IYGB GCE

Core Mathematics C2

Advanced Subsidiary

Practice Paper Z

Difficulty Rating: 4.1267/2.1352

Time: 2 hours

Candidates may use any calculator allowed by the Regulations of the Joint Council for Qualifications.

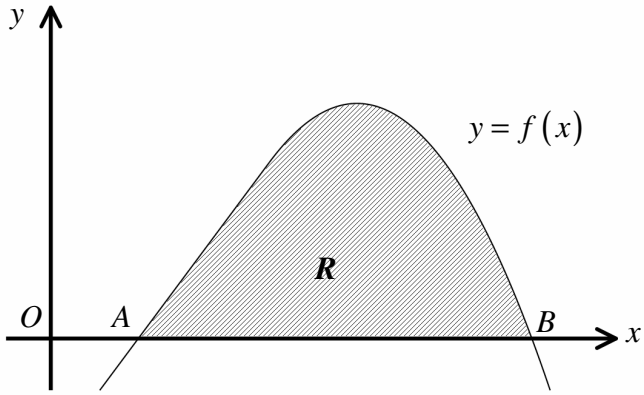
Information for Candidates

This practice paper follows the Edexcel Syllabus.
The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 9 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Question 1



The figure above shows the graph of the curve with equation

$$f(x)=\begin{cases} 4x-8 & x<5 \\ -x^2+14x-33 & x\geq 5 \end{cases}.$$

The curve meets the x axis at the points A and B.

The finite region R, shown shaded in the figure above, is bounded by the curve and the x axis.

Find the area of R. NC integration **(10)**

Question 2

A circle passes through the points $(0,0)$, $(8,0)$ and $(0,6)$. A B C Find eqs of perp bis, find it

Determine the coordinates of the centre of the circle and the size of its radius. **(3)**

PB, AB, midpoint = (4,0) ∴ ⇒ $x=4$ 4,3
PB, AC, midpoint = (0,3) ∴ ⇒ $y=3$ C = (3,4)

(3,4) to (0,0) = $\sqrt{3^2+4^2}$ = 5, r = 5 ✓

Question 3

$$f(x)=(1+2x)^5,\ x\in\mathbb{R}.$$

a) Find the binomial expansion of $f(x)$. **(3)**

b) Hence state the binomial expansion of $f(-x)$. **(1)**

c) Find the two non zero solutions of the equation

$$f(x)-f(-x)=64x. \quad \textbf{(5)}$$

Question 4

Solve the following logarithmic equation

$$\log_4 x-2\log_x 4=1. \quad \textbf{(5)}$$

Question 5

The first three terms of a geometric series are

$$(2x+4),\ (3x+2) \text{ and } \left(x^2-11\right),$$

where x is a constant.

a) Show that x is a solution of the cubic equation

$$2x^3-5x^2-34x-48=0. \quad \textbf{(3)}$$

b) Show that $x=6$ is the only real solution of the above equation. **(3)**

c) Determine the sum of the first eight terms of the geometric series. **(3)**

$\sin\tan^2(2\sin+3)+\tan^2=0$
$\tan^2(\sin(2\sin+3)+1)=0$
$\tan^2(2\sin^2+3\sin+1)=0$

$\tan^2=0$ | $2\sin^2+3\sin+1=0$
$\theta=0°,180°$ | $(2\sin+1)(\sin+1)=0$
| $\sin\theta=-1,-0.5$
| $\theta=210°,270°,330°$

Question 6

Solve the following trigonometric equation for $0 \leq \theta < 360°$

$$\sin\theta\tan^2\theta(2\sin\theta+3)+\tan^2\theta=0. \qquad \textbf{(8)}$$

$\theta = 0°, 180°, 210°, 270°, 330°$

Question 7

$$y=\left(\frac{1}{2}\right)^x.$$

a) Describe the geometric transformation which maps the graph of the curve with equation $y=2^x$, onto the graph of the curve with equation $y=\left(\frac{1}{2}\right)^x$. **(2)**

