

# AQA Level 2 Certificate in FURTHER MATHEMATICS (8365/2)

Paper 2

Specimen 2020

Time allowed: 1 hour 45 minutes

## Materials

For this paper you must have:

- mathematical instruments

You may use a calculator



## Instructions

- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Fill in the boxes at the bottom of this page.
- Answer **all** questions.
- You must answer the questions in the space provided. Do not write outside the box around each page or on blank pages.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- In all calculations, show clearly how you work out your answer.

## Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer booklet.

Please write clearly, in block capitals, to allow character computer recognition.

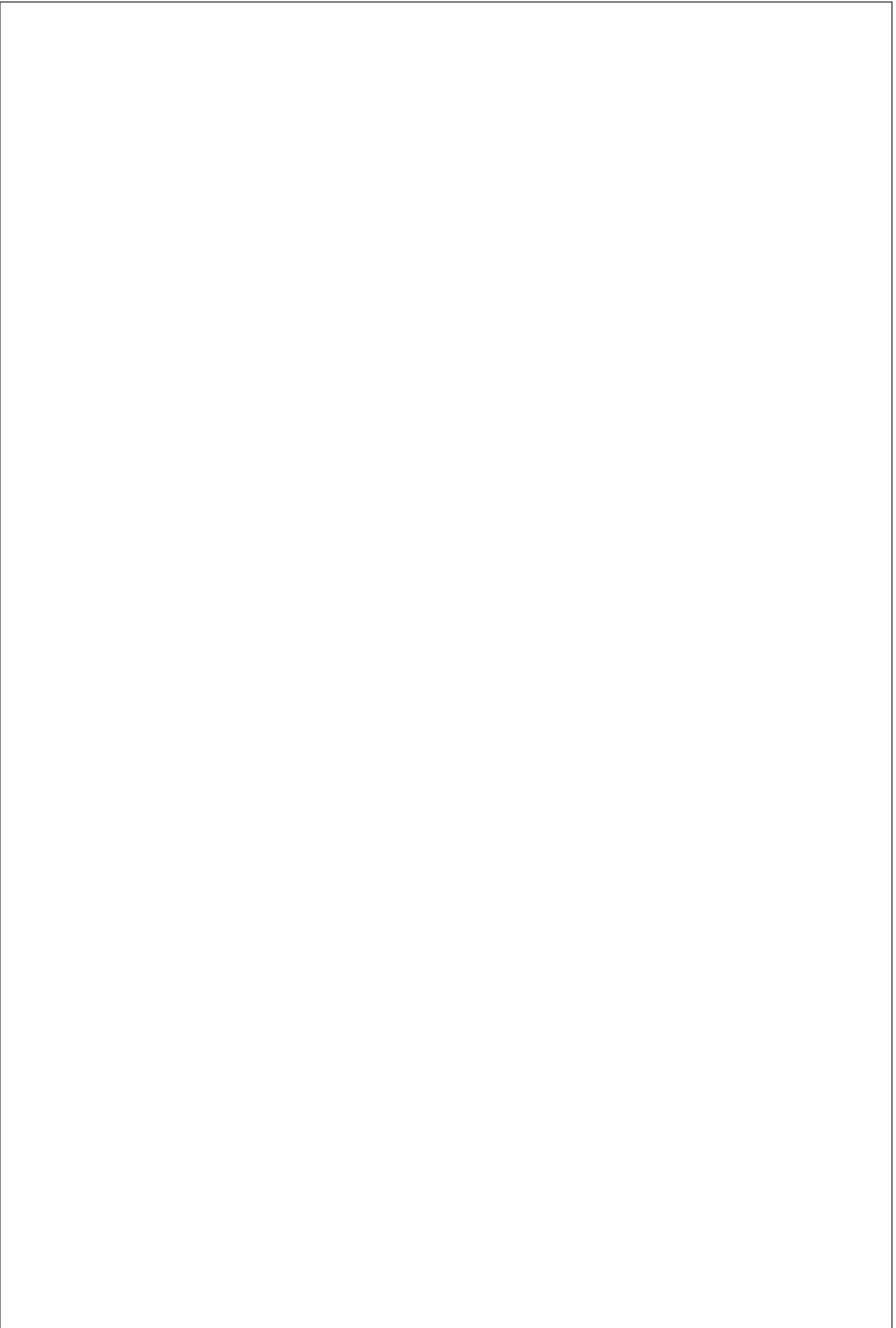
Centre number

Candidate number

Surname

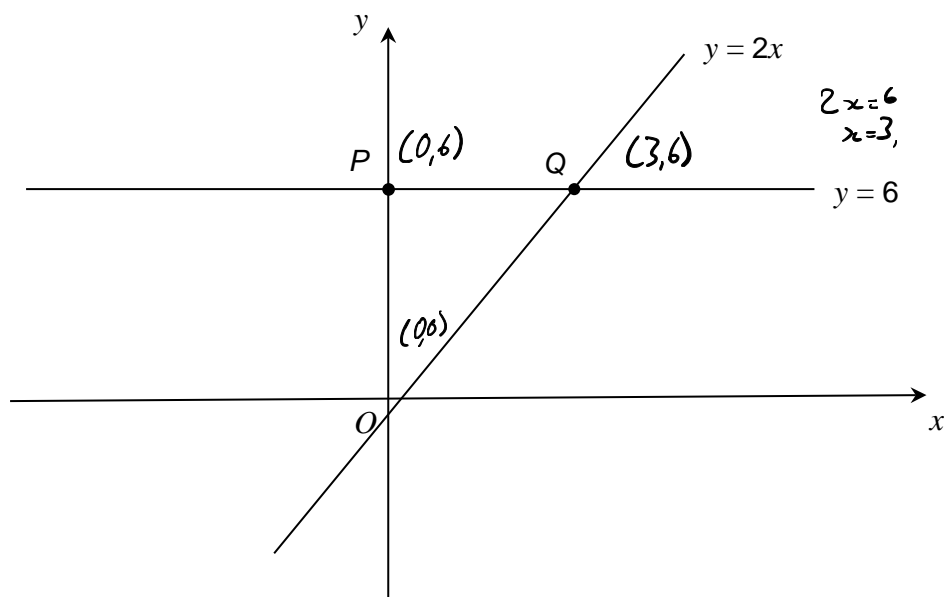
Forename(s)

Candidate signature \_\_\_\_\_



Answer **all** questions in the spaces provided.

