ABM: Minecraft Redstone with Graph and Set Theory

Due on June 26, 2024

Mr. Rios

John Fleming

1 Foundations

Definition 1.1. *(Max)*
Let X be any $X \subseteq \mathbb{N}_0$. Then

$$Max(X) \coloneqq \{x \in X \mid \forall y \in X(x \geq y)\} \tag{1}$$

Definition 1.2. *(Number Set Notation)*

$$\mathbb{Z}^+ \coloneqq \mathbb{N}_0 \setminus \{0\} \tag{2}$$
$$\mathbb{Z}^- \coloneqq \{n \in \mathbb{Z} \mid n < 0\} \tag{3}$$
$$\mathbb{Z}_0^- \coloneqq \mathbb{Z}^- \cup \{0\} \tag{4}$$
$$\tag{5}$$

Definition 1.3. *(Signal Sets)*

$$S \coloneqq \{0, 1, 2, ..., 15\} \tag{6}$$
$$S^+ \coloneqq \{x \in S \mid x > 0\} \tag{7}$$
$$S_0 \coloneqq \{0\} \tag{8}$$

Definition 1.4. *(Max Delay)*

$$D_m \coloneqq 4 \tag{9}$$

Max delay of any component.

Definition 1.5. *(Component Set)*

$$C \coloneqq \{T_c, R1_c, R2_c, R3_c, R4_c, O_c, I_c\} \tag{10}$$

The set of supported redstone components.

Definition 1.6. *(Vertices)*
Let V be a set.
Then V is a set of vertices if

$$V \neq \varnothing \tag{11}$$

Definition 1.7. *(Directed Edges)*
Let V be any set of vertices.
Then E is a set of directed edges on V if

$$E \subseteq (V \times V) \tag{12}$$

Definition 1.8. *(Graph)*
Let V be any set of vertices, and let E be any set of directed edges on V.
Then G is a digraph on V and E if

$$G = (V, E) \tag{13}$$

Definition 1.9. *(Propegation)*
Let $G = (V, E)$ be any digraph. Then

$$\omega^+ : V \times \mathcal{P}(G) \to \mathcal{P}(V) \tag{14}$$

Where

$$\omega^+(v, G) = \{u \in V \mid (u, v) \in E\}$$

And

$$\omega^- : V \times \mathcal{P}(G) \to \mathcal{P}(V) \tag{15}$$

Where

$$\omega^-(v, G) = \{v \in V \mid \exists(u, v) \in E\}$$

Definition 1.10. *(Behavior functions)*

$$\beta_s : C \to (\mathcal{P}(S) \times S \times \mathbb{N}_0 \to S) \tag{16}$$

$$\beta_o : C \to (S \times \mathbb{N}_0 \to S) \tag{17}$$

Definition 1.11. *(Redstone Digraph)*
Let:

- $G = (V, E)$ *be any digraph*
- $\Sigma : V \times \mathbb{N}_0 \to S \times \mathbb{N}_0$
- $\lambda : V \to C$

Then G_r is a redstone digraph if

$$G_r = (G, \lambda, \Sigma) \tag{18}$$

Definition 1.12. *(State)*
Let $G_r = (G, \lambda, \Sigma)$ be any redstone digraph where $G = (V, E)$.
For any vertex $v \in V$ at an arbitrary tick $t \in \mathbb{N}_0$, I_v is a set of Input Signals if

$$I_v = \{((\beta_o \circ \lambda)(v) \circ \Sigma)(u, t) \mid u \in \omega^-(v, G)\}$$

and D_v is a set of Input Durations if

$$D_v = \{\varphi \in S \mid (\varphi, \delta) \in \Sigma(v, t), \delta \in \mathbb{N}_0\} \cup \{\Phi\} \tag{19}$$

Where

$$\Phi = (\beta_s \circ \lambda)(v)(I_v, \Sigma(v, t)) \tag{20}$$

And

$$\Delta = \begin{cases} \delta + 1,\ (\varphi, \delta) \in \Sigma(v, t) & \textit{if } (D_v \subseteq S^+) \vee (D_v \subseteq S_0) \\ 0 & \textit{otherwise} \end{cases} \tag{21}$$

Then

$$\Sigma(v, t + 1)|_{V \times \mathbb{Z}^+} = (\Phi, \Delta) \tag{22}$$

Note: the state mapping isn't defined for $t = 0$ to allow the implementation to define a custom mapping for all vertices at $t = 0$.

2 Redstone Objects

Definition 2.1. *(Redstone Torch)*
Let

$$T_s : \mathcal{P}(S) \times S \times \mathbb{N}_0 \to S \tag{23}$$
$$T_o : S \times \mathbb{N}_0 \to S \tag{24}$$

Where

$$T_s(I, \varphi, \delta) = \begin{cases} 0 & \text{if } Max(I) \subseteq S_0 \\ 15 & \text{otherwise} \end{cases} \tag{25}$$

$$T_o(\varphi, \delta) = \begin{cases} 15 & \text{if } \varphi \in S_0 \\ 0 & \text{otherwise} \end{cases} \tag{26}$$

