AQA

Level 2 Certificate in Further Mathematics
January 2013
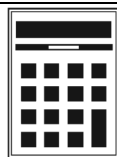
Further Mathematics 8360/2

Level 2

Paper 2   Calculator

Tuesday 29 January 2013   1.30 pm to 3.30 pm

For this paper you must have:
- a calculator
- mathematical instruments.

Time allowed
- 2 hours

Instructions
- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- In all calculations, show clearly how you work out your answer.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 105.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer book.
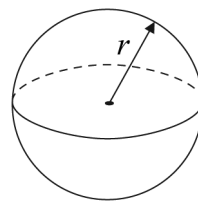- The use of a calculator is expected but calculators with a facility for symbolic algebra must **not** be used.

Formulae Sheet

Volume of sphere $= \frac{4}{3}\pi r^3$

Surface area of sphere $= 4\pi r^2$

Volume of cone $= \frac{1}{3}\pi r^2 h$

Curved surface area of cone $= \pi r l$
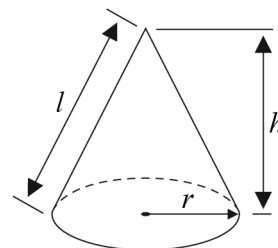
In any triangle ABC

Area of triangle $= \frac{1}{2}ab\sin C$

Sine rule $\quad \dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine rule $\quad a^2 = b^2 + c^2 - 2bc\cos A$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$
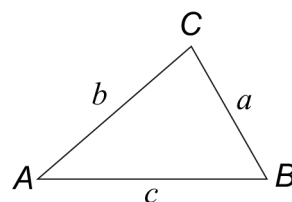
The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$, where $a \neq 0$, are given by

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

Trigonometric Identities

$$\tan\theta \equiv \frac{\sin\theta}{\cos\theta} \qquad \sin^2\theta + \cos^2\theta \equiv 1$$

Answer **all** questions in the spaces provided.

1 A sketch of $2x + 3y = 12$ is shown.

(handwritten working:)
$3y = 12 - 2x$
$y = 4 - \frac{2}{3}x$

$0 = 4 - \frac{2}{3}x$

$\frac{2}{3}x = 4$

$x = 6$

(graph with y-axis and x-axis, line passing through R on y-axis at 4 and S on x-axis at 6; markings showing 2, 3, 6)

1 (a) Work out the coordinates of *R*.

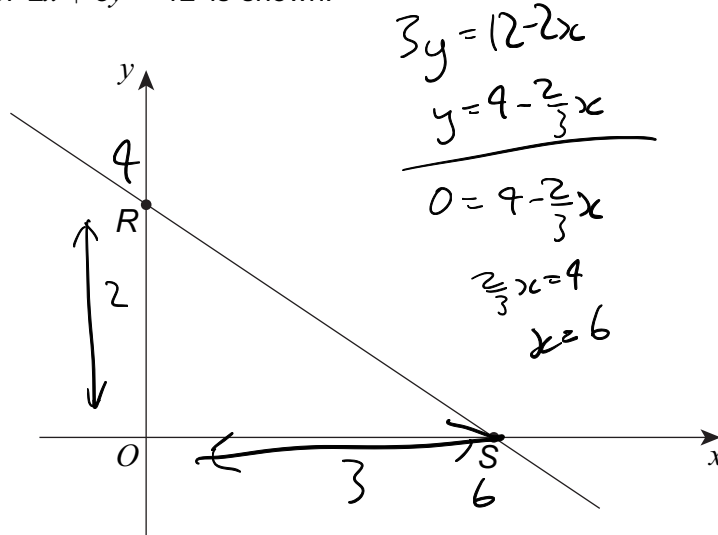...

Answer (........**0**........, ...**4**........)

(1 mark)

1 (b) Work out the coordinates of the midpoint of *RS*.

....

....

....

Answer (........**3**........, ...**2**........)

