AQA

Please write clearly in block capitals.

Centre number	
Candidate number	
Surname	
Forename(s)	
Candidate signature	

Level 2 Certificate
FURTHER MATHEMATICS

Paper 2 Calculator

Monday 19 June 2017 Morning Time allowed: 2 hours

Materials
For this paper you must have:
- a calculator
- mathematical instruments.

Instructions
- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- Do all rough work in this book. Cross through any work you do not want to be marked.
- In all calculations, show clearly how you work out your answer.

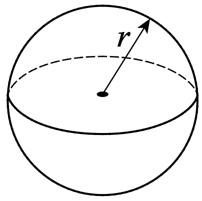
Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 105.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer book.
- The use of a calculator is expected but calculators with a facility for symbolic algebra must **not** be used.

For Examiner's Use	
Pages	Mark
3	
4 – 5	
6 – 7	
8 – 9	
10 – 11	
12 – 13	
14 – 15	
16 – 17	
18 – 19	
20 – 21	
22 – 23	
24 – 25	
26 – 27	
28 – 29	
30	
TOTAL	

Formulae Sheet

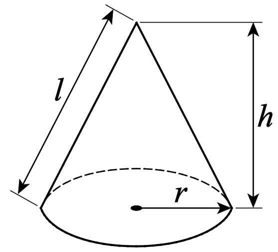
Volume of sphere $= \frac{4}{3}\pi r^3$

Surface area of sphere $= 4\pi r^2$

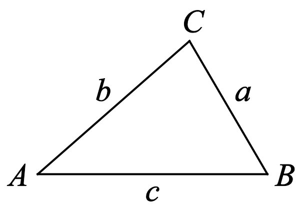


Volume of cone $= \frac{1}{3}\pi r^2 h$

Curved surface area of cone $= \pi r l$



In any triangle ABC



Area of triangle $= \frac{1}{2}ab \sin C$

Sine rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule $a^2 = b^2 + c^2 - 2bc \cos A$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$, where $a \neq 0$, are given by $x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$

Trigonometric Identities

$\tan\theta \equiv \frac{\sin\theta}{\cos\theta}$ $\qquad \sin^2\theta + \cos^2\theta \equiv 1$

Do not write outside the box

Answer **all** questions in the spaces provided.

1 (a) The nth term of a sequence is $\frac{3-5n}{2}$

Work out the difference between the 20th term and the 8th term.

[2 marks]

so abs

$= \frac{3-5\times8}{2} - \frac{3-5\times20}{2} = \frac{-37}{2} - \frac{-97}{2} = \frac{60}{2} = 30$

Answer 30

1 (b) The nth term of another sequence is $\frac{3n}{1-2n}$

Write down the limiting value of the sequence as $n \to \infty$

[1 mark]

Answer $n \to -1.5$

Turn over for the next question

3

Do not write outside the box

2 $\mathbf{A} = \begin{pmatrix} 4 & -1 \\ 3 & -2 \end{pmatrix}$ $\mathbf{B} = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$

2 (a) Work out $\mathbf{A}^2$

[2 marks]

$= \begin{pmatrix} 4 & -1 \\ 3 & -2 \end{pmatrix}\begin{pmatrix} 4 & -1 \\ 3 & -2 \end{pmatrix}$

$= \begin{pmatrix} 13 & -2 \\ 6 & 1 \end{pmatrix}$

Answer $\begin{pmatrix} 13 & -2 \\ 6 & 1 \end{pmatrix}$

2 (b) $k\mathbf{B} = \begin{pmatrix} 11-3k \\ 11-6k \end{pmatrix}$ where k is a constant.

Work out the value of k.

[2 marks]

$\begin{pmatrix} 3k \\ 2k \end{pmatrix} = \begin{pmatrix} 11-3k \\ 11-6k \end{pmatrix}$

OR

$5k = 11-3k$
$8k = 11$
$k = \frac{11}{8}$

$2k = 11-6k$
$8k = 11$
$k = \frac{11}{8}$

Answer $k = \frac{11}{8}$

Do not write outside the box

2 (c) Give a reason why it is **not** possible to work out **BA**

[1 mark]

? needs to be AB

Bcols != A rows etc.

Turn over for the next question

5

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3 (a) p, q and r are all integers greater than 1

$pqr = 1365$

Work out one possible set of values for p, q and r.

[2 marks]

F $1365 = 3 \times 5 \times 7 \times 13$

$p =$ 13 $q =$ 7 $r =$ 15

3 (b) a and b are both **square** numbers greater than 1

$ab - 11b$ is also a **square** number.

By factorising $ab - 11b$, work out one possible pair of values for a and b.
You **must** show your working.

[2 marks]

$= b(a-11)$, $b = a - 11$, squares: 1, 4, 9, 16, 25, 36

$25 = 36 - 11$

$a =$ 36 $b =$ 25

Do not write outside the box

4 Solve $\frac{56}{\sqrt[3]{x}} = 4$

[2 marks]

$\frac{56}{x^{\frac{3}{2}}} = 4$ wut $-\frac{1}{3}$ No $\frac{56^3}{x} = 4^3$ etc.