- 1 A sketch of the lines  $y = 2x$  and  $y = 6$  is shown.



Work out the area of triangle  $OPQ$ .

$$A = \frac{bh}{2}$$

$$= \frac{3 \times 6}{2} = 9$$

[3 marks]

Answer \_\_\_\_\_ units<sup>2</sup>

- 2 A circle, centre  $(0, 0)$  has circumference  $20\pi$

Work out the equation of the circle.

$$P = \pi d$$

$$P = 20\pi$$

$$d = 20$$

$$r = 10, r^2 = 100$$

[2 marks]

Answer  $x^2 + y^2 = 100$

- 3  $M$  is the midpoint of the line  $AB$ .

$A(-2, 3)$

Not drawn  
accurately

$M(p, -1)$

$B(7, r)$

Work out the values of  $p$  and  $r$ .

 [2 marks]

$$p: \frac{-2+7}{2} = x = \frac{5}{2} = 2.5$$

$$r: \frac{3+r}{2} = -1, 3+r = -2, r = -5$$

$$p = 2\frac{1}{2} \quad r = -5$$

- 4 (a) Circle the solution of  $-3x < -18$

$x > -6$

$x < -6$

$x > 6$

$x < 6$

 [1 mark]

- 4 (b) Circle the solution of  $x^2 \geq 16$

$x \geq -4 \text{ or } x \leq 4$

$x \leq -4 \text{ or } x \geq 4$

$x \geq -4 \text{ or } x \geq 4$

$x \leq -4 \text{ or } x \leq 4$

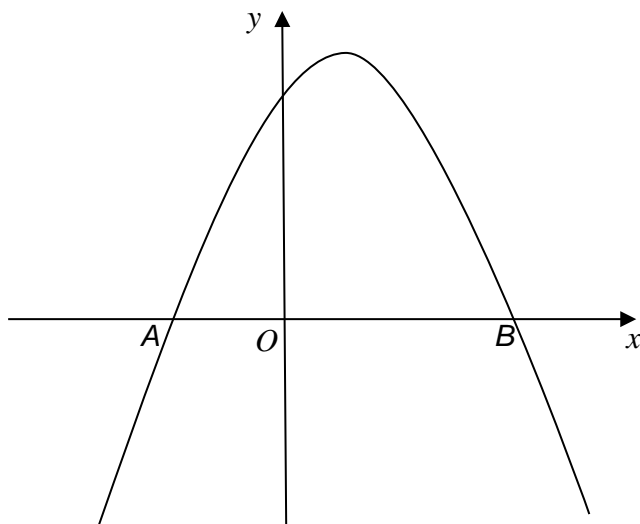
 [1 mark]

