AQA

Level 2 Certificate
FURTHER MATHEMATICS

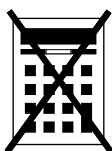Paper 1 Non-Calculator

Tuesday 19 June 2018   Morning   Time allowed: 1 hour 30 minutes

Materials
For this paper you must have:
• mathematical instruments.
You must **not** use a calculator.

Instructions
• Use black ink or black ball-point pen. Draw diagrams in pencil.
• Fill in the boxes at the top of this page.
• Answer **all** questions.
• You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
• Do all rough work in this book. Cross through any work you do not want to be marked.
• In all calculations, show clearly how you work out your answer.

Information
• The marks for questions are shown in brackets.
• The maximum mark for this paper is 70.
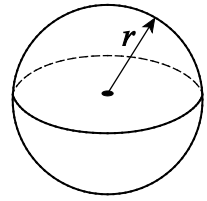• You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer book.
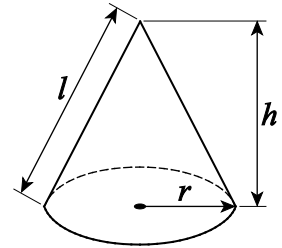
Formulae Sheet

Volume of sphere $= \dfrac{4}{3}\pi r^3$

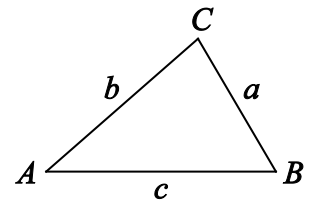**Surface area of sphere** $= 4\pi r^2$

Volume of cone $= \dfrac{1}{3}\pi r^2 h$

Curved surface area of cone $= \pi r l$

In any triangle ABC

Area of triangle $= \dfrac{1}{2}ab \sin C$

Sine rule $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine rule $a^2 = b^2 + c^2 - 2bc \cos A$

$$\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$$

The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$, where $a \neq 0$, are given by $x = \dfrac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$

Trigonometric Identities

$\tan \theta \equiv \dfrac{\sin \theta}{\cos \theta}$ $\sin^2 \theta + \cos^2 \theta \equiv 1$

Answer **all** questions in the spaces provided.

1 $y = \dfrac{x^6}{2} + \dfrac{x^4}{4}$

Work out $\dfrac{dy}{dx}$

Simplify your answer.

$$\dfrac{dy}{dx} = \dfrac{6x^5}{2} + \dfrac{4x^3}{4}$$

[2 marks]

Answer $3x^5 + x^3$

Turn over for the next question

Turn over ►

2

2   *P* is the point $(-12, b)$

Q is the point $(a, 4)$

R is the point $(6, -2)$

Q is the midpoint of *PR*.

Work out the values of a and b.

[3 marks]

$$\frac{-12+6}{2} = a \qquad \frac{b-2}{2} = 4$$

$$\frac{-6}{2} = a \qquad b-2 = 8$$

$$a = -3 \qquad b = 10$$

$a =$ __-3__ $b =$ __10__

3      $A = \begin{pmatrix} 2 & 4 \\ 3 & -1 \end{pmatrix}$ and $B = \begin{pmatrix} -2 & 6 \\ 2 & 1 \end{pmatrix}$

Work out **AB**.

[2 marks]

$$\begin{pmatrix} 2 & 4 \\ 3 & -1 \end{pmatrix} \begin{pmatrix} -2 & 6 \\ 2 & 1 \end{pmatrix} = \begin{pmatrix} 4-8 & 12+4 \\ -6-2 & 18-1 \end{pmatrix}$$

$$= \begin{pmatrix} -4 & 16 \\ -8 & 17 \end{pmatrix}$$

Answer _____

Turn over for the next question

4 $P = 4x$ and $Q = 7x$

   P **increases** by 25%

   Q **decreases** by 40%

Now, P is 28 greater than Q.

Work out the value of x.

[4 marks]

(marking: circled 2)

$\times 0.6$

$4x \times 1.25 = 7x \div 1.4 + 28$

$\dfrac{4x \times 5}{4} = 7x \div \dfrac{7}{5} + 28$

$5x = \dfrac{7x \times 5}{7} + 28$

$5x = 5x + 28$???

$5x - 4.2x = 28$

$x = 35$

Answer _____

5 In the expansion and simplification of $(x-3)(x^2+5x+k)$ the coefficient of x^2 is equal to the coefficient of x.

k is a constant.

Work out the value of k.