$4x^{\frac{3}{2}} = 56$

$x^{\frac{3}{2}} = 14$

$x = 14^{\frac{2}{3}} = 5.808$

2744

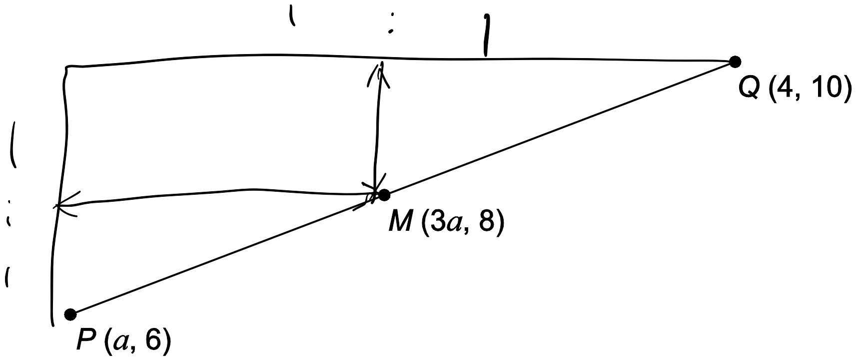
$x =$ 5.81

Turn over for the next question

6

Turn over ▶

5 M is the midpoint of PQ.



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Work out the value of a.

[3 marks]

$\frac{a+4}{2} = 3a$, $a+4 = 6a$, $4 = 5a$, $a = \frac{4}{5}$

Answer $a = 0.8$

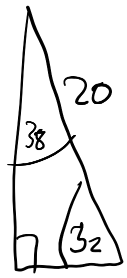
Do not write outside the box

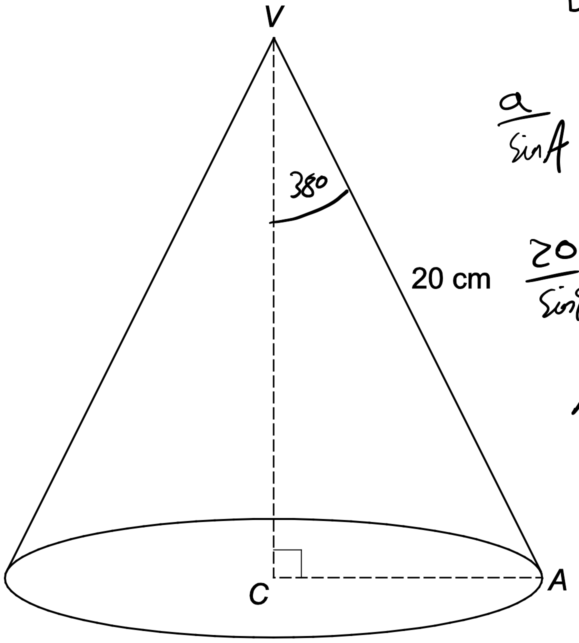
6 A cone has vertex *V*.

C is the centre of the base.

The slant height, *VA*, is 20 cm

The angle between *VA* and *VC* is 38°





$\frac{a}{\sin A} = \frac{b}{\sin B}$

$\frac{20}{\sin 90} = \frac{AC}{\sin 38}$

$AC = 20 \times \sin 38$

$= 12.313$

Work out the radius of the base.

[3 marks]

Answer 12.3 cm

6

Turn over ►

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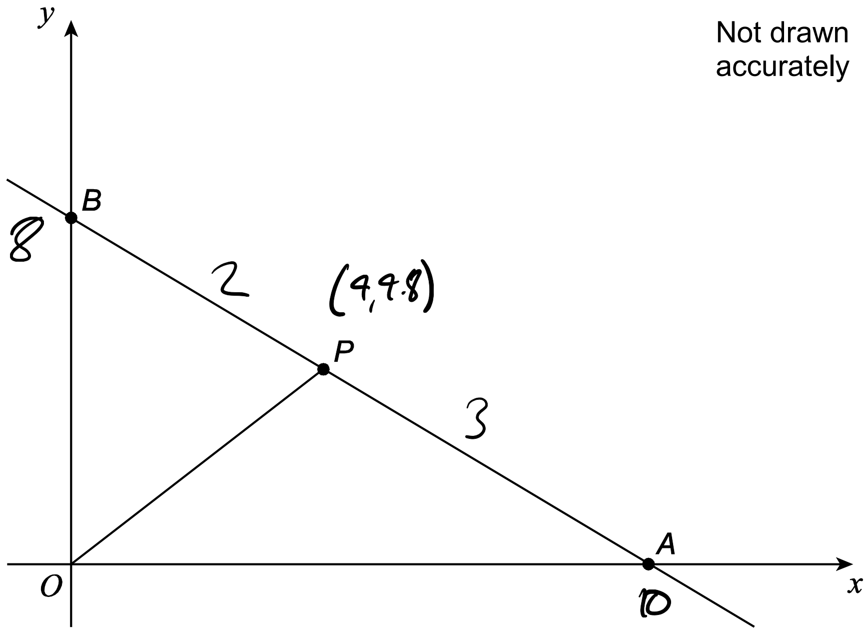
7 The equation of the line through B, P and A is $4x + 5y = 40$

$BP : PA = 2 : 3$ 6:1=5



Work out the area of triangle OBP.

