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# LEVEL 2 CERTIFICATE

# **Further Mathematics**

8360/2 – Paper 2 Calculator

Mark scheme

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June 2018

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Version/Stage: 1.0 Final

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Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from [aqa.org.uk](http://aqa.org.uk)

## Glossary for Mark Schemes

GCSE examinations are marked in such a way as to award positive achievement wherever possible. Thus, for GCSE Mathematics papers, marks are awarded under various categories.

If a student uses a method which is not explicitly covered by the mark scheme the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

|                |  |
|----------------|--|
| <b>M</b>       | Method marks are awarded for a correct method which could lead to a correct answer.  |
| <b>M dep</b>   | A method mark dependent on a previous method mark being awarded.   |
| <b>A</b>       | Accuracy marks are awarded when following on from a correct method. It is not necessary to always see the method. This can be implied. |
| <b>B</b>       | Marks awarded independent of method.   |
| <b>B dep</b>   | A mark that can only be awarded if a previous independent mark has been awarded.   |
| <b>ft</b>      | Follow through marks. Marks awarded following a mistake in an earlier step.  |
| <b>SC</b>      | Special case. Marks awarded within the scheme for a common misinterpretation which has some mathematical worth.                        |
| <b>oe</b>      | Or equivalent. Accept answers that are equivalent.<br><br>eg, accept 0.5 as well as $\frac{1}{2}$                                      |
| <b>[a, b]</b>  | Accept values between a and b inclusive.   |
| <b>3.14...</b> | Accept answers which begin 3.14 eg 3.14, 3.142, 3.1416   |

Examiners should consistently apply the following principles.

***Diagrams***

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

***Responses which appear to come from incorrect methods***

Whenever there is doubt as to whether a candidate has used an incorrect method to obtain an answer, as a general principle, the benefit of doubt must be given to the candidate. In cases where there is no doubt that the answer has come from incorrect working then the candidate should be penalised.

***Questions which ask candidates to show working***

Instructions on marking will be given but usually marks are not awarded to candidates who show no working.

***Questions which do not ask candidates to show working***

As a general principle, a correct response is awarded full marks.

***Misread or miscopy***

Candidates often copy values from a question incorrectly. If the examiner thinks that the candidate has made a genuine misread, then only the accuracy marks (A or B marks), up to a maximum of 2 marks are penalised. The method marks can still be awarded.

***Further work***

Once the correct answer has been seen, further working may be ignored unless it goes on to contradict the correct answer.

***Choice***

When a choice of answers and/or methods is given, mark each attempt. If both methods are valid then M marks can be awarded but any incorrect answer or method would result in marks being lost.

***Work not replaced***

Erased or crossed out work that is still legible should be marked.

***Work replaced***

Erased or crossed out work that has been replaced is not awarded marks.

***Premature approximation***

Rounding off too early can lead to inaccuracy in the final answer. This should be penalised by 1 mark unless instructed otherwise.

***Continental notation***

Accept a comma used instead of a decimal point (for example, in measurements or currency), provided that it is clear to the examiner that the candidate intended it to be a decimal point.

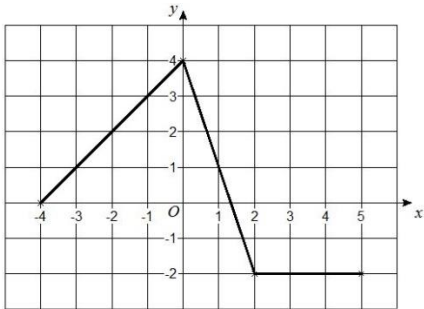
| Q | Answer | Mark | Comments |
|---|--------|------|----------|
|---|--------|------|----------|

|       |  |    |                        |
|-------|--|----|------------------------|
| 1 (a) | $1420 - 5n = 0$ or $5n = 1420$<br>or $\frac{1420}{5}$                  | M1 | oe eg $5(284 - n) = 0$ |
|       | 284  | A1 |                        |
|       | <b>Additional Guidance</b>   |    |                        |
|       | $\frac{1420 - 5n}{1420 + 5n} = 0$                                      |    | Zero                   |
|       | $1420 - 5n = 0(1420 + 5n)$   |    | Zero                   |
|       | $n = 284$  |    | M1A1                   |
|       | $1420 - 5n = 0$ and $1420 + 5n = 0$ with correct equation not selected |    | Zero                   |
|       | $\pm 284$ is A0  |    |                        |
|       | Embedded answer  |    | M1A0                   |

|       |   |    |    |
|-------|---|----|----|
| 1 (b) | -1  | B1 |    |
|       | <b>Additional Guidance</b>                      |    |    |
|       | $-\frac{5}{5}$                                  |    | B0 |
|       | $-1 \quad n \rightarrow \infty$                 |    | B1 |
|       | $-1 \rightarrow \infty$                         |    | B0 |
|       | $x \rightarrow -1$ (any letter other than $n$ ) |    | B1 |

| Q | Answer   | Mark | Comments  |
|---|--|------|---|
| 2 | Any pair of integer values for $a$ and $b$ for which $b = 12a + 26$  | B2   | B1 Correct equation in any form<br>eg $\frac{b-10}{a-3} = 12$ or $b + 10 = 12(a + 3)$<br>or $\frac{y-10}{x-3} = 12$ or $y + 10 = 12(x + 3)$<br>or $b = 12a + c$ and $c = 26$<br>or $y = 12x + c$ and $c = 26$<br>or<br>$-3 + k$ and $-10 + 12k$ where $k$ is a non-zero integer |
|   | <b>Additional Guidance</b>   |      |   |
|   | Examples of B2 responses<br>$a = -4$ and $b = -22$<br>or $a = -2$ and $b = 2$<br>or $a = -1$ and $b = 14$<br>or $a = 0$ and $b = 26$<br>or $a = 1$ and $b = 38$<br>or $a = 2$ and $b = 50$<br>or $a = 3$ and $b = 62$<br>or $a = 4$ and $b = 74$ |      | B2  |
|   | $a = -3$ and $b = -10$ is point $P$ so will not score B2 (B1 possible)   |      |   |
|   | $-3 + 1$ and $-10 + 12$  |      | B1  |
|   | $-3 + 2$ and $-10 + 24$  |      | B1  |

| Q    | Answer  | Mark  | Comments  |
|------|---|-------|---|
| 3(a) | $-0.112$ or $-\frac{14}{125}$   | B1    | oe fraction   |
|      | <b>Additional Guidance</b>  |       |   |
|      | Ignore incorrect conversion between fraction and decimal if correct value seen  |       |   |
|      | Ignore rounding or truncation after correct value seen  |       |   |
|      | Answer $-\frac{3.5}{31.25}$   |       | B0  |
| 3(b) | $2(m^2 + 1) = m + 2$<br>or $2m^2 + 2 = m + 2$<br>or $2m^2 = m$<br>or $2m^2 - m = 0$   | M1    | oe equation without fractions   |
|      | $m(2m - 1) (= 0)$<br>or<br>$m(1 - 2m) (= 0)$<br>or<br>$\frac{- -1 \pm \sqrt{(-1)^2 - 4 \times 2 \times 0}}{2 \times 2}$   | M1dep | oe eg $\frac{1}{4} \pm \sqrt{\frac{1}{16}}$<br>may be implied by both correct solutions |
|      | $0.5 \quad 0$ or $\frac{1}{2} \quad 0$  | A1    | oe  |
|      | <b>Additional Guidance</b>  |       |   |
|      | $0.5 \quad 0$ in working but only one of these on answer line   |       | M2A0  |
|      | Equation left in terms of $p$ and $m$   |       | Zero  |
|      | Answers only of $0.5 \quad 0$   |       | M2A1  |
|      | Answer only of $0.5$  |       | Zero  |
|      | Answer only of $0$  |       | Zero  |
|      | If using formula with an error seen the maximum mark is M1M0A0<br>eg $2m^2 - m = 0 \quad \frac{- -1 \pm \sqrt{(-1)^2 - 4 \times 1 \times 0}}{2 \times 2}$ Answers $0.5 \quad 0$ |       | M1M0A0  |