[3 marks]

$$x^3 + 5x^2 + kx - 3x^2 - 15x - 3k$$

$$x^3 + 2x^2 + (k-15)x - 3k$$

$$2 = k - 15$$

$$k = 17$$

Answer _____

Turn over for the next question

Turn over ►

6 A circle has centre $(-1, 2)$ and radius 5

Which of these is the equation of the circle?

Tick **one** box.

[1 mark]

$(x + 1)^2 + (y - 2)^2 = 5$ ☐

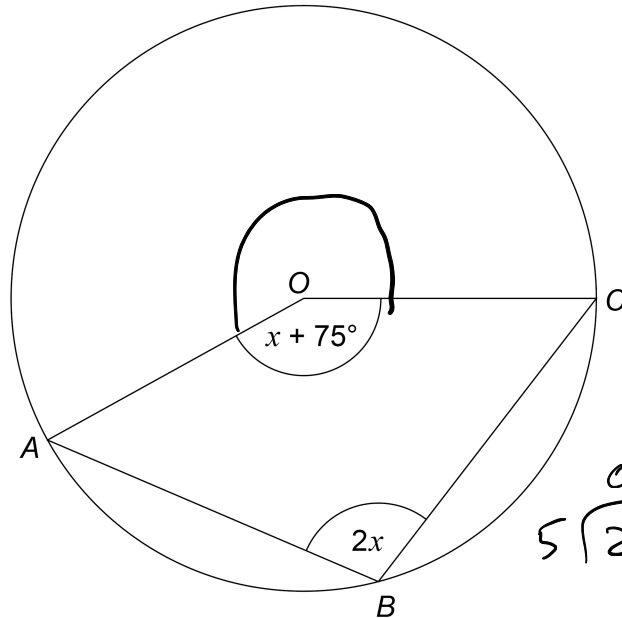$(x - 1)^2 + (y + 2)^2 = 5$ ☐

$(x + 1)^2 + (y - 2)^2 = 25$ ✓

$(x - 1)^2 + (y + 2)^2 = 25$ ☐

7 Points *A*, *B* and *C* lie on a circle, centre *O*.

Angle $AOC = x + 75°$

Angle $ABC = 2x$

Not drawn accurately

057
$5\overline{)2\overset{3}{8}5}$

Work out the value of *x*.

[3 marks]

$360 - x - 75 = 2 + 2x$

$285 = 5x$

$x = 57°$

Answer _____ degrees

Turn over for the next question

Turn over ▶

8 Write $(1 + 2\sqrt{5})(4 - \sqrt{5})$ in the form $a + b\sqrt{5}$ where a and b are integers.

[2 marks]

$$= 4 - \sqrt{5} + 8\sqrt{5} - 2\sqrt{5}\sqrt{5}$$
$$= 4 + 7\sqrt{5} - 10$$
$$= -6 + 7\sqrt{5}$$

Answer _____

9 $f(x) = 14 - x^2$ for all real values of x.

Solve $f(2x) = 5$

You **must** show your working.

[4 marks]

$$5 = 14 - (2x)^2$$
$$4x^2 - 9 = 0$$
$$(2x+3)(2x-3) = 0$$
$$2x = \pm 3$$

or $4x^2 = 9$

$x^2 = \frac{9}{4}$

$x = \pm \frac{3}{2}$

Answer $\pm \dfrac{3}{2}$

10   Rearrange $\dfrac{1}{xy} = 4 - \dfrac{3}{y}$ to make x the subject.

[3 marks]

$$1 = 4xy - \frac{3xy}{y}$$

$$1 = 4xy - 3x$$

$$1 = x(4y - 3)$$

$$x = \frac{1}{4y - 3}$$

Answer _____

Turn over for the next question

Turn over ▶

9

11 A curve has equation $y = 2x^2 + 3x - 9$

At a point *P* on the curve, the tangent is parallel to the line $y = 4 - 5x$

Work out the coordinates of *P*.   where $m = -5$

You **must** show your working.