[4 marks]

$4x + 5y = 40$

$x = 0,\ 5y = 40,\ y = 8$ | $y = 0,\ 4x = 40,\ x = 10$

$\vec{BA} = \begin{pmatrix} 10 \\ -8 \end{pmatrix}$, $\frac{2}{5}\vec{BA} = \begin{pmatrix} 4 \\ -3.2 \end{pmatrix}$, $P = \begin{pmatrix} 4 \\ 4.8 \end{pmatrix}$

$BP = \sqrt{4^2 + 3.2^2} = \frac{4\sqrt{41}}{5} = 5.12\ldots$ vor A

$OP = \sqrt{4^2 + 4.8^2} = \frac{4\sqrt{61}}{5} = 6.24$ vor B

$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

$\hat{OBP} = \cos^{-1}\left(\frac{5.12^2 + 8^2 - 6.24^2}{2 \times 5.12 \times 8}\right) = \cos^{-1}\left(\frac{4\sqrt{41}}{41}\right) = 51.3$ vor C

$A = \frac{1}{2}ab\sin C$

$= \frac{5.12 \times 8 \times \sin(51.3)}{2}$

$= 16$

or just $\boxed{\frac{8 \times 10}{2} \times \frac{2}{2+3} = 16}$

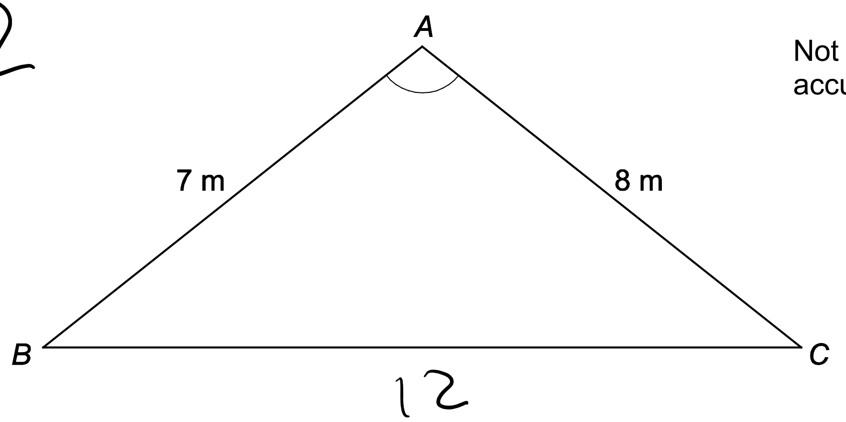
but that works too

Answer 16 square units

Do not write outside the box

8 The perimeter of a triangular flower bed, *ABC*, is marked out using 27 metres of rope.

$27 - 7 - 8 = 12$



Work out the size of angle *BAC*.

[4 marks]

$\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

$B\hat{A}C = \cos^{-1}\left(\dfrac{7^2 + 8^2 - 12^2}{2 \times 7 \times 8}\right)$

$= \cos^{-1}\left(-\dfrac{31}{112}\right)$

$= 106.06$

Answer 106 degrees

Turn over for the next question

8

Turn over ►

Do not write outside the box

9 $-11 < 5x \leqslant 5$ and $6x + 7 \leqslant 4x + 4$

Show that there is **exactly** one integer that x can be.

[5 marks]

$-2\frac{1}{5} < x \leq 1$ | $2x + 7 \leq 4$

$2x \leq -3$

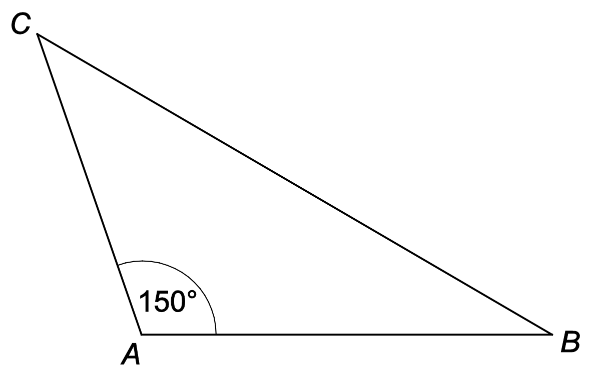
$x \leq 1.5$

x must be > -2.2 and ≤ 1.5, so it can only be -2.

