Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Pages	Mark
3	
4 – 5	
6 – 7	
8 – 9	
10 – 11	
12 – 13	
14 – 15	
16	
TOTAL	

AQA

Level 2 Certificate in Further Mathematics
June 2014

Further Mathematics 8360/1

Level 2

Paper 1 Non-Calculator

Monday 16 June 2014 9.00 am to 10.30 am

For this paper you must have:
- mathematical instruments.

You may **not** use a calculator.

Time allowed
- 1 hour 30 minutes

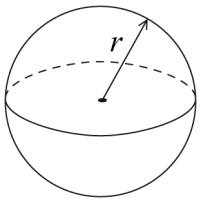
Instructions
- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- In all calculations, show clearly how you work out your answer.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 70.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer book.

Formulae Sheet

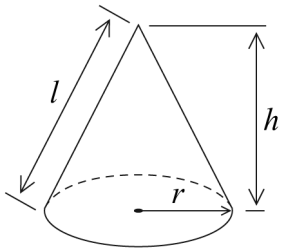
Volume of sphere $= \frac{4}{3}\pi r^3$

Surface area of sphere $= 4\pi r^2$



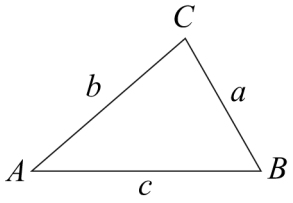
Volume of cone $= \frac{1}{3}\pi r^2 h$

Curved surface area of cone $= \pi r l$



In any triangle ABC

Area of triangle $= \frac{1}{2}ab\sin C$



Sine rule $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine rule $a^2 = b^2 + c^2 - 2bc\cos A$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$, where $a \neq 0$, are given by

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

Trigonometric Identities

$\tan\theta \equiv \dfrac{\sin\theta}{\cos\theta}$ $\qquad \sin^2\theta + \cos^2\theta \equiv 1$

Answer **all** questions in the spaces provided.

1 A straight line has gradient -2 and passes through the point $(-3, 10)$.

Work out the equation of the line.
Give your answer in the form $y = mx + c$

[2 marks]

$y = -2x + c$

$10 = -2 \times -3 + c$

$c = 4$

Answer $y = -2x + 4$

2 $y = 4x^3 - 7x$

Work out $\frac{dy}{dx}$

[2 marks]

Answer $\frac{dy}{dx} = 12x^2 - 7$

Turn over for the next question

3 A transformation is given by the matrix **M**, where $\mathbf{M} = \begin{pmatrix} 1 & a \\ 0 & 2 \end{pmatrix}$

The image of the point $(b, 5)$ under **M** is $(5, b)$.

Work out the values of a and b.

[3 marks]

$\begin{pmatrix} 1 & a \\ 0 & 2 \end{pmatrix}\begin{pmatrix} b \\ 5 \end{pmatrix} = \begin{pmatrix} 5 \\ b \end{pmatrix}$

$\begin{pmatrix} 5+ab \\ 2b \end{pmatrix} = \begin{pmatrix} 5 \\ b \end{pmatrix}$ $3+5a$ 10 $b=5a=5, b=10$

??

$a =$ ..... , $b =$ .....

4 Solve $20 + w < 3(w + 2)$

1

[3 marks]

$20+w < 3w+6$

$14 < 2w$

$7 < w$

$w > 7$

Answer .....

3

5 $f(x) = 10 - x^2$ for all values of x.

$g(x) = (x + 2a)(x + 3)$ for all values of x.

5 (a) Circle the correct value of $f(-4)$ **[1 mark]**

26 −6 36 16 196

5 (b) Write down the range of $f(x)$. **[1 mark]**

Answer $f(x) \leqslant 10$

5 (c) $g(0) = 24$

Show that $a = 4$ **[1 mark]**

$24 = (2a)(3)$

$24 = 6a$

$a = 4$

5 (d) Hence solve $f(x) = g(x)$ **[4 marks]**

$10 - x^2 = (x + 8)(x + 3)$

$10 - x^2 = x^2 + 11x + 24$

$0 = 2x^2 + 11x + 14$

$(2x + 7)(x + 2) = 0$

Answer $-4, -\frac{3}{2}$

6 The nth term of a sequence is $\dfrac{2n^2+7}{3n^2-2}$

$=50\times1-1$ 99
$=50\times2-2$ 98
$=50\times3-3$ 197

6 (a) Work out the 7th term.
Give your answer as a fraction in its simplest form.

