ABM: Minecraft Redstone with Graph and Set Theory

Due on June 26, 2024

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1 Foundations

Definition 1.1. *(Max)*
Let X be any $X \subseteq \mathbb{N}_0$. Then

$$Max(X) \coloneqq \{x \in X \mid \forall y \in X(x \geq y)\} \tag{1}$$

Definition 1.2. *(Number Set Notation)*

$$\mathbb{Z}^+ \coloneqq \mathbb{N}_0 \setminus \{0\} \tag{2}$$
$$\mathbb{Z}^- \coloneqq \{n \in \mathbb{Z} \mid n < 0\} \tag{3}$$
$$\mathbb{Z}_0^- \coloneqq \mathbb{Z}^- \cup \{0\} \tag{4}$$
$$\tag{5}$$

Definition 1.3. *(Signal Sets)*

$$S \coloneqq \{0, 1, 2, ..., 15\} \tag{6}$$
$$S^+ \coloneqq \{x \in S \mid x > 0\} \tag{7}$$
$$S_0 \coloneqq \{0\} \tag{8}$$

Definition 1.4. *(Component Set)*

$$C \coloneqq \{T_c, R1_c, R2_c, R3_c, R4_c, O_c, I_c\} \tag{9}$$

The set of supported redstone components.

Definition 1.5. *(Vertices)*
Let V be a set.
Then V is a set of vertices if

$$V \neq \varnothing \tag{10}$$

Definition 1.6. *(Directed Edges)*
Let V be any set of vertices.
Then E is a set of directed edges on V if

$$E \subseteq (V \times V) \tag{11}$$

Definition 1.7. *(Graph)*
Let V be any set of vertices, and let E be any set of directed edges on V.
Then G is a digraph on V and E if

$$G = (V, E) \tag{12}$$

Definition 1.8. *(Propegation)*
Let $G = (V, E)$ be any digraph. Then

$$\omega^+ : V \times \mathcal{P}(G) \to \mathcal{P}(V) \tag{13}$$

Where

$$\omega^{+}(v, G) = \{u \in V \mid (u, v) \in E\}$$

And

$$\omega^{-} : V \times \mathcal{P}(G) \to \mathcal{P}(V) \tag{14}$$

Where

$$\omega^{-}(v, G) = \{v \in V \mid \exists (u, v) \in E\}$$

Definition 1.9. *(Behavior functions)*

$$\beta_s : C \to (\mathcal{P}(S) \times S \times \mathbb{N}_0 \to S) \tag{15}$$
$$\beta_o : C \to (S \times \mathbb{N}_0 \to S) \tag{16}$$

Definition 1.10. *(Redstone Digraph)*
Let:

- $G = (V, E)$ *be any digraph*
- $\Sigma : V \times \mathbb{N}_0 \to S \times \mathbb{N}_0$
- $\lambda : V \to C$

Then G_r *is a redstone digraph if*

$$G_r = (G, \lambda, \Sigma) \tag{17}$$

Definition 1.11. *(State)*
Let $G_r = (G, \lambda, \Sigma)$ *be any redstone digraph where* $G = (V, E)$*.*
Now for an arbitrary tick $t \in \mathbb{N}_0$*, and any* $v \in V$*, let*

$$I = \{u \in \omega^{-}(v, G) \mid ((\beta_o \circ \lambda)(v) \circ \Sigma)(u, t)\}$$

And

$$\Phi = (\beta_s \circ \lambda)(v)(I, \Sigma(v, t)) \tag{18}$$

Now let

$$D = \{\varphi \in S \mid (\varphi, \delta) \in \Sigma(v, t), \delta \in \mathbb{N}_0\} \cup \{\Phi\} \tag{19}$$

And let

$$\Delta = \begin{cases} \delta + 1,\ (\varphi, \delta) \in \Sigma(v, t) & \textit{if } (D \subseteq S^{+}) \vee (D \subseteq S_0) \\ 0 & \textit{otherwise} \end{cases} \tag{20}$$