Do not write outside the box

10 *ABC* is an isosceles triangle with $AB = AC$

The area of *ABC* is 57.76 cm^2



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Work out the length of *AB*.

[3 marks]

$A = \frac{1}{2} ab \sin C$

$57.76 = AB^2 \times \sin 150 \times \frac{1}{2}$

$AB^2 = 57.76 \div \frac{1}{2}$

$AB^2 = \frac{2888}{25}$

$AB = \frac{38\sqrt{2}}{5} = 10.74$

15.2

Answer 10.7 cm

Turn over for the next question

8

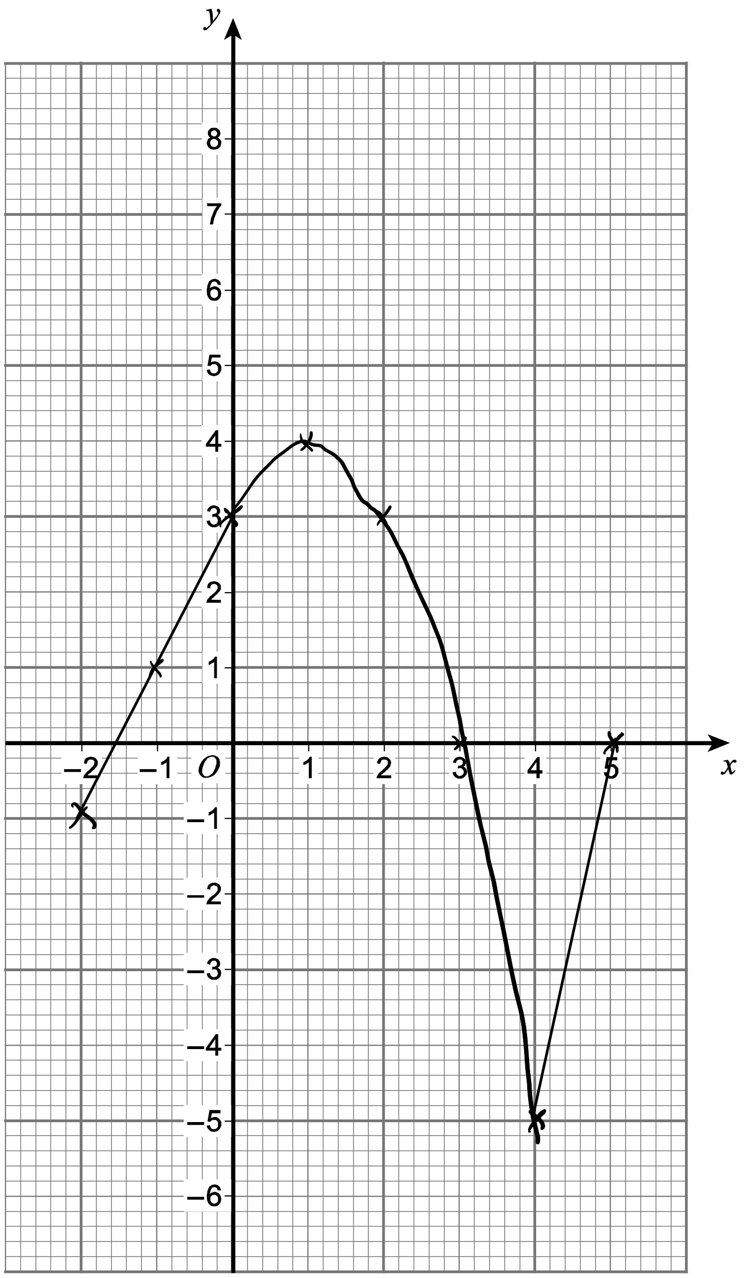
Do not write outside the box

11 A function $f(x)$ is defined as

$$f(x) = 3 - 2x \qquad -2 \leqslant x < 0$$
$$= (1 + x)(3 - x) \qquad 0 \leqslant x < 4$$
$$= 5x - 25 \qquad 4 \leqslant x \leqslant 5$$

11 (a) Draw the graph of $y = f(x)$ on the axes below.

[4 marks]



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11 (b) State the range of $f(x)$

≤ 7, because of the 3rd part

[2 marks]

Answer $-5 \leq x \leq 7$

12 (a) Factorise fully $75 - 3x^2$

[2 marks]

$= -3(x^2 - 25)$

$= -3(x+5)(x-5)$

Answer

12 (b) Simplify fully $(3n + 1)^2 - (3n - 1)^2$

[2 marks]

$= 3n^2 + 6n + 1 - (3n^2 - 6n + 1)$

$= 3n^2 - 3n^2 + 6n + 6n + 1 - 1$

$= 12n$

Answer $12n$

10

Turn over ►

Do not write outside the box

13 Simplify fully $\dfrac{8a}{3a+6} \times \dfrac{5a+10}{3a^2} \div \dfrac{4}{15a^3}$

[3 marks]

