IYGB GCE

Core Mathematics C1

Advanced Subsidiary

Practice Paper R

Difficulty Rating: 3.1333/1.3953

Time: 1 hour 30 minutes

Calculators may NOT be used in this examination.

Information for Candidates

This practice paper follows the Edexcel Syllabus.
The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 12 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

$x^2 + 9 = \frac{15}{2}x$

$x^2 - \frac{15}{2}x + 9 = 0$

$2x^2 - 15x + 18 = 0$

$(2x - 3)(x - 6)$

$\boxed{x = 6, \frac{3}{2}}$

Question 1

Solve the equation

$$x + \frac{9}{x} = \frac{15}{2}, \quad x \neq 0.$$

(4)

Question 2

a) Evaluate the following indicial expression, giving the final answer as an exact simplified fraction.

$$4^{\frac{3}{2}} + 4^{-\frac{1}{2}}.$$

$= \sqrt{4}^3 + \frac{1}{\sqrt{4}} = 8 + \frac{1}{2} = \boxed{8\frac{1}{2}}$ ✓

(2)

b) Simplify fully the following expression

$-5 - -2 = -3$

$$\frac{12y^{-5}}{3y^{-2}}. = \boxed{4y^{-7}}$$

(2)

Question 3

$$y = x\left(6x - 5\sqrt{x}\right), \quad x \geq 0.$$

By showing all steps in the workings, find an expression for

$$\int y \, dx.$$

$\boxed{NC}$

(5)

Question 4

The k^{th} term of a sequence is given by

$$a_k = 5k - 3.$$

By showing clearly all the steps in the calculation, evaluate the sum

$$\sum_{k=1}^{100} a_k .$$

(5)

$a_1 = 5-3 = 2$

$d = 5$

$n = 100$

$S_n = \frac{n}{2}(2a + (n-1)d)$

$= 50(4 + 99 \times 5)$

$= 50 \times 4 + 50 \times 5 \times 99$

$= 200 + 250 \times 99$

$= 200 + 2250$

$\boxed{= 2450}$

Question 5

It is given that

$$f(x) = x^2 - kx + (k+3),$$

where k is a constant.

If the equation $f(x) = 0$ has real roots find the range of the possible values of k. **(6)**

$0 = x^2 - kx + k - 3$

$b^2 - 4ac \geqslant 0$

$(-k)^2 - 4 \times 1 \times (k-3) \geqslant 0$

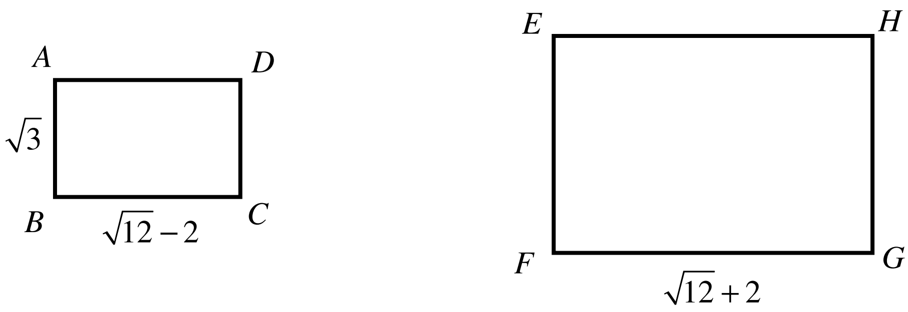
$k^2 - 4k + 12 \geqslant 0$

$(k-6)(k+2) \geqslant 0$

$\boxed{k \leq -2 \quad k \geqslant 6}$

Question 6

The two rectangles shown in the figure below are similar.



$k(\sqrt{12}-2) = \sqrt{12}+2$

It is further given that in suitable units

$k = \frac{\sqrt{12}+2}{\sqrt{12}-2} = \frac{12+4\sqrt{12}+4}{12-4} = \frac{16+8\sqrt{3}}{8}$

$$|AB| = \sqrt{3}, \quad |BC| = \sqrt{12}-2 \quad \text{and} \quad |FG| = \sqrt{12}+2.$$

$= 2+\sqrt{3}$

Find the exact length of EF. $\boxed{3+2\sqrt{3}}$ **(6)**

$= \sqrt{3}(2+\sqrt{3})$

$= 2\sqrt{3}+3$

Question 7

$$f(x) = \sqrt{27x^3+1}, \; x \geq -\tfrac{1}{3}.$$

The graph of $f(x)$ is stretched horizontally by scale factor 3, to produce the graph of $g(x)$.

$f(\tfrac{1}{3}x) = g(x)$

Determine in its simplest form the equation of $g(x)$. **(2)**

$\sqrt{x^3+1}$

$= \sqrt{27 \times \left(\frac{x}{3}\right)^3 + 1}$

$= \sqrt{27 \times \frac{x^3}{27} + 1}$

$\boxed{= \sqrt{x^3+1}}$

Question 8

$m = \frac{6-4}{2-4} = \frac{2}{6} = \frac{1}{3}$

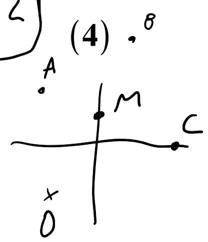
$m_T = -3$

The points A and B have coordinates $(-4,4)$ and $(2,6)$, respectively.

The straight line L_1 passes through the point B and is perpendicular to the straight line which passes through A and B.

$y - y_1 = m(x - x_1)$

a) Find an equation of L_1. $y - 6 = -3(x-2)$ $\quad y - 6 = -3x + 6$ $\quad \boxed{y = -3x + 12}$ **(4)**



L_1 meets the y axis at the point C.