[4 marks]

$$\frac{dy}{dx} = 4x + 3 \qquad 4x + 3 = -5$$
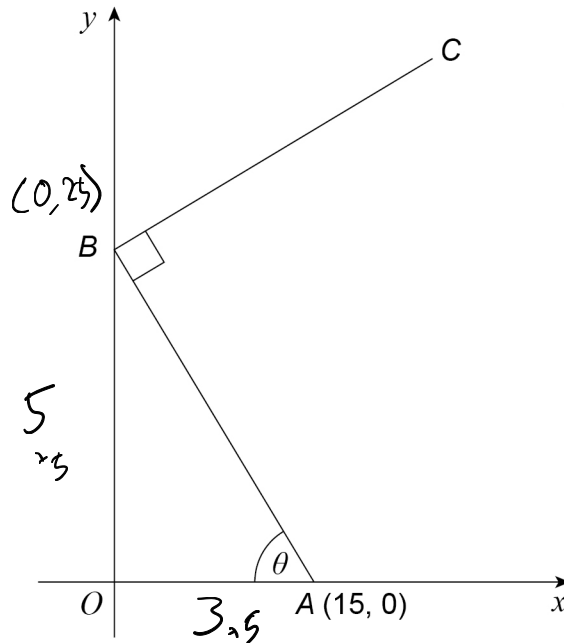
$$4x = -8$$

$$x = -2$$

$$y = 2x^2 + 3x - 9$$

$$x = 2 \quad y = 8 - 6 - 9 = -7$$

Answer (-2 , -7)

12 In the diagram,

A is the point (15, 0) and *B* lies on the *y*-axis.

Angle *ABC* = 90° and tan $\theta = \dfrac{5}{3}$

(0,25)

B

5
×5

Not drawn accurately

$\tan \theta = \dfrac{O}{A}$

θ

O   3×5   A (15, 0)   x

Work out the equation of the line *BC*.

[4 marks]

$3x = 15$

$x = 5$

$M_{AB} = -\dfrac{5}{3}$, $M_{BC} = \dfrac{3}{5}$ $c = 25$

$y = \dfrac{3x}{5} + 25$

Answer _____

Turn over for the next question

Turn over ▶

13 Solve the simultaneous equations

$xy = 2$ and $y = 3x + 5$

Do **not** use trial and improvement.

You **must** show your working.

[6 marks]

$x = \dfrac{2}{y}$

$y = \dfrac{2}{x}$

$\dfrac{2}{x} = 3x + 5$

$2 = 3x^2 + 5x$

$3x^2 + 5x - 2 = 0$

$(3x - 1)(x + 2) = 0$

$x = -2, \; \tfrac{1}{3}$

$y = \dfrac{2}{x}$

$= \dfrac{2}{-2} \qquad = 2 \div \tfrac{1}{3}$

$= -1 \qquad = 6$

Answer $\;(-2, -1)\;\left(\tfrac{1}{3}, 6\right)$

14 Work out the value of $\left(3^{\frac{1}{2}} + 3^{\frac{3}{2}}\right)^2$ $= \left(\sqrt{3} + 3\sqrt{3}\right)^2$

You **must** show your working.

$= \left(4\sqrt{3}\right)^2$

[3 marks]