5 Here is a sketch of  $y = f(x)$  where  $f(x)$  is a quadratic function.

The graph

intersects the  $x$ -axis at  $A(-1, 0)$  and  $B$

has a maximum point at  $(0.5, 6)$



Not drawn  
accurately

5 (a) Work out the coordinates of  $B$ .

?

[1 mark]

Answer ( 2 , 0 )

5 (b) The equation  $f(x) = k$  has exactly **one** solution.

Write down the value of  $k$ .

$x$  then  $y$

[1 mark]

Answer 0.5 6

$$6 \quad \mathbf{A} = \begin{pmatrix} 4 & -1 \\ -7 & 2 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} s \\ -5 \end{pmatrix} \quad \mathbf{C} = \begin{pmatrix} -1 \\ t \end{pmatrix} \quad \mathbf{D} = \begin{pmatrix} 2 & 1 \\ 7 & u \end{pmatrix}$$

$s$ ,  $t$  and  $u$  are constants.

6 (a)  $\mathbf{AB} = \mathbf{C}$

Work out the values of  $s$  and  $t$ .

$$\begin{pmatrix} 4 & -1 \\ 7 & 2 \end{pmatrix} \begin{pmatrix} s \\ -5 \end{pmatrix} = \begin{pmatrix} -1 \\ t \end{pmatrix} \quad \left| \quad \begin{pmatrix} 4s+5 \\ 7s-10 \end{pmatrix} = \begin{pmatrix} -1 \\ t \end{pmatrix} \right.$$

$$4s+5 = -1$$

$$4s = -6$$

$$s = -1.5$$

$$7s-10 = t$$

$$-7-10 = t$$

$$t = -17$$

[3 marks]

$$s = -1.5$$

$$t = -17$$

6 (b)  $AD = I$ Work out the value of  $u$ .

$$\begin{pmatrix} 4 & -1 \\ -7 & 2 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 7 & u \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

[1 mark]

$$\begin{pmatrix} 1 & 4-u \\ 0 & -7+2u \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{array}{l} 4-u=0 \\ u=4 \end{array} \quad \text{OR} \quad \begin{array}{l} -7+2u=1 \\ 2u=8 \\ u=4 \end{array}$$

$$u = 4$$

7

Work out the equation of the straight line that is  
parallel to the line  $2y = x$ ,  $y = \frac{x}{2}$ ,  $y = \frac{1}{2}x$   
and  
intersects the  $x$ -axis at  $(4, 0)$

[3 marks]

$$y = \frac{x}{2} + c$$

$$0 = \frac{4}{2} + c$$

$$c = -2$$

Answer  $y = \frac{x}{2} - 2$

**8 (a)** Work out  $\frac{ab}{cd} \div \frac{bc}{ad}$

Give your answer as a single fraction in its simplest form.

[2 marks]

$$= \frac{ab}{cd} \times \frac{ad}{bc} = \frac{a^2 \cancel{b} \cancel{d}}{c^2 \cancel{b} \cancel{d}} = \frac{a^2}{c^2}$$

Answer \_\_\_\_\_

**8 (b)** Work out  $\frac{7}{2x^2} + \frac{4}{3x}$

Give your answer as a single fraction in its simplest form.

[2 marks]