2

$$= \frac{8a}{3(\cancel{a+2})} \times \frac{5(\cancel{a+2})}{3a^2} \times \frac{15a^3}{4}$$

$$= \frac{\cancel{40}^{10}}{_{1}\cancel{3a}} \times \frac{^{5}\cancel{15}a^{\cancel{3}^2}}{\cancel{4}_1} = 5a^2$$

$$= \frac{50a^2}{3} \ ?$$

Answer $5a^2$

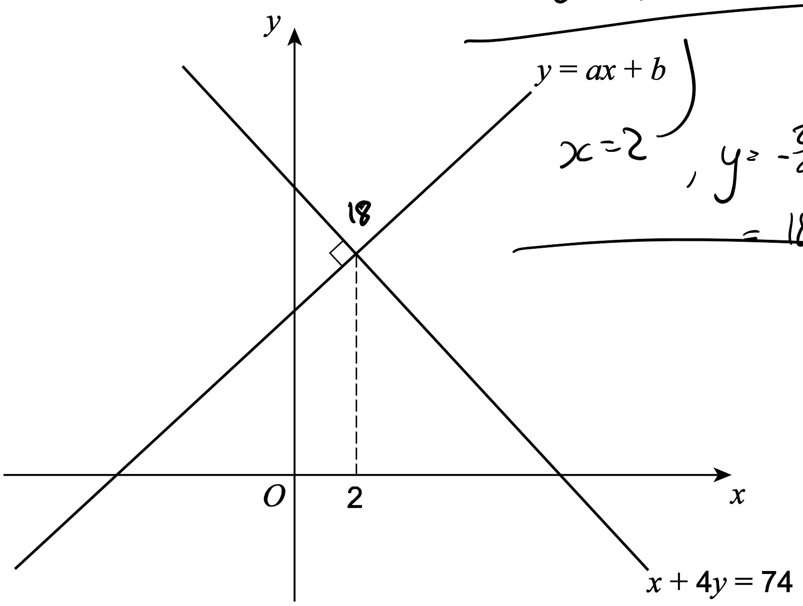
Do not write outside the box

14 The line $y = ax + b$ is perpendicular to the line $x + 4y = 74$

The lines intersect at the point where $x = 2$

$4y = -x + 74$

$y = -\frac{1}{4}x + 18\frac{1}{2}$



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$x = 2$, $y = -\frac{2}{4} + 18\frac{1}{2}$

$= 18$

Work out the values of a and b.

[5 marks]

neg inv $= 4$, $y = 4x + c$

$18 = 4 \times 2 + c$

$c = 10$

$a =$ 4 $b =$ 10

8

Turn over ►

Do not write outside the box

15 Rearrange $w = \dfrac{8x - y}{y}$ to make y the subject.

[3 marks]

3

$wy = 8x - y$

$wy + y = 8x$

$y(w+1) = 8x$

$y = \dfrac{8x}{w+1}$

Answer ______________________

Do not write outside the box

16 (a) $a = 3^{2b}$

Circle the correct expression for $\frac{1}{a}$

[1 mark]

3^{2b-1} $\quad$ 3^{-2b} $\quad$ -3^{2b} $\quad$ $\left(\frac{1}{3}\right)^{-2b}$

16 (b) $y = 5^x$ $\quad$ $5^x \times 25 = 5^2 \times 5^x$

Circle the correct expression for $25y$

[1 mark]

5^{x+2} $\quad$ 25^x $\quad$ 5^{2x} $\quad$ 125^x

16 (c) $w = 2^m$ $\quad$ $(2^m)^3 = 8^{3m}$

Circle the correct expression for w^3

[1 mark]

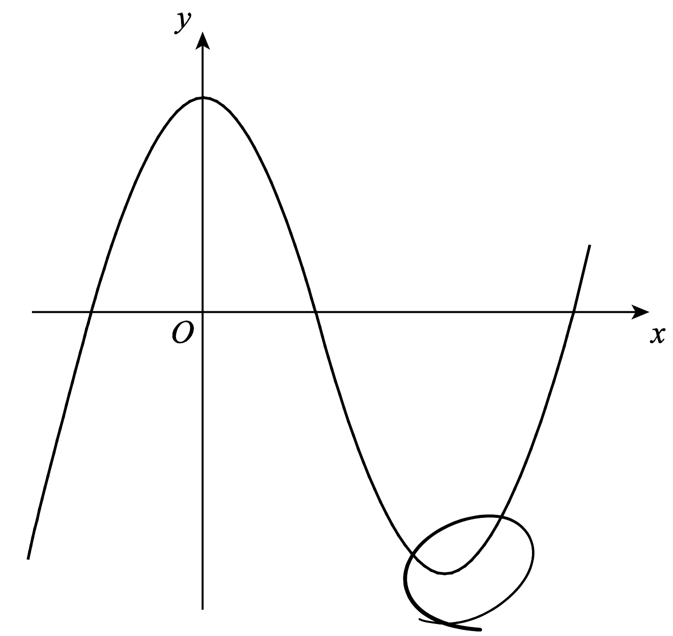
8^{3m} $\quad$ 6^m $\quad$ 2^{m+3} $\quad$ 2^{3m}

Turn over for the next question

6

Do not write outside the box

17 Here is a sketch of $y = x^3 - 6x^2 + 7$



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17 (a) Use differentiation to work out the coordinates of the stationary point that is a minimum.
You **must** show your working.

