementation and comparison of two new, modern computationally efficient solutions based on well-known optimisation techniques. It should also be noted that attempts based on genetic algorithms are not so frequent in the literature due to the associated difficulty.

The rest of this document is organised as follows. Section 2 comprises the general description of the frequency assignment (planning) problem addressed in this paper. Section 3 comprises the corresponding formal problem statement. Section 4 comprises the mathematical formulation and Sections 5 and 6 the description of the computationally efficient algorithms. Section 7 comprises results and Section 8 concluding remarks.

## 2. Problem Description

In this section we describe the problem version that is addressed in this paper. Our starting point is the general problem description of Figure 3(a). Issues that will be specified in more detail are the inputs of the problem, the objective function and the constraints. The output of this section is the problem description that is depicted in Figure 3(b).

A first assumption regarding the inputs of the problem is that the offered traffic and mobility conditions (see Figure 3(a)) that are encountered in each cell are translated, through appropriate pre-processing as depicted in Figure 3(b), into the number of frequencies required in each cell. This number of frequencies should be sufficiently large for ensuring that the QoS levels in each cell will exceed some pre-defined thresholds. The exact realisation of this phase is beyond the scope of this paper, as it is methodologically independent from the rest of our approach. In summary, the translation process may rely on analytical methods or simulation, and the aspects that are taken into account are the load per service per cell, the mobility conditions in each cell, and models of the multiple access scheme according to which services will access the transmission medium (air-interface).

Several factors are assumed to contribute to the interference conditions of the system (Figure 3(b)). First, the *co-channel interference*, which is caused by the use of the same frequency in neighbouring (or “near-by” in general) cells. Second, the *adjacent channel interference*, which is caused by the use of adjacent frequencies in neighbouring (or “near-by”) cells. Third, external factors, which may be caused by other operators or other systems.

The *hard co-channel interference* constraints describe a binary relation between cells that are strictly not allowed to use the same frequency. In the same way the *hard adjacent channel interference* constraints provide pairs of cells that are strictly not allowed to use adjacent frequencies. As will be shown in Section 4 our model can readily be extended to include  $\pm n$  adjacent channel affects. In this respect, this aspect will not be addressed in detail so as to preserve the notation simplicity. Moreover, the above can be used for modelling constraints that derive from the fact that BTSs share the same site.

Interference conditions, as well as other technical limitations, impose a minimum separation between frequencies that are assigned to a cell. This is called a *combiner separation* constraint. Finally, in order to account for external factors that contribute to the interference levels we allow each cell to have an associated set of *forbidden frequencies*.

The last aspect on interference condition modelling provides the *soft co-channel interference* and the *soft adjacent channel interference* values. The former provide a measure of the interference that is induced to cell  $i$  by the allocation to cell  $j$  of a frequency that is used in  $i$ . In principle, the level of this interference is not prohibitively high; otherwise it can be represented by a hard co-channel interference constraint. The soft adjacent channel interference values provide a measure of the interference induced to cell  $i$  by the allocation to cell  $j$  of a frequency that is adjacent to one used in  $i$ .

Since our version of the problem falls in the interference-oriented category, the objective function of the problem will involve the interference levels in the system. These are a function of the soft co-channel and adjacent channel interference values.

The constraints of our problem impose the following: (i) the allocation of the frequencies required in each cell for handling the expected traffic; (ii) the preservation of the constraints that are related to the hard co-channel and adjacent channel interference, combiner separation requirement and forbidden frequencies sets.

Based on the discussion so far the description of the problem version addressed in this paper is the following.

*Problem 1. [Interference-Oriented Frequency Assignment Problem – IOFAP]:* Given

- (a) the set of cells in the system,
- (b) the set of available frequencies,

- (c) the number of frequencies that are required in each cell for adequately coping with the offered traffic,
- (d) the pairs of cells that should not use the same frequency (hard co-channel interference constraints),
- (e) the pairs of cells that should not use adjacent frequencies (hard adjacent channel interference constraints),
- (f) the minimum separation required among frequencies that are assigned to the same cell (combiner separation constraints),
- (g) for each cell the associated set of forbidden frequencies,
- (h) for each pair of cells the interference levels that will be induced in the system if they acquire the same frequency (soft co-channel interference values), and adjacent frequencies (soft adjacent channel interference values),

find an allocation of frequencies to cells that optimises a quality criterion associated with the (soft) interference levels in the system, allocates the number of frequencies required in each cell, and respects the constraints on the hard co-channel and adjacent channel interference, combiner separation requirement, and forbidden frequencies sets.

### 3. Formal Problem Statement

The set of cells is denoted as  $V$ . The set of available frequencies is denoted as  $F$ . Each cell requires a number of frequencies for coping with the offered traffic and the mobility conditions. This requirement is denoted as  $r(i)$  ( $\forall i \in V$ ). The combiner separation constraint of each cell is denoted as  $c(i)$  ( $\forall i \in V$ ). The set of frequencies that should not be used in (are forbidden for) cell  $i \in V$  is denoted as  $FF(i)$  ( $FF(i) \subseteq F$ ).

In the sequel, we define, for each  $i \in V$ , the set of cells  $CC(i)$  ( $CC(i) \subseteq V$ ), which contains the cells that are not allowed to use a frequency also allocated to cell  $i$ . In a similar manner, the set of cells that should not use a frequency that is adjacent to a frequency that is allocated to cell  $i$ , is denoted as  $AC(i)$  ( $AC(i) \subseteq V$ ). Hence, the various  $CC(i)$  and  $AC(i)$  sets model the hard co-channel and adjacent channel interference constraints of our problem.

The next aspect that should be modelled is the soft interference values. To this end, we define for every  $(i, j) \in V^2$ , a value  $S_c(i, j)$ , representing the co-channel interference that will be imposed in cell  $i$  by cell  $j$ , if they are allocated the same frequency. Similarly, the value  $S_a(i, j)$  represents the adjacent channel interference that will be imposed in cell  $i$  by cell  $j$ , if they are allocated adjacent frequencies.

The aim of the problem is to find an allocation of frequencies to cells  $A = \{A(i) | i \in V\}$ . The set  $A(i)$  ( $A(i) \subseteq F$ ) comprises the frequencies that are allocated to cell  $i$  ( $i \in V$ ). The aim of the allocation is to minimise a cost function, denoted as  $C(A)$ , which is related to the (soft) interference levels in the system.

The constraints of the problem address the following aspects. The first set of constraints reflects that each cell should be allocated as many frequencies as required. Hence, the condition  $|A(i)| = r(i)$ , for all  $i \in V$ , should hold. The second set of constraints should impose the preservation of the necessary combiner separation constraints. Therefore, for all  $i \in V$  and for all the pairs  $(f, g) \in (A(i) \times A(i))$  the condition  $|f - g| \geq c(i)$  should hold. The third set of constraints should reflect that no cell should use a forbidden frequency. Hence, for all  $i \in V$  the condition  $A(i) \cap FF(i) = \emptyset$  should hold.

The fourth set of constraints should ensure that the co-channel interference constraints are met. This means that for every  $i \in V$  the condition  $A(i) \cap A(j) = \emptyset$  if  $j \in CC(i)$ , should hold. The fifth set of constraints should ensure that the adjacent frequency interference constraints are met. This means that for every  $i \in V$ , and for every  $f \in A(i)$ , the conditions  $(f + 1) \notin A(j)$  and  $(f - 1) \notin A(j)$ , if  $j \in AC(i)$ , should hold.

Based on the above definitions the resulting problem may be formally stated as follows:

**Problem 1 [Interference-Oriented Frequency Assignment Problem – IOFAP]:** Given the set of cells  $V$ , the set of available frequencies  $F$ , for each cell  $i \in V$ , the number of frequencies required in the cell,  $r(i)$ , the minimum distance that should separate the frequencies assigned to the cell (combiner separation),  $c(i)$ , the set of frequencies that should not be used in (are forbidden for) the cell,  $FF(i)$  ( $FF(i) \subseteq F$ ), the set of cells that should not use a frequency that is used in  $i$ ,  $CC(i)$  ( $CC(i) \subseteq V$ ), the set of cells that should not use a frequency that is adjacent to one that is used in  $i$ ,  $AC(i)$  ( $AC(i) \subseteq V$ ), and for every  $(i, j) \in V^2$  the soft co-channel and adjacent channel interference values,  $S_c(i, j)$  and  $S_a(i, j)$ , respectively, find an allocation of frequencies to cells  $A = \{A(i) | i \in V\}$  ( $A(i) \subseteq F$ ) that optimises the cost function  $C(A)$  (related to the interference levels in the system), subject to the restrictions: (i) for all  $i \in V$ ,  $|A(i)| = r(i)$ ,  $A(i) \cap FF(i) = \emptyset$ , and  $A(i) \cap A(j) = \emptyset$  if  $j \in CC(i)$ , (ii) for all  $i \in V$  and for all  $(f, g) \in (A(i) \times A(i)) |f - g| \geq c(i)$ , and (iii) for every  $i \in V$ , and for every  $f \in A(i)$ ,  $(f + 1) \notin A(j)$  and  $(f - 1) \notin A(j)$ , if  $j \in AC(i)$ .

#### 4. Mathematical Formulation

In this section we provide the mathematical formulation of the frequency assignment problem of this paper. As a first step, and in order to describe in a more convenient manner the allocation  $A$  we define the decision variables  $x_{if}$  take the value 1 if frequency  $f$  is allocated to cell  $i$  and 0 otherwise. The problem of establishing the best allocation of frequencies to cells may be reduced to the following mathematical programming problem.