| Q | Answer  | Mark | Comments   |
|---|---|------|--|
| 4 | 3rd box ticked  | B1   |  |
|   | <b>Additional Guidance</b>  |      |  |
|   |   |      |  |
| 5 | Line from $(-4, 0)$ to $(0, 4)$   | M1   | mark intention   |
|   | Line from $(0, 4)$ to $(2, -2)$   | M1   | lines do not have to be straight but must pass through all integer points  |
|   | Line from $(2, -2)$ to $(5, -2)$  | M1   | only condone the first instance of a line that extends beyond the given domain   |
|   | Straight line from $(-4, 0)$ to $(0, 4)$ and<br>straight line from $(0, 4)$ to $(2, -2)$ and<br>straight line from $(2, -2)$ to $(5, -2)$   | A1   | all straight lines must be the correct length with no other lines<br>graph must be accurate<br>SC3 $(-4, 0)$ and $(-3, 1)$ and $(-2, 2)$ and $(-1, 3)$ and $(0, 4)$ and $(1, 1)$ and $(2, -2)$ and $(3, -2)$ and $(4, -2)$ and $(5, -2)$ plotted (any other points plotted must be correct ones for the graph)<br>SC2 $(-4, 0)$ and $(0, 4)$ and $(2, -2)$ and $(5, -2)$ plotted (any other points plotted must be correct ones for the graph) |
|   | <b>Additional Guidance</b>  |      |  |
|   |  <p>(crosses do not have to be shown)</p>  |      | M3A1   |
|   | Dashed or dotted lines can score up to M3A0   |      |  |
|   | Points may be implied by a correct line   |      |  |
|   | M mark examples<br>eg1 2 correct lines and 1 extended line (but otherwise correct)<br>eg2 1 correct line and 2 extended lines (but otherwise correct)<br>eg3 3 extended lines (but otherwise correct) |      | M3A0<br>M2A0<br>M1A0   |



| Q    | Answer   | Mark | Comments  |
|------|--|------|---|
| 6(a) | $f(x) \geq -7$ or $-7 \leq f(x)$   | B1   |   |
|      | <b>Additional Guidance</b>   |      |   |
|      | $f(x)$ may be replaced by $y$ or $f$ or $fx$ or $g(x)$ or $g$ or $gx$ or $x^2 - 7$   |      |   |
|      | $x \geq -7$  |      | B0  |
|      | $\geq -7$  |      | B0  |
|      | Condone $-7 \leq f(x) < \infty$ or $-7 \leq f(x) \leq \infty$ or $-7 \leq f(x) <$ or $-7 \leq f(x) \leq$   |      | B1  |
|      | $[-7, \infty)$ or $[-7, \infty]$   |      | B0  |
| 6(b) | $-11 \leq g(x) \leq 13$<br>or<br>$13 \geq g(x) \geq -11$   | B2   | B1 $g(x) \geq -11$ or $g(x) \leq 13$ on their own<br>or embedded within an inequality<br>or<br>$-11 < g(x) < 13$<br>or $[-11, 13]$<br>or $-11 \leq x \leq 13$ |
|      | <b>Additional Guidance</b>   |      |   |
|      | $g(x)$ may be replaced by $y$ or $g$ or $gx$ or $f(x)$ or $f$ or $fx$ or $1 - 3x$<br>in B2 or B1 responses   |      |   |
|      | $g(x) \geq -11$ $g(x) \leq 13$   |      | B1  |
|      | $-11$ to $13$ inclusive ('inclusive' must be seen)<br>Do not allow if 24 also seen   |      | B1  |
|      | B1 may be seen with an incorrect inequality<br>eg1 $-11 < g(x) \leq 13$<br>eg2 $-11 \leq g(x) < 13$<br>eg3 $0 < g(x) \leq 13$<br>eg4 $13 \leq g(x) \geq -11$ |      | B1<br>B1<br>B1<br>B1  |
|      | $[-11, 13)$ or $(-11, 13]$ or $(-11, 13)$  |      | B0  |
|      | $-11 < x \leq 13$ or $-11 \leq x < 13$ or $-11 < x < 13$   |      | B0  |
|      | $\{-11, -10, -9, \dots, 0, 1, 2, 3, \dots, 12, 13\}$   |      | B0  |

| Q    | Answer  | Mark | Comments   |
|------|---|------|--|
| 6(c) | $2x^2 - 14$   | M1   |  |
|      | $2x^2 + 3x - 15 (= 0)$<br>or $-2x^2 - 3x + 15 (= 0)$<br>or $2x^2 + 3x = 15$<br>or $-2x^2 - 3x = -15$                                    | A1   |  |
|      | $\frac{-3 \pm \sqrt{3^2 - 4 \times 2 \times -15}}{2 \times 2}$ or $\frac{-3 \pm \sqrt{9 + 120}}{4}$<br>or $\frac{-3 \pm \sqrt{129}}{4}$ | M1   | oe eg $-\frac{3}{4} \pm \sqrt{\frac{15}{2} + \left(\frac{3}{4}\right)^2}$<br>correct method to solve their 3-term quadratic<br>implied by correct solutions to their 3-term quadratic to at least 2 dp |
|      | 2.089    -3.589   | A1ft | correct or ft M1A0M1 or M0A0M1<br>must both be rounded to 3 decimal places   |
|      | <b>Additional Guidance</b>  |      |  |
|      | 2nd M1 Allow correct factorisation of their 3-term quadratic if it does factorise   |      |  |
|      | 2nd M1 Allow correct use of formula even if discriminant is negative  |      |  |
|      | Two 'correct' solutions to at least 2 decimal places implies M1A1M1<br>eg 2.09 and -3.59  |      | M1A1M1A0   |
|      | 2.089 and -3.589 in working but only one on answer line   |      | M1A1M1A0   |
|      | Answers only 2.089    -3.589  |      | M1A1M1A1   |
|      | Answer only 2.089   |      | Zero   |
|      | Answer only -3.589  |      | Zero   |
|      | $2x^2 - 7$ from incorrect expansion leading to 1.386    -2.886  |      | M0A0M1A1ft   |
|      | $x^2 - 14$ from incorrect expansion leading to 2.653    -5.653  |      | M0A0M1A1ft   |
|      | $2x^2 - 14$ and $2x^2 + 3x - 13 (= 0)$<br>Answers 1.908    -3.408   |      | M1A0<br>M1A1ft   |

| Q | Answer   | Mark | Comments  |
|---|--|------|---|
| 7 | $\frac{1}{2} \times (8 + 4) \times a (= 63)$<br>or $\frac{1}{2} \times 12 \times a (= 63)$<br>or $6a (= 63)$<br>or $63 \div 6$ | M1   | any letter<br>oe eg $12a = 126$<br>or $\frac{1}{2} \times 3 \times a + 4 \times a + \frac{1}{2} \times 1 \times a (= 63)$ |
|   | 10.5 or $10\frac{1}{2}$ or $\frac{21}{2}$  | A1   |   |
|   | <b>Additional Guidance</b>   |      |   |
|   | M1 is for a full area calculation (= 63)   |      |   |

| Q | Answer  | Mark  | Comments   |
|---|---|-------|--|
| 8 | <b>Alternative method 1</b>   |       |  |
|   | ( $x$ -coordinate of $P$ ) 5.5<br>and<br>( $y$ -coordinate of $P$ ) 2.25  | B2    | oe may be seen on diagram<br>B1 ( $x$ -coordinate of $P$ ) 5.5<br>or<br>( $y$ -coordinate of $P$ ) 2.25<br>or<br>$x$ -coordinate of $P$ = 2.25<br>and $y$ -coordinate of $P$ = 5.5 |
|   | $(9 - \text{their } 5.5)^2 + (12 - \text{their } 2.25)^2$<br>or $3.5^2 + 9.75^2$<br>or $12.25 + 95.06(25)$<br>or $12.25 + 95.063$<br>or $107.3(125)$ or $107.313$ | M1    | oe eg $\sqrt{3.5^2 + 9.75^2}$ or $\frac{1717}{16}$<br><br>$1 < \text{their } 5.5 < 7$<br>$1 < \text{their } 2.25 < 6$  |
|   | 10.36   | A1ft  | correct or ft their 5.5 and/or their 2.25<br>must be rounded to 4 sig figs   |
|   | <b>Alternative method 2</b> Uses $AC = 10$ , $BC = \sqrt{125}$ or $5\sqrt{5}$ or 11.18... and $AB = \sqrt{61}$ or 7.81...   |       |  |
|   | $\cos^{-1} \frac{10^2 + 7.81...^2 - 11.18...^2}{2 \times 10 \times 7.81...}$<br>or [76.67, 76.71]   | M1    | oe eg $\cos^{-1} 0.23(0...)$ or $\cos^{-1} 0.231$<br>may be on diagram<br>angle $BAC$  |
|   | $(0.75 \times 7.81...)^2 + 10^2$<br>$- 2 \times (0.75 \times 7.81...) \times 10 \times \cos \text{their}$<br>[76.67, 76.71]                                       | M1dep |  |
|   | [107.3, 107.4]  | A1    |  |
|   | 10.36   | A1    |  |