[2 marks]

$= \dfrac{2\times99+7}{3\times99-2} = \dfrac{105}{195} = \dfrac{21}{29}$

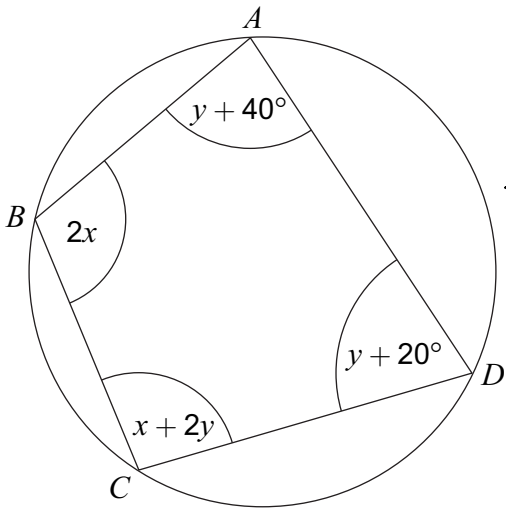
Answer $\dfrac{21}{29}$

6 (b) Show that the limiting value of $\dfrac{2n^2+7}{3n^2-2}$ as $n \to \infty$ is $\dfrac{2}{3}$

[2 marks]

$\lim_{n\to\infty} = \dfrac{2n^2}{3n^2} = \dfrac{2}{3}$

7 $ABCD$ is a cyclic quadrilateral.



Not drawn accurately

$= 2x + 3y + 90$
$= 68 + 72 + 90$
$= 180$

$x + 2y + y + 90 = 180$

Work out the values of x and y.

[5 marks]

(A) $2x + y + 20 = 180$

(B) $x + 3y + 40 = 180$

(2B) $2x + 6y + 80 = 360$

$-5y - 60 = -180$

$5y = 120$

$y = 24$

$x + 3y + 40 = 180$

$x + 72 = 140$

$x = 68$

$x =$ 68 , $y =$ 24

9

8 (a) Factorise fully $3x^2 - 12$

[2 marks]

$= 3(x^2 - 4)$

$= 3(x + 2)(x - 2)$

Answer .....

8 (b) Factorise $5x^2 + 4xy - 12y^2$

[3 marks]

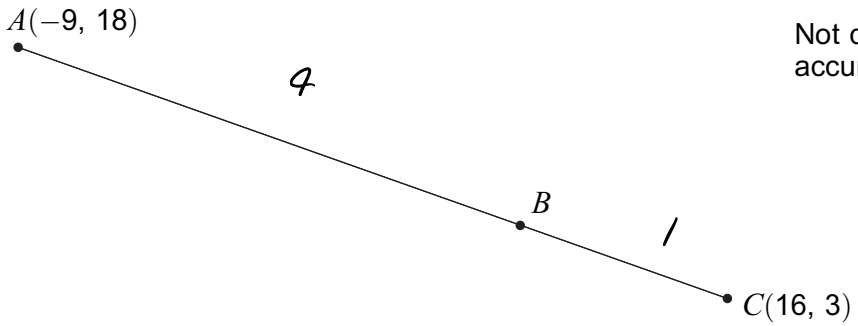
$= 5x^2 + 4y(x - 3y)$

$(5x + 6y)(x \pm 2y)$??

$(5x - 6y)(x + 2y)$

Answer .....

9 ABC is a straight line.
BC is 20% of AC.



Not drawn accurately

Work out the coordinates of B.