[4 marks]

$\frac{dy}{dx} = 3x^2 - 12x$

$\frac{d^2y}{dx^2} = 6x - 12$

$y = x^3 - 6x^2 + 7, \; x = 4$

$= 64 - 96 + 7$

$= -25$

$3x^2 - 12 = 0$

$(x)(3x - 12) = 0$

$x = 0, \; 4$

discard 0
because looking for min

Answer (4 , -25)

Do not write outside the box

17 (b) The three roots of $x^3 - 6x^2 + 7 = 0$ are the x-coordinates of the points where the graph intersects the x-axis.

Show that $x = -1$ is one root of $x^3 - 6x^2 + 7 = 0$

[1 mark]

$x = -1,\quad 0 = -1 - 6 + 7 = -7 + 7 = 0$

17 (c) Hence, work out the other two roots of $x^3 - 6x^2 + 7 = 0$

Give your answers to 2 decimal places.
You **must** show your working.

[5 marks]

$= (x+1)(x^2 + bx + 7)$

$= x^3 + x^2 + bx^2 + bx + 7x + 7$

$= x^3 + (b+1)x^2 + (b+7)x + 7$

OR

$b + 7 = 0$ | $b + 1 = -6$

$b = -7$ | $b = -7$

$= (x+1)(x^2 - 7x + 7)$

solve $x^2 - 7x + 7 = 0$

$x^2 - 7x = -7$

$(x - 3.5)^2 - 12\frac{1}{4} = -7$

$(x - 3.5)^2 = 5\frac{1}{4} = \frac{21}{4}$

$x - 3.5 = \pm\frac{\sqrt{21}}{2}$

$x = \frac{\sqrt{21}}{2} + 3.5$ or $3.5 - \frac{\sqrt{21}}{2}$

Answer 5.79, 1.21

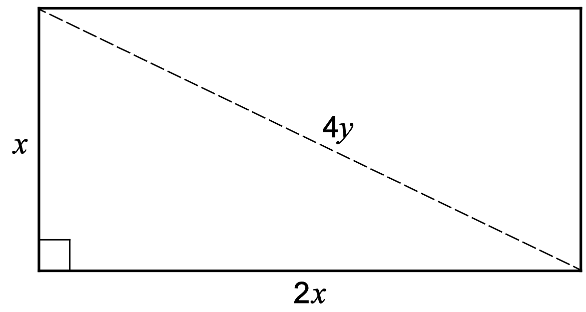
10

Turn over ▶

Do not write outside the box

18 The diagram shows a rectangle with a diagonal drawn.

The given expressions for the measurements are in centimetres.



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Work out an expression for the area of the rectangle, in cm^2
Give your answer in its simplest form, in terms of y.

[4 marks]

$\sqrt{x^2 + 4x^2} = 4y$

$4y = \sqrt{5x^2}$

$16y^2 = 5x^2$

$x^2 = \frac{16}{5}y^2$

$x = \sqrt{\frac{16y^2}{5}}$

$A = 2x^2$

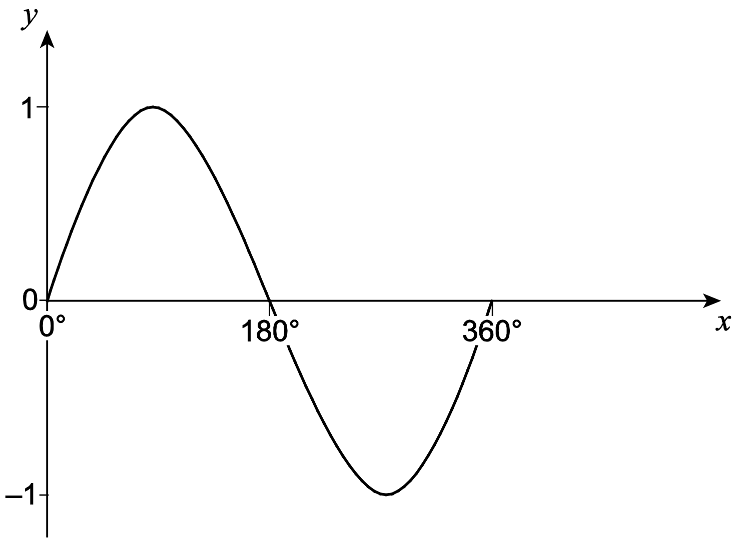
$= 2 \times \left(\sqrt{\frac{16y^2}{5}}\right)^2$

$= \frac{32y^2}{5}$

Answer $\frac{32y^2}{5}$ cm^2

Do not write outside the box

19 Here is a sketch of $y = \sin x$ for $0° \leqslant x \leqslant 360°$



α is an acute angle measured in degrees.