Mark scheme and additional guidance continues on the next page

| Q | Answer   | Mark  | Comments   |
|---|--|-------|--|
| 8 | <b>Alternative method 3</b> Uses $AC = 10$ , $BC = \sqrt{125}$ or $5\sqrt{5}$ or $11.18\dots$ and $AB = \sqrt{61}$ or $7.81\dots$                  |       |  |
|   | $\cos^{-1} \frac{7.81\dots^2 + 11.18\dots^2 - 10^2}{2 \times 7.81\dots \times 11.18\dots}$<br>or [60.49, 60.66]                                    | M1    | oe eg $\cos^{-1} [0.49, 0.4925]$<br>may be on diagram<br>angle $ABC$ |
|   | $(0.25 \times 7.81\dots)^2 + 11.18\dots^2$<br>$- 2 \times (0.25 \times 7.81\dots) \times 11.18\dots$<br>$\times \cos \text{their } [60.49, 60.66]$ | M1dep |  |
|   | [107.3, 107.4]   | A1    |  |
|   | 10.36  | A1    |  |
|   | <b>Additional Guidance</b>   |       |  |
|   | If 5.5 is from gradient $BC$   |       | B0   |
|   | Alt 1 $P(4.5, 3.75)$<br>$(9 - 4.5)^2 + (12 - 3.75)^2$<br>9.397   |       | B0<br>M1<br>A1ft   |

| Q | Answer  | Mark  | Comments  |
|---|---|-------|---|
| 9 | $\frac{2x^6}{3}$ or $\frac{2}{3}x^6$<br>or<br>$\frac{15x}{3}$ or $5x$                 | M1    | implied by $\frac{2x^6 + a}{3}$ or $\frac{b + 15x}{3}$<br>a can be numerical or algebraic<br>b can be numerical or algebraic<br>allow 0.66... or 0.67 for $\frac{2}{3}$ |
|   | $6 \times \frac{2x^5}{3}$ or $\frac{12x^5}{3}$ or $4x^5$<br>or<br>$\frac{15}{3}$ or 5 | M1dep | correct differentiation of one correct term<br>implied by $\frac{6 \times 2x^5 + a}{3}$ or $\frac{b + 15}{3}$   |
|   | $4x^5 + 5 = 133$<br>or $4x^5 = 128$<br>or $x^5 = 32$<br>or $\sqrt[5]{32}$             | A1    | oe<br>both correct terms differentiated and simplified correctly and equated to 133   |
|   | 2   | A1    |   |
|   | <b>Additional Guidance</b>  |       |   |
|   | $\frac{14x^6 + 30x}{3}$   |       | Zero  |

| Q  | Answer  | Mark | Comments                                       |
|----|---|------|--|
| 10 | $\begin{pmatrix} a & b \\ 2a & 3b \end{pmatrix} \begin{pmatrix} 1 \\ -3 \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$ | M1   | oe<br>implied by a correct equation            |
|    | $a - 3b = 1$  | A1   | oe<br>may be implied by correct answers        |
|    | $2a - 9b = 4$   | A1   |  |
|    | Correct elimination of a variable from their 2 linear equations with both equations having the same two variables             | M1   | eg $3a - 2a = 3 - 4$<br>or $-6b - -9b = 2 - 4$ |
|    | $a = -1 \quad b = -\frac{2}{3}$   | A1   | must be exact values                           |

**Additional guidance continues on the next page**

| Q  | Answer  | Mark                                     | Comments |
|----|---|--|----------|
| 10 | <b>Additional Guidance</b>  |  |          |
|    | $\begin{pmatrix} a-3b \\ 2a-9b \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$ implies M1 but does not imply any correct equations  |  |          |
|    | If the same method is used for both $a$ and $b$ (eg equates coefficients and eliminates a variable), mark the attempt that favours the student  |  |          |
|    | Ignore commas and lines within matrices   |  |          |
|    | Allow missing brackets if arrays are correct dimensions   |  |          |
|    | Answers only $a = -1$ and $b = -\frac{2}{3}$  | 5 marks                                  |          |
|    | Allow use of $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$ as a misread<br>If solution $a = -1$ and $b = \frac{2}{3}$  | M1A0A0<br>M1A1ft<br>(A1ft after misread) |          |
|    | Transposing $\begin{pmatrix} 1 \\ -3 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 4 \end{pmatrix}$ is not a misread (could still score 2nd M1)  |  |          |
|    | $\begin{pmatrix} 1 \\ -3 \end{pmatrix} \begin{pmatrix} a & b \\ 2a & 3b \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$ followed by $a - 3b = 1$ and $2a - 9b = 4$<br>(could still score 2nd M1)      | M1A1A1                                   |          |
|    | $\begin{pmatrix} 1 \\ -3 \end{pmatrix} \begin{pmatrix} a & b \\ 2a & 3b \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$ with only one of $a - 3b = 1$ and $2a - 9b = 4$<br>(could still score 2nd M1) | M1A1A0                                   |          |
|    | $\begin{pmatrix} 1 \\ -3 \end{pmatrix} \begin{pmatrix} a & b \\ 2a & 3b \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$ with neither equation correct #<br>(could still score 2nd M1)                 | M0A0A0                                   |          |



| Q  | Answer   | Mark  | Comments   |
|----|--|-------|--|
| 11 | <b>Alternative method 1</b> expands $(x + 2)(x + 3)$ first |       |  |
|    | $x^2 + 3x + 2x + 6$ or $x^2 + 5x + 6$                      | M1    | oe<br>must have a term in $x^2$<br>allow one error but no omissions or extras<br>implied by $x^2 + 5x + k$ or $ax^2 + 5x + 6$  |
|    | $x^3 + 5x^2 + 6x + 4x^2 + 20x + 24$                        | M1dep | oe eg<br>$x^3 + 3x^2 + 2x^2 + 6x + 4x^2 + 12x + 8x + 24$<br>allow one further error but no omissions or extras                 |
|    | $x^3 + 9x^2 + 26x + 24$                                    | A1    |  |
|    | <b>Alternative method 2</b> expands $(x + 3)(x + 4)$ first |       |  |
|    | $x^2 + 3x + 4x + 12$ or $x^2 + 7x + 12$                    | M1    | oe<br>must have a term in $x^2$<br>allow one error but no omissions or extras<br>implied by $x^2 + 7x + k$ or $ax^2 + 7x + 12$ |
|    | $x^3 + 7x^2 + 12x + 2x^2 + 14x + 24$                       | M1dep | oe eg<br>$x^3 + 3x^2 + 4x^2 + 12x + 2x^2 + 6x + 8x + 24$<br>allow one further error but no omissions or extras                 |
|    | $x^3 + 9x^2 + 26x + 24$                                    | A1    |  |

Mark scheme and additional guidance continues on the next page

| Q  | Answer   | Mark  | Comments  |
|----|--|-------|---|
| 11 | <b>Alternative method 3</b> expands $(x + 2)(x + 4)$ first   |       |   |
|    | $x^2 + 4x + 2x + 8$ or $x^2 + 6x + 8$  | M1    | oe<br>must have a term in $x^2$<br>allow one error but no omissions or extras<br>implied by $x^2 + 6x + k$ or $ax^2 + 6x + 8$ |
|    | $x^3 + 6x^2 + 8x + 3x^2 + 18x + 24$  | M1dep | oe eg<br>$x^3 + 4x^2 + 2x^2 + 8x + 3x^2 + 12x + 6x + 24$<br>allow one further error but no omissions or extras                |
|    | $x^3 + 9x^2 + 26x + 24$  | A1    |   |
|    | <b>Additional Guidance</b>   |       |   |
|    | For M marks terms may be seen in a grid (+ signs not needed)   |       |   |
|    | Correct answer followed by further work  |       | M2A0  |
|    | Ignore further simplification after 4 terms seen<br>eg Alt 1 $x^2 + 3x + 2x + 6 = x^2 + 6x + 6$<br>$(x^2 + 6x + 6)(x + 4) \rightarrow x^3 + 4x^2 + 6x^2 + 24x + 6x + 18$ (error) |       | M1<br>M1depA0   |
|    | Second M1<br>Must be the product of a two term bracket and a three or four term bracket  |       |   |
|    | Missing brackets may be recovered  |       |   |