(2 marks)

Turn over ▶

2 In triangle *PQR*, *X* is a point on *PQ*.
RX is perpendicular to *PQ*.

R

Not drawn accurately

20 cm

34 cm

16 cm

P

X

30

Q
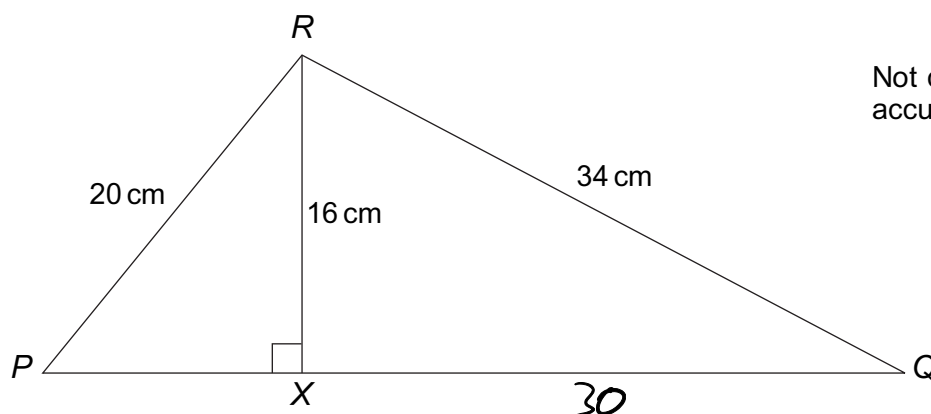
Work out the ratio *PX* : *XQ*
Give your answer in its simplest form.

$a = \sqrt{c^2 - b^2}$

$a = \sqrt{c^2 - b^2}$

$XQ = \sqrt{34^2 - 16^2}$

$PX = \sqrt{20^2 - 16^2}$

$= \sqrt{900}$

$= \sqrt{144}$

$= 30$

$= 12$

$= 12 : 30$

$= 2 : 5$

Answer   2   :   5   (4 marks)

3   Solve $5d - 3 > d + 17$

$4d > 20$

$d > 5$

Answer .... *(2 marks)*

4   Match each statement with an equation.
You will **not** use all of the equations.

One has been done for you.

| A curve passing through $(0, 0)$ |
| A curve passing through $(1, 0)$ |
| A circle centre $(2, -1)$ |
| A circle passing through $(3, 1)$ |

| $x^2 + y^2 = 10$ |
| $(x + 2)^2 + (y - 1)^2 = 1$ |
| $y = x^3$ |
| $y = x^3 + x - 2$ |
| $(x - 2)^2 + (y + 1)^2 = 1$ |
| $y = x^2 - 2$ |

$3^2 + 1^2 = 10$

$x^3 + x - 2 = 1 + 1 - 2 = 0 \qquad (1, 0)$

(3 marks)

9

Turn over ▶

5 A parallelogram and a trapezium are shown.
All lengths are in centimetres.

5

$x + 3$

x

Not drawn accurately

6

12
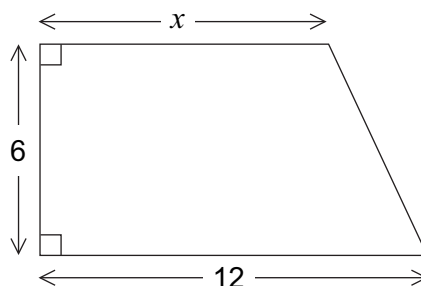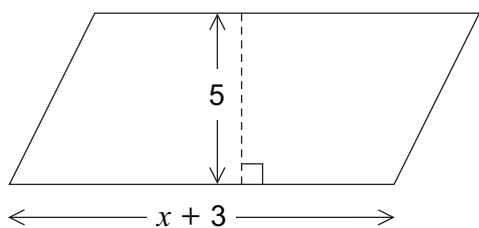
The area of the parallelogram is equal to the area of the trapezium.

Work out the value of x.

$$5(x+3) = 6\frac{(x+12)}{2}$$

$$5(x+3) = 3(x+12)$$

$$5x+15 = 3x+36$$

$$2x = 21$$

$$x = 10\tfrac{1}{2}$$

$x =$ .... cm *(4 marks)*

6 A function f(x) is defined as

$$f(x) = 4 \qquad\qquad x < -2$$
$$= x^2 \qquad\qquad -2 \leqslant x \leqslant 2$$
$$= 12 - 4x \qquad\qquad x > 2$$

