Simulated Isobaric Annealing

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ABSTRACT

A Simulated Annealing algorithm for wire placement is proposed in which annealing is carried out under conditions of constant interconnection pressure rather than constant area. This is achieved by modifying the Metropolis criterion so that changes in the Gibbs free energy of the wires are calculated. This enables the area of the array devoted to wiring to decrease as the average wire length is reduced during the annealing schedule. The definition of interconnection pressure follows naturally from the thermodynamic interpretation of Simulated Annealing and it is shown to be equivalent to a wiring figure of merit called the channel density. The paper ends by speculating on the use of wire pressure to determine the phase transition temperature between the "gas" and "liquid" state of the system.

I. Introduction

Imagine for a moment a course offered by a physics department on wire placement algorithms! The course notes might describe the Simulated Annealing (SA) algorithm [1] as *"implementing a isothermal ensemble of one-dimensional particles whose linear extent varies in proportion to their energy, under conditions of constant area. The entropy of these particles is maximized by employing Metropolis' algorithm [2] at successively lower temperatures until the ground state of the system is reached."*

Fortunately, perhaps, such a course in unlikely to be offered. For most computer scientists and electrical engineers the view offered by statistical mechanics obscures the more important fact that the SA algorithm is used because it is works in a cost-effective manner, both in terms of speed and memory. Nonetheless, this perspective will be retained in this paper in order to introduce some additional concepts which will be used to extend the scope of the SA algorithm.

Consider the properties of a two-dimensional balloon of area A and temperature T. To a first order approximation the state of the system is defined by the ideal gas law:

$$P = kT\frac{N_p}{A} \quad \text{(newton/cm)}, \tag{1}$$

where N_p is the number of particles, k is Boltzmann's constant, and P is the gas pressure. This indicates that under isobaric conditions any change in temperature must be accompanied by a proportional change in area. Comparison with experiment indicates that this prediction is accurate when the specific area of a particle is large enough so that inter-particle forces can be ignored.

It is proposed that isobaric conditions be incorporated into the SA algorithm. Under these circumstances the area devoted to wiring is free to vary in order to accommodate the decreasing amount of wire resulting from the progression of the annealing schedule. The next section introduces the concept of the ideal interconnection gas which enables the definition of interconnection pressure and defines the required modifications to the SA algorithm. In Section III the results of implementing the algorithm are presented and compared with a standard constant-area algorithm. Modifications to the algorithm to incorporate non-idealities such as non-zero wire width are discussed in Section IV, which leads to a discussion of the possibility of a phase diagram for computer wiring. The main concepts are then summarized in Section V.

II. The Ideal Interconnection Gas

The performance of modern computer systems is largely determined by the transfer of information between processing elements. It is therefore convenient to discard the perception of a computer as an array of devices interconnected by wires and instead perceive it as array wires that must be excited by devices. It is in this spirit that a model of a processing element array, which may represent a gate array or multi-chip module, is introduced in which wires are represented as one-dimensional particles routed to processing elements represented as point objects of zero area. The equivalent of the particle kinetic energy is defined by

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the Manhattan distance between terminating processing elements.

It may be shown that to a first-order approximation the state of this ideal interconnection gas composed of N_p wires at an effective annealing temperature T is defined by

$$P = \kappa T \frac{N_p}{A} \quad (\text{cm}^{-1}) \tag{2}$$

where κ plays the role of Boltzmann's constant and A is the area of the array. It is convenient to set κ to unity and to make it dimensionless so that temperature and energy have the same units of length. By invoking the equipartition of energy theorem it is proposed that the average wire length is simply related to the annealing temperature by

$$\bar{\ell} = \kappa T \quad (\text{cm}). \tag{3}$$

This may be interpreted as the thermodynamic justification of the SA algorithm since the purpose of the algorithm is to reduce the average wire length by reducing the annealing temperature.