| Q     | Answer  | Mark  | Comments   |
|-------|---|-------|--|
| 12(a) | Valid common denominator with at least one numerator correct                  | M1    | eg $\frac{7x}{9x^2}$ and $\frac{a}{9x^2}$<br>or $\frac{7x+a}{9x^2}$<br>or $\frac{b}{9x \times 3x^2}$ and $\frac{2 \times 9x}{9x \times 3x^2}$<br>numerators and denominators may be seen as products<br>a can be numerical or algebraic<br>b can be numerical or algebraic |
|       | Valid common denominator with both numerators correct                         | M1dep | $\frac{7x}{9x^2}$ and $\frac{6}{9x^2}$<br>or $\frac{7 \times 3x^2}{9x \times 3x^2}$ and $\frac{2 \times 9x}{9x \times 3x^2}$<br>numerators and denominators may be seen as products  |
|       | $\frac{7x+6}{9x^2}$ or $\frac{7x+6}{(3x)^2}$<br>with no further work          | A1    |  |
|       | <b>Additional Guidance</b>  |       |  |
|       | $\frac{21x^2+18x}{27x^3}$ or $\frac{21x+18}{27x^2}$ or $\frac{7x^2+6x}{9x^3}$ |       | M2A0   |
|       | $\frac{7x^{-1}+6x^{-2}}{9}$   |       | M2A0   |
|       | $7x + 6 / 9x^2$   |       | M2A0   |

| Q     | Answer   | Mark | Comments   |
|-------|--|------|--|
| 12(b) | Changes division to multiplication<br>and inverts to $\frac{3x+12}{x^2}$   | M1   | may be implied   |
|       | $(3x+12 \Rightarrow) 3(x+4)$   | M1   | may be implied   |
|       | Correct expression written as a single fraction or a product<br>must have<br>factor $(x+4)$ in a numerator and denominator $x+4$<br>or<br>correct expression written as a single fraction or a product<br>must have denominator $x^3$ or $x^2$ or $x$ or 1 | A1   | may be implied by final A1<br>eg $\frac{3x(x+2)(x+4)}{x+4}$ or $\frac{(3x^2+6x)(x+4)}{x+4}$<br>or $\frac{x}{x+4} \times \frac{x+2}{1} \times 3(x+4)$<br>or $\frac{x}{x+4} \times 3(x+2)(x+4)$<br>or $\frac{3x^4(x+2)}{x^3}$ or $x^4 \times \frac{x+2}{x} \times \frac{3}{x^2}$<br>or $\frac{(x+2)}{x^3} \times 3x^4$<br>or $\frac{3x^3(x+2)}{x^2}$<br>or $\frac{3x^2(x+2)}{x}$<br>or $\frac{3x(x+2)}{1}$<br>or $x \times (x+2) \times 3$<br>or $3x(x+2)$ |
|       | $3x^2+6x$  | A1   | SC2 $\frac{x(x+2)(3x+12)}{x+4}$  |
|       | <b>Additional Guidance</b>   |      |  |
|       | The list of examples in the first A1 is not exhaustive   |      |  |
|       | $3x^2+6x$ with no incorrect working  |      | 4 marks  |

| Q     | Answer  | Mark | Comments  |
|-------|---|------|---|
| 13(a) | 1   | B1   | allow in words  |
|       | <b>Additional Guidance</b>                      |      |   |
|       |   |      |   |
| 13(b) | 0   | B1   | allow in words eg none or zero  |
|       | <b>Additional Guidance</b>                      |      |   |
|       |   |      |   |
| 13(c) | (0, 1) (90, 0) (270, 0)<br>with no other points | B2   | B1 two answers, both correct<br>or three answers, two correct<br>or four answers, three correct |
|       | <b>Additional Guidance</b>                      |      |   |
|       | Condone 0, 1 for (0, 1) etc                     |      |   |
|       | 0, 90, 270                                      |      | B0  |
|       | (1, 0) (0, 90) (0, 270)                         |      | B0  |

| Q     | Answer   | Mark | Comments   |
|-------|--|------|--|
| 14(a) | $6pq^2r(2q - 3r + 4)$  | B2   | B1 correct factorised expression with a common factor involving at least two variables<br>eg $pq(12q^2r - 18qr^2 + 24qr)$<br>or $2q^2r(6pq - 9pr + 12p)$<br>or<br>common factor $6pq^2r$ with two out of the three terms in the bracket correct<br>eg $6pq^2r(2q - 3r + 4p)$ |
|       | <b>Additional Guidance</b>   |      |  |
|       | B2 answer followed by further work   |      | B1   |
|       | $6pq^2r(2q - 3r + 4)$ in working with $6qp^2r(2q - 3r + 4)$ on answer line       |      | B1   |
|       | B1 answer followed by further work   |      | B1   |
|       | $2q^2r(6pq - 9pr + 12p)$ in working with $2p^2r(6pq - 9pr + 12p)$ on answer line |      | B1   |
|       | Use of multiplication signs scores a maximum of B1                               |      |  |
|       | $qpq(12qr - 18r^2 + 24r)$  |      | B1   |
|       | $6pqrq(2q - 3r + 4)$   |      | B1   |

| Q     | Answer   | Mark | Comments   |
|-------|--|------|--|
| 14(b) | Correct factorised expression with a common factor   | M1   | eg $(y + 3) [6(y + 3)^4 + 4(y + 3)^3]$<br>or $2[3(y + 3)^5 + 2(y + 3)^4]$<br>or $2(y + 3)^2 [3(y + 3)^3 + 2(y + 3)^2]$ |
|       | $2(y + 3)^4 [3(y + 3) + 2]$<br>or $2(y + 3)^4 (3y + 9 + 2)$<br>or $(y + 3)^4 [6(y + 3) + 4]$<br>or $(y + 3)^4 (6y + 18 + 4)$<br>or $(y + 3)^4 (6y + 22)$ | A1   |  |
|       | $2(y + 3)^4 (3y + 11)$   | A1   |  |
|       | <b>Additional Guidance</b>   |      |  |
|       | Use of multiplication signs scores a maximum of M1A1A0   |      |  |
|       | Any combination of bracket shape may be used   |      |  |
|       | Correct answer followed by further work  |      | M1A1A0   |
|       | Incorrect notation eg $(y + 3)^4 2(3y + 11)$   |      | M1A1A0   |
|       | $(2)(y + 3)^4 (3y + 11)$ or $(2(y + 3)^4)(3y + 11)$  |      | M1A1A1   |
|       | Allow substitution eg $n = (y + 3)$ for M1A1 but must revert to $(y + 3)$ for final mark   |      |  |
|       | Missing brackets must be recovered eg $(y + 3)^4 6y + 22$ with M1 not seen   |      | Zero   |

| Q     | Answer  | Mark | Comments  |
|-------|---|------|---|
| 14(c) | $3(4 + 5x)(4 - 5x)$<br>or $3(-4 - 5x)(5x - 4)$<br>or $-3(4 + 5x)(5x - 4)$<br>or $-3(-4 - 5x)(4 - 5x)$                       | B2   | B1 Partial factorisation<br>eg $3(16 - 25x^2)$ or $-3(25x^2 - 16)$<br>or $(12 + 15x)(4 - 5x)$ or $(12 - 15x)(4 + 5x)$ |
|       | <b>Additional Guidance</b>  |      |   |
|       | Brackets in either order for B2 or B1   |      |   |
|       | $-(75x^2 - 48)$   |      | B0  |
|       | $(-5x + 4)$ is equivalent to $(4 - 5x)$ etc   |      |   |
|       | Incorrect notation eg $(4 + 5x)3(4 - 5x)$   |      | B1  |
|       | Use of surds<br>eg $(\sqrt{48} + \sqrt{75}x)(\sqrt{48} - \sqrt{75}x)$ or $(4\sqrt{3} + 5\sqrt{3}x)(4\sqrt{3} - 5\sqrt{3}x)$ |      | B1  |
|       | Use of multiplication signs scores a maximum of B1<br>eg $3 \times (4 + 5x)(4 - 5x)$  |      | B1  |
|       | B2 answer followed by further work  |      | B1  |
|       | B1 answer followed by further work  |      | B1  |
|       | Missing brackets must be recovered eg $3 \times 16 - 25x^2$   |      | B0  |