$= 16 \times 3$

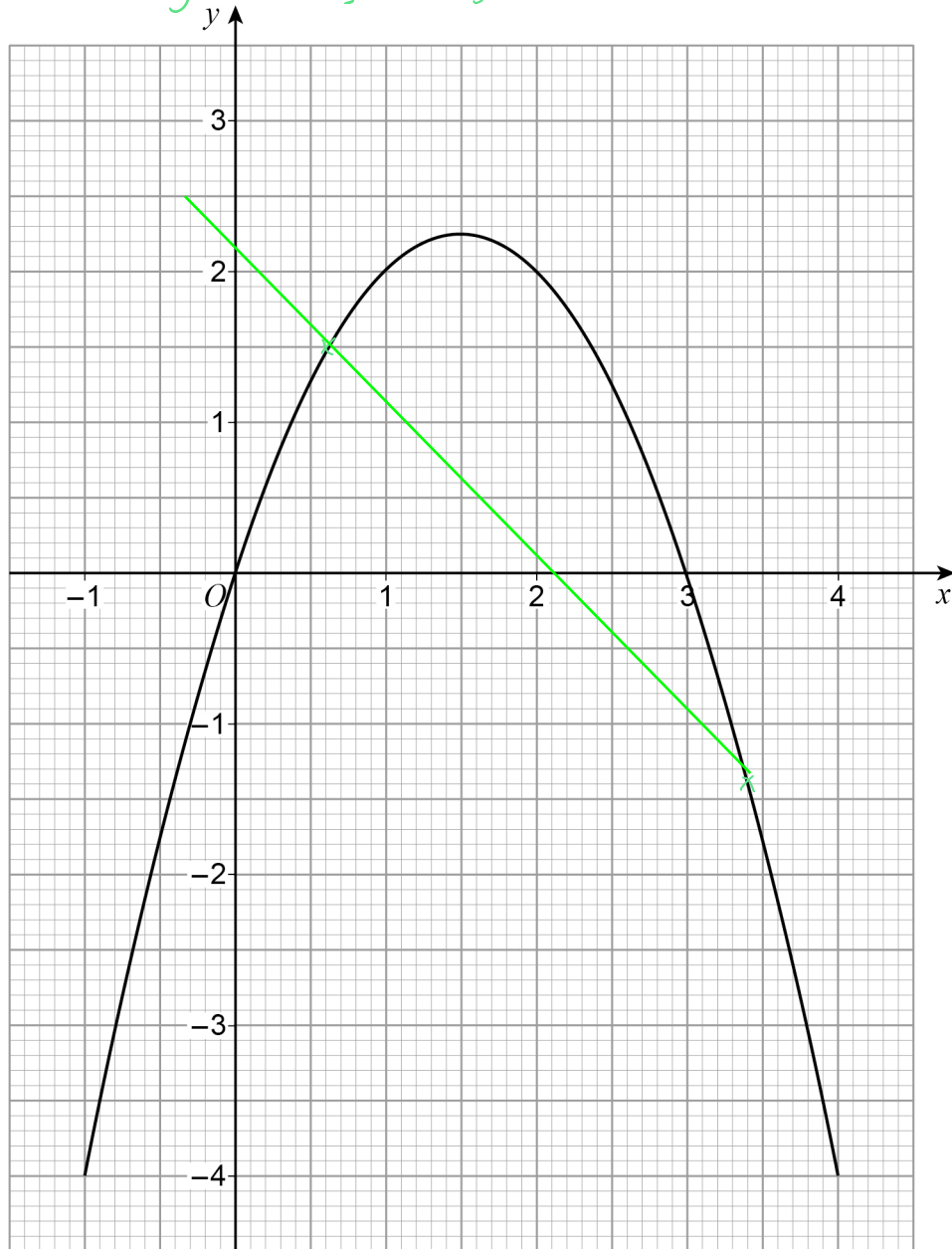$= 48$

Answer _____

Turn over for the next question

9

Turn over ▶

15 Here is the graph of $y = 3x - x^2$ for values of x from -1 to 4

$y = x^2 - 9x + 2$, So draw $y = 2 - x$

By drawing a suitable **linear** graph on the grid, work out approximate solutions to

$$x^2 - 4x + 2 = 0$$

[4 marks]

Answer _____

Turn over for the next question

Turn over ▶

16 $y = f(x)$ is a cubic curve with a maximum and a minimum stationary point.

$$\frac{dy}{dx} = x^2 + 2x - 3$$

[handwritten: so a pos x 3]

[handwritten: factorise to $(x+3)(x-1)$]