$\sin\alpha = k$ where k is a constant.

Write the answers to each of the following in terms of k, without involving trigonometric functions.

19 (a) $\sin(180° - \alpha)$

[1 mark]

Answer ?, k

19 (b) $\sin(360° - \alpha)$

[1 mark]

Answer ? −k

19 (c) $\cos\alpha$

[2 marks]

$k^2 + \cos^2 a = 1$

$1 - k^2$

Answer ? K×0.5

8

Turn over ►

20 Two circles overlap.

A, *B* and *E* lie on the circle, centre *O*.

B, *C*, *D* and *E* lie on the other circle.

AOBC and *AED* are straight lines.

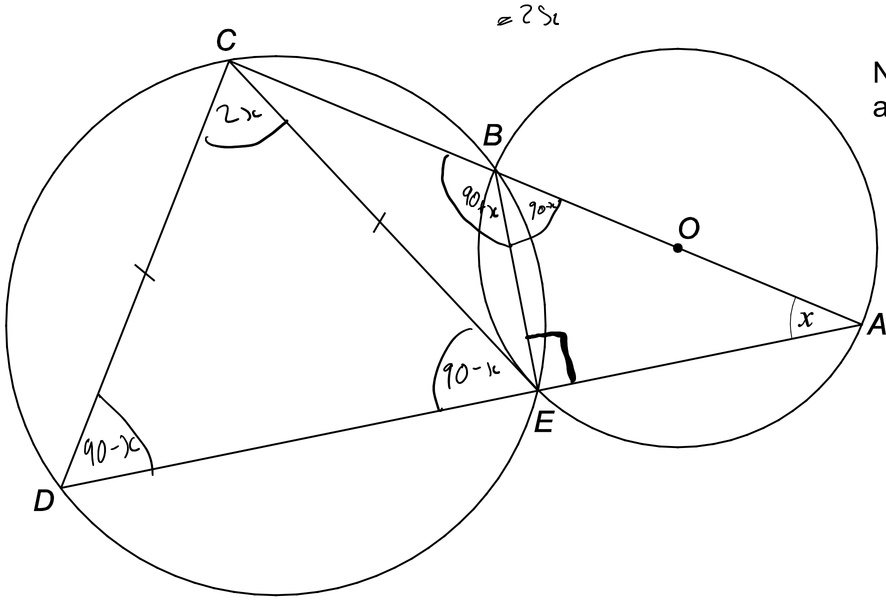
$CD = CE$

angle $BAE = x$

$= 180 - (90 - x)$
$= 90 + x$

$= 180 - (90 + x)$
$= 90 - x$

$= 180 - 2(90 - x)$
$= 180 - 180 + 2x$
$= 2x$



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20 (a) Give a reason why angle $BEA = 90°$

[1 mark]

Cyclic Tri with line on centre, hyp angle is 90°

Do not write outside the box

20 (b) Prove that angle $DCE = 2x$

[4 marks]

See ↑

Turn over for the next question

5

Do not write outside the box

21 Here is a sketch of $y = (x + 2)(4 - x)$ $= (x+2)(x-4)(-1) = -(x^2-2x-8)$

$= -x^2+2x+8$

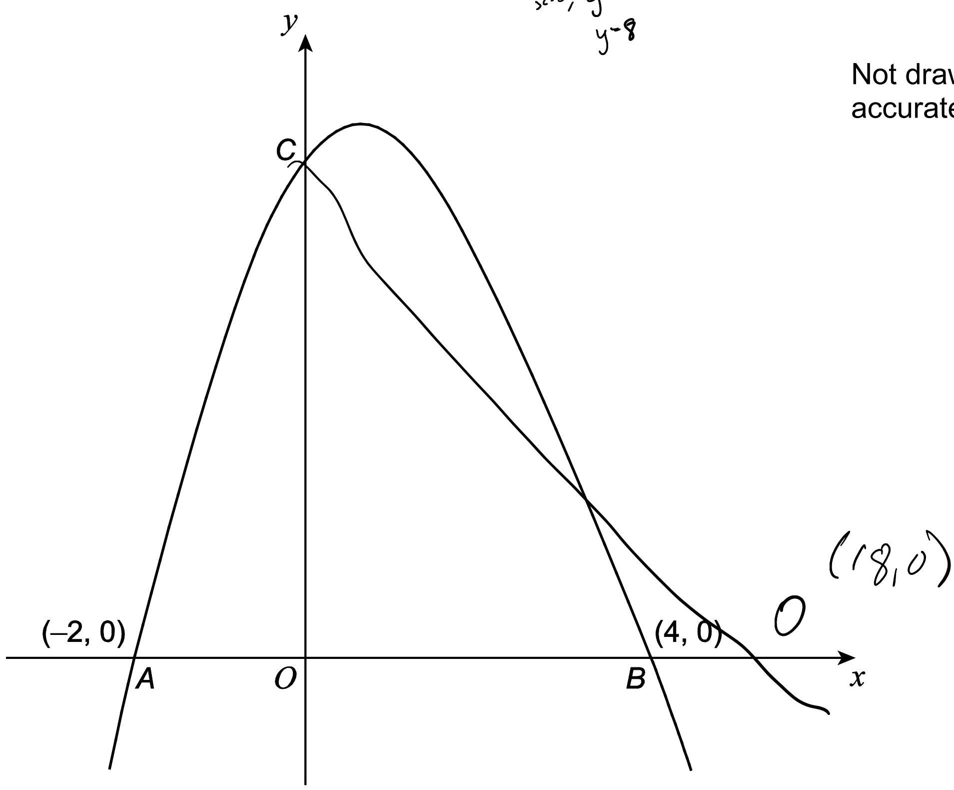
The graph intersects the axes at *A* (–2, 0), *B* (4, 0) and *C*.