In many construction techniques the wiring is routed through channels whose width is determined by the total amount of wire to be routed. Since the area devoted to the processing elements is a relatively small and constant, the size of the array is approximated by the sum of the channel widths. The channel width may be estimated by one of several theories each based upon the assumption that the average wire length of the system is known with some degree of certainty. In practice this is not the case and it must be estimated [4]. This leads to wiring inefficiencies since any over-estimate is amplified by a feedback effect; a longer average wire length increases the channel width which makes wires longer. The observed efficiency of the wiring is commonly measured by the channel density or wire capacity, defined as the length of wire associated with each processing element [5]

$$D = \frac{\bar{r}}{W^2} \quad (\text{cm}^{-1}), \tag{4}$$

where $\bar{r}$ and W^2 are the average wire length and area associated with each processing element, respectively. The channel density gives an indication of the length of wire that can be packed into a given area of the array surface and forms a useful wiring figure of merit.

In constant-area annealing a new state of the system is determined by swapping two processing elements and calculating the change in total wire length. If the wire length decreases the swap is kept. If the total wire length increases the swap is kept according to a conditional probability given by Maxwell-Boltzmann statistics [2]

$$Q\,|_{A,T,N_p} = e^{-\Delta L/T}, \tag{5}$$

where ΔL is the change in wire length and T is the annealing temperature.

Isobaric conditions are implemented by redefining the conditional swap probability in terms of changes in the Gibbs free energy [6]

$$Q\,|_{P,T,N_p} = e^{-(\Delta L + P\Delta A - N_p \ln \Delta A)/T}, \tag{6}$$

where the changes in area are randomly generated within some pre-defined range at the same time the processing elements are swapped.

This unusual form of the Metropolis criterion may be interpreted at the circuit level by re-writing the equation of state in terms of wiring parameters. By using the equipartition of energy theorem (Eqn. 3) the average wire length may be substituted for the annealing temperature:

$$P = \frac{N_p}{A}\bar{\ell}. \tag{7}$$

The average wire length $\bar{\ell}$ may then be replaced by the average wire length per processing element since $\bar{\ell} = N_g\bar{r}/N_p$, where N_g is the number of processing elements in the array. The interconnection pressure is then given by

$$P = \frac{\bar{r}}{A/N_g} = \frac{\bar{r}}{W^2}, \tag{8}$$

which is recognized as the definition of channel density given by Eqn. 4. The isobaric annealing schedule is therefore interpreted as carrying out wire optimization under conditions of constant channel density. The results of this strategy are presented in the next section.

III. Algorithmic implementation

The latter stages of an isobaric annealing process are illustrated in Fig. 1. Each processing element is allocated a shaded square according to the total length of wire originating from that element. The shading is normalized at a temperature of $T = 5$ cm so that the element with the most wire is shaded white and the element with the least wire is shaded black. For this array the total number of wires is $N_p = 149$ and the number of processing elements is $N_g = 100$, which implies that on average each processing element is associated with approximately three wires. As the frame sequence is traversed from left to right and top to bottom the state of the array is captured at different points in the annealing schedule. The array area is observed to decrease as the temperature is reduced in order to maintain a fixed pressure of 10 cm^{-1}. This is equivalent to stating that the algorithm attempts to find the minimum wire length consistent with packing 10 cm of wire into every square cm of array surface. The final frame indicates that at a temperature of 1 cm the area has shrunk to $A = 14.97$ cm^2 and the average wire length