The y-coordinate of the minimum point is $2\frac{1}{3}$

The y-coordinate of the maximum point is 13
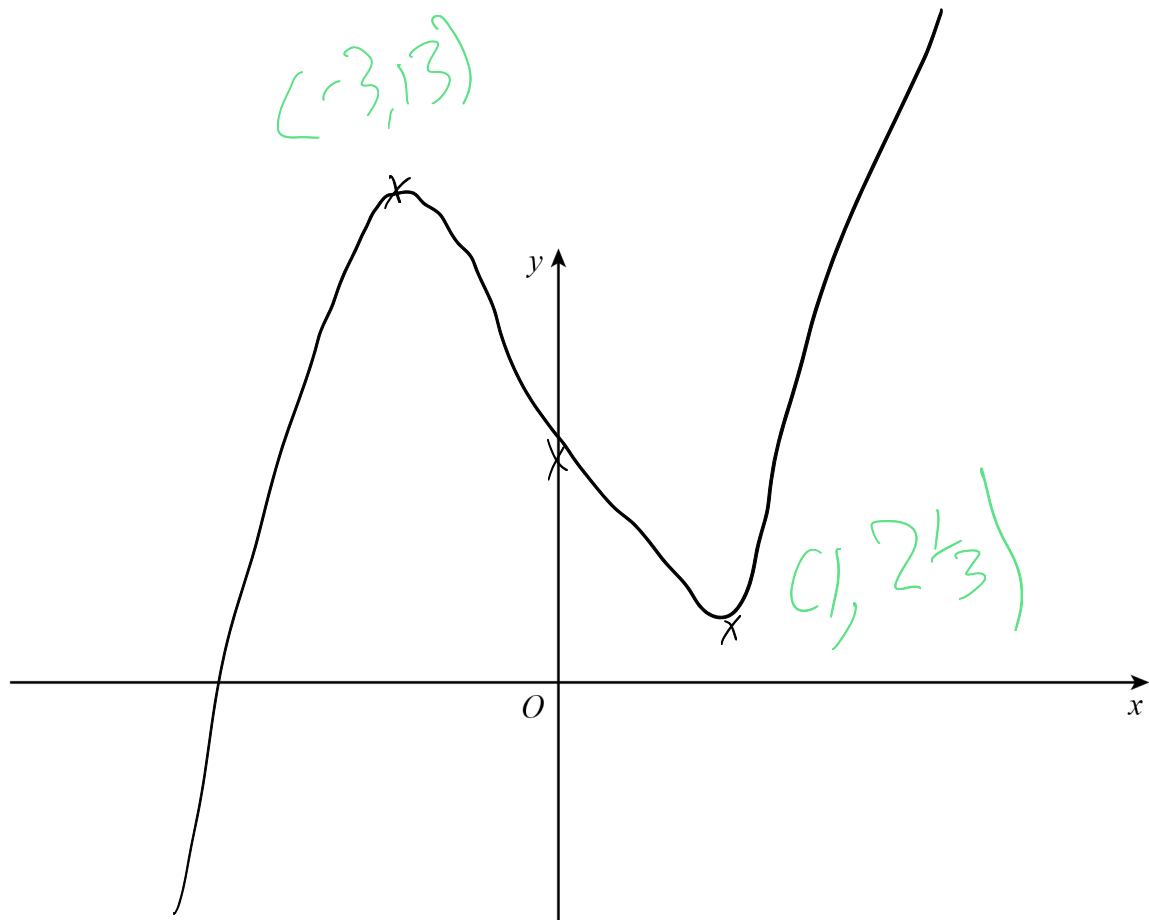
(0, 4) is a point on the curve.

The tangent at (0, 4) has a negative gradient.

Sketch the curve on the grid below.

Show the coordinates of the stationary points.

[4 marks]

[handwritten: ? No it done]

[handwritten labels on graph: $(-3, 13)$ and $(1, 2\frac{1}{3})$]

17 (a) Use the factor theorem to show that $(x - 2)$ is a factor of $\quad x^3 + 8x^2 + 5x - 50$

[1 mark]

$$F(x) = x^3 + 8x^2 + 5x - 50$$

$$F(2) = 0 = 2^3 + 8 \cdot 2^2 + 5 \cdot 2 - 50 = 8 + 32 + 10 - 50 = 0$$

$(x-2)$ a factor

17 (b) Hence, factorise fully $\quad x^3 + 8x^2 + 5x - 50$

[3 marks]

$$(x-2)(x^2 + 10x + 25)$$

use $\checkmark$ as an in between

$$(x-2)(x+5)^2$$

Answer _____

Turn over for the next question

Turn over ▶

18   *D*, *E*, *F* and *S* are points on a circle.

RST is a tangent.

The straight line *EDT* is parallel to *FS*.