| Q  | Answer   | Mark  | Comments  |
|----|--|-------|---|
| 15 | $x^4 - 9x^2$   | M1    |   |
|    | $4x^3$ or $(-2) \times 9x$ or $(-18)x$   | M1    | differentiates at least one of their terms<br>their term must be a function of $x$ after an attempt to expand brackets<br>$4x^3 - 18x$ implies M2 |
|    | $4 \times (-2)^3 - 18 \times -2$<br>or $4 \times -8 - 18 \times -2$<br>or $-32 + 36$   | M1dep | oe<br>dep on 2nd M1<br>substitutes $-2$ into their $4x^3 - 18x$<br>their $4x^3 - 18x$ must be two terms, each a function of $x$                   |
|    | 4  | A1    |   |
|    | <b>Additional Guidance</b>   |       |   |
|    | Allow recovery of brackets   |       |   |
|    | $4 \times -2^3$ is allowed for $4 \times (-2)^3$   |       |   |
|    | 3rd M can still be awarded even if further substitution seen<br>eg1 $4x^3 - 18x$ $4 \times (-2)^3 - 18 \times -2$ and $4^4 - 9 \times 4^2 = 112$<br>Answer 112<br>eg2 $4x^3 - 18x$ $4 \times (-2)^3 - 18 \times -2 = 4$ and $4 \times 4^3 - 18 \times 4 = 184$<br>Answer 184 |       | M3<br>A0<br>M3<br>A0  |
|    | Only substituting $x = -2$ into second derivative can score a maximum of M1M1M0A0  |       |   |
|    | 4 followed by answer $y = 4$   |       | M3A1  |
|    | 4 followed by answer $y = 4x$  |       | M3A0  |
|    | Do not regard substitution of $x = 2$ as a misread   |       |   |
|    | Beware that finding the gradient of the line through $(-2, -20)$ and $(3, 0)$ gives answer 4   |       | Zero  |
|    | Beware that $(-18)x$ may come from wrong method<br>eg1 $2x(2x - 9) = 4x^2 - 18x$<br>eg2 $2x(x^2 - 9) = 2x^3 - 18x$   |       | Zero<br>Zero  |

| Q  | Answer   | Mark  | Comments  |
|----|--|-------|---|
| 16 | <b>Alternative method 1</b>  |       |   |
|    | $2(2 - 5x) + 3(3x - 1)$<br>or $4 - 10x$ or $9x - 3$  | M1    |   |
|    | $4 - 10x + 9x - 3 = 1 - x$   | M1dep |   |
|    | $(1 - x)^2 = 1 - 2x + x^2$   | A1    | must see working for M2   |
|    | $2 - 5x + 3x - 1 + x^2 = 1 - 2x + x^2$   | B1    |   |
|    | <b>Alternative method 2</b>  |       |   |
|    | $4(2 - 5x)^2 + 6(2 - 5x)(3x - 1)$<br>$+ 6(2 - 5x)(3x - 1) + 9(3x - 1)^2$   | M1    | oe<br>allow $+ 12(2 - 5x)(3x - 1)$ for<br>$+ 6(2 - 5x)(3x - 1) + 6(2 - 5x)(3x - 1)$   |
|    | $4(4 - 10x - 10x + 25x^2)$<br>$+ 6(6x - 2 - 15x^2 + 5x)$<br>$+ 6(6x - 2 - 15x^2 + 5x)$<br>$+ 9(9x^2 - 3x - 3x + 1)$<br>$= 16 - 40x - 40x + 100x^2 + 36x - 12$<br>$- 90x^2 + 30x + 36x - 12 - 90x^2$<br>$+ 30x + 81x^2 - 27x - 27x + 9$ | M1dep | oe<br>must see expansions<br>must see working for 1st M1<br>allow $+ 12(6x - 2 - 15x^2 + 5x)$ for<br>$+ 6(6x - 2 - 15x^2 + 5x)$<br>$+ 6(6x - 2 - 15x^2 + 5x)$ |
|    | $1 - 2x + x^2$   | A1    | must see working for M2   |
|    | $2 - 5x + 3x - 1 + x^2 = 1 - 2x + x^2$   | B1    |   |

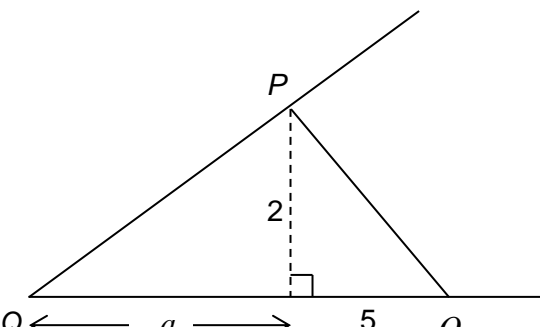
**Mark scheme and additional guidance continues on the next page**

| Q  | Answer  | Mark  | Comments                  |
|----|---|-------|---------------------------|
| 16 | <b>Alternative method 3</b>   |       |                           |
|    | $2(2 - 5x) + 3(3x - 1)$<br>or $4 - 10x$ or $9x - 3$   | M1    | oe                        |
|    | $(4 - 10x + 9x - 3)^2$<br>$= 16 - 40x + 36x - 12 - 40x + 100x^2$<br>$- 90x^2 + 30x + 36x - 90x^2 + 81x^2$<br>$- 27x - 12 + 30x - 27x + 9$ | M1dep | oe<br>must see expansions |
|    | $1 - 2x + x^2$  | A1    | must see working for M2   |
|    | $2 - 5x + 3x - 1 + x^2 = 1 - 2x + x^2$  | B1    |                           |
|    | <b>Additional Guidance</b>  |       |                           |
|    | Allow working down both sides of an equation/identity   |       |                           |
|    | M2A1 is for working on $(2A + 3B)^2$  |       |                           |
|    | B1 is for working on $A + B + C$  |       |                           |
|    | $1 - 2x + x^2$ with working for M2 seen and $2 - 5x + 3x - 1 + x^2 = x^2 - 2x + 1$  |       | 4 marks                   |
|    | $1 - x^2 = 1 - 2x + x^2$ (do not allow missing brackets even if recovered)  |       |                           |

| Q     | Answer                                  | Mark | Comments   |  |
|-------|---|------|--|--|
| 17(a) | $(-5)^2 + 2^2 = 29$                     | B1   | oe involving use of $-5$ and $2$<br>eg $(-5 - 0)^2 + (2 - 0)^2 = 29$<br>or $(0 - -5)^2 + (0 - 2)^2 = 29$<br>or $\sqrt{(-5)^2 + 2^2} = \sqrt{29}$<br>or $29 - (-5)^2 = 2^2$<br>or $29 - 2^2 = (-5)^2$<br>or $\sqrt{29 - (-5)^2} = 2$<br>or $\sqrt{29 - 2^2} = -5$ |  |
|       | Additional Guidance                     |      |  |  |
|       | $25 + 4 = 29$                           |      | B0   |  |
|       | $-5^2 + 2^2 = 29$                       |      | B0   |  |
|       | Allow 29 to be written as $\sqrt{29}^2$ |      |  |  |

| Q     | Answer  | Mark  | Comments   |
|-------|---|-------|--|
| 17(b) | <b>Alternative method 1</b> Using gradients   |       |  |
|       | (gradient $OP =$ )<br>$\frac{2-0}{-5-0}$ or $-\frac{2}{5}$ or $-0.4$  | M1    | oe<br>may be implied<br>eg $y = -\frac{2}{5}x$<br>or<br>gradient of tangent $= \frac{5}{2}$ (with gradient $OP$ not seen)  |
|       | (gradient tangent =)<br>their $-\frac{1}{2}$ or $\frac{5}{2}$ or $2.5$  | M1    | oe<br>correct or ft their $-\frac{2}{5}$   |
|       | $y - 2 = \text{their } \frac{5}{2}(x - -5)$<br>or<br>$0 - 2 = \text{their } \frac{5}{2}(x - -5)$<br>or<br>$2 = \text{their } \frac{5}{2} \times -5 + c$ | M1dep | oe<br>dep on 2nd M1<br>equation of their tangent with or without substitution of $y = 0$<br>implied by $y = \frac{5}{2}x + \frac{29}{2}$ oe<br>or $0 = \frac{5}{2}x + \frac{29}{2}$ oe |
|       | $-\frac{29}{5}$ or $-5.8$   | A1    | oe<br>allow $\left(-\frac{29}{5}, 0\right)$<br>SC2 answer $-10$ (grad tangent $= \frac{2}{5}$ )<br>SC2 answer $-\frac{21}{5}$ or $-4.2$ oe<br>(grad tangent $= -\frac{5}{2}$ )         |