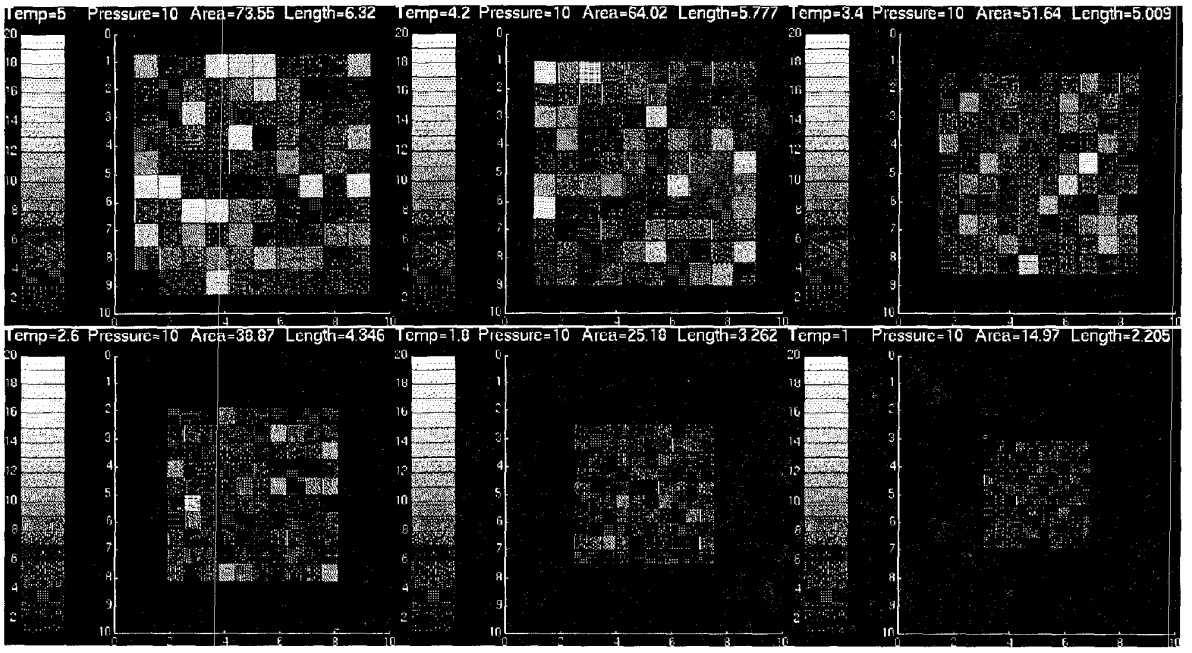

Figure 1: Sampled stages from simulated isobaric annealing algorithm. The shading represents amount of wire associated with each processing element indexed by the legend on the left of each frame.