$$= \frac{7(3x) + 4(2x^2)}{(2x^2)(3x)}$$

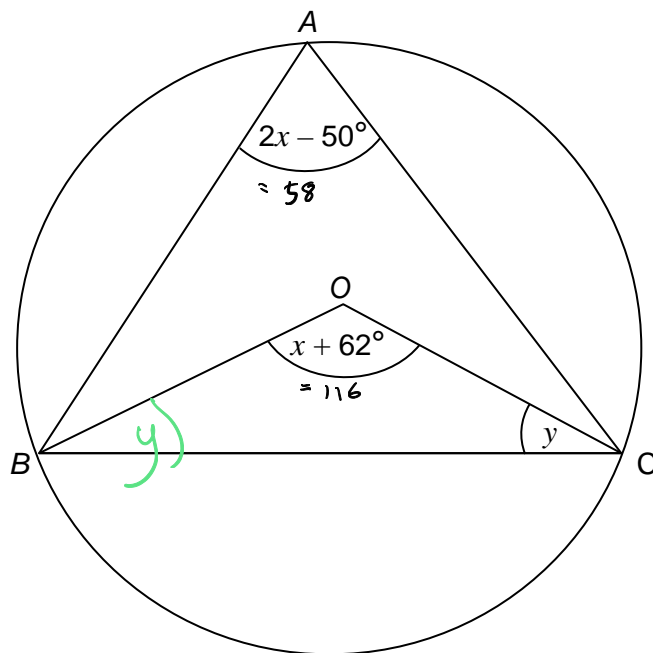
$$= \frac{21x + 8x^2}{6x^3} = \frac{8x + 21}{6x^2}$$

Answer \_\_\_\_\_



9

$A$ ,  $B$  and  $C$  are points on a circle, centre  $O$ .



Not drawn  
accurately

Work out the size of angle  $y$ .

[5 marks]

$$2(2x - 50) = x + 62$$

$$4x - 100 = x + 62$$

$$3x = 162$$

$$x = 54$$

???

$$\frac{180 - 62 - 54}{2} = 32$$

Answer \_\_\_\_\_ degrees

10

$$y = \frac{6x^9 + x^8}{2x^4}$$

$$\frac{6x^9}{2x^4} + \frac{x^8}{2x^4}$$

$$3x^5 + \frac{1}{2}x^4, \text{ then diff}$$

Work out the value of  $\frac{d^2y}{dx^2}$  when  $x = 0.5$

[5 marks]

$$y = (6x^9 + x^8) \times \frac{1}{2}x^{-4}$$

$$\frac{dy}{dx} = (54x^8 + 8x^7) \times -2x^{-5}$$

$$\frac{d^2y}{dx^2} = (432x^7 + 56x^6) \times -10x^{-6}$$

$$= \frac{-4320x^1 - 560x^0}{x^0} = -4320x - 560, x=0.5, = 1600$$

Answer

1600

11 For sequence A,  $n$ th term =  $\frac{n}{14n+30}$

For sequence B,  $n$ th term =  $\frac{2}{n}$

The  $k$ th term of sequence A equals the  $k$ th term of sequence B.

Work out the value of  $k$ .

You **must** show your working.

[4 marks]

$$\frac{2}{k} = \frac{k}{14k+30}$$

$$2(14k+30) = k^2$$

$$-k^2 + 28k + 60 = 0$$

$$k^2 - 28k - 60 = 0$$

$$(k-30)(k+2) = 0$$

$k = 30, -2$ , cannot be neg so discard  $-2$

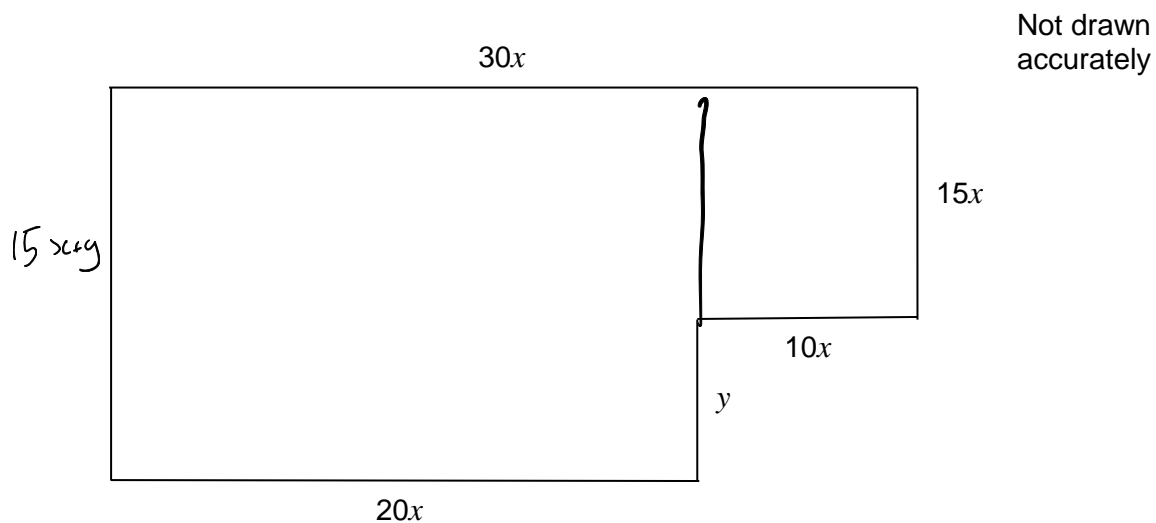
30

Answer

12

This shape is made from two rectangles.