Mark scheme and additional guidance continues on the next page

| Q     | Answer   | Mark  | Comments   |
|-------|--|-------|--|
| 17(b) | <b>Alternative method 2</b> Using similar triangles (see diagram in Additional Guidance)             |       |  |
|       | $\frac{a}{2} = \frac{2}{5}$  | M1    | oe equation<br>any letter  |
|       | $a = \frac{2}{5} \times 2$ or $a = \frac{4}{5}$  | M1dep |  |
|       | $-5 - \text{their } \frac{4}{5}$   | M1dep | dep on M2  |
|       | $-\frac{29}{5}$ or $-5.8$  | A1    | oe<br>allow $\left(-\frac{29}{5}, 0\right)$<br>SC2 answer $-10$ (grad tangent = $\frac{2}{5}$ )<br>SC2 answer $-\frac{21}{5}$ or $-4.2$ oe<br>(grad tangent = $-\frac{5}{2}$ ) |
|       | <b>Additional Guidance</b>   |       |  |
|       | Alt 1 2nd M mark is not dependent but there must be a numerical value for grad $OP$ to ft            |       |  |
|       | grad $OP = -0.4$ and grad tangent = $-0.4$   |       | M1M0M0A0   |
|       | $\left(0, -\frac{29}{5}\right)$  |       | M3A0   |
|       | Ignore any incorrect conversion between fraction and decimal after correct answer seen               |       |  |
|       | Alt 2 diagram<br> |       |  |

| Q     | Answer  | Mark  | Comments                                  |
|-------|---|-------|---|
| 18(a) | $-5 - 3 < 4x \leq 13 - 3$<br>or $-8 < 4x \leq 10$<br>or $-1.25 < x + 0.75 \leq 3.25$<br>or $x \leq 2.5$<br>or $x > -2$<br>or $x \leq 2$<br>or $x \geq -1$   | M1    | could be embedded eg $-2 \leq x \leq 2.5$ |
|       | $\frac{\text{their } -8}{4} < x \leq \frac{\text{their } 10}{4}$<br>or<br>their $-1.25 - 0.75 < x \leq \text{their } 3.25 - 0.75$<br>or $-2 < x \leq 2.5$<br>or $-2 < x \leq 2$<br>or $-1 \leq x \leq 2.5$<br>or $-1 \leq x \leq 2$<br>or $x \leq 2.5$ and $x > -2$<br>or $x \leq 2$ and $x > -2$<br>or $x \leq 2.5$ and $x \geq -1$<br>or $x \leq 2$ and $x \geq -1$ | M1dep | oe eg $(-2, 2.5]$ or $[-1, 2.5]$          |
|       | $-1 \quad 0 \quad 1 \quad 2$<br>with no incorrect working   | A1    |   |
|       | <b>Additional Guidance</b>  |       |   |
|       | Answer only $-1 \quad 0 \quad 1 \quad 2$  |       | M2A1                                      |
|       | Answer only $-1 \quad 1 \quad 2$  |       | Zero                                      |
|       | $x = 2.5$ and $x = -2$ (from solving equations) followed by $-1 \quad 0 \quad 1 \quad 2$  |       | M2A1                                      |
|       | $x = 2.5$ and $x = -2$ (from solving equations)   |       | Zero                                      |
|       | $-1 \quad 0 \quad 1 \quad 2$ with no incorrect working and a correct inequality on answer line  |       | M2A1                                      |
|       | $-1 \quad 0 \quad 1 \quad 2$ in working but $-1 \quad 1 \quad 2$ on answer line   |       | M2A0                                      |
|       | Ignore repeated integers eg Answer only $-1 \quad 0 \quad 1 \quad 1 \quad 2 \quad 2$  |       | M2A1                                      |

| Q     | Answer  | Mark | Comments  |
|-------|---|------|---|
| 18(b) | $(x-4)(x-7)$<br>or $\frac{-11 \pm \sqrt{(-11)^2 - 4 \times 1 \times 28}}{2 \times 1}$<br>or $\frac{11}{2} \pm \sqrt{\frac{9}{4}}$ | M1   | oe  |
|       | Identifies 4 and 7  | A1   | may be on a graph or implied by an inequality using 4 and 7 |
|       | $x < 4 \quad x > 7$   | A1   | do not allow incorrect notation<br>eg $4 > x > 7$           |
|       | <b>Additional Guidance</b>  |      |   |
|       | $x < 4$ with M1 not scored  |      | Zero  |
|       | $x > 7$ with M1 not scored  |      | Zero  |
|       | Both $x < 4$ and $x > 7$ in working but only one on answer line   |      | M1A1A0  |
|       | $x < 4$ and $x > 7$   |      | M1A2  |
|       | $x < 4$ or $x > 7$  |      | M1A2  |



| Q  | Answer  | Mark | Comments   |
|----|---|------|--|
| 19 | <b>Alternative method 1 C(BA)</b>   |      |  |
|    | $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ and<br>$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ and<br>indication that $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ is the identity matrix   | B5   | for B5, products must be seen in correct order and results of products must be correct<br>B4 a B5 response with no indication that $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ is the identity matrix<br>B3 (reflection in $y = -x$ ) $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$<br>and (rotation) $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$<br>and (reflection in $x$ -axis) $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$<br>B2 Any two of the above<br>B1 Any one of the above |
|    | <b>Alternative method 2 (CB)A</b>   |      |  |
|    | $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ and<br>$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ and<br>indication that $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ is the identity matrix | B5   | for B5, products must be seen in correct order and results of products must be correct<br>B4 a B5 response with no indication that $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ is the identity matrix<br>B3 (reflection in $y = -x$ ) $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$<br>and (rotation) $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$<br>and (reflection in $x$ -axis) $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$<br>B2 Any two of the above<br>B1 Any one of the above |

Mark scheme continues on the next page

| Q  | Answer   | Mark | Comments  |
|----|--|------|---|
| 19 | <b>Alternative method 3</b> transforms a general point   |      |   |
|    | $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -y \\ -x \end{pmatrix}$ and<br>$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} -y \\ -x \end{pmatrix} = \begin{pmatrix} x \\ -y \end{pmatrix}$ and<br>$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} x \\ -y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$ and<br>indication that $\begin{pmatrix} x \\ y \end{pmatrix}$ has mapped to itself  | B5   | for B5, products must be seen in correct order and results of products must be correct<br>B4 a B5 response with no indication that $\begin{pmatrix} x \\ y \end{pmatrix}$ has mapped to itself<br>B3 (reflection in $y = -x$ ) $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$<br>and (rotation) $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$<br>and (reflection in $x$ -axis) $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$<br>B2 Any two of the above<br>B1 Any one of the above                        |
| 19 | <b>Alternative method 4</b> transforms the unit square   |      |   |
|    | $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$ $= \begin{pmatrix} 0 & 0 & -1 & -1 \\ 0 & -1 & -1 & 0 \end{pmatrix}$ and<br>$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & -1 & -1 \\ 0 & -1 & -1 & 0 \end{pmatrix}$ $= \begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & -1 & -1 \end{pmatrix}$ and<br>$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & -1 & -1 \end{pmatrix}$ $= \begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$ and<br>indication that unit square has mapped to itself | B5   | columns in 2 by 4 matrices can be in any order<br>for B5, products must be seen in correct order and results of products must be correct<br>B4 a B5 response with no indication that unit square has mapped to itself<br>B3 (reflection in $y = -x$ ) $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$<br>and (rotation) $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$<br>and (reflection in $x$ -axis) $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$<br>B2 Any two of the above<br>B1 Any one of the above |

Additional guidance continues on the next page

| Q  | Answer   | Mark | Comments |
|----|--|------|----------|
| 19 | <b>Additional Guidance</b>   |      |          |
|    | For B3, B2 and B1 the matrices must not be the answers to a product  |      |          |
|    | Must use matrix multiplication   |      |          |
|    | Ignore commas and lines within matrices  |      |          |
|    | Allow missing brackets if arrays are correct   |      |          |
|    | Examples of indication<br>Alt 1 or 2 $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$ or $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ which is the identity matrix<br>Alt 3 $\begin{pmatrix} x \\ y \end{pmatrix}$ which is the same as the original<br>Alt 4 $\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$ which is the same as the original |      |          |
|    | Alts 1 and 2 Indications may be seen at the start of the solution  |      |          |
|    | Alt 3 $\begin{pmatrix} x \\ y \end{pmatrix}$ must be algebraic   |      |          |

| Q     | Answer   | Mark  | Comments   |
|-------|--|-------|--|
| 20(a) | <b>Alternative method 1</b>  |       |  |
|       | $12^2 + \left(\frac{10}{2}\right)^2$ or $12^2 + 5^2$<br>or $144 + 25$ or $169$ | M1    | oe<br>$RM^2$   |
|       | $\sqrt{\text{their } 169}$ or $13$   | M1dep | oe<br>$RM$<br>may be seen on diagram<br>13 implies M2      |
|       | $\tan x = \frac{7}{\text{their } 13}$  | M1dep | any letter<br>oe eg $\tan^{-1} \frac{7}{\text{their } 13}$ |
|       | $28(.3\dots)$  | A1    |  |