**Problem 1 [Interference-Oriented Frequency Assignment Problem – IOFAP]:**

Minimise

$$C(A) = \sum_{i \in V} \sum_{f \in F} x_{if} \cdot [I_c(i, f) + I_a(i, f)]. \quad (1)$$

Subject to:

$$I_c(i, f) = \sum_{j \in V} S_c(i, j) \cdot x_{jf} \quad \forall i \in V, \quad \forall f \in F \quad (2)$$

$$I_a(i, f) = \sum_{j \in V} S_a(i, j) \cdot [x_{j, (f-1)} + x_{j, (f+1)}] \quad \forall i \in V, \quad \forall f \in F \quad (3)$$

$$\sum_{f \in F} x_{if} = r(i) \quad \forall i \in V \quad (4)$$

$$|f \cdot x_{if} - g \cdot x_{ig}| \geq c(i) \quad \forall i \in V, \quad \forall (f, g) \in F^2 \quad (5)$$

$$x_{if} \cdot \left( \sum_{j \in CC(i)} x_{jf} + \sum_{j \in AC(i)} (y_{j(f-1)} + x_{j(f+1)}) \right) = 0 \quad \forall i \in V, \quad \forall f \in F \quad (6)$$

$$x_{if} \in \{0, 1\} \quad \forall i \in V, \quad \forall f \in F \quad (7)$$

$$A = \{x_{if} | i \in V, f \in F\}. \quad (8)$$

Relation (1) expresses the interference levels in the system, which are a function of the co-channel interference,  $I_c(i, f)$ , and the adjacent frequency (channel) interference,  $I_a(i, f)$ , that each cell  $i$  will sense on each allocated frequency  $f$ . The co-channel interference,  $I_c(i, f)$ , depends on whether other cells  $j$ , imposing soft co-channel interference on  $i$  (value  $S_c(i, j)$ ), use also frequency  $f$ . This is expressed by Relation (2). Similarly, the adjacent channel interference,  $I_a(i, f)$ , depends on whether other cells  $j$ , imposing soft adjacent channel interference on  $i$  (value  $S_a(i, j)$ ), use frequency  $f - 1$  or  $f + 1$ . This is expressed by Relation (3). Relation (4) ensures that each cell will be allocated as many frequencies as required. Relation (5) ensures that if two frequencies  $f, g$  are allocated to cell  $i$  the separation between them will be greater than, or equal to, the combiner separation. Relation (6) ensures that if cell  $i$  is allocated frequency  $f$ , then no cell  $j \in CC(i)$  will be allocated frequency  $f$ , and no cell  $j \in AC(i)$  will be allocated an adjacent frequency, namely  $f - 1$  or  $f + 1$ .

#### 4.1. EXTENSIONS TO THE IOFAP

As already stated the IOFAP can be extended to include supplementary features. In this subsection we provide some indicative examples.

Our first point is the inclusion of  $\pm n$  adjacent channel (frequency) constraints. Specifically, each cell  $i \in V$  can have an associated set of cells  $AC_n(i)$  ( $n = 1, \dots, |F|$ ). This set can comprise cells  $j \in V$ , which should not use frequency  $f \pm k$ , where  $k = 1, 2, \dots, n$ , in case cell  $i$  uses frequency  $f$ . The corresponding mathematical expression of the constraint is

$$x_{if} \cdot \left( \sum_{j \in CC(i)} x_{jf} + \sum_{j \in AC_n(i)} \sum_{k=1}^n (x_{j(f-k)} + x_{j(f+k)}) \right) = 0 \quad (6a)$$

$$\forall i \in V, \forall f \in F, n = 1, \dots, |F|.$$

Our second point is the description of the approach for including constraints that should be satisfied when cells are served by BTSs that share the same site. Specifically, in this case a usual measure is to impose a set of  $\pm n$  adjacent channel (frequency) constraints to the, so-called, co-sited cells. This feature can be straightforwardly accomplished by appropriately exploiting the various  $AC_n(i)$  sets.

A third point that merits some discussion is the determination of the beacon frequencies (or Broadcast Common Control Channels – BCCHs) [1, 2] that are required in each cell. In principle, the allocation of BCCHs and traffic channels can be done through two consecutive solutions of the IOFAP. The first application can be targeted to the allocation of BCCHs and considers only a portion of the available spectrum (e.g., a subset of the overall set of frequencies  $F$ ). The second application allocates the traffic channels by taking into account the outcome of the BCCH allocation phase.



## 5. Algorithm Based on Simulated Annealing

### 5.1. SIMULATED ANNEALING FUNDAMENTALS

Annealing is the physical process in which a crystal is cooled from the liquid to the solid state. Careful cooling brings the crystal to the solid state. In analogy, a simulated annealing algorithm considers each solution of the optimisation problem as a state, the cost of each solution as the energy of the state, and the optimal solution as the minimum energy state. During each phase of the algorithm a new solution is generated by minimally altering the currently best solution (in other words, the new solution is chosen among those that are “neighbouring” to the currently best one). If the cost value that corresponds to the new solution is smaller (i.e., the difference between the cost of the old and the new solution,  $\Delta c$ , is positive) the new solution becomes the currently best solution. Solutions that increase the cost may also be accepted with probability  $e^{-(\Delta c/CT)}$ . This is a mechanism that assists in escaping from local optima.  $CT$  is a control parameter, which may be perceived as the physical analogous of the temperature in the physical process. This parameter is decreased as the algorithm proceeds, according to the cooling schedule. The algorithm ends when either  $CT = 0$  or when a significant number of moves have been made without improving the cost function.

A formal description of the algorithm is provided in Section 5.3, after presenting in the next sub-section some decisions that have been made for the simulated annealing algorithm that we developed for the IOFAP.

### 5.2. FEATURES OF THE SIMULATED ANNEALING-BASED ALGORITHM FOR THE IOFAP

The development of a simulated annealing based procedure means that the following aspects have to be addressed: configuration space, cost function, neighborhood structure and cooling schedule (i.e., manner in which the temperature will be reduced).

The following apply with respect to our problem. The configuration space is the set of feasible solutions  $\{x_{if} | i \in V, f \in F\}$  (i.e., allocations of frequencies to cells) that satisfy the constraints (2)–(8). The cost function is the one introduced by Relation (1).

Altering the value of two decision variables  $x_{if}$  and  $x_{if'}$ , while preserving the set of constraints (2)–(8) produces the neighbourhood structure. The cell that will change frequency and the frequency that will be released is selected according to a random process that follows a uniform distribution. The new frequency assignment should not violate the interference constraints.

There are various options regarding the cooling schedule. A recent advanced survey on this aspect may be found in [40]. The first (simpler and more well-known) scheme identified therein is the geometric cooling scheme. It follows the relations  $T_{new} = r \cdot CT$  and  $CT = T_{new}$ , where  $CT$  is the temperature in a certain phase of the algorithm (current temperature) and  $r$  is a number that usually ranges from 0.95 to 0.99. The second scheme, which was used in this paper, is the, so-called, “re-heating as a function of the cost”. As stated in [40] the scheme was introduced in [41, 42]. The re-heating scheme may increase the temperature of the algorithm, and therefore, prolong the run-time of the algorithm. It is an advanced mechanism for escaping from local optima. Finally, in our paper re-heating was applied in conjunction with adaptive cooling, which the third cooling scheme identified in [40]. This is a sophisticated scheme for reducing the temperature of the algorithm, suitable for intensifying the search at certain areas of the solution space.

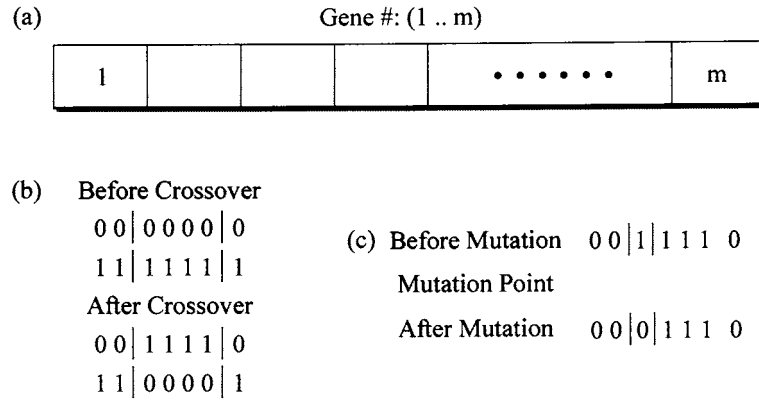


Figure 4. Genetic algorithm fundamentals [29]: (a) Chromosome, i.e., sequence of genes, representing a solution of the problem; (b) General application of the crossover operator; (c) General application of the mutation operator.

### 5.3. FORMAL ALGORITHM DESCRIPTION

The simulated annealing-based algorithm may be described as follows:

*Algorithm 1: [Simulated Annealing-Based Algorithm for the IOFAP]*

- Step 0.* Initialisation. Get an initial solution,  $IS$ , and an initial temperature value  $T$ . The currently best solution ( $CBS$ ) is  $IS$ , i.e.,  $CBS = IS$ , and the current temperature value ( $CT$ ) is  $T$ , i.e.,  $CT = T$ .
- Step 1.* If  $CT = 0$ , or if the stop criterion is satisfied, the procedure ends and a transition to *Step 6* is performed.
- Step 2.* A new solution ( $NS$ ) that is neighbouring to  $CBS$  is found.
- Step 3.* The difference of the costs of the two solutions,  $CBS$  and  $NS$  is found, i.e., the quantity  $\Delta c = C(CBS) - C(NS)$  is computed.
- Step 4.* If  $\Delta c < 0$  then the new solution becomes the currently best solution, i.e.,  $CBS = NS$ . Otherwise, if  $\Delta c \geq 0$ , then if  $e^{-(\Delta c/CT)} > rand[0, 1]$ , the new solution becomes the currently best solution, i.e.,  $CBS = NS$ .
- Step 5.* The cooling schedule, which is a combination of the re-heating as a function of cost, the adaptive and geometric cooling schemes, is applied, in order to calculate the new current temperature value  $CT$ , and a transition to *Step 1* is performed.
- Step 6.* End.