R reflect in $x=0$ or y axis ✓

b) Sketch the graph of $y=3\times2^x$. **(2)**

✓

The curve with equation $y=\left(\frac{1}{2}\right)^x$ intersects the curve with equation $y=3\times2^x$ at the point P.

c) Determine, as an exact simplified surd, the y coordinate of P. **(4)**

$\left(\frac{1}{2}\right)^x=3\times2^x$, $2^{-x}=3\times2^x$, $\frac{1}{2^x}=3\times2^x$, $1=3\times2^{x^2}=3\times2^{2x}$, $2^{2x}=\frac{1}{3}$, $\frac{1}{\sqrt{3}}=2^x$, $x=\log_2\frac{\sqrt{3}}{3}$

$y=3\times2^{\log_2\frac{\sqrt{3}}{3}}=3\times\frac{\sqrt{3}}{3}=\sqrt{3}$ ✓

Question 8

A tank is in the shape of a closed right circular cylinder of radius r m and height h m.

The tank has a volume of 16π m^3 and is made of thin sheet metal.

$16\pi=\pi r^2h$, $h=\frac{16}{r^2}$

Given the surface area of the tank is a minimum, determine the value of r and the value of h. **(10)**

$SA=2\pi r^2+2\pi rh=2\pi r^2+2\pi r\frac{16}{r^2}=2\pi r^2+32\pi r^{-1}$

$\frac{dA}{dr}=4\pi r-32\pi r^{-2}$, $4\pi r-32\pi r^{-2}=0$

$r=8r^{-2}$

$r^3=8$

$r=2$ ✓

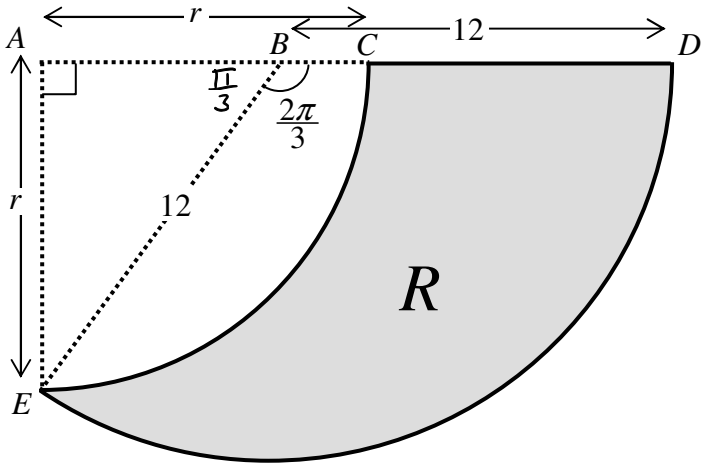
$h=\frac{16}{r^2}=\frac{16}{2^2}=\frac{16}{4}$

$h=4$ ✓

$A_{EBD} = \frac{1}{2}\theta r^2 = \frac{1}{2} \times \frac{2\pi}{3} \times 12^2 = 48\pi$

$R = EBD - (ACE - ABE)$

Question 9



The figure above is constructed as follows.

- EBD is a circular sector with centre at B and radius 12 units, subtending an angle of $\frac{2\pi}{3}$ radians at B.
- EAC is a quarter circle with centre at A and radius r units, so that $ABCD$ is a straight line and CAE is a right angle.

The shaded region R is bounded by the arcs ED and EC, and the straight line CD.

Show that the area of R is

$$3\left(7\pi + 6\sqrt{3}\right) \text{ square units.} \qquad \textbf{(13)}$$

$\frac{1}{2} \times 12^2 \times \frac{2\pi}{3} = 48\pi$

$A\hat{B}E = \pi - \frac{2\pi}{3} = \frac{\pi}{3}$

$\frac{\pi}{3} = \frac{"r"}{12}$

$"r" = 6\sqrt{3}$

$\frac{1}{4}\pi (6\sqrt{3})^2 = 27\pi$

$r = 6$

$\frac{1}{2} \times 6 \times 6\sqrt{3} = 18\sqrt{3}$

$18\sqrt{3} + 48\pi - 27\pi$

$3(7\pi - 6\sqrt{3})$