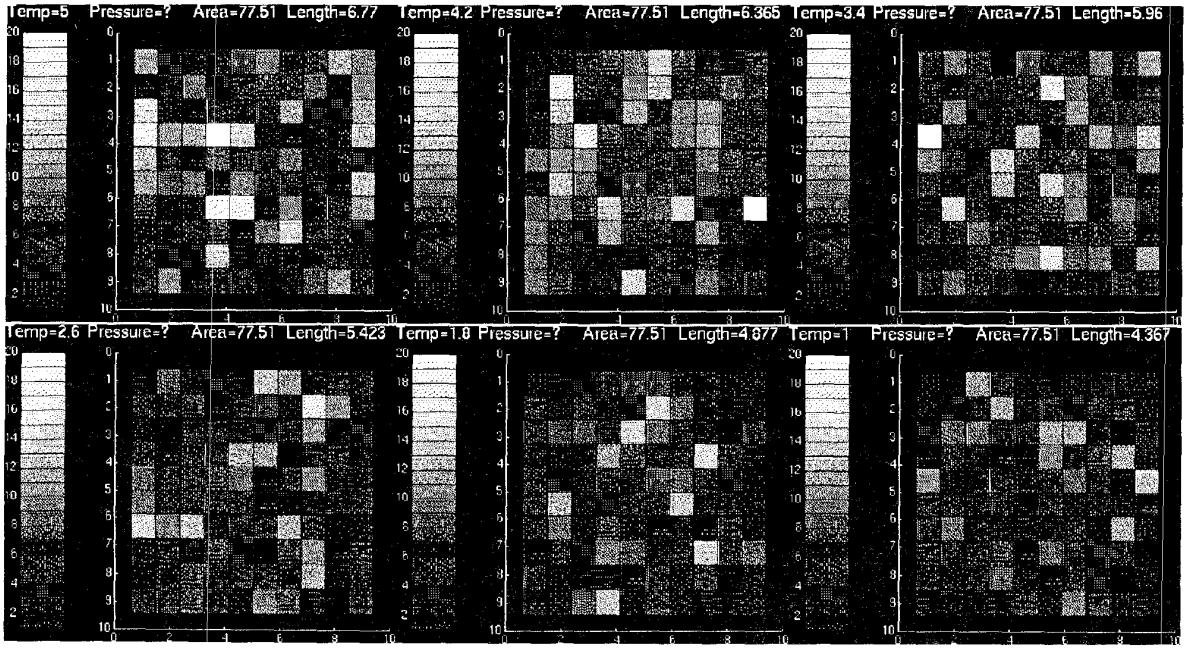

Figure 2: Sampled stages from simulated constant-area annealing algorithm. The shading indicates amount of wire associated with each processing element indexed by the legend on the left of each frame.

is $\bar{\ell} = 2.21$ cm. For comparison, the latter stages of the standard constant-area annealing process are illustrated in Fig. 2. This achieves a reduction in the average wire length from $\bar{\ell} = 6.77$ cm to 4.367 cm over the same temperature range. For this frame sequence the chip area was fixed at $A = 77.51$ cm^2 in order to approximate the area associated with the temperature and pressure of frame 1 of Fig. 1.

IV. Non-Idealities

The most obvious modification to the ideal gas model is to endow the wires with non-zero width. This generates interactions between wires and leads to potential energy contributions to the total energy of the ensemble. Initial investigations have been restricted to a "hard shell" wire model. This model does not permit swaps which cause the sum of the wire widths in any channel segment to exceed the channel width. In effect there is an infinite interaction energy if wires overlap.

Fig. 3 shows the experimentally determined phase diagram for the array of Figs. 1 and 2 with the wire width set to 1 mm. The phase transition pressure at a given temperature was determined by observing the isothermal logarithmic pressure/area characteristic, shown inset at a temperature of $T = 100$ cm. The abrupt change in gradient from the ideal gas value of -1 to zero at a transition pressure of approximately $P_t = 100$ cm^{-1} indicates that the array has changed from a gas to a liquid phase at a transition temperature of $T_t = 100$ cm.

Consideration of Eqn. 2 indicates that the expression for the transition region is given by

$$P_t = \frac{N_p}{A_t} T_t,$$

where A_t is the minimum area of the array at the transition point. Since the gradient of the transition region is independent of temperature and approximately equal to unity the minimum chip area is independent of annealing temperature and is approximately equal to N_p cm^2. This suggests that new annealing schedules should be investigated which combine changes in temperature with changes in pressure.

V. Conclusions

The concept of interconnection pressure has been introduced in order to implement isobaric conditions within the Simulated Annealing algorithm. The equivalence between pressure and channel capacity leads to the interpretation of isobaric conditions as those which maintain a constant amount of wire length per processing element.

If wires are endowed with non-zero widths then the algorithm is easily modified so that placement and routing are accomplished in a unified manner. The details of the routing algorithm are contained within the potential profile of the wire. For simplicity a hard-shell potential profile was selected. However, it is anticipated that other potential profiles will have beneficial effects on routing efficiency and congestion.

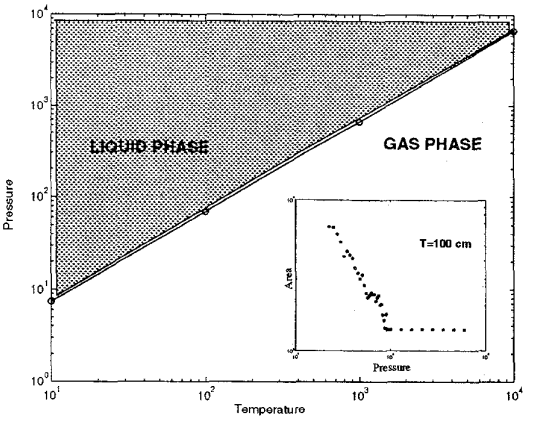

Figure 3: Phase diagram for hard-shell wire model. Inset diagram shows isotherm at a temperature of $T = 100$ cm.

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