Various alternatives may be applied for realising the stop criterion in *Step 1*. The algorithm may stop when no improvement has been made after a given number of temperature decreases, which may involve applications of the re-heating criterion. Finally, the cooling schedule that was mainly used was a combination of the re-heating and the adaptive cooling schemes.

## 6. Genetic Algorithm

### 6.1. GENETIC ALGORITHM FUNDAMENTALS

In general, genetic algorithms maintain a set of problem solutions, which may be seen as the equivalent of a population of individuals. A string, also called a *chromosome* (Figure 4(a)),

is used for representing a solution. During each algorithm iteration, or *generation* in more strict genetic algorithm terms, the solutions are rated with respect to their quality, or *fitness*. As a result of this evaluation some solutions will be selected and used for the generation of a new population. This generation relies on the so-called genetic algorithm *operators*. In general, genetic algorithms use the *selection (reproduction)*, *crossover*, *mutation* and *replacement* operators.

The selection operator aims at selecting the solutions that will reproduce. The usual choice is to select the solutions that exhibit the best performance with respect to the fitness function (in analogy with real life, where individuals with higher fitness have a bigger probability to reproduce). A technique for guaranteeing the convergence of a genetic algorithm is to retain in the new population the best solutions of the previous population.

The crossover operator is applied with probability  $p_c$  after the reproduction operator. The basic idea of this phase is to select two “parent” solutions from the current set of solutions and to combine them so as to create two children as illustrated in Figure 4(b). The mutation operator is applied after the crossover operator with probability  $p_m$ . The mutation operator produces a new solution by modifying one or more gene values of an existing solution as illustrated in Figure 4(c).

In general, the crossover and mutation operators generate solutions from the already available population of solutions. The replacement operator is applied to the population of the already available and the generated solutions so as to create the new population of available solutions.

A formal description of the algorithm is provided in Subsection 6.3, after presenting in the next sub-section some decisions that have been made for the genetic algorithm that we developed for the IOFAP.

## 6.2. FEATURES OF THE GENETIC ALGORITHM FOR THE IOFAP

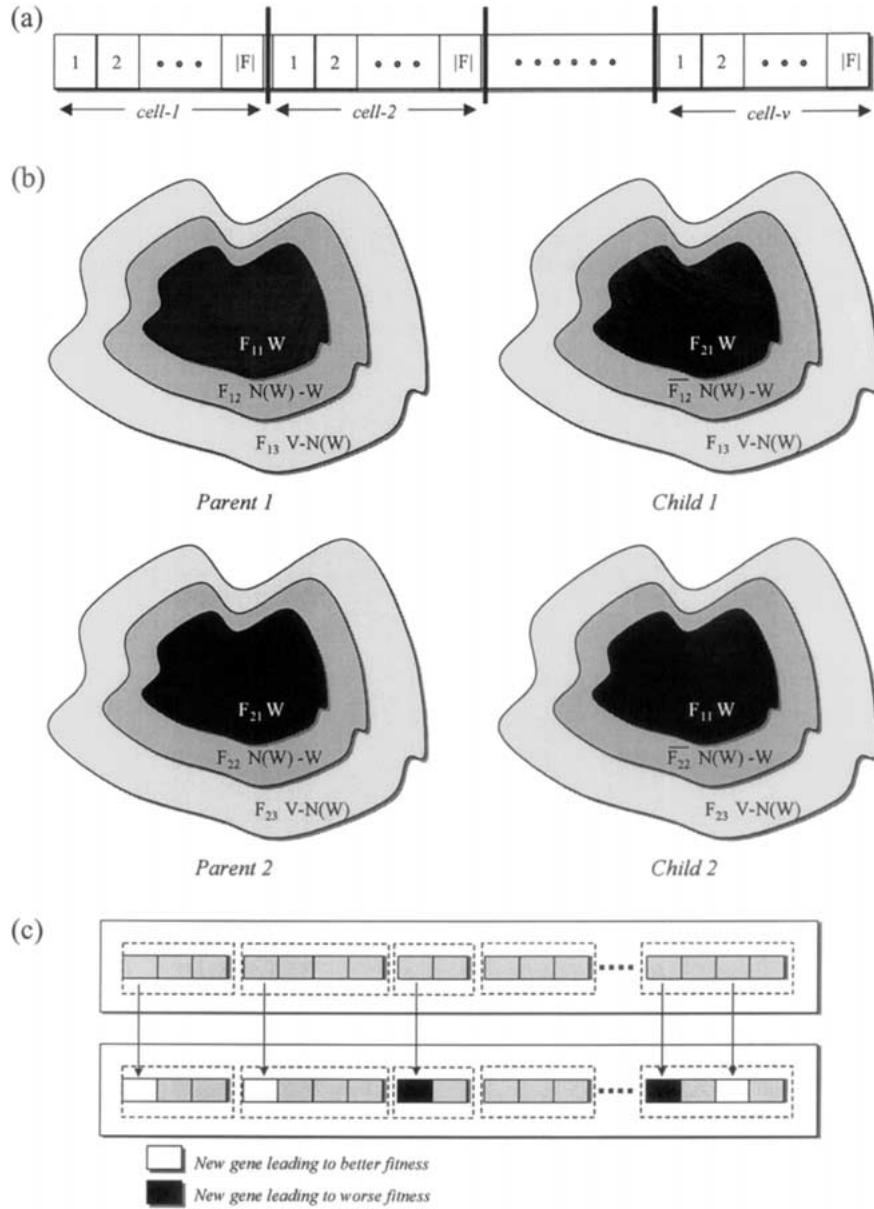
The construction of a genetic algorithm requires that the following points be addressed. First, the aspects that are represented by the genetic chromosome should be chosen. Second, the set of genetic operators should be chosen. Third, the fitness function should be defined. Fourth, the probabilities for controlling the genetic operators should be determined.

The chromosome in our case consists of a sequence of  $C_{fit}(c)$  genes (Figure 5(a)). The first  $C_{fit}(c)$  genes provide the frequencies assigned to cell 1, etc.

The fitness function  $C_{fit}(c)$  of a solution (allocation of frequencies to cells), or chromosome, is taken as the inverse of the objective function (1), i.e.,  $C_{fit}(c) = C(c)$ . This is done, so as to straightforwardly express that solutions that yield lower objective function values are seen as more “fit” from the algorithm point of view.

The following definitions are useful prior to the detailed description of the decisions that have been taken regarding the operators of the genetic algorithm for the interference oriented frequency assignment problem.

*Definition 1.* Given a set of cells  $W = \{u_1, \dots, u_k\}$  ( $W \subseteq V$ ), we define the set of cells that are “neighbouring” to  $W$ ,  $N(W)$ , as follows,  $N(W) = CC(u_1) \cup AC(u_1) \cup \dots \cup CC(u_k) \cup AC(u_k)$ .



*Figure 5.* Features of the genetic algorithm for the interference oriented frequency assignment problem: (a) Chromosome: The first  $|F|$  genes provide the frequencies assigned to cell 1, etc.; Only  $r(1)$  of these genes will be set to 1; (c) Crossover operator: The  $W$  set of cells in child 1 (2) obtains the frequencies of parent 2 (1), i.e., the  $F_{21}$  ( $F_{11}$ ) set of frequencies; (c) Mutation operator.

*Definition 2.* Given a solution (allocation of frequencies to cells), or chromosome, we define the age of the chromosome as the number of generations (i.e., loops from *Step 1* to *Step 5* in algorithm 2) for which the chromosome has remained in the population of solutions.

In the sequel, we describe in more detail the decisions regarding the specific genetic algorithm operators.

*Selection.* It is based on a random process that favours the selection (for reproduction) of solutions that have higher fitness function values. Other sophisticated versions of this operator proved to have negligible contribution to the performance of the frequency assignment problem.

*Crossover.* The technique works as illustrated in (Figure 5(b)). The cell layout in these solutions is separated in three sectors, namely, sets  $W$ ,  $N(W) - W$ , and  $V - (N(W) \cup W)$ . After the application of the crossover operator the cells in set of child 1 (2) will obtain the frequencies of the cells in set  $W$  of parent 2 (1). Unchanged remain the frequencies of the cells of set  $V - (N(W) \cup W)$ . The frequencies that will be assigned to  $N(W) - W$  may derive from the application of a suitably adapted heuristic algorithm, like those described in [43, 44].

*Replacement.* The aim is twofold. On the one hand, to replace solutions that score small fitness function values. On the other hand, to replace adult chromosomes, which are among the worse (in terms of fitness) in the population and whose age exceeds a given threshold  $a_{max,r}$ . More specifically, the conditions for replacing the adult chromosome  $c_2$  with respect to the younger one  $c_1$  are: either  $C_{fit}(c_1) > C_{fit}(c_2)$ , or  $|C_{fit}(c_1) - C_{fit}(c_2)|/C_{fit}(c_1) < f_{replace}$  and  $a(c_2) > a_{max,r}$ , where  $f_{replace}$  is the fitness function value related threshold for the replacement operator.