Then

$$\beta_s(T_c) = T_s \tag{27}$$
$$\beta_o(T_c) = T_o \tag{28}$$

Definition 2.2. *(Repeater(s))*
Let

$$n \in \{1, 2, 3, 4\}, Rn_s : \mathcal{P}(S) \times S \times \mathbb{N}_0 \to S \tag{29}$$
$$n \in \{1, 2, 3, 4\}, Rn_o : S \times \mathbb{N}_0 \to S \tag{30}$$

Where

$$Rn_s(I, \varphi, \delta) = \begin{cases} 15 & \text{if } Max(I) \subseteq S^+ \\ 15 & \text{if } (\varphi \in S^+) \wedge (Max(I) \subseteq S_0) \wedge (\delta < n) \\ 0 & \text{otherwise} \end{cases} \tag{31}$$

$$Rn_o(\varphi, \delta) = \begin{cases} 15 & \text{if } (|\varphi \cap S^+| > 1) \wedge (\delta >= n) \\ 15 & \text{if } (\varphi \subseteq S_0) \wedge (\delta < n) \\ 0 & \text{otherwise} \end{cases} \tag{32}$$

Then

$$n \in 1, 2, 3, 4, \beta_s(Rn_c) = Rn_s \tag{33}$$
$$n \in 1, 2, 3, 4, \beta_o(Rn_c) = Rn_o \tag{34}$$

Definition 2.3. *(Lamp (Output))*
Let

$$O_s : \mathcal{P}(S) \times S \times \mathbb{N}_0 \to S \tag{35}$$
$$O_o : S \times \mathbb{N}_0 \to S \tag{36}$$

Where

$$O_s(I, \varphi, \delta) = \begin{cases} 0 & \text{if } Max(I) \subseteq S_0 \\ 15 & \text{otherwise} \end{cases} \tag{37}$$

$$O_o(\varphi, \delta) = \begin{cases} 15 & \text{if } (\varphi \cap S^+| > 1) \\ 15 & \text{if } (\varphi \subseteq S_0) \wedge (\delta = 0) \\ 0 & \text{otherwise} \end{cases} \tag{38}$$

Then

$$\beta_s(O_c) = O_s \tag{39}$$
$$\beta_o(O_c) = O_o \tag{40}$$

Definition 2.4. *(Lever (Input))*
Let

$$I_s : \mathcal{P}(S) \times S \times \mathbb{N}_0 \to S \tag{41}$$
$$I_o : S \times \mathbb{N}_0 \to S \tag{42}$$

Where

$$I_s(I, \varphi, \delta) = \varphi \tag{43}$$
$$I_o(\varphi, \delta) = \varphi \tag{44}$$

Then

$$\beta_s(I_c) = I_s \tag{45}$$
$$\beta_o(I_c) = I_o \tag{46}$$

Proof. Clock proof

- Let $G_r = (G, \lambda, \Sigma)$ be a redstone digraph where $G = (V, E)$.
- Let $V = \{t_1, t_2, t_3\}$ and $E = \{(t_1, t_2), (t_2, t_3), (t_3, t_1)\}$.
- Let $\forall v \in V(\lambda(v) = T_c)$.
- Let $\forall v \in V(\Sigma(v, 0) = (0, D_m))$.

For t_3 (vertex) at $t = 1$ (tick), I_{t_3} is a set of Input Vertices where

$$\begin{aligned}
I_{t_3} &= \{((\beta_o \circ \lambda)(t_3) \circ \Sigma)(t_2, 0)\} \\
&= \{(\beta_o \circ T_c \circ \Sigma)(t_2, 0)\} \\
&= \{T_o(\Sigma(t_2, 0)\} \\
&= \{T_o(0, 4)\} \\
&= \{15\}
\end{aligned}$$

Now let

$$\begin{aligned}
\Phi &= (\beta_s \circ \lambda)(v_3)(I_{v_3}, \Sigma(v_3, 0)) \\
&= (\beta_s \circ T_c)(\{15\}, 0, 4) \\
&= T_s(\{15\}, 0, 4) \\
&= 15
\end{aligned}$$

Then D_{t_3} is a set of Input Durations where

$$\begin{aligned}D_{t_3} &= \{\phi \in S \mid (\phi, \delta) \in \Sigma(v_3, 0)\} \cup \{\Phi\} \\ &= \{0\} \cup \{15\} \\ &= \{0, 15\}\end{aligned}$$

Now let

$$\Delta = 0$$

Then

$$\begin{aligned}\Sigma(t_3, 1) &= (\Phi, \Delta) \\ &= (15, 0)\end{aligned}$$

And the output at $t = 1$ is then

$$\begin{aligned}(\beta_o \circ \lambda)(t_3)(\Sigma(t_3, 1) &= (\beta_o \circ T_c)(\Sigma(t_3, 1)) \\ &= T_o(\Sigma(t_3, 1)) \\ &= T_o(15, 0) \\ &= 0\end{aligned}$$

And the output at $t = 0$ is

$$\begin{aligned}(\beta_o \circ \lambda)(t_3)(\Sigma(t_3, 0) &= (\beta_o \circ T_c)(\Sigma(t_3, 0)) \\ &= T_o(\Sigma(t_3, 0)) \\ &= T_o(0, 4) \\ &= 15\end{aligned}$$

Finally, it is then proven that

t_3	$t = 0$	$t = 1$
0	15	0

□