**Mark scheme and additional guidance continues on the next page**

| Q     | Answer   | Mark  | Comments  |
|-------|--|-------|---|
| 20(a) | <b>Alternative method 2</b>  |       |   |
|       | $12^2 + \left(\frac{10}{2}\right)^2 + 7^2$<br>or $12^2 + 5^2 + 7^2$<br>or $144 + 25 + 49$ or 218                           | M1    | oe<br>$UM^2$  |
|       | $\sqrt{\text{their } 218}$ or [14.76, 14.8]  | M1dep | oe<br>$UM$<br>may be seen on diagram<br>[14.76, 14.8] implies M2  |
|       | $\sin x = \frac{7}{\text{their } [14.76, 14.8]}$<br>or<br>$\cos x = \frac{\sqrt{12^2 + 5^2}}{\text{their } [14.76, 14.8]}$ | M1dep | any letter<br>oe eg $\sin^{-1} \frac{7}{\text{their } [14.76, 14.8]}$<br>or<br>$\sin x = \frac{\sin 90}{\text{their } [14.76, 14.8]} \times 7$<br>or<br>$\cos x = \frac{12^2 + 5^2 + \text{their } [14.76, 14.8]^2 - 7^2}{2 \times \sqrt{12^2 + 5^2} \times \text{their } [14.76, 14.8]}$ |
|       | 28(.3...)  | A1    |   |
|       | <b>Additional Guidance</b>   |       |   |
|       | Allow $\tan = \frac{7}{\text{their } 13}$ etc  |       |   |
|       | Do not allow $\tan \frac{7}{\text{their } 13}$ etc unless recovered  |       |   |
|       | If using sine or cosine rule, must rearrange to make $\sin x$ or $\cos x$ the subject                                      |       |   |
|       | Allow up to M1M1dep from either alt 1 or alt 2 even if not subsequently used   |       |   |

| Q     | Answer  | Mark | Comments   |
|-------|---|------|--|
| 20(b) | $\tan \alpha = \frac{10 \div 2}{12}$<br>or<br>$\tan \alpha = \frac{5}{12}$<br>or<br>$\sin \alpha = \frac{10 \div 2}{\text{their } 13}$<br>or<br>$\sin \alpha = \frac{5}{\text{their } 13}$<br>or<br>$\cos \alpha = \frac{12}{\text{their } 13}$ | M1   | any letter<br>oe eg $\tan^{-1} \frac{5}{12}$<br>their 13 = <i>RM</i> from (a)<br>may be seen on diagram<br>oe eg $\sin \alpha = \frac{\sin 90}{\sqrt{12^2 + 5^2}} \times 5$<br>or $\sin \alpha = \frac{\sin 90}{\text{their } 13} \times 5$<br>or $\cos \alpha = \frac{12^2 + 5^2 + 12^2 - 5^2}{2 \times 12 \times \sqrt{12^2 + 5^2}}$<br>or $\cos \alpha = \frac{\text{their } 13^2 + 12^2 - 5^2}{2 \times 12 \times \text{their } 13}$ |
|       | 22.6...   | A1ft | allow 23 with working<br>correct or ft<br>ft answers correct to at least 1 dp  |
|       | <b>Additional Guidance</b>  |      |  |
|       | Allow $\tan = \frac{5}{12}$ etc   |      |  |
|       | Do not allow $\tan \frac{5}{12}$ etc unless recovered   |      |  |
|       | If using sine or cosine rule, must rearrange to make $\sin \alpha$ or $\cos \alpha$ the subject   |      |  |

| Q  | Answer   | Mark | Comments  |
|----|--|------|---|
| 21 | $(-1, 3)$ or $(2, 1)$  | M1   |   |
|    | $(-1, 3)$ max(imum)  | A1   |   |
|    | $(2, 1)$ (point of) inflection   | A1   | SC1 $(3, -1)$ max(imum)<br>and $(1, 2)$ (point of) inflection |
|    | <b>Additional Guidance</b>   |      |   |
|    | One correct point and nature   |      | M1A1  |
|    | Ignore reference to 'stationary points' or 'turning points' or 'local' |      |   |
|    | Condone p o i for point of inflection                                  |      |   |

| Q     | Answer  | Mark | Comments  |
|-------|---|------|---|
| 22(a) | <b>Alternative method 1</b>   |      |   |
|       | Divides trigonometric expression by $\cos x$<br>or<br>rearranges equation                                       | M1   | eg<br>$8 \frac{\cos x}{\cos x} + 5 \frac{\sin x}{\cos x}$ or $8 + 5 \frac{\sin x}{\cos x}$<br>or $8 \cos x = -5 \sin x$<br>or $\frac{\cos x}{\sin x} = -\frac{5}{8}$<br>or $8 \frac{\cos x}{\cos x} = -5 \frac{\sin x}{\cos x}$<br>or $5 \tan x = -8$ |
|       | $\tan x = -\frac{8}{5}$ or $\tan x = -1.6$<br>or<br>$\cos x = -\frac{5}{\sqrt{89}}$<br>or $-57.9\dots$ or $-58$ | A1   | oe eg $\tan^{-1} -1.6$<br>may be implied by final answer  |
|       | 122.(0...)<br>with no other angle   | A1   |   |

Mark scheme and additional guidance continues on the next page



| Q     | Answer   | Mark   | Comments  |
|-------|--|--------|---|
| 22(a) | <b>Alternative method 2</b>  |        |   |
|       | Divides trigonometric expression by $\cos x$<br>or<br>rearranges equation                    | M1     | eg<br>$8 \frac{\cos x}{\cos x} + 5 \frac{\sin x}{\cos x}$ or $8 + 5 \frac{\sin x}{\cos x}$<br>or $8 \cos x = -5 \sin x$<br>or $\frac{\cos x}{\sin x} = -\frac{5}{8}$<br>or $8 \frac{\cos x}{\cos x} = -5 \frac{\sin x}{\cos x}$<br>or $5 \tan x = -8$ |
|       | $\sin x = \frac{8}{\sqrt{89}}$<br>or 57.9... or 58   | A1     | oe eg $\sin^{-1} \frac{8}{\sqrt{89}}$<br>may be implied by final answer   |
|       | 122.(0...)<br>with no other angle  | A1     |   |
|       | <b>Additional Guidance</b>   |        |   |
|       | Allow division of expression by $k \cos x$<br>eg ( $k = 8$ ) $1 + \frac{5 \sin x}{8 \cos x}$ | M1     |   |
|       | Answer only 122.(0...)   | M1A2   |   |
|       | Embedded answer 122.(0...)   | M1A1A0 |   |
|       | Answer only 121.9  | Zero   |   |
|       | If working seen, use the alt method for the working seen                                     |        |   |
|       | Answer only -58 (BOD alt 1)  | M1A1A0 |   |
|       | Answer only 58 (BOD alt 2)   | M1A1A0 |   |
|       | Allow cos for $\cos x$ etc   |        |   |
|       | Allow c for $\cos x$ etc   |        |   |

| Q     | Answer  | Mark | Comments                             |
|-------|---|------|--------------------------------------|
| 22(b) | <b>Alternative method 1</b>   |      |                                      |
|       | $6(1 - \cos^2 x) + 4 \cos^2 x$<br>or $6 - 6 \cos^2 x + 4 \cos^2 x$<br>or $2(1 - \cos^2 x) + 4$<br>or $6 - 2 \cos^2 x$ | M1   | oe expression in terms of $\cos^2 x$ |
|       | A = 6 and B = -2<br>with no incorrect working   | A1   |                                      |
|       | <b>Alternative method 2</b>   |      |                                      |
|       | $A \sin^2 x + A \cos^2 x + B \cos^2 x$<br>and A = 6 and A + B = 4   | M1   |                                      |
|       | A = 6 and B = -2<br>with no incorrect working   | A1   |                                      |
|       | <b>Additional Guidance</b>  |      |                                      |
|       | A = 6 and B = -2 with no working  |      | M1A1                                 |