*Mutation.* This operator is applied for changing the value of a number of genes ( $g_m$ ) of a given chromosome. In principle, these changes will improve the chromosome. Moreover, we extend the application field of the operator, so as to eventually enable the improvement of solutions that have aged and are not among the best in the population. In other words, an *elitistic* strategy is followed so as to protect good, old solutions. However, old chromosomes that are not among the best in the population are allowed to mutate towards versions that have worse fitness function values, on condition that the fitness degradation is not beyond a certain threshold. Hence, the condition for allowing the mutation of an old chromosome  $c$  towards version  $c_m$  is:  $p_m > rand[0, 1]$  and  $|C_{fit}(c_m) - C_{fit}(c)|/C_{fit}(c) < f_{mutate}$  and  $a(c) > a_{max,m}$ .

In summary, the following important parameters of the algorithm have to be configured. First, the size of the current population of solutions. Second, the parameters that are associated with the crossover operator, i.e., the size of the generated population of solutions, the various  $W$  sets, and the  $p_c$  value. Third, those that are associated with the replacement operator, i.e., the  $A_{max}$  value, and the fitness function value related threshold ( $f_{replace}$ ) that will be imposed to solutions that have got old. Fourth, those that are associated with the mutation operator, i.e., the  $g_m$  and  $p_m$  values, and the age threshold ( $a_{max,m}$ ) and fitness function value related threshold ( $f_{mutate}$ ) that will be imposed to solutions that have got old.

### 6.3. FORMAL ALGORITHM DESCRIPTION

Based on the discussion above a general description of the genetic algorithm is the following:

#### *Algorithm 2: [Genetic Algorithm for the IOFAP]*

*Step 0.* Initialisation. The various  $W$  sets are selected and suitably adapted heuristic algorithm, like those described in [43, 44] are applied so as to select the initial population of solutions ( $IPS$ ). The current population of solutions ( $CPS$ ) is the  $IPS$ , i.e.,  $CPS = IPS$ .

- Step 1.* If the stop criterion is satisfied the algorithm stops and a transition to *Step 6* is performed.
- Step 2.* The reproduction operator is applied to the solutions of *CPS*, so as to select the set of the mating solutions, *MS*, that will be allowed to reproduce. The solutions in the *MS* set are selected based on a random process that favours solutions that exhibit the best fitness function values.
- Step 3.* The crossover and mutation criteria are applied to the set of the best (mating) solutions *MS*, so as to obtain the generated population of solutions *GPS*. Moreover, the mutation operator is applied to some old solutions, so as to eventually enable them to improve, as explained in the previous sub-section.
- Step 4.* The generated solutions are temporarily appended to the current population of solutions. This yields the temporary population of solutions  $TPS = CPS \cup GPS$ . Subsequently, the replacement operator is applied to the *TPS* set. This yields the set of solutions, *RS*, which should be removed from the *TPS* set. It holds that  $|RS| = |GPS|$ . Solutions that score small fitness function values, or others that are old (and are among the worse), are replaced. The new *CPS* set is  $CPS = TPS - RS$ .
- Step 5.* A transition to *Step 1* is performed.
- Step 6.* End.

## 7. Results

In this section we provide indicative results on the performance of the computationally efficient algorithms that were presented in the previous sections. The service area and cell layouts considered in our test cases are depicted in Figure 6. The service area is covered by 49 hexagonal cells, which are organised in a  $7 \times 7$  configuration. The choice of this cell structure does not simplify the computational effort. Any other topology of the same size and connectivity degree could have been chosen instead. The base station is assumed to be located at the center of each cell. The radius of each cell is assumed to be  $R_0 = 1$  kilometer (Km).

A typical assumption has been made regarding the hard co-channel constraints of each cell. More specifically, we assumed that for each  $i \in V$  the sets  $CC(i)$  and  $AC(i)$  are formed as depicted in Figure 6(b)). In other words, we assumed that the minimum allowable co-channel reuse distance is 2 cells (i.e., two cells can utilise the same frequency if between them there are at least two cells that do not use it) and that the minimum allowable adjacent channel reuse distance is 1 cell. Nevertheless, our scheme is completely independent from the manner in which the hard co-channel constraints derive. The combiner separation constraint has been set equal to 3, which is a typical value used in real frequency assignment exercises. No cell is assumed to have a set of forbidden frequencies. The soft co-channel interference values  $S_c(i, j)$  and the soft adjacent channel interference values  $S_a(i, j)$  are taken to be a function of the distance among cells. More specifically,  $S_c(i, j) \approx x_c \cdot [d(i, j)]^{-4}$  and  $S_a(i, j) \approx x_a \cdot [d(i, j)]^{-4}$ . In the context of our experiments we arbitrarily chose  $x_c = 32.000 \cdot [1/R_0]^{-4}$  and  $x_a = 4.000 \cdot [1/R_0]^{-4}$  [11]. This corresponds to an adjacent channel interference suppression of 9 dB at the receiver filter. These choices also do not affect our schemes. They just attribute a relatively higher importance to the co-channel with respect to the adjacent channel interference phenomena.

Our algorithms are applied to three test cases. In all three test cases 41 frequencies are assumed to be available in the system. Our aim through these test cases is twofold. First,

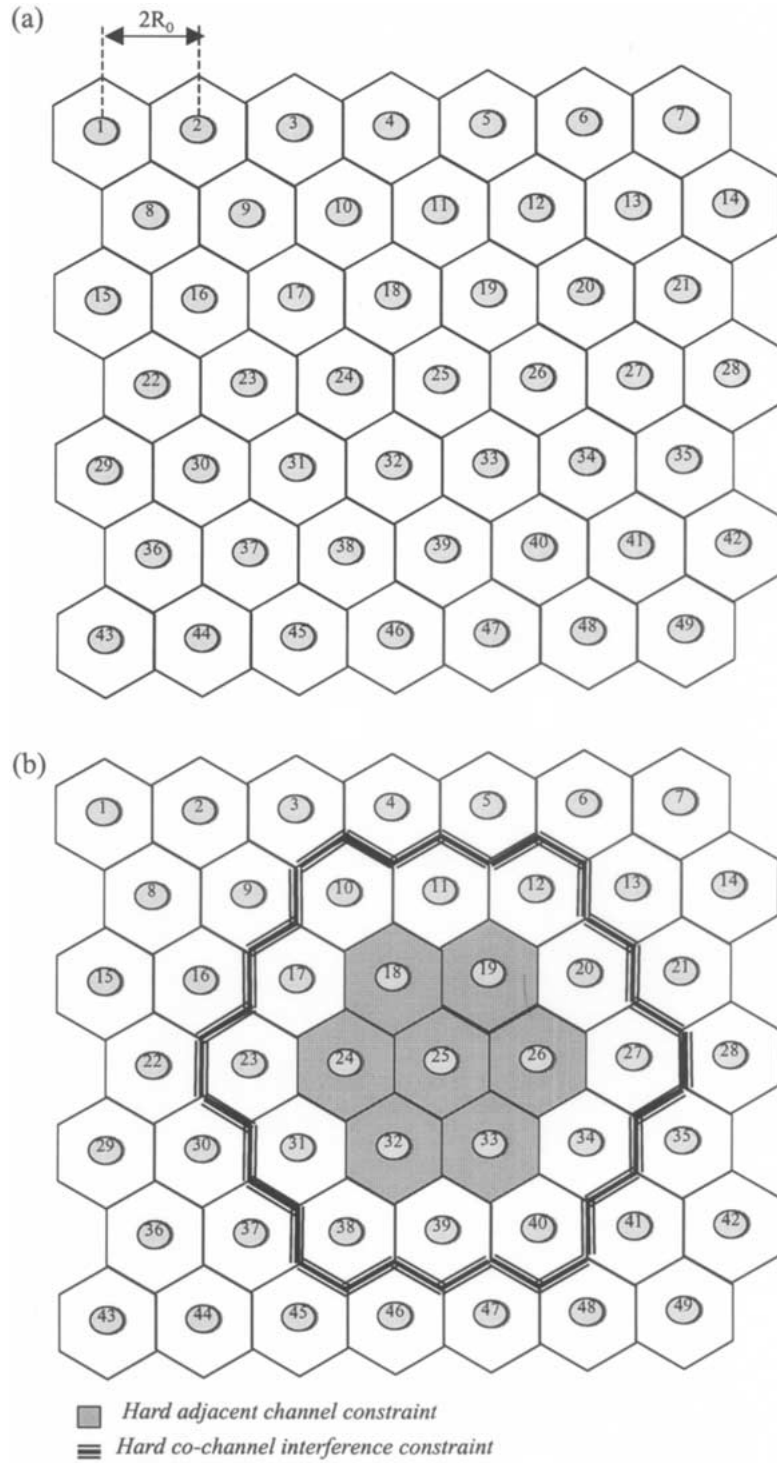


Figure 6. (a) Service area and cell layout of our test case; (b) Set of cells that belong to the hard adjacent channel constraints set and the hard co-channel constraint set of cell 25.

