AQA

**Level 2 Certificate in Further Mathematics
June 2015**
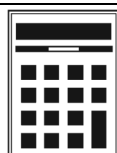
Further Mathematics   8360/2

Level 2

Paper 2   Calculator

Friday 19 June 2015   9.00 am to 11.00 am

For this paper you must have:
- a calculator
- mathematical instruments.

Time allowed
- 2 hours

Instructions
- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- In all calculations, show clearly how you work out your answer.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 105.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer book.
- The use of a calculator is expected but calculators with a facility for symbolic algebra must **not** be used.

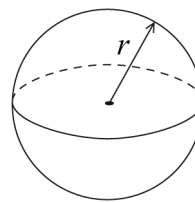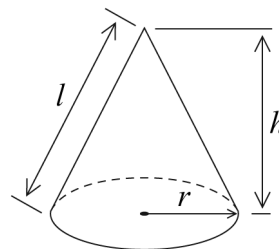**8360/2**

Formulae Sheet

Volume of sphere $= \frac{4}{3}\pi r^3$

Surface area of sphere $= 4\pi r^2$

Volume of cone $= \frac{1}{3}\pi r^2 h$

Curved surface area of cone $= \pi r l$

In any triangle ABC

Area of triangle $= \frac{1}{2}ab\sin C$

Sine rule $\quad \dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine rule $\quad a^2 = b^2 + c^2 - 2bc\cos A$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$
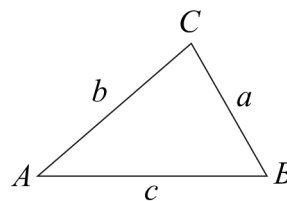
The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$, where $a \neq 0$, are given by

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

Trigonometric Identities

$$\tan\theta \equiv \frac{\sin\theta}{\cos\theta} \qquad \sin^2\theta + \cos^2\theta \equiv 1$$

Answer all questions in the spaces provided.

1 A circle, centre $(0, 0)$, has circumference 12π

Work out the equation of the circle.

$C = 2\pi r$
$12\pi = 2\pi r$
$r = 6$

[2 marks]

Answer $x^2 + y^2 = 36$

2 $a : b : c = 5 : 3 : 2$

Work out $4a - c : 3b$
Give your answer in its simplest form.

[2 marks]

$5a = 36 \quad 3b = 2c$
$2.5a = 1.5b = 1c$
$= 4 \times 2.5 - 1 : 3 \times 1.5$
$= 9 : 4.5$
$= 2 : 1$

Answer $2 : 1$

4

0 3

3 The distance between the points $(2, 5p)$ and $(2, -10)$ is 30 units.

Work out the **two** possible values of p. *Straight*

[3 marks]

$$5p --10 = 30$$
$$5p = 20$$
$$p = 4$$

$$-10 - 5p = 30$$
$$-5p = 40$$
$$p = -8$$

Answer .... 4 .... and .... -8 ....

4 The first term of a sequence is $1 - a$

The term-to-term rule of a sequence is

add $2a$ then multiply by 3

4 (a) Show that the second term is $3 + 3a$

[1 mark]

$= 3\left((1-a) + 2a\right)$

$= 3(1 + a)$

$= 3 + 3a$

4 (b) The third term is 16

Work out the value of a.

[3 marks]