[4 marks]

$B_x = -9 + \frac{(16 - -9) \times 4}{5} = -9 + \frac{25 \times 4}{5} = -9 + 20 = 11$

$B_y = 18 + \frac{(3 - 18) \times 4}{5} = 18 - \frac{15 \times 4}{5} = 18 - 12 = 6$

Answer (11 , 6)

Turn over for the next question

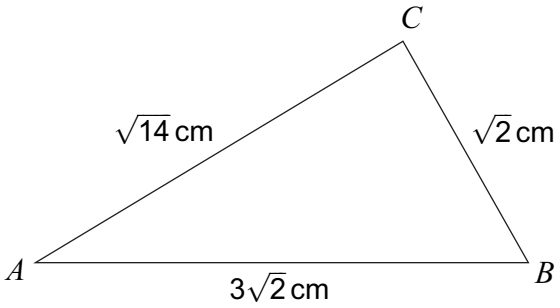
10 Rationalise the denominator of $\dfrac{8}{3-\sqrt{5}}$

Give your answer in the form $a+b\sqrt{5}$ where a and b are integers. **[3 marks]**

$= \dfrac{8}{3-\sqrt{5}} \times \dfrac{3+\sqrt{5}}{3+\sqrt{5}} = \dfrac{24+8\sqrt{5}}{9-5} = \dfrac{4(6+2\sqrt{5})}{4}$

Answer $6+2\sqrt{5}$

11 (a) Here is triangle ABC.



Not drawn accurately

Show that angle $B = 60°$

[3 marks]

$a^2 = b^2 + c^2 - 2bc\cos A$

$\cos A = \frac{b^2+c^2-a^2}{2bc}$

$\cos B = \frac{a^2+c^2-b^2}{2ac}$

$= \frac{2+18-14}{2\times 3\sqrt{2}\times\sqrt{2}} = \frac{6}{12} = \frac{1}{2}$

$B = \cos^{-1}\left(\frac{1}{2}\right)$

$B = 60°$

11 (b) Hence work out the area of triangle ABC.

[3 marks]

$A = \frac{1}{2}ab\sin C$

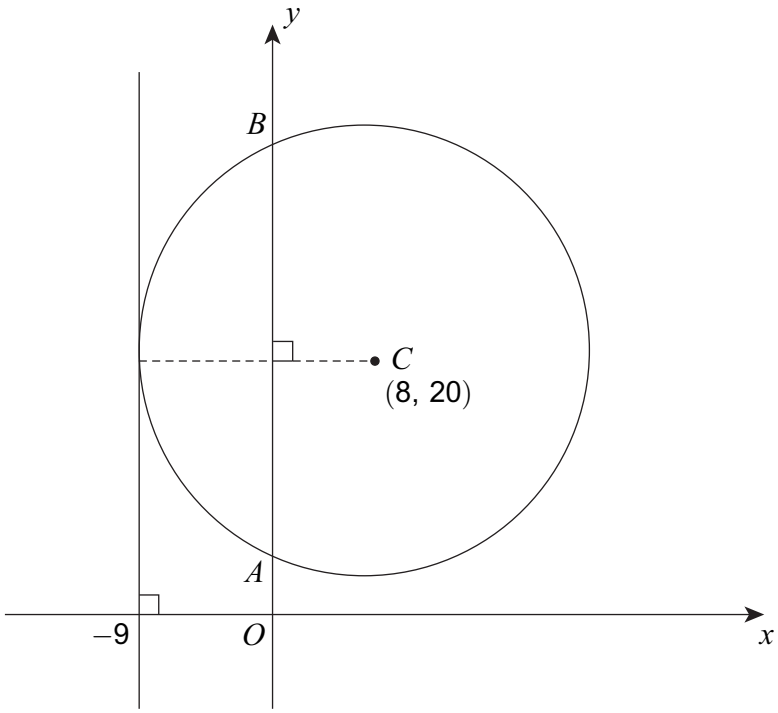
$= \frac{1}{2}ac\sin B$

$= \frac{3\sqrt{2}\times\sqrt{2}}{2}\times\frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{2}$

Answer ..... cm^2

9

12 The line $x = -9$ is a tangent to the circle, centre $C(8, 20)$



Not drawn accurately

12 (a) Show that the radius of the circle is 17.