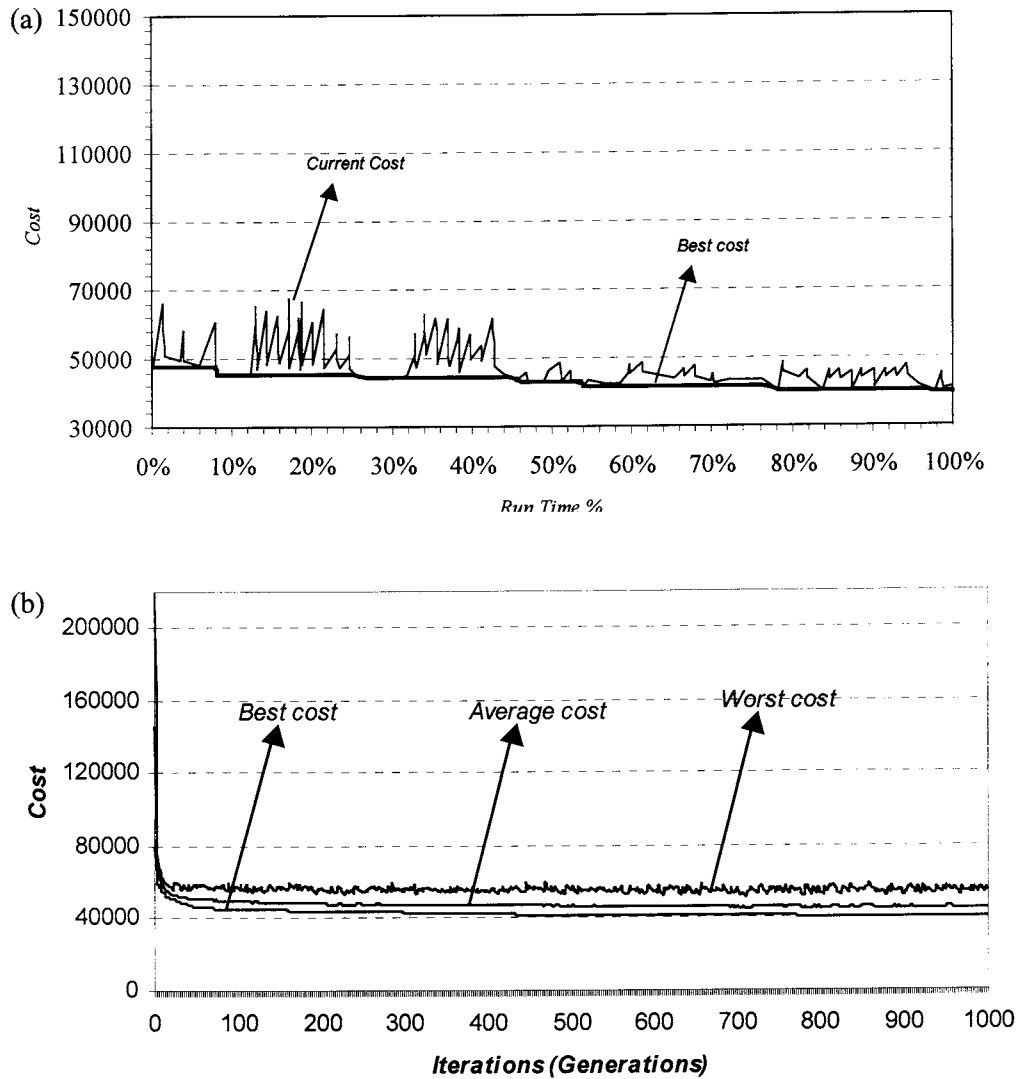


Figure 7. Performance of the algorithms in the first test case: (a) Evolution of the objective function value versus the run-time of the simulated annealing-based algorithm; (b) Evolution of the objective function value versus the number of iterations (generations) of the genetic algorithm.

to highlight the performance of the algorithms from a computational complexity viewpoint. In this respect, Figure 7, Figure 9 and Figure 12 depict the performance (objective function values) of the algorithms versus measures that indicate the required computation time. The second aim of the experiments is to analyse the results from a communications viewpoint. In this respect, we provide Figure 8, Figure 10, and Figure 13, which depict measures that are indicative of the anticipated interference levels in the system.

In the first test case we assume that the frequency requirements are homogeneous. More specifically, we assume that each cell requires 2 frequencies. The overall cost that the algorithms accomplished in the first test case is around 40,000 for both the simulated annealing based algorithm and the genetic algorithm. Details on the corresponding interference levels that will be imposed on the network are presented in the sequel. Figure 7 depicts the evolution



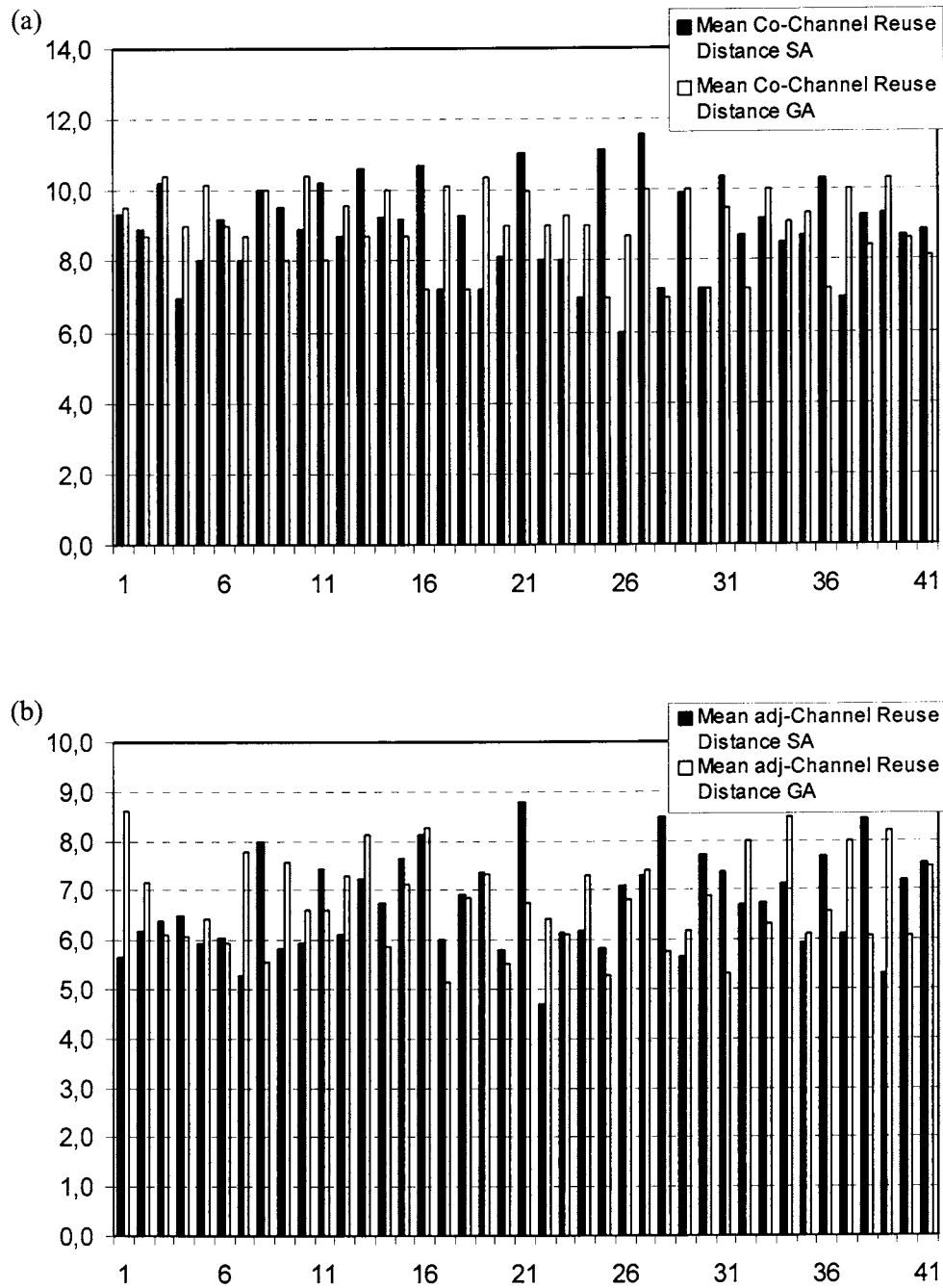


Figure 8. First test case: (a) Mean co-channel reuse distance achieved by the simulated annealing and the genetic algorithm for each frequency; (b) Mean adjacent channel reuse distance achieved by the simulated annealing and the genetic algorithm for each frequency.