6 (a) Draw the graph of $y = f(x)$ for $-4 \leqslant x \leqslant 4$
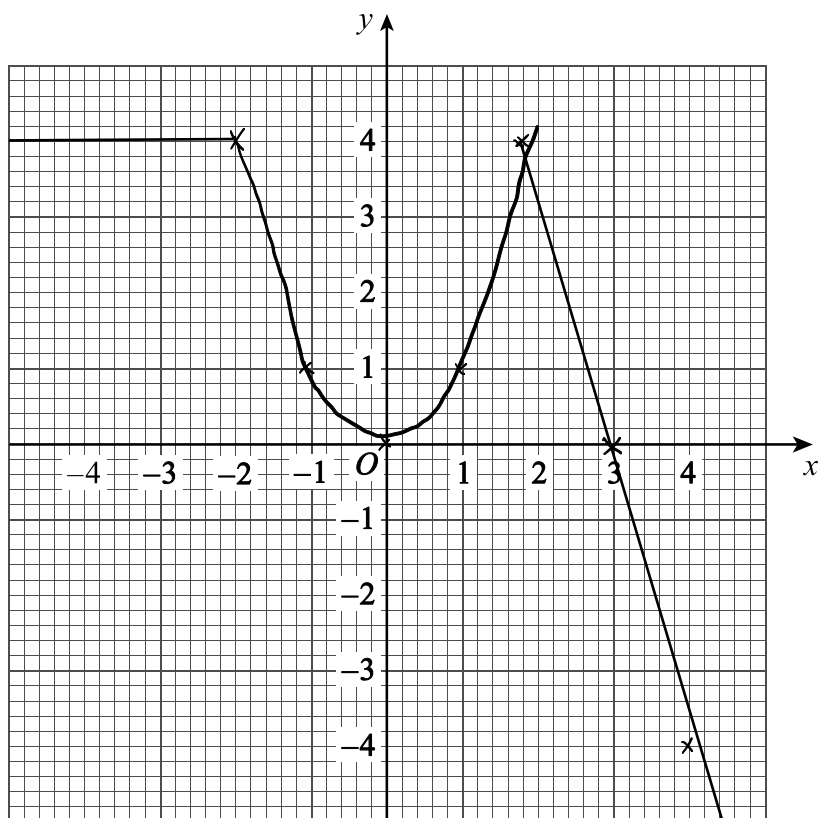
(3 marks)

6 (b) Use your graph to write down **how many** solutions there are to f(x) = 3

Answer ....3.......... *(1 mark)*

6 (c) Solve f(x) = -10

x^2 non neg, $x! = 4$ so $f(x) = 12 - 4x$

$12 - 4x = 10$

$-4x = -22$

$4x = 22$

$x =$

$x =$ ....5.5.......... *(2 marks)*

Turn over ▶

7 Here are the first four terms of a sequence.

$$\overset{+5a}{-1} \quad \overset{+5a}{4a} \quad \overset{+5a}{9a} \quad \overset{+5a}{14a} \quad 19a$$

The nth term of the sequence is $\dfrac{10n - 2}{3}$

Work out the value of a.

$5an - 1a = \dfrac{10n-2}{3}$

$15an - 3a = 10n - 2$

$a = \dfrac{2}{3}$

$a = \underline{\quad 2/3 \quad}$ *(2 marks)*

8 (a) Factorise fully $5m^2 - 20p^2$

$5(m+2p)(m-2p)$

Answer $5(m^2 - 9p^2)$ *(3 marks)*

8 (b) You are given that $p = 15$ and $5m^2 - 20p^2 = 0$

Using your answer to part (a), or otherwise, work out the values of m.

$5m^2 - 20p^2 = 0$

$5m^2 = 20 \times 225$

$m^2 = 9 \times 225$

$m^2 = 900$

$m = \pm 30$

Answer $\underline{\hspace{6cm}}$ *(2 marks)*

9 (a) Expand $(x + m)(x + n)$

....

....

Answer $x^2 + (m+n)x + mn$ .... *(1 mark)*

9 (b) $x^2 + qx + r \equiv (x + m)(x + n)$

Use your answer to part (a) to write q and r in terms of m and n.

$q = $ .... $m + n$ ....

$r = $ .... mn .... *(2 marks)*

9 (c) r is an odd integer.