$0 = -3x + 12$

$x = 4$

b) Show by calculation that

$\boxed{NC}$

$$|AB| = |BC|.$$ **(3)**

The quadrilateral $ABCD$ is a square whose diagonals intersect at the point M.

$M = M_{AC} = \left(\frac{4 + -4}{2}, \frac{0+4}{2}\right) = (0, 2)$ $\quad C = (0,12)$?

$M - B = (0-2, 2-6)$
$= (-2, -4) = E$

$B + 2E = D$
$= (2,6) + (-4,-8)$
$= (-2,-2)$

c) Determine …

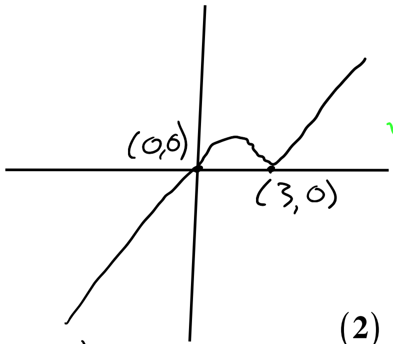
i. … the coordinates of M. $\boxed{(0,2)}$ $(-2,8)$ **(2)**

ii. … the coordinates of D. $\boxed{(-2,-2)}$ $(-6,10)$ **(2)**

Question 9

The curve C has equation

$$y = x^3 - 6x^2 + 9x.$$



a) Express y as a product of linear factors. **(2)**

$y = x(x^2 - 6x + 9)$

$\boxed{= x(x-3)^2}$

b) Sketch the graph of C. **(3)**

The sketch must include the coordinates of all the points where the curve meets the coordinate axes.

The graph of C is rotated by $180°$ about the origin onto another curve C'.

c) Determine the equation of C'. **(2)**

$y = +x^3 + 6x^2 + 9x$

Question 10

Andrew is planning to pay money into a pension scheme for the next 40 years.

He plans to pay into the pension scheme £800 in the first year and each successive year thereafter, an extra £100 compared to the previous year. $U_n = 100n + 800$

9

$= 100 \times 10 + 800$

a) Calculate the amount Andrew will pay into the scheme on the tenth year. **(2)** $\boxed{= £1800}$

b) Find the total amount Andrew will have paid into the scheme after 20 years. **(3)**

Not 40

$S_n = \frac{n}{2}(2a + (n-1)d)$

$= 20 \times (1600 + 19 \times 100)$

$= 20 \times 5500$

$\boxed{= £110{,}000}$

35,000

Beatrice is also planning to pay money into a pension scheme for the next 40 years.

She plans to pay £1580 in the first year and each successive year thereafter, to pay an extra £d compared to the previous year.

c) Find the value of d, if both Andrew and Beatrice paid into their pension schemes the same amount of money over the next 40 years. **(4)**

$S_n = \frac{n}{2}(2a + (n-1)d)$

$110{,}000 = 20(2 \times 1580 + 39d)$

$d = \left(\frac{110{,}000}{20} - 3160\right) \div 39$

$= 2340 \div 39$

$\boxed{= £60}$

Question 11

$$f(x) = 4x^2 + 12kx, \ x \in \mathbb{R},$$

where k is a constant.

a) Show clearly that the equation $f(x) = 9$ has two distinct real roots for all values of k. **(3)**

$9 = 4x^2 + 12kx$

$4x^2 + 12kx - 9 = 0$

$b^2 - 4ac = (12k)^2 - 4 \times 4 \times -9$

$= 144k^2 + 144 = 144(k^2+1)$

k real $\therefore k^2 > 0 \therefore 144(k^2+1) > 0 \therefore$ 2 roots

b) Hence find the solutions of the equation $f(x) = 9$, giving the answers in the form $pk \pm p\sqrt{k^2+1}$, where p is a constant to be found. **(3)**

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-12k \pm \sqrt{144(k^2+1)}}{2 \times 4}$$

$$= \frac{-12k \pm 12\sqrt{k^2+1}}{8} = -\frac{3}{2}k \pm \frac{3}{2}\sqrt{k^2+1}$$

Question 12

The point $P(2,9)$ lies on the curve C with equation

$$y = x^3 - 3x^2 + 2x + 9,\ x \in \mathbb{R},\ x \geq 1.$$

$\frac{dy}{dx} = 3x^2 - 6x + 2$

$m_2 = 3 \times 4 - 6 \times 2 + 2$

$= 2$

$m_T = -\frac{1}{2}$

NOT NORMAL

a) Find an equation of the tangent to C at P, giving the answer in the form $y = mx + c$, where m and c are constants. **(4)**

$y - y_1 = m(x - x_1)$

$y - 9 = -\frac{1}{2}(x - 2)$

$y - 9 = -\frac{x}{2} + 1$

$y = -\frac{1}{2}x + 10$

The point Q also lies on C so that the tangent to C at Q is perpendicular to the tangent to C at P.

$\frac{-1}{3x^2 - 6x + 2} = 2$

b) Show that the x coordinate of Q is

$$\frac{6 + \sqrt{6}}{6}.$$

(6)

$-1 = 6x^2 - 12x + \cancel{2}\ 4$

$6x^2 - 12x + \cancel{2}\ 5 = 0$

$2x^2 - 4x = -1$

$x^2 - 2x = -\frac{1}{2}$

$(x - 1)^2 - 1 = -\frac{1}{2}$

$(x - 1)^2 = \frac{1}{2}$

$x - 1 = \pm \frac{1}{\sqrt{4}}$

$x = 1 \pm \frac{\sqrt{4}}{4}$

$x = ???$