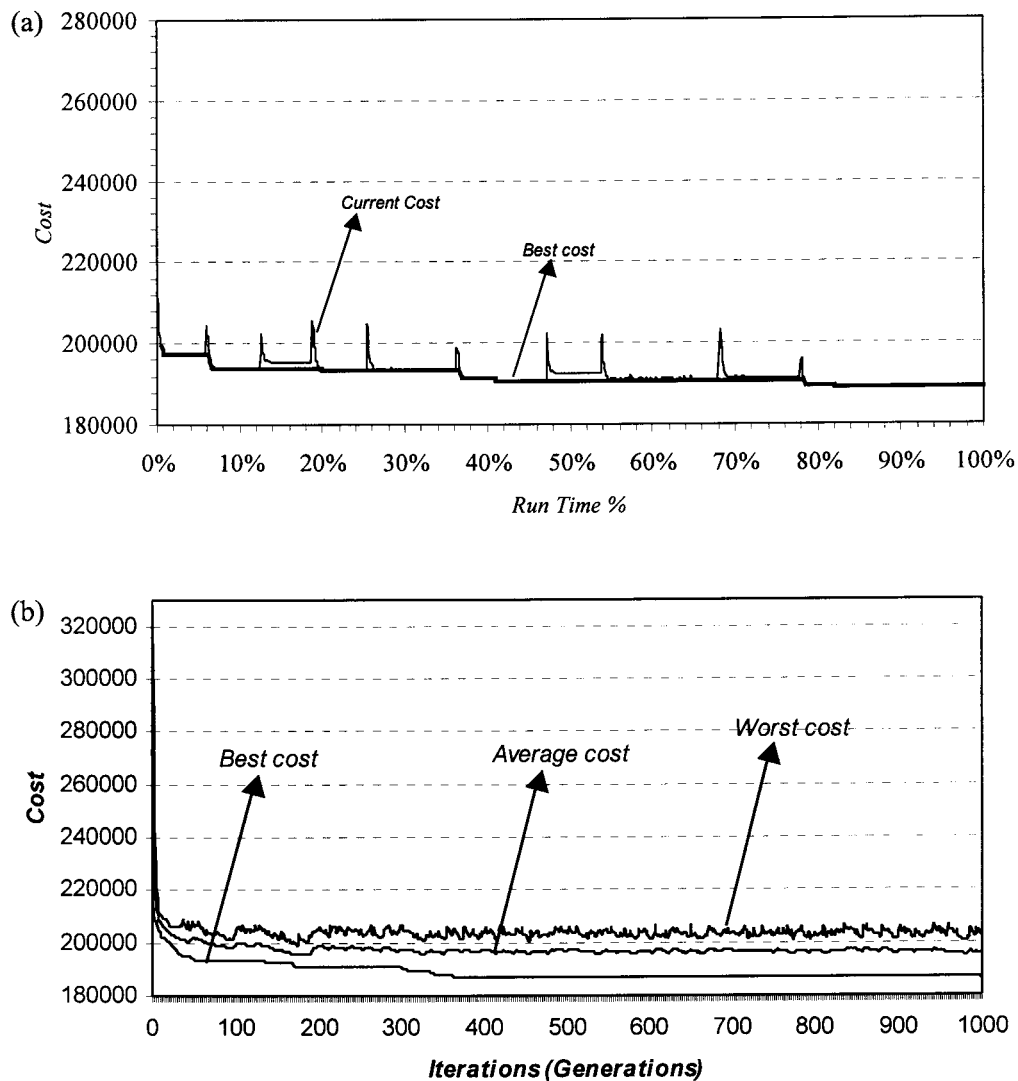


Figure 9. Performance of the algorithms in the second test case: (a) Evolution of the objective function value versus the run-time of the simulated annealing-based algorithm; (b) Evolution of the objective function value versus the number of iterations (generations) of the genetic algorithm.

of the performance of the simulated annealing-based algorithm with respect to the run-time of the algorithm, and the performance of the genetic algorithm with respect to the number of iterations (generations) conducted.

Figure 7(a) depicts the evolution of the cost of the best solution ever obtained, and the cost of the “currently best solution” (CBS, see Section 5), versus the run-time of the simulated annealing based algorithm. The currently best solution is used in the generation of neighbouring solutions. However, since the simulated annealing technique uses a mechanism for escaping from local optima, there is a probability that the best solution ever obtained is disregarded at some phase of the algorithm. This aspect requires the storage of the best solutions, even if solutions of higher cost are accepted as the currently best solutions at some phase of the

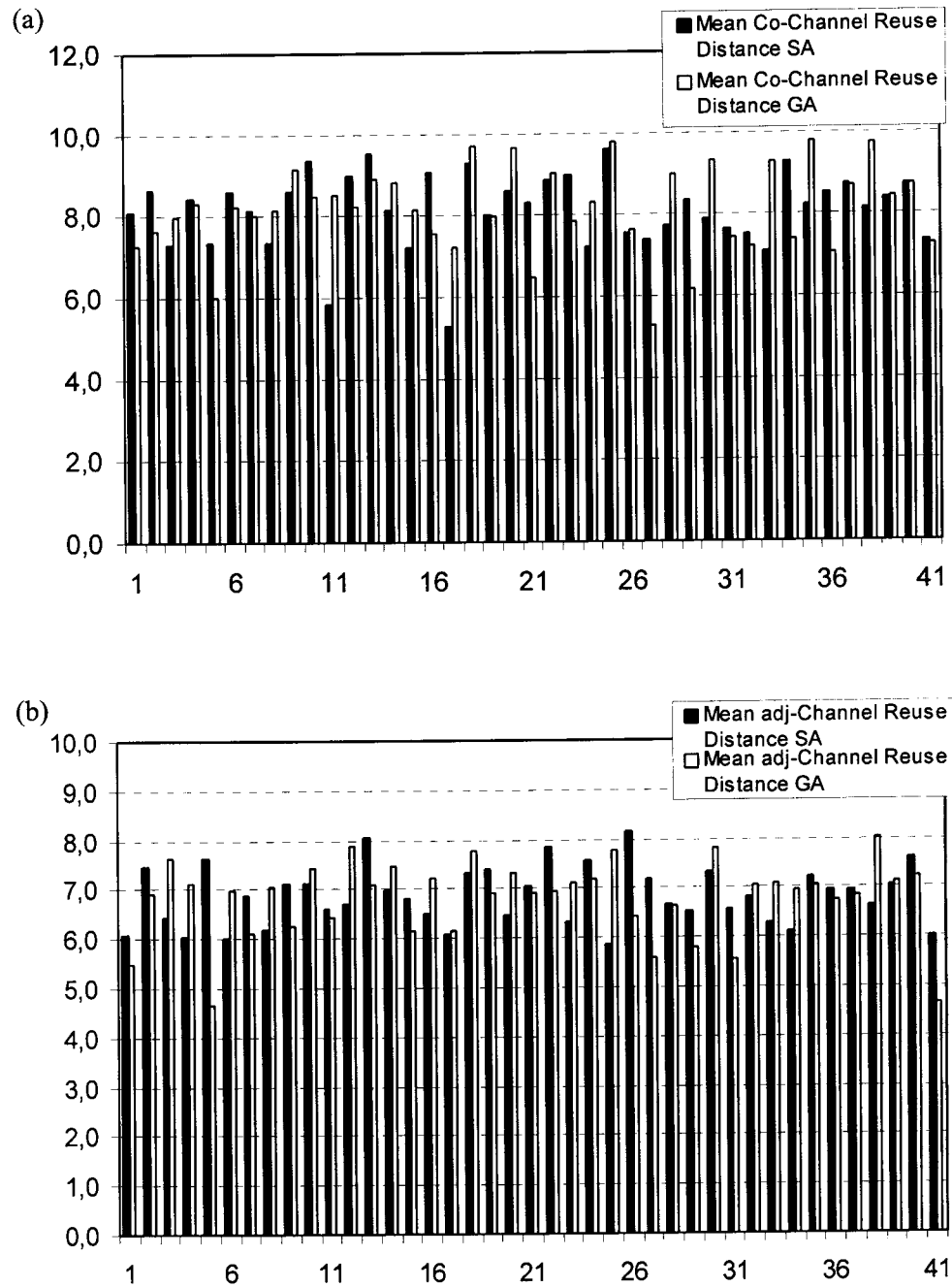


Figure 10. Second test case: (a) Mean co-channel reuse distance achieved by the simulated annealing and the genetic algorithm for each frequency; (b) Mean adjacent channel reuse distance achieved by the simulated annealing and the genetic algorithm for each frequency.

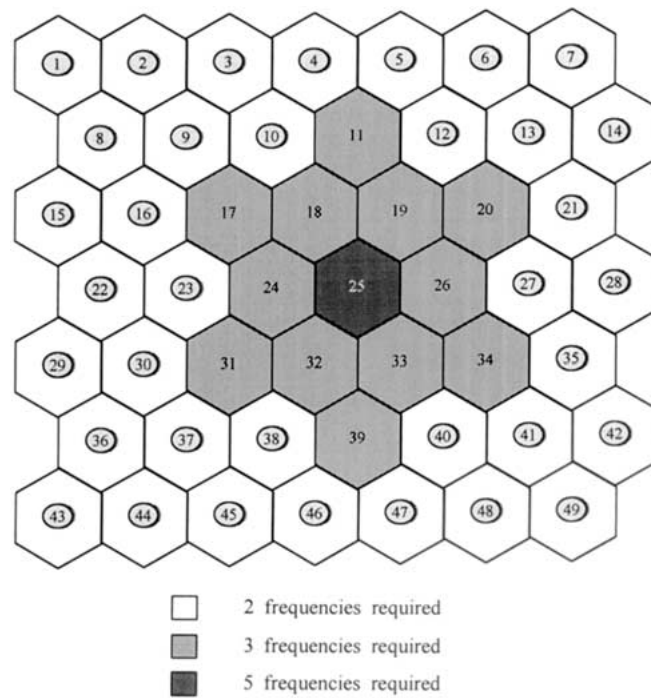


Figure 11. Third test case – “hot spot scenario”: channel requirement per cell.

algorithm. Nevertheless, at the final phases of the algorithm there is a convergence among the two solutions.

Figure 7(b) depicts the evolution of the cost of the best and worst solution in the population of the genetic algorithm, and the evolution of the average cost of all the solutions in the population, versus the number of iterations (generations) of the algorithm.

The equivalence of the results obtained by the two independent algorithms, i.e., the fact that the objective function values achieved are similar, should be noted. The equivalent performance is strong (or even, the only possible) evidence on the quality of the two algorithms. In general, a comparison with the optimal algorithm would be absolutely convincing on the quality of the solutions of the heuristic algorithms. However, an optimal algorithm to the IOFAP exhibits a prohibitively high complexity. This means that the provision of solutions to problem instances as those of this paper may not be achieved. Hence, the contribution of this paper, i.e., the development of two different computationally efficient algorithms, and the realisation of comparative studies in which equivalent results are obtained, is a solid basis (or the only way) for being convinced on the quality of the solutions. These remarks are also valid for the second and third test case as can be noted later in this section.