$y = -x^2+2x+8$

$x=0,\ y=-0+0+8$

$y=8$

Not drawn accurately



21 (a) Work out the coordinates of *C*.

[1 mark]

Answer (0 , 8)

Do not write outside the box

21 (b) Work out the gradient function of the curve.

[3 marks]

$y = -x^2 + 2x + 8$

$\frac{dy}{dx} = -2x + 2$

Answer $2 - 2x$

21 (c) The normal to the curve at *C* intersects the x-axis at *D*.

Show that length $BD = 2 \times$ length AB

[5 marks]

$m_C = 2 - 2x$, where $x = 0$, $= 2$

neg inv $= -\frac{1}{2}$

$y = mx + c$

$8 = -\frac{2}{2} + c$

$c = 9$

$y = -\frac{x}{2} + 9$

$0 = -\frac{x}{2} + 9$

$9 = \frac{x}{2}$

$x = 18$

$BD = 18 - 4 = 14$

$AB = 4 + 2 = 6$, $\times 2 = 12$

???

9

Do not write outside the box

22 The equation of a circle is $(x-2)^2 + (y-1)^2 = 16$, $x^2-4x+4+y^2-2y+1-16=0$

$x^2-4x-11 = 2y-y^2$

The equation of a line is $y = 2x + 1$

$x^2-4x-11 = 2(2x+1)-(2x+1)^2$

The circle and the line intersect at two points.

$x^2-4x-11 = 4x+2-(2x^2+4x+1)$

$x^2-4x-11 = 4x+2-2x^2-4x-1$

$x^2-4x-11=-2x^2+1$

Work out the coordinates of the two points.
You **must** show your working.
Do **not** use trial and improvement.

$3x^2-4x-10=0$

$3x^2-4x=10$

$3(x^2-\frac{4}{3}x)=10$

$x^2-\frac{4}{3}x=\frac{10}{3}$

$(x-\frac{2}{3})^2-\frac{16}{36}=\frac{10}{3}$

$\frac{16}{36}=\frac{8}{18}=\frac{4}{9}$

[5 marks]

$y=2x+1$

$(x-\frac{2}{3})^2=\frac{34}{9}$

$x-\frac{2}{3}=\frac{\pm\sqrt{34}}{3}$

$x=\frac{2\pm\sqrt{34}}{3}$

$x=\frac{2\pm\sqrt{34}}{3}$

$y=\frac{7+2\sqrt{34}}{3}$

Answer ($\frac{2+\sqrt{34}}{3}$, $\frac{7+2\sqrt{39}}{3}$) and ($\frac{2-\sqrt{34}}{3}$, $\frac{7-2\sqrt{39}}{3}$)

Do not write outside the box

23 In this question, $\tan x \neq 0$ and $\sin x \neq 0$

Show that $\dfrac{1}{\tan^2 x} - \dfrac{1}{\sin^2 x}$ is a constant.

[3 marks]

2

$$= \frac{1}{\frac{\sin^2 x}{\cos^2 x}}$$

$$= \frac{\cos^2 x}{\sin^2 x} - \frac{1}{\sin^2 x}$$

$$= \frac{\cos^2 x - 1}{\sin^2 x}$$

$$= \frac{-(1 - \cos^2 x)}{\sin^2 x}$$

$$= -\frac{\sin^2 x}{\sin^2 x}$$

$$= -1$$

Turn over for the next question

8

Do not write outside the box

24 Write $12x^2 - 60x + 5$ in the form $a(bx + c)^2 + d$ where a, b, c and d are integers.

[5 marks]

$12x^2 - 60x = -5$

$3(4x^2 - 20x) = -5$

$3((2x - 5)^2 - 25) = -5$

$3(2x - 5)^2 - 75 = -5$

$3(2x - 5)^2 - 70 = 0$

Answer ____________________

END OF QUESTIONS

5

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