Use your answer to part (b) to explain why q is an even integer.

To get an odd integer, you must multiply together two odd numbers ∴ m and n are both odd.

Two odd numbers added always make an even number ∴ q is even.

....

....
(2 marks)

12

Turn over ▶

10 $S = \dfrac{a}{1-r}$

10 (a)   Show that $r = \dfrac{S-a}{S}$

$$S - Sr = a$$
$$-Sr = a - S$$
$$Sr = S - a$$
$$r = \dfrac{S-a}{S}$$

(3 marks)

10 (b)   Work out the value of r when $S = 10a$

$$r = \dfrac{10a - a}{10a}$$
$$= \dfrac{9\cancel{a}}{10\cancel{a}}$$

$r =$ .... $\dfrac{9}{10}$ .... *(2 marks)*

11   In the diagram, $AB = BC$
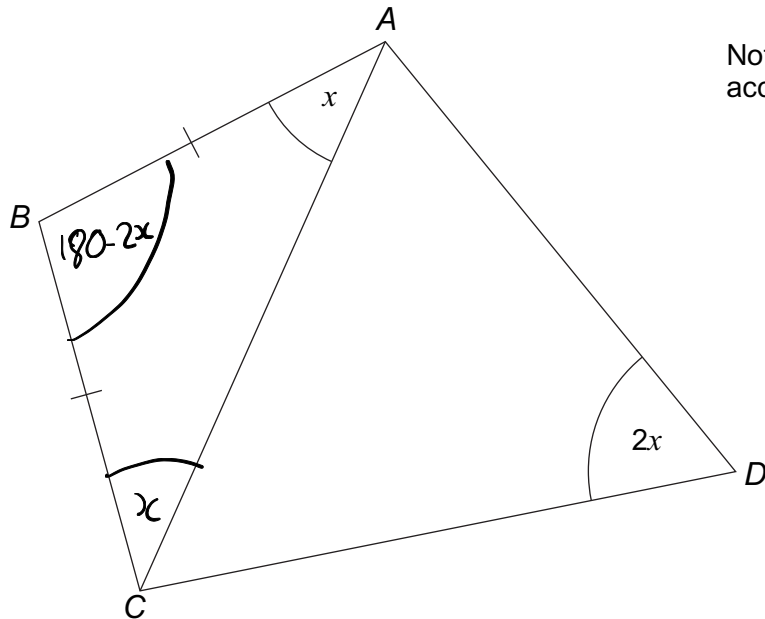
Not drawn
accurately

Prove that *ABCD* is a cyclic quadrilateral.
Give reasons for any statements you make.

Isosceles Triangle — $A\hat{B}C$ = Sum of int (180°) — other
angles (2x) = 180-2x.

In cyclic quadrilateral, opposite angles add up to 180°
so $A\hat{B}C$ = 180 - other side (2x).

(3 marks)

Turn over ▶

8

12 $f(x) = \sin x$ $180° \leqslant x \leqslant 360°$

$g(x) = \cos x$ $0° \leqslant x \leqslant \theta$

12 (a)   Calculate the value of f(210°).

$= \sin (210)$

$= -\frac{1}{2}$

Answer .... *(1 mark)*

12 (b)   Complete this inequality for the range of f(x).

Answer .... $\leqslant f(x) \leqslant$ .... -1 .... *(2 marks)*

0

OWA!
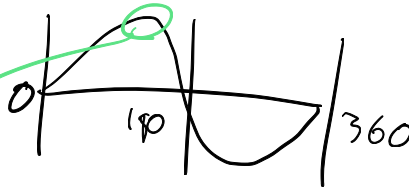
12 (c)   You are given that $0 \leqslant g(x) \leqslant 1$

Work out the value of θ.