Figure 8 analyses the results from a communications viewpoint by depicting the mean co-channel reuse distance and the mean adjacent channel reuse distance that is achieved by the simulated annealing based algorithm and the genetic algorithm. The mean co-channel reuse distance is a measure of the density of the use of each frequency. This may be seen as an important measure of the co-channel interference that will be imposed by the frequency assignment pattern. It may be computed as follows: Assume that  $n$  cells use a given frequency  $f$ . The mean co-channel reuse distance is provided by adding the minimum distances among all pairs of cells that use the frequency (each pair of cells being taken once into account), and

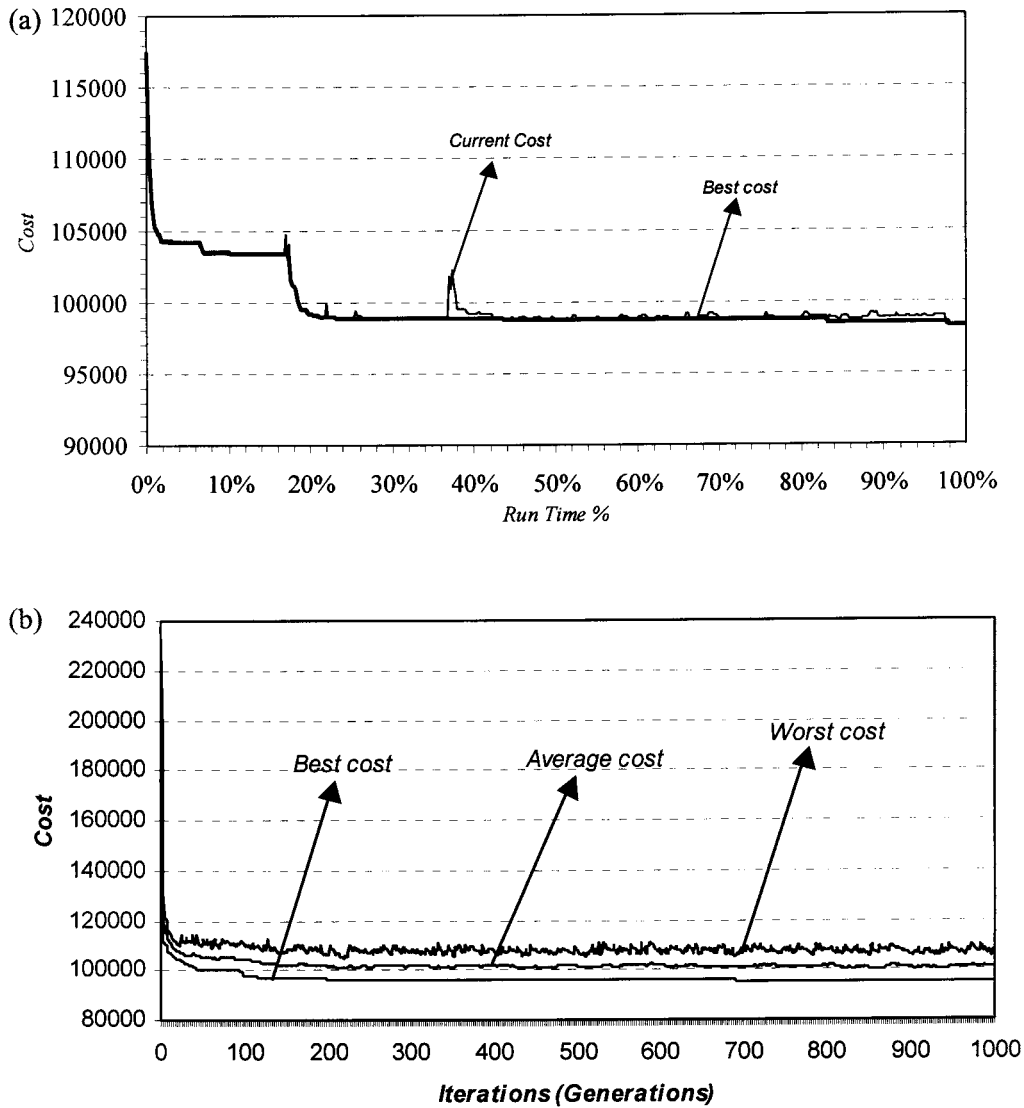


Figure 12. Performance of the algorithms in the third test case: (a) Evolution of the objective function versus the run-time of the simulated annealing-based algorithm; (b) Evolution of the objective function value versus the number of iterations (generations) of the genetic algorithm.

then by dividing the sum with the  $n(n - 1)/2$  quantity (which represents the total number of possible values).

In a similar manner the mean adjacent channel reuse distance can be computed. This quantity is a measure of the density of the use of adjacent frequencies, and hence, an important indicator of the adjacent-channel interference that is imposed by the assignment. Assuming again that  $n$  cells use a given frequency  $f$ , the mean adjacent channel reuse distance is provided by adding the minimum distances of the pairs of cells that use adjacent frequencies, and then by dividing the sum with the  $n(n - 1)/2$  quantity.

As may be observed in Figure 8 the mean co-channel reuse distance of the two algorithms is on the average around  $8-9R_0$ . At the same time the adjacent channel reuse distance ranges

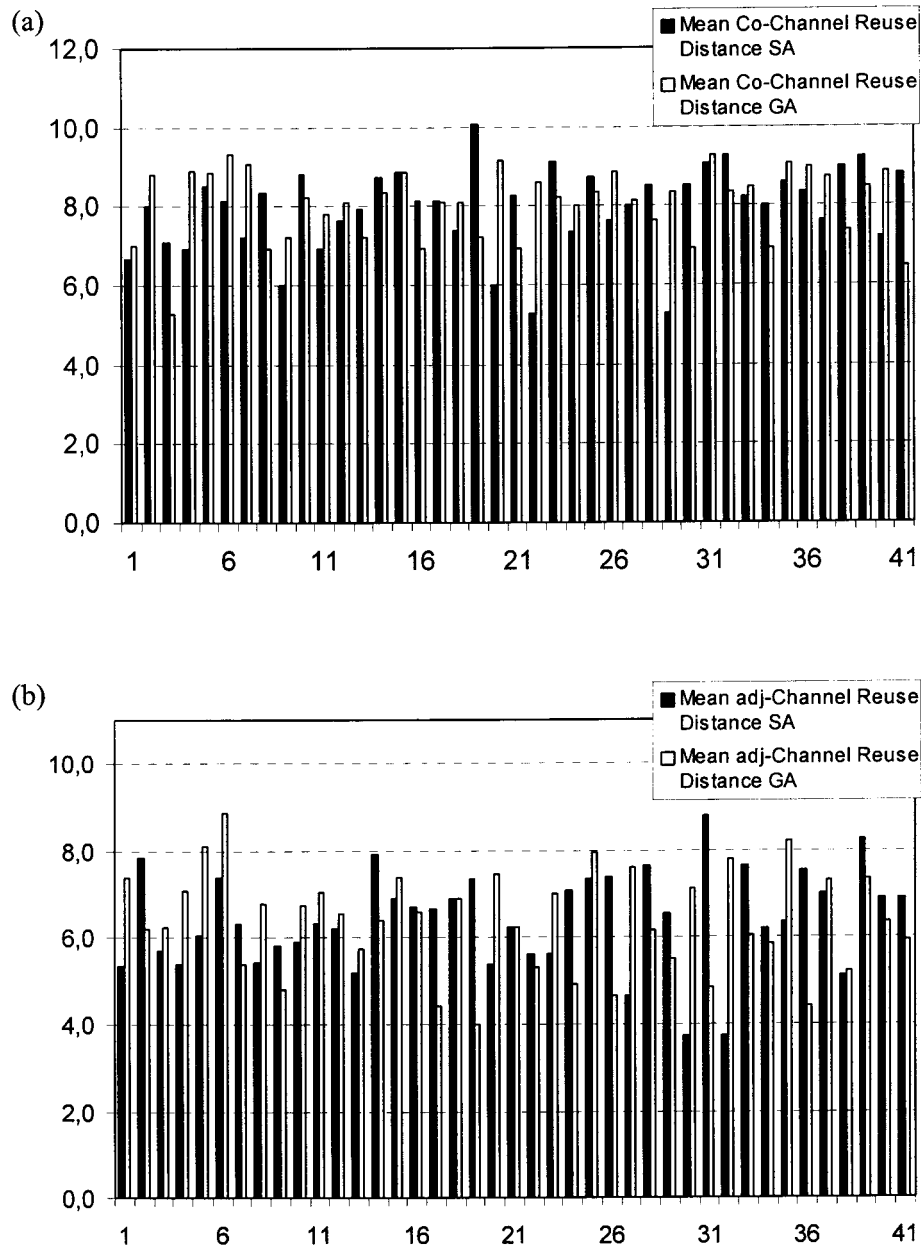


Figure 13. Third test case: (a) Mean co-channel reuse distance achieved by the simulated annealing and the genetic algorithm for each frequency; (b) Mean adjacent channel reuse distance achieved by the simulated annealing and the genetic algorithm for each frequency.

of the two algorithms is around  $6-7R_0$ . We remind that the minimum allowable co-channel reuse distance is around  $5R_0$  (or 2 cells) and that the minimum allowable adjacent channel reuse distance is  $3R_0$ . This means that the average co-channel reuse distance that is achieved by the algorithms is approximately 50% higher than the minimum allowable co-channel reuse distance. Likewise, the average adjacent channel reuse distance is significantly higher than the minimum allowable adjacent channel reuse distance. Therefore, the anticipated co-channel

and adjacent channel interference levels are significantly lower than the maximum tolerable ones.