All dimensions are in centimetres.



12 (a)

The perimeter of the shape is 252 cm

Show that  $y = 126 - 45x$

[2 marks]

$$30x + 20x + 10x + 15x + 15x + y + y = 252$$

$$90x + 2y = 252$$

$$2y = 252 - 90x$$

$$y = 126 - 45x$$

12 (b) The area of the shape is  $A \text{ cm}^2$

Show that  $A = 2520x - 450x^2$

[2 marks]

$$\begin{aligned}
 A &= (20x)(15x+10) + (15x)(10x) \\
 &= 300x^2 + 20xy + 150x^2 \\
 &= 950x^2 + 20x(126-15x) \\
 &= 450x^2 + 2520x - 900x^2 \\
 &= 2520x - 450x^2
 \end{aligned}$$

12 (c) Use differentiation to work out the maximum value of  $A$  as  $x$  varies.

[3 marks]

$$\frac{dy}{dx} = 2520 - 900x \quad -x^2 \text{ so only 1 max and no min pt}$$

$$\begin{aligned}
 2520 - 900x &= 0 \\
 2520 &= 900x
 \end{aligned}$$

$$x = 2.8$$

$$\begin{aligned}
 A &= 2520x - 450x^2, \quad x = 2.8 \\
 &= 7056 - 3528 \\
 &= 3528
 \end{aligned}$$

Answer \_\_\_\_\_

13  $f(x) = 3x^2 + 6$  for all  $x$   
 $g(x) = \sqrt{x-5}$   $x \geq 5$

13 (a) Work out the value of  $gf(4)$

[2 marks]

$$\begin{aligned} &= g(3 \times 4^2 + 6) = g(98 + 6) \\ &= \sqrt{98 + 6} \\ &= \sqrt{104} \\ &= 7 \end{aligned}$$

Answer 7

13 (b) Show that  $fg(x)$  can be written in the form  $a(x - a)$  where  $a$  is an integer.

[2 marks]

$$= f(\sqrt{x-5})$$

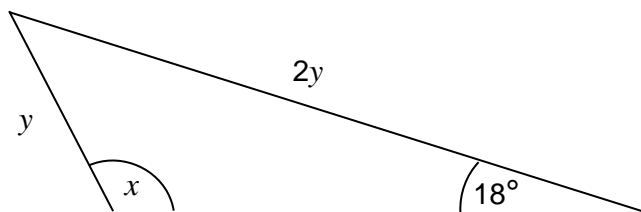
$$= 3(\sqrt{x-5}) + 6$$

$$= 3x - 15 + 6 = 3x - 9$$

$$= 3(x - 3)$$

Answer \_\_\_\_\_

14

Use the sine rule to work out the size of obtuse angle  $x$ .Not drawn  
accurately

[3 marks]

$$\frac{\sin A}{a} = \frac{\sin B}{b}, \quad \sin(x) = \frac{\sin(18)}{2y} y$$

$$x = \sin^{-1}\left(\frac{\sin(18)}{2}\right) \quad \text{not } \frac{1}{2}, \text{ but } \times 2$$