$= \cos^{-1}(1)$

$= 0°$ or $360°$

can't be 0°

$90° = 1$

$\theta =$ .... 360 90 .... degrees   *(1 mark)*

13 (a) Show that $\dfrac{4}{x} + \dfrac{2}{x-1}$ simplifies to $\dfrac{6x-4}{x(x-1)}$

$$= \frac{4(x-1) + 2(x)}{x(x-1)}$$

$$= \frac{4x - 4 + 2x}{x(x-1)}$$

$$= \frac{6x - 4}{x(x-1)}$$

(2 marks)

13 (b) Hence, or otherwise, solve $\dfrac{4}{x} + \dfrac{2}{x-1} = 3$

$$x(x-1) = x^2 - x$$

Give your solutions to 3 significant figures.

$$\frac{6x - 4}{x(x-1)} = 3$$

$$\Rightarrow 6x - 4 = 3x^2 - 3x$$

$$\Rightarrow 3x^2 - 3x - 6x + 4 = 0$$

$$3x^2 - 9x + 4 = 0$$

$$0 = 3(x^2 - 3x) + 4$$

$$= 3((x-1.5)^2 - 2.25) + 4$$

$$= 3(x-1.5)^2 - 6.75 + 4$$

$$= 3(x-1.5)^2 - 2.75$$

$$(x-1.5)^2 = \frac{2.75}{3} = \frac{11}{12}$$

$$x - 1.5 = \pm\sqrt{\frac{11}{12}}$$

$$x = 1.5 \pm \sqrt{\frac{11}{12}}$$

$$\sqrt{\frac{11}{12}} = \frac{\sqrt{11}}{\sqrt{12}} \cdot \frac{\sqrt{12}}{\sqrt{12}} = \frac{\sqrt{132}}{12} = \frac{2\sqrt{33}}{12} = \frac{\sqrt{33}}{6}$$

Answer $x = 1.5 \pm \dfrac{\sqrt{33}}{6}$ *(5 marks)*

but instead, go $\dfrac{9 \pm \sqrt{33}}{6}$

14 The value of x is 50% **more** than the value of t.
The value of y is 10% **less** than the value of w.

$x = y$

Work out $\dfrac{t}{w}$

Give your answer as a decimal.

$1.5t = x$

$w = 0.9y$

$\dfrac{10w}{9} = y$

$0.9w$ not $0.9y$

$= \dfrac{1.5}{1.1111}$

$= \dfrac{27}{20}$

FO M /ECF

③

$\dfrac{t}{w} =$ .... *(4 marks)*

15 Describe fully the **single** transformation represented by the matrix $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$
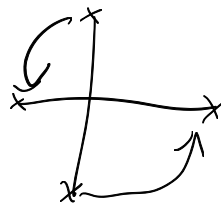
Rotate 90° Anti-Clockwise about (0,0)

2

(3 marks)

16 $y = (x^3 - 1)^2 + (\sqrt{x})^8$

Work out $\dfrac{dy}{dx}$.

$= (x^3 - 1)(x^3 - 1)$

$= x^6 - 2x^3 + 1$

$y = x^6 - 2x^3 + 1 + \left(x^{\frac{1}{2}}\right)^8$

$= x^6 + x^4 - 2x^3 + 1$

$\dfrac{dy}{dx} = 6x^5 + 4x^3 - 6x^2$

$\dfrac{dy}{dx} = $ ....   *(5 marks)*

Turn over for the next question

17 $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ represents a reflection in the y-axis.

$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ represents a reflection in the line $y = x$

Work out the matrix that represents a reflection in the y-axis followed by a reflection in the line $y = x$

$$y=x \qquad x=0$$

$$= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

Answer $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ *(2 marks)*

18 Express $1 - \tan\theta \sin\theta \cos\theta$ in terms of $\cos\theta$.

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$= 1 - \frac{\sin\theta}{\cos\theta} \sin\theta \cos\theta$

$= 1 - \frac{\sin^2\theta}{\cos\theta} \cos\theta$

$= 1 - \sin^2\theta$

$= \cos^2\theta$

Answer .... *(3 marks)*

19 A cubic function f(x) has domain $-4 \leqslant x \leqslant 4$

$POS x^3$

The curve $y = $ f(x)

- has a minimum point at $(-2, 0)$
- has a maximum point at $(1, 4)$
- meets the x-axis at $(4, 0)$.
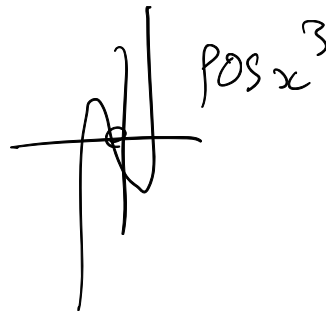
Sketch the graph of $y = $ f(x) on these axes.
Label any points where the graph meets the x-axis.