$= 3(3 + 3a + 2a)$

$= 3(3 + 5a)$

$= 9 + 15a$

$9 + 15a = 16$

$15a = 7$

$a = 7/15$

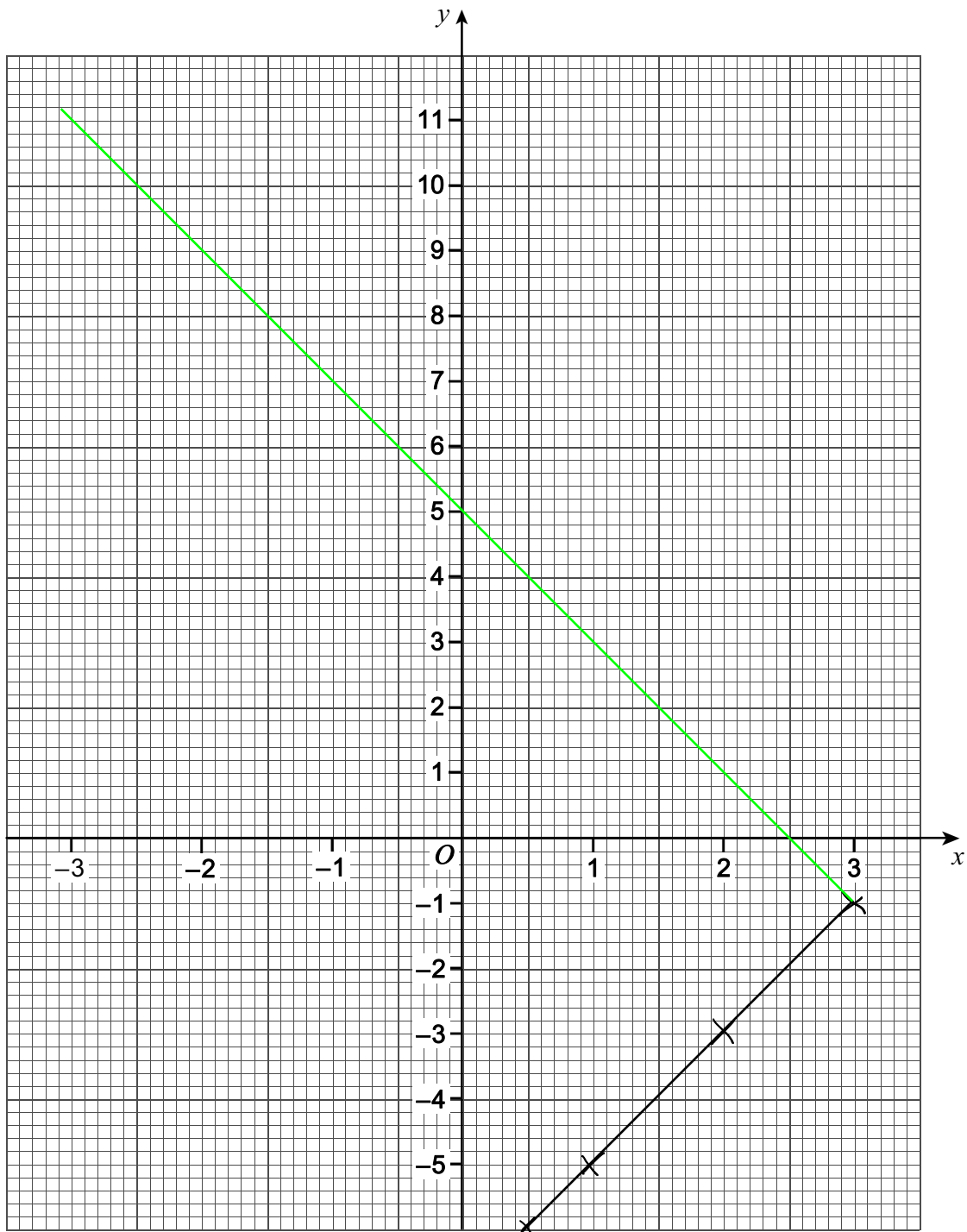Answer ....

7

Turn over ▶

5 A straight line L

is parallel to the straight line $y = 1 - 2x$
passes through $(3, -1)$ $m = -2$ OWA

On the grid below, draw the straight line L for values of x from −3 to 3.

[4 marks]

....

....

....

6   Write $\dfrac{15x^8 - 18x^7}{3x^2}$ in the form $ax^n - nx^a$ where a and n are integers.

[2 marks]

$$= \frac{3x^2(5x^6 - 6x^5)}{3x^2}$$

$$= 5x^6 - 6x^5$$

7 $y = \frac{2}{3}x^6 - 8x^3$

Work out the rate of change of y with respect to x when $x = -1$

[3 marks]

$$\frac{dy}{dx} = 4x^5 - 24x^2$$

$$x = -1 \quad 4 =$$

Answer -28

8 (a) $f(x) = x^4$

The domain of $f(x)$ is $x \geqslant 2$

Work out the range of $f(x)$.

[1 mark]

Answer $x \geqslant 16$

8 (b) $g(x) = x^2 - 1$

The domain of $g(x)$ is $-2 \leqslant x \leqslant 3$

Work out the range of $g(x)$.

[2 marks]

Answer $3 \leqslant x \leqslant 8$

8 (c) $h(x) = 5x - 3$

The **range** of $h(x)$ is $-2 < h(x) < 1$

Work out the domain of $h(x)$.

[2 marks]

$5x - 3 = -2$ | $5x - 3 = 1$

$5x = 1$ | $5x = 4$

$x = 0.2$ | $x = 0.8$

Answer $0.2 < x < 0.8$

9 (a) Solve $6(2y - 3) - 10 = 2y$

[3 marks]

$12y - 18 - 10 = 2y$

$14y = 28$

$y = 2$

$y = $ ....