DS = *DT*

Not drawn
accurately
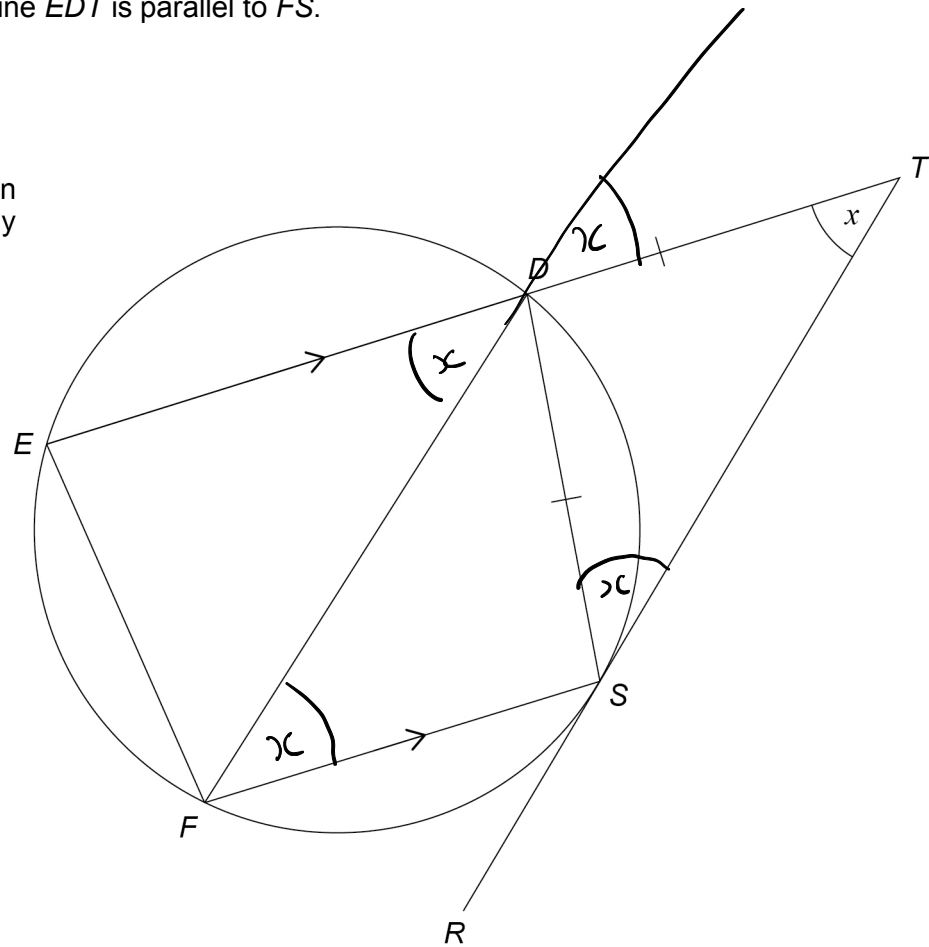
Prove that *FD* is parallel to *RST*.

Use angle *DTS* as x to help you.

[5 marks]

TOS = 180 - 2x (from triangle S; = 180)

TOS = 180 - 2x (bracket angles)

add to 180°

FD

Turn over for the next question

5

19   Write $\quad 2x^2 - 16x + 13 \quad$ in the form $\quad a(x + b)^2 + c \quad$ where a, b and c are integers.

[4 marks]

$$= 2(x^2 - 8x) + 13$$

$$= 2((x - 4)^2 - 16) + 13$$

$$= 2(x - 4)^2 - 32 + 13$$

$$= 2(x - 4)^2 + 19$$

Answer _____

20 In triangle *ABC*,

AB = 6 √2 cm, angle *ABC* = 45° and angle *ACB* = 60°
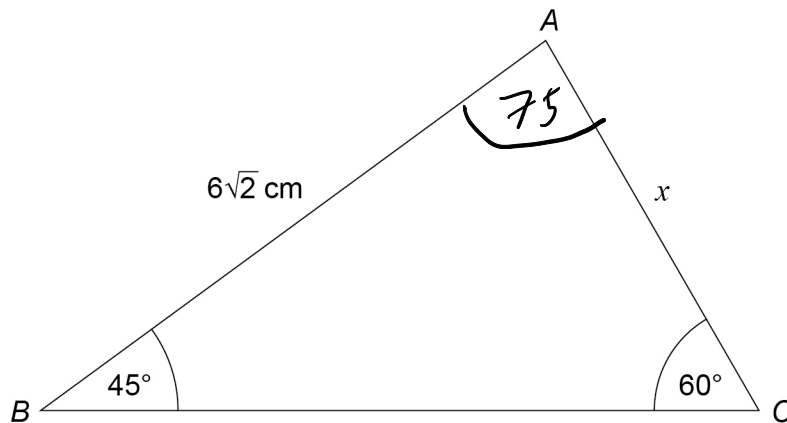
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Work out the value of x.

Give your answer in the form $a\sqrt{b}$, where a and b are integers.

You **must** show your working.

[5 marks]

$$\frac{a}{\sin A} = \frac{b}{\sin B}, \quad \frac{6\sqrt{2}}{\sin 60} = \frac{x}{\sin 45}, \quad x = \frac{6\sqrt{2}}{\sqrt{3} \div 2} \times \frac{1}{\sqrt{2}}$$

$$x = \frac{6\sqrt{2}}{1} \times \frac{2}{\sqrt{3}} \times \frac{1}{\sqrt{2}} = \frac{12}{\sqrt{3}} = \frac{12\sqrt{3}}{3} = 4\sqrt{3}$$

Answer _____ cm

END OF QUESTIONS

9

Do not write outside the box

There are no questions printed on this page

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ANSWER IN THE SPACES PROVIDED