[1 mark]

dist $(-9, 8) = 17$

12 (b) The circle intersects the y-axis at A and B.

Show that the length AB is 30.

[3 marks]

$(x-8)^2 + (y-20)^2 = 17^2$

$(-8)^2 + y^2 - 40y + 400 = 289$

$y^2 - 40y + 400 + 64 - 289 = 0$

$y^2 - 40y + 275 = 0$

$y = 20 \pm 5\sqrt{5}$

$AB = 20 + 5\sqrt{5} - (20 - 5\sqrt{5})$

$= 0 + 10\sqrt{5}$??

$(-8)^2 + (y-20)^2 = 17^2$

$(y-20)^2 = 15^2$

$y - 20 = \pm 15$

$y = 35, 5$

$AB = 30$

17 × 17 = 289 (17 × 7 = 119; + 170)

400 − 289 = 275

13 A curve has equation $y = x^3 - 3x^2 + 5$

13 (a) Show that the curve has a minimum point when $x = 2$

[4 marks]

$\frac{dy}{dx} = 3x^2 - 6x$, $3x^2 - 6x = 0$

$3x(x-2) = 0$

$x = 0, 2$

$\frac{d^2y}{dx^2} = 6x - 6$

$6x = 6$, $x = 2$, $= 12 - 6 = 6$ – pos so min pt

13 (b) Show that the tangent at the minimum point meets the curve again when $x = -1$

[3 marks]

$m, x=2 = 0$, $y, x=2$, $= 8 - 12 + 5 = 1$

$y = 0x + c$

$1 = c$ $y = 1$ $dy/dx = 2$ on ?, and $y = 1$

$y = 1$ meets $y = x^3 - 3x^2 + 5$

$1 = x^3 - 3x^2 + 5$

$x^3 - 3x^2 + 4 = 0$??

14 $(x - a)$ is a factor of $x^3 + 2ax^2 - a^2x - 16$

14 (a) Show that $a = 2$ **[2 marks]**

$x - a = 0$
$x = a$

a is a solution for $y = 0$

$0 = a^3 + 2a^3 - a^3 - 16$

$16 = 2a^3$

$a^3 = 8$

$a = 2$

14 (b) Solve $x^3 + 4x^2 - 4x - 16 = 0$ **[4 marks]**

$f(x) = x^3 + 4x^2 - 4x - 16$ $(x - 2)$ a factor

$f(1) = 1 + 4 - 4 - 16 \neq 0$

$f(-2) = -8 + 16 - 8 - 16 = 0$

$(x - 2)(x^2 + \ldots + 8) = 0$???

$(x - 2)(x^2 + 6x + 8) = 0$

$2, -2, -4$

Answer .....

15 Prove that $\dfrac{\sin\theta - \sin^3\theta}{\cos^3\theta} \equiv \tan\theta$

[3 marks]

$\equiv \dfrac{\sin\theta - \sin^3\theta}{\cos^3\theta}$

$\equiv \dfrac{\sin\theta(1 - \sin^2\theta)}{\cos\theta(\cos^2\theta)}$

$\equiv \dfrac{\sin\theta \cos^2\theta}{\cos\theta \cos^2\theta}$

$\equiv \tan\theta$

Turn over for the next question

16 $2x^2 - 2bx + 7a \equiv 2(x - a)^2 + 3$

Work out the **two** possible pairs of values of a and b.

[6 marks]

$2x^2 - 2bx + 7a \equiv 2(x^2 - 2ax + a^2) + 3$

$2x^2 - 2bx + 7a \equiv 2x^2 - 4ax + 2a^2 + 3$

$-2bx = -4ax$ $\qquad 7a = 2a^2 + 3$

$2b = 4a$ $\qquad 2a^2 - 7a + 3 = 0$

$b = 2a$ $\qquad (2a - 1)(a - 3) = 0$

$a = 3, 0.5$

$a =$ 3, $b =$ 6

and

$a =$ 0.5, $b =$ 1

END OF QUESTIONS

6