9 (b) Solve $\dfrac{\sqrt{w + 4}}{2} = 6$

[3 marks]

$\sqrt{w+4} = 12$

$w + 4 = 144$

$w = 180$

$w = $ ....

9 (c) Solve $3m^{\frac{1}{5}} + 9 = 0$

[2 marks]

$3m^{1/5} = -9$

$m^{1/5} = -3$

$\sqrt[5]{m} = -3$

$m = \sqrt[5]{-3}$ $= (-3)^5$

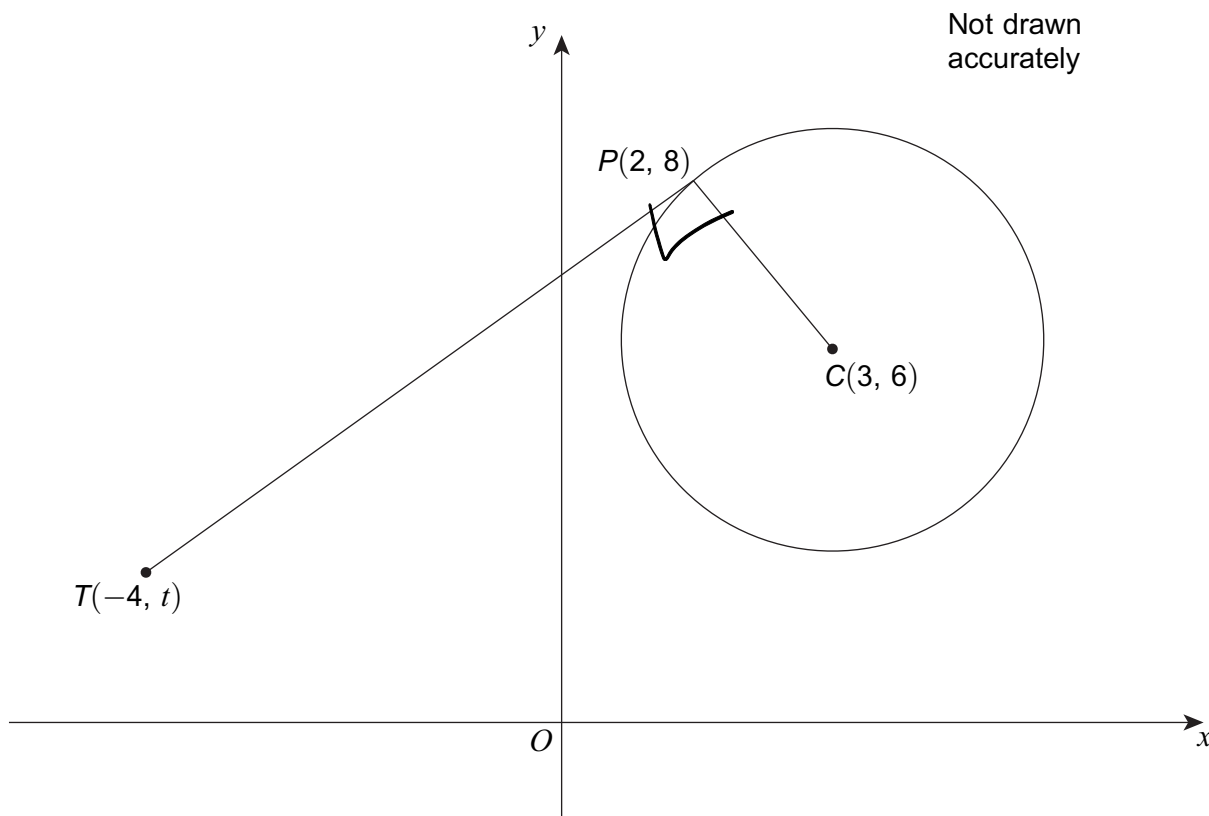$m = $ .... -243

10 The diagram shows a circle, centre *C*.
TP is a tangent to the circle at *P*.

Not drawn accurately

P(2, 8)

C(3, 6)

T(−4, *t*)

O

y

x

Work out the value of *t*.

[4 marks]

$$M_{PC} = \frac{6-8}{3-2} = \frac{-2}{1} = -2, \quad M_{PT} = 1/2$$

$$y = mx + c, \quad 8 = \frac{2}{2} + c$$

$$8 = 1 + c$$

$$c = 7 \Rightarrow y = x/2 