(4 marks)

9
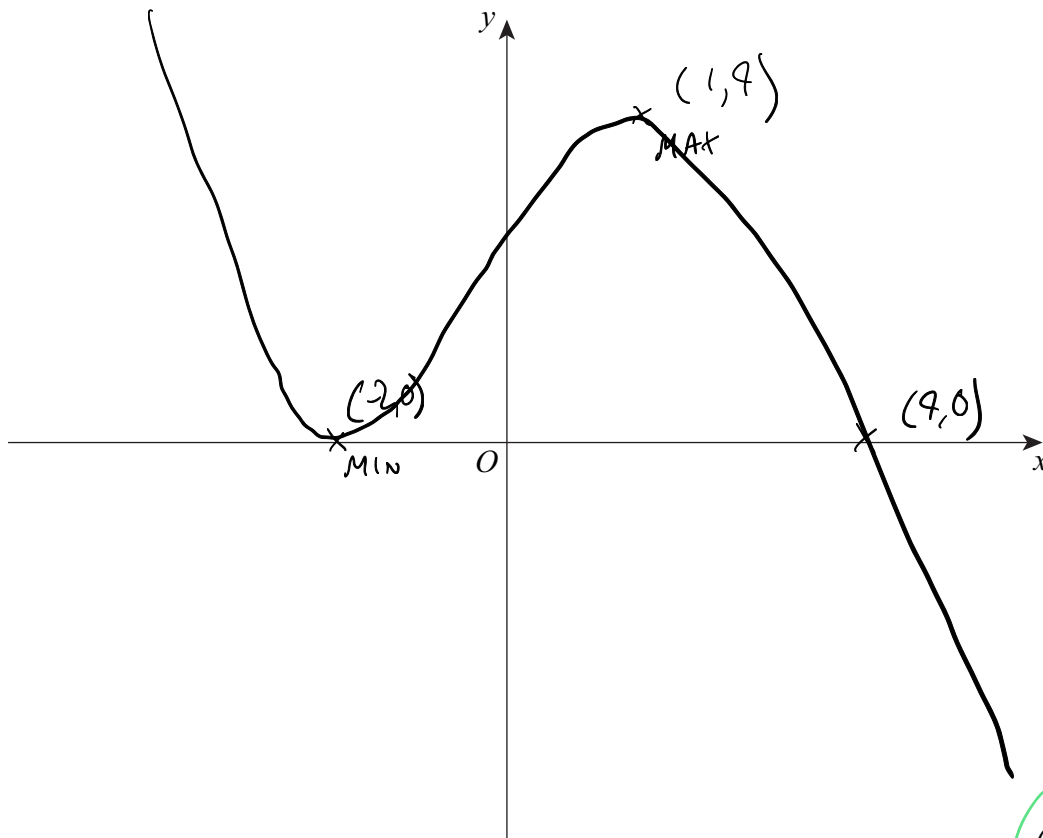
Turn over ▶

20 The area of this triangle is $18\,\text{cm}^2$.

Not drawn accurately

$w = 6$

y

$30°$

$2w = 12$
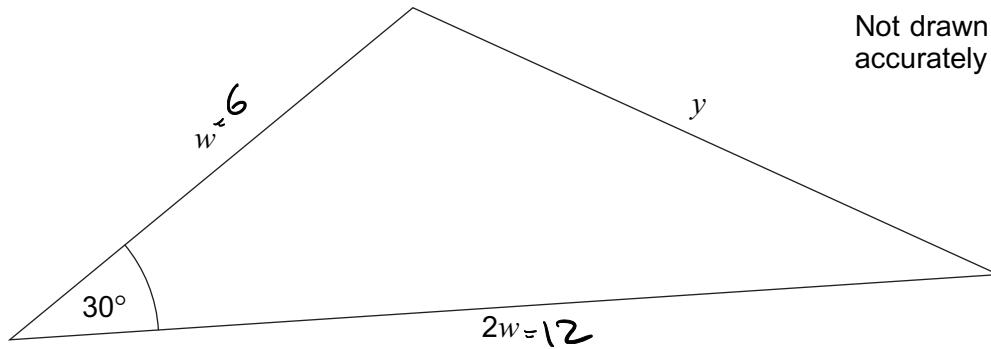
Work out y.