$$= \sin^{-1}(0.154\dots)$$

$$= 8.888$$

Answer

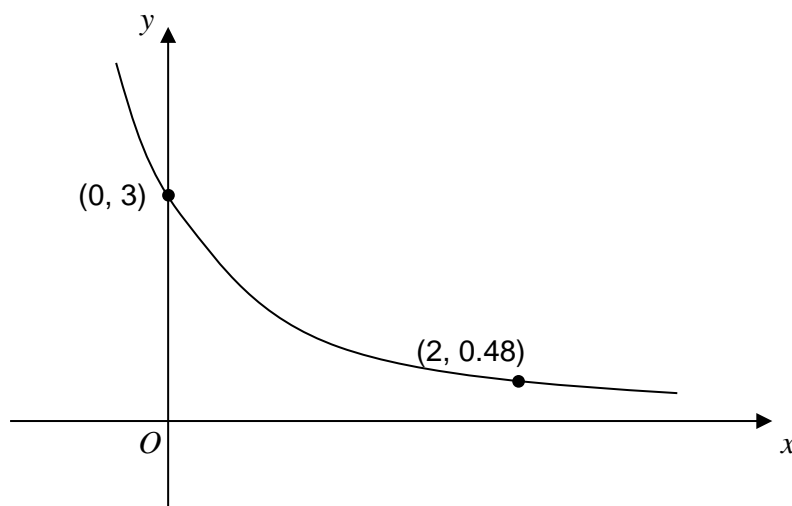
8.89

degrees

15

Here is a sketch of the curve  $y = ab^{-x}$  where  $a$  and  $b$  are positive constants.

$(0, 3)$  and  $(2, 0.48)$  lie on the curve.



Not drawn  
accurately

Work out the values of  $a$  and  $b$ .

[4 marks]

$$\begin{aligned} (0, 3), 3 &= ab^{-0} \quad 3 = a \\ (2, 0.48) \quad 0.48 &= 3 \times b^{-2} \\ b^{-2} &= \frac{9}{25} \\ b^2 &= \frac{25}{9} \\ b &= \sqrt{\frac{25}{9}} = 2.5 \end{aligned}$$

$$\begin{aligned} a &= 3 \\ b &= 2.5 \end{aligned}$$



16

Simplify  $\frac{8x^3 - 50x}{2x(6x^2 - x - 35)}$

Give your answer in the form  $\frac{ax+b}{cx+d}$  where  $a, b, c$  and  $d$  are integers.

[5 marks]

$$= \frac{\cancel{2x}(4x^2 - 25)}{\cancel{2x}(2x-5)(3x+7)} = \frac{\cancel{(2x-5)}(2x+5)}{\cancel{(2x-5)}(3x+7)} = \frac{2x+5}{3x+7}$$

Answer \_\_\_\_\_

17

By multiplying both sides of the equation by  $x^{\frac{1}{2}}$

Solve  $2x^{\frac{3}{2}} - 3x^{\frac{1}{2}} = 7x^{-\frac{1}{2}}$  for  $x > 0$

Give your answer to 3 significant figures.

You **must** show your working.

[4 marks]

$$(2x^{\frac{3}{2}} - 3x^{\frac{1}{2}})x^{\frac{1}{2}} = (7x^{-\frac{1}{2}})x^{\frac{1}{2}}$$

$$2x^2 - 3x = 7x^0 \quad \checkmark$$

$$2(x^2 - 1.5x) = 7$$

$$x^2 - 1.5x = 3.5$$

$$(x - 0.75)^2 - 0.5625 = 3.5$$

$$(x - 0.75)^2 = 4.0625$$

[only for  $x > 0$ , so can just 5]

$$x - 0.75 = 4.0625$$

$$x = 4.8125$$

Answer

18

How many odd numbers greater than 30 000 can be formed from these digits

2

4

6

7

8

with no repetition of any digit?

start

[3 marks]

$$\begin{aligned}
 &= 3 \times 3 \times 2 \times 1 \\
 &= 4 \times 4 \times 3 \times 2 \times 1 \\
 &= 6912
 \end{aligned}$$

Answer \_\_\_\_\_

19  $f(x) = 3x^3 - 2x^2 - 7x - 2$

19 (a) Use the factor theorem to show that  $(3x + 1)$  is a factor of  $f(x)$ .

[2 marks]

$$3x + 1 = 0$$

$$x = -\frac{1}{3}$$

$$\begin{aligned} f\left(-\frac{1}{3}\right) &= 3x - \frac{1}{2} + 2 \times \frac{1}{9} - 7x - \frac{1}{3} - 2 \\ &= -\frac{1}{9} - \frac{2}{9} + 2\frac{1}{3} - 2 \\ &= 0 \end{aligned}$$

19 (b) Factorise  $f(x)$  fully.