The results of the first test case are promising with respect to the mean co-channel and adjacent channel reuse that is achieved by the algorithms. This is partly due to the lightly loaded network, which entails that few frequencies are required per cell. In order to test our schemes under more difficult conditions we realise the second test case. In the second test case we maintain the assumption that the network is uniformly loaded. However, we assume the load is higher, and that 3 frequencies are required per cell, in order to handle it.

The overall cost that the algorithms accomplished in the second test case was around 187.000 for both algorithms. Details on the corresponding interference levels that will be imposed on the network are presented in the sequel. Figure 9 depicts the evolution of the performance of the two algorithms, with respect to the run-time of the simulated annealing based algorithm, and the number of iterations (generations) of the genetic algorithm. Again the two algorithms yield almost equivalent results. The cost of the solutions is higher because of the heavier load that has to be accommodated. This requires a more dense frequency reuse, which means that the associated interference levels will be higher.

Figure 9(a) depicts the evolution of the cost of the best solution ever obtained, and the cost of the “currently best solution”, versus the run-time of the algorithm. The algorithm stops because of the insignificant improvement in the objective function values. Figure 9(b) depicts the evolution of the cost of the best and worst solution in the population of the genetic algorithm, and the evolution of the average cost of all the solutions in the population, versus the number of iterations (generations) of the algorithm.

The results of the algorithms for the second test case are analysed in Figure 10 that depicts the mean co-channel and the mean adjacent channel reuse distance that is achieved by the simulated annealing and the genetic algorithm. Figure 10(a) indicates that the mean co-channel reuse distance achieved by the simulated annealing-based algorithm and the genetic algorithm is around  $7R_0$ . At the same time the adjacent channel reuse distance ranges of the two algorithms is around  $6R_0$  as may be observed in Figure 10(b).

The fact that the average reuse distances in the second case are smaller with respect to those of the first test case indicates that the interference levels in the network will be higher in the second test case compared to the first test case. This will be caused by the more dense frequency reuse, which is required so as to cope with the frequency requirement of the second test case.

In the first two test cases we considered that the network is uniformly loaded, and hence, that the same number of frequencies is required per cell. In the third test case we alter this assumption and assume that the frequencies that are required per cell are as presented in Figure 11. In this way we simulate a, so-called, hot spot scenario, in which the communication traffic in the centre of the network is higher with respect to that in cells in the surroundings (periphery of the network). As will be observed similar remarks as previously can be drawn from this last test case.

The overall cost that the algorithms accomplished in the third test case is somehow less than 100.000 for both algorithms. Figure 12(a) depicts the evolution of the costs of the best solution and the “currently best solution” of the simulated annealing based algorithm. Figure 12(b) depicts the best, worst and average costs of the genetic algorithm versus the number of iterations (generations) of the algorithm. The main remark made in the previous two test cases, i.e., the similar performance of the algorithms, is valid for this test case as well. Figure 13 depicts the

*Table 1.* Best cost obtained by the simulated annealing algorithm and the genetic algorithm in the three test cases considered in this paper.

Test cast scenario	Best cost of simulated annealing algorithm	Best cost of genetic algorithm
First test-case:		
Uniform load with 2 channel required per cell	39.540	40.538
Second test-case:		
Uniform load with 2 channel required per cell	188.924	186.665
Third test-case:		
Hot spot scenario	98.232	95.061

corresponding mean co-channel reuse and the mean adjacent channel reuse distances that are achieved by the two algorithms.

Finally, this section concludes by providing some additional insight regarding the comparison of results obtained with simulated annealing and genetic algorithms. In this respect, Table 1 provides the best volume of the cost function obtained in each test case by the two algorithms. The table depicts the equivalent performance of the two approaches in the three diverse test-cases (light uniform load, higher uniform load and non-uniform load). As already expressed the equivalent performance is evidence on the quality of the two algorithms.

## 8. Conclusions

In this paper an interference-oriented version of the frequency assignment problem was defined. The objective function was associated with the interference levels that are imposed by the frequency allocation, while the constraints were related to the allocation of the frequencies required in each cell and the prevention of some unacceptable interference situations. The problem was formally stated, mathematically formulated and solved by means of two computationally efficient algorithms. These algorithms were applied to test cases and corresponding results were presented. A promising aspect regarding the quality of the two algorithms is that they provided equivalent results to the test cases.

The algorithms presented herewith are suitable for application in the design, or the re-engineering, phases of the system. An interesting issue for future study is to direct our work towards schemes that may dynamically handle the interference situations that occur in the network. This extension of our work will lead to algorithms that are suitable for use in the management domain.

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**Serafim A. Kotrotsos** was born in Athens, Greece, in 1974. He received the diploma in electrical and computer engineering from the National Technical University of Athens (NTUA) in 1997. From 1997–2000 he has been a senior research engineer at the Telecommunications Laboratory in NTUA working in the area of the design and performance evaluation of mobile and broadband networks, optimization algorithms, business level management software and software engineering. During that period he has been actively involved in a number of national and international research and development programs of the EU (ACTS SCREEN, IST MOEBIUS). He is a member of the Technical Chamber of Greece.

**George V. Kotsakis** was born in Kiparissia Messinias, Greece in 1971. He received the diploma (with highest honours) and the Ph.D. degrees in electrical and computer engineering from the National Technical University of Athens (NTUA) in 1995 and 1999, respectively. Since 1995 he is a senior research engineer at the Telecommunications Laboratory in NTUA. He has been actively involved in a number of national and international research and development programs, e.g., ACTS COBUCO (AC031 – Cordless Business Communication System) and ACTS EXODUS (AC013 – Experiments on the Deployment of UMTS). For four consecutive years, from 1995 until 1999, he was awarded with prizes from the National Scholarship Foundation. His research interests include the design and performance evaluation of mobile and broadband networks, the design of communications' protocols, algorithm and complexity theory and software engineering. He is a member of the Technical Chamber of Greece.



**Panagiotis P. Demestichas** was born in Athens, Greece, in 1967. He received the diploma and the Ph.D. degree in electrical and computer engineering from the National Technical University of Athens (NTUA). Currently, he is a senior research engineer at the Telecommunications Laboratory in NTUA. Since 1993, he has been actively involved in a number of national and international research and development programs, e.g., EURET EURATN (European Aeronautical Telecommunications Network), RACE II MoNet (R2066-Mobile Networks) RACE II TOMQAT (R2166-Total Quality Management in B-ISDN), BRITE/EURAM FANSTIC (Future ATM, New Systems and Technologies Integration in the Cockpit), ACTS STORMS (AC016-Software Tools for the Optimisation of Resources in Mobile Systems), ACTS SCREEN (AC227-Service Creation Engineering Environment), and ACTS MONTAGE (AC325 – Mobile Intelligent Agents in Accounting Charging and Personal Mobility Support). Most of his current activities are focused in the IST projects SHUFFLE (agent based approach for controlling resources in UMTS) and MOEBIUS (Mobile Extranet Based Integrated User Services). His research interests include the design and performance evaluation of mobile and broadband networks, service and software engineering, service and network management, algorithms and complexity theory and queueing theory. He has over 50 publications in these areas in international journals and refereed conferences. He has held positions in the private sector for several years, and has been a member of the Hellenic Navy for one year, working as a software engineer. He is a member of the IEEE and of the Technical Chamber of Greece.



**Evangelia C. Tzifa** was born in Athens, Greece, in 1972. She received the diploma and the Ph.D. degrees in electrical and computer engineering from the National Technical University of Athens (NTUA) in 1995, and 1998, respectively. Currently, she is a senior research engineer at the Telecommunications Laboratory in NTUA. Since 1995, she has been actively involved in a number of research and development programs of the EU (BRITE/EURAM FANSTIC, EURET EURATN, ACTS DOLMEN, ACTS MONTAGE). Her research interests include mobile and broadband telecommunications service and software engineering, algorithms and complexity theory and queueing theory. She is a member of Technical Chamber of Greece since 1995.



**Vasiliki P. Demesticha** was born in Athens, Greece, in 1970. She received the diploma in physics from the University of Athens in 1994 and the Ph.D. degrees in electrical and computer engineering from the National Technical University of Athens (NTUA) in 1998. From 1994–2000 she has been a senior research engineer at the Telecommunications Laboratory in NTUA working in the area of the mobile and broadband communications, algorithm and complexity theory and software engineering, hardware design, and the design and administration of high performance computing systems. In that period she has been actively involved in a number of national and international research and development programs of the EU (BRITE/EURAM FANSTIC, RACE II TOMQAT, ACTS STORMS). In March 2000 she joined IBM Greece. She is a member of IEEE since 1998.



**Miltiades E. Anagnostou** was born in Athens on 21 June 1958. He received the electrical engineer's diploma from the National Technical University of Athens (NTUA) in 1981. During the period 1982–1987 he was a teaching assistant at NTUA. In 1987 he received his Ph.D. in the area of computer networks. In 1987–1989 he served the greek army in the Air Force Research and Development Center. In 1989–1991 he was a lecturer, in 1991–1996 an assistant professor, in 1996–1999 an associate professor, and since February 2000 he is a full professor in the Computer Science Division, Department of Electrical and Computer Engineering of the NTUA.

He teaches courses on modern telecommunications, formal specification of protocols, stochastic processes, and queueing theory. Prof. Anagnostou has been involved in research programs both in the areas of ATM based broadband networks, mobile and personal communications, service engineering, mobile agent technology, communications for civil aviation and air traffic control.

He is author of more than 100 scientific papers in the areas of computer networks, protocols, queueing theory, algorithms, mobile and personal communications, and B-ISDN. He is a member of the Technical Chamber of Greece, IEEE and ACM.