$$A = \frac{1}{2}ab\sin C$$

$$18 = \frac{2w^2 \times \sin 30}{2}$$

$$4 \times 18 = 2w^2$$

$$36 = w^2$$

$$w = 6$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$a = \sqrt{6^2 + 12^2 - 2 \times 6 \times 12 \times \cos 30}$$

$$= \sqrt{180 - 144 \times \frac{\sqrt{3}}{2}}$$

$$= \sqrt{180 - 72\sqrt{3}}$$

$$= 7.43588$$

$y =$ .... 7.99 .... cm **(5 marks)**

21 Work out the equation of the normal to the curve $y = x^2 + 4x + 5$ at the point where $x = -3$

$$\frac{dy}{dx} = 2x + 4 \qquad m = -6 + 4 = -2$$

$$\text{recip} = \frac{1}{2}$$

$$y = 9 - 12 + 5$$
$$= 2$$

$$y = mx + c$$
$$2 = \frac{-3}{2} + c$$
$$c = 3.5$$

Answer $\qquad y = \frac{x}{2} + 3.5 \qquad$ **(5 marks)**

22 $f(x) = x^3 + ax^2 + bx + 24$ for all values of x.

$$x = 2, -3$$

Two of the factors of $f(x)$ are $(x - 2)$ and $(x + 3)$.

Work out the values of a and b.

$$f(2) = 8 + 4a + 2b + 24 \quad \therefore \text{①} \; 4a + 2b = -32 \qquad 4a + 2b = 32$$
$$f(-3) = 27 + 9a - 3b + 24 \quad \therefore \text{②} \; 9a - 3b = 3 \qquad 2b = 32 - 4 \times 3.4$$
$$\text{③A} \; 12a + 6b = -96 \qquad 2b = 18.4$$
$$+ \; \text{②B} \; 18a - 6b = 6 \qquad b = 9.2$$
$$30a = 102$$
$$a = 3.4$$

SIGNS!

???

$a = \underline{\quad 3.4 \quad}$ $\qquad$ $b = \underline{\quad 9.2 \quad}$ $\qquad$ **(5 marks)**

Turn over ▶

23   The diagram shows a cuboid *ABCDPQRS* and a pyramid *PQRSV*.
V is directly above the centre, *X*, of *ABCD*.
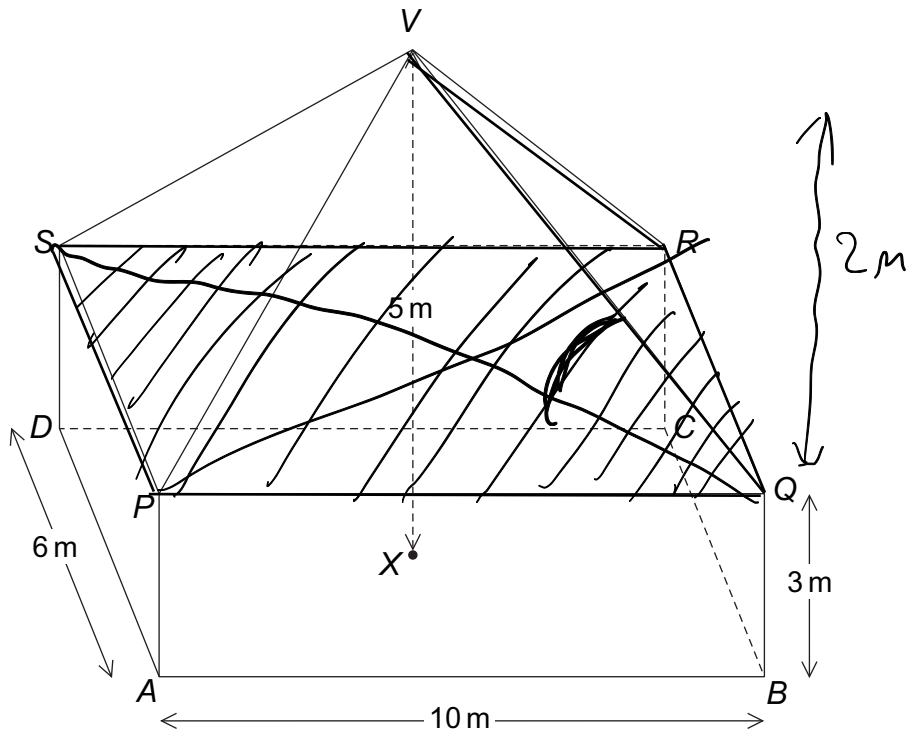
The total height, *VX*, is 5 metres.