Then

$$\Sigma(v, t+1)|_{V \times \mathbb{Z}^{+}} = (\Phi, \Delta) \tag{21}$$

Note: the state mapping isn't defined for $t = 0$ *to allow the implimentation to define a custom mapping for all vertices at* $t = 0$*.*

2 Redstone Objects

Definition 2.1. *(Redstone Torch)*
Let

$$T_s : \mathcal{P}(S) \times S \times \mathbb{N}_0 \to S \tag{22}$$
$$T_o : S \times \mathbb{N}_0 \to S \tag{23}$$

Where

$$T_s(I, \varphi, \delta) = \begin{cases} 0 & \textit{if } Max(I) \subseteq S_0 \\ 15 & \textit{otherwise} \end{cases} \tag{24}$$

$$T_o(\varphi, \delta) = \begin{cases} 15 & \textit{if } \varphi \in S_0 \\ 0 & \textit{otherwise} \end{cases} \tag{25}$$

Then

$$\beta_s(T_c) = T_s \tag{26}$$
$$\beta_o(T_c) = T_o \tag{27}$$

Definition 2.2. *(Repeater(s))*
Let

$$n \in \{1, 2, 3, 4\}, Rn_s : \mathcal{P}(S) \times S \times \mathbb{N}_0 \to S \tag{28}$$
$$n \in \{1, 2, 3, 4\}, Rn_o : S \times \mathbb{N}_0 \to S \tag{29}$$

Where

$$Rn_s(I, \varphi, \delta) = \begin{cases} 15 & \textit{if } Max(I) \subseteq S^+ \\ 15 & \textit{if } (\varphi \in S^+) \wedge (Max(I) \subseteq S_0) \wedge (\delta < n) \\ 0 & \textit{otherwise} \end{cases} \tag{30}$$

$$Rn_o(\varphi, \delta) = \begin{cases} 15 & \textit{if } (|\varphi \cap S^+| > 1) \wedge (\delta >= n) \\ 15 & \textit{if } (\varphi \subseteq S_0) \wedge (\delta < n) \\ 0 & \textit{otherwise} \end{cases} \tag{31}$$

Then

$$n \in 1, 2, 3, 4, \beta_s(Rn_c) = Rn_s \tag{32}$$
$$n \in 1, 2, 3, 4, \beta_o(Rn_c) = Rn_o \tag{33}$$

Definition 2.3. *(Lamp (Output))*
Let

$$O_s : \mathcal{P}(S) \times S \times \mathbb{N}_0 \to S \tag{34}$$
$$O_o : S \times \mathbb{N}_0 \to S \tag{35}$$

Where

$$O_s(I, \varphi, \delta) = \begin{cases} 0 & \textit{if } Max(I) \subseteq S_0 \\ 15 & \textit{otherwise} \end{cases} \tag{36}$$

$$O_o(\varphi, \delta) = \begin{cases} 15 & \textit{if } (\varphi \cap S^+| > 1) \\ 15 & \textit{if } (\varphi \subseteq S_0) \wedge (\delta = 0) \\ 0 & \textit{otherwise} \end{cases} \tag{37}$$

Then

$$\beta_s(O_c) = O_s \tag{38}$$
$$\beta_o(O_c) = O_o \tag{39}$$

Definition 2.4. *(Lever (Input))*
Let

$$I_s : \mathcal{P}(S) \times S \times \mathbb{N}_0 \to S \tag{40}$$
$$I_o : S \times \mathbb{N}_0 \to S \tag{41}$$

Where

$$I_s(I, \varphi, \delta) = \varphi \tag{42}$$
$$I_o(\varphi, \delta) = \varphi \tag{43}$$

Then

$$\beta_s(I_c) = I_s \tag{44}$$
$$\beta_o(I_c) = I_o \tag{45}$$