[3 marks]

$$= (3x + 1)(x^2 + bx - 2)$$

$$= 3x^3 + 3bx^2 - 6x + x^2 + bx - 2$$

$$= 3x^3 + (3b + 1)x^2 + (b - 6)x - 2$$

$$\begin{aligned} b - 6 &= -7 \\ b &= -1 \\ \text{OR } 3b + 1 &= -2 \\ 3b &= -3 \\ b &= -1 \end{aligned}$$

$$= (3x + 1)(x^2 - x - 2)$$

$$= (3x + 1)(x - 2)(x + 1)$$

Answer \_\_\_\_\_

20

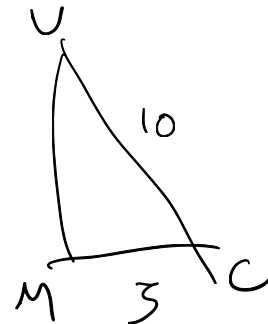
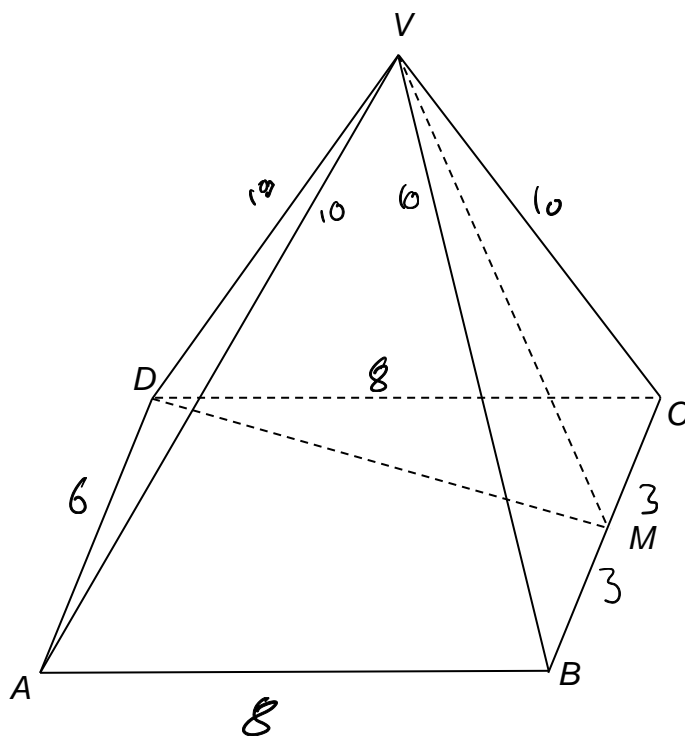
$VABCD$  is a pyramid with a horizontal rectangular base  $ABCD$ .

$V$  is directly above the centre of the base.

$$VA = VB = VC = VD = 10 \text{ cm}$$

$$AB = 8 \text{ cm} \quad BC = 6 \text{ cm}$$

$M$  is the midpoint of  $BC$ .



Work out the size of angle  $VMD$ .

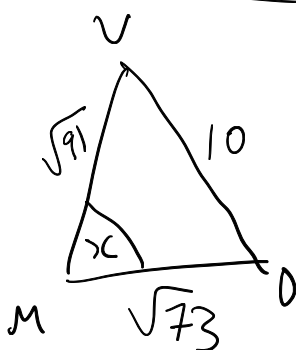
$$a^2 + b^2 = c^2$$

$$DM = \sqrt{3^2 + 8^2} = \sqrt{73}$$

$$c^2 - b^2 = a^2$$

$$MV = \sqrt{10^2 - 3^2} = \sqrt{91}$$

[5 marks]



$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\angle VMD = \cos^{-1} \left( \frac{91 + 73 - 100}{2 \times \sqrt{91} \times \sqrt{73}} \right) = \cos^{-1} \left( \frac{64}{163.009} \right) = 66.88$$

Turn over ►

Answer

66.9

degrees

$$\begin{array}{c} 1 \\ 1 \ 1 \\ 1 \ 2 \ 1 \\ 1 \ 2 \ 2 \ 1 \end{array}$$

21

Show that  $(2n+3)^3 + n^3$  is divisible by 9 for all integer values of  $n$ .

[4 marks]

$$= (2n)^3 + 2(2n)^2(3) + 2(2n)(3)^2 + (3)^3 + n^3$$

$$= 8n^3 + (2 \times 4 \times 3)n^2 + (2 \times 2 \times 9)n + 9 + n^3$$

$$= 9n^3 + 24n^2 + 36n + 9$$

$$\begin{array}{c} 99 \\ 11 \end{array}$$

END OF QUESTIONS