VA √BABCD

$$\frac{\sqrt{10^2+6^2}}{2} = \sqrt{34}$$

$$A = \frac{bh}{2} = \frac{\sqrt{34} \times 5}{2}$$

$$A = \frac{1}{2}ab\sin C$$

$$\frac{\sqrt{34} \times 5}{2} = \frac{\sqrt{34} \times \sqrt{34} \times \sin\theta}{2}$$

$$5 = \sqrt{34} \times \sin\theta$$
$$\frac{5\sqrt{34}}{34} = \sin\theta$$
$$\theta = 40.6$$
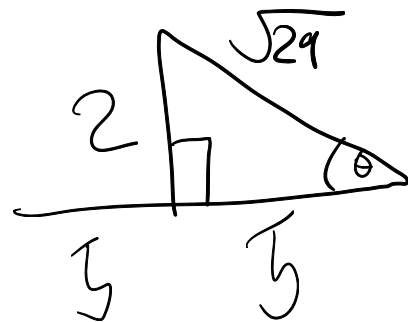
$$A = \frac{bh}{2} = 5$$

$$= \frac{1}{2}ab\sin C$$

$$5 = \frac{5\sqrt{29} \times \sin\theta}{2}$$

$$10 = 5\sqrt{29} \times \sin\theta$$

$$\frac{2\sqrt{29}}{29} = \sin\theta$$

$$\theta = 21.8°$$

23 (a) Work out the angle between the line *VA* and the plane *ABCD*.

....

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Answer ....40.6........................... degrees *(4 marks)*

23 (b) Work out the angle between the planes *VQR* and *PQRS*.

....

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....

....

Answer ....21.8........................... degrees *(2 marks)*

6

Turn over ▶

24   Solve $3\cos^2\theta - 1 = 0$ for $0° \leqslant \theta \leqslant 180°$

$0 = 3(\sin^2\theta - 1) - 1$

$= 3\sin^2\theta - 4$

$\cos^2\theta = 1/3$

$\cos\theta = \pm\sqrt{1/3}$

$54.7, \ 125.2$

??

Answer .... *(4 marks)*

25   Here are two number machines.

Input

a → -4 → $\times k$ → Output b

Input

$a+2$ → $\times 3$ → $-k$ → Output $b+2$
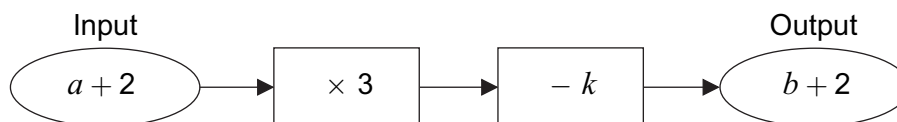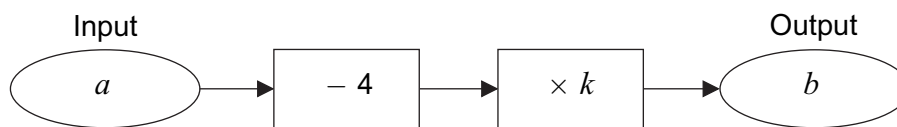
Work out a in terms of k.

$$b = k(a-4)$$
$$b+2 = 3(a+2)-k$$
$$b = 3(a+2)-k-2$$
$$k(a-4) = 3(a+2)-k-2$$
$$ka-4k = 3a+6-k-2$$
$$ka-3a = 6-k+4k-2$$
$$a(k-3) = 3k+4$$
$$a = \frac{3k+4}{k-3}$$

$a =$ .... *(6 marks)*

END OF QUESTIONS

10

There are no questions printed on this page

**DO NOT WRITE ON THIS PAGE
ANSWER IN THE SPACES PROVIDED**