| Q  | Answer  | Mark    | Comments   |
|----|---|---------|--|
| 23 | $(a^2 + 3) \times k$ or $ka^2 + 3k$   | M1      | oe eg $b = ka^2 + 3k$<br>may be seen on diagram  |
|    | $(ak)^2 + 3$ or $a^2k^2 + 3$  | M1      | oe eg $b = a^2k^2 + 3$<br>may be seen on diagram   |
|    | $ka^2 + 3k = a^2k^2 + 3$  | M1dep   | oe<br>equates and expands brackets correctly<br>dep on M2<br>may include $-b$ on each side   |
|    | $a^2(k - k^2) = 3 - 3k$<br>or $ka^2(1 - k) = 3 - 3k$<br>or $ka^2 - a^2k^2 = 3(1 - k)$<br>or $a^2(k - k^2) = 3(1 - k)$<br>or $ka^2(1 - k) = 3(1 - k)$<br>or $a^2(k^2 - k) = 3k - 3$<br>or $ka^2(k - 1) = 3k - 3$<br>or $k^2a^2 - ka^2 = 3(k - 1)$<br>or $a^2(k^2 - k) = 3(k - 1)$<br>or $ka^2(k - 1) = 3(k - 1)$ | M1dep   | oe eg $(a^2 =) \frac{3 - 3k}{k - k^2}$ or $(a =) (\pm) \sqrt{\frac{3 - 3k}{k - k^2}}$<br>or $(a^2 =) \frac{3k - 3}{k^2 - k}$ or $(a =) (\pm) \sqrt{\frac{3k - 3}{k^2 - k}}$<br>collects terms in $a^2$ and factorises correctly<br>on at least one side<br>must use $a^2$ as a factor if awarding mark<br>for factorising $ka^2 - a^2k^2$<br>dep on M3 |
|    | $(a^2 =) \frac{3(1 - k)}{k(1 - k)}$ or $(a^2 =) \frac{3}{k}$<br>or $(a =) (\pm) \sqrt{\frac{3(1 - k)}{k(1 - k)}}$   | M1dep   | oe eg $(a^2 =) \frac{3(k - 1)}{k(k - 1)}$<br>correct fraction with numerator and<br>denominator factorised correctly<br>dep on M4  |
|    | $(a =) \sqrt{\frac{3}{k}}$ or $(a =) \left(\frac{3}{k}\right)^{\frac{1}{2}}$  | A1      | oe eg $(a =) \frac{\sqrt{3}}{\sqrt{k}}$ or $(a =) \left(\frac{k}{3}\right)^{-\frac{1}{2}}$<br>$(a =) \pm \sqrt{\frac{3}{k}}$ M5A0 $(a =) -\sqrt{\frac{3}{k}}$ M5A0   |
|    | <b>Additional Guidance</b>  |         |  |
|    | Only one machine fully correct  | M1 only |  |
|    | Missing brackets must be recovered  |         |  |

| Q  | Answer  | Mark  | Comments   |
|----|---|-------|--|
| 24 | <b>Alternative method 1</b> Powers of 3   |       |  |
|    | $(3^2)^{0.5p}$ or $(3^3)^{2p-1}$<br>or<br>$3^{2 \times 0.5p+4}$                               | M1    | oe powers of 3<br>eg $3^p$ or $3^{6p-3}$<br>or<br>$3^{p+4}$<br>brackets not needed if intention clear<br>eg $3^{2^{0.5p}}$ |
|    | $(3^2)^{0.5p}$ and $3^4$ and $(3^3)^{2p-1}$<br>or<br>$3^{2 \times 0.5p+4}$ and $(3^3)^{2p-1}$ | M1dep | oe powers of 3<br>eg $3^p$ and $3^4$ and $3^{6p-3}$<br>or<br>$3^{p+4}$ and $3^{6p-3}$                                      |
|    | $2 \times 0.5p + 4 = 3(2p - 1)$<br>or<br>$p + 4 = 6p - 3$                                     | M1dep | oe equation<br>dep on M2   |
|    | 1.4 or $\frac{7}{5}$  | A1    | oe   |
|    | <b>Alternative method 2</b> Powers of 9   |       |  |
|    | $9^{0.5p+2}$ or $(9^{1.5})^{2p-1}$  | M1    | oe power of 9<br>eg $9^{3p-1.5}$<br>brackets not needed if intention clear<br>eg $9^{1.5^{2p-1}}$                          |
|    | $9^2$ and $(9^{1.5})^{2p-1}$<br>or<br>$9^{0.5p+2}$ and $(9^{1.5})^{2p-1}$                     | M1dep | oe powers of 9<br>eg $9^2$ and $9^{3p-1.5}$<br>or<br>$9^{0.5p+2}$ and $9^{3p-1.5}$   |
|    | $0.5p + 2 = 1.5(2p - 1)$<br>or<br>$0.5p + 2 = 3p - 1.5$                                       | M1dep | oe equation<br>dep on M2   |
|    | 1.4 or $\frac{7}{5}$  | A1    | oe   |

Mark scheme continues on the next page

| Q  | Answer  | Mark  | Comments  |
|----|---|-------|---|
| 24 | <b>Alternative method 3</b> Powers of 27  |       |   |
|    | $\left(27^{\frac{2}{3}}\right)^{0.5p}$  | M1    | oe power of 27<br>eg $27^{\frac{2}{3} \times 0.5p}$ or $27^{\frac{1}{3}p}$<br>brackets not needed if intention clear<br>eg $27^{\frac{2^{0.5}p}{3}}$  |
|    | $\left(27^{\frac{2}{3}}\right)^{0.5p}$ and $27^{\frac{4}{3}}$                                   | M1dep | oe powers of 27<br>eg $27^{\frac{2}{3} \times 0.5p}$ and $27^{\frac{4}{3}}$<br>or<br>$27^{\frac{1}{3}p}$ and $27^{\frac{4}{3}}$<br>M2 $27^{\frac{2}{3} \times 0.5p + \frac{4}{3}}$ or $27^{\frac{1}{3}p + \frac{4}{3}}$ |
|    | $\frac{2}{3} \times 0.5p + \frac{4}{3} = 2p - 1$<br>or<br>$\frac{1}{3}p + \frac{4}{3} = 2p - 1$ | M1dep | oe equation<br>dep on M2  |
|    | 1.4 or $\frac{7}{5}$  | A1    | oe  |

Mark scheme and additional guidance continues on the next page

| Q  | Answer  | Mark  | Comments  |
|----|---|-------|---|
| 24 | <b>Alternative method 4 Powers of 81</b>  |       |   |
|    | $(81^{0.5})^{0.5p}$ or $(81^{0.75})^{2p-1}$<br>or<br>$81^{0.5 \times 0.5p+1}$   | M1    | oe powers of 81<br>eg $81^{0.25p}$ or $81^{1.5p-0.75}$<br>or $81^{0.25p+1}$<br>brackets not needed if intention clear<br>eg $81^{0.5^{0.5p}}$ |
|    | $(81^{0.5})^{0.5p}$ and $(81^{0.75})^{2p-1}$<br>or<br>$81^{0.5 \times 0.5p+1}$ and $(81^{0.75})^{2p-1}$   | M1dep | oe powers of 81<br>eg $81^{0.25p}$ and $81^{1.5p-0.75}$<br>or<br>$81^{0.25p+1}$ and $81^{1.5p-0.75}$  |
|    | $0.5 \times 0.5p + 1 = 0.75(2p - 1)$<br>or<br>$0.25p + 1 = 1.5p - 0.75$   | M1dep | oe equation<br>dep on M2  |
|    | 1.4 or $\frac{7}{5}$  | A1    | oe  |
|    | <b>Additional Guidance</b>  |       |   |
|    | Mark positively if potentially more than one scheme used  |       |   |
|    | Answer 1.4  |       | M3A1  |
|    | Correct equation implies M3   |       |   |
|    | Just seeing expressions not in an equation and not as powers scores zero<br>eg Alt 1 $6p - 3$ and $p + 4$ not in an equation and not as powers of 3 |       | M0M0M0  |
|    | Allow recovery of missing brackets  |       |   |
|    | Use of logs with answer not 1.4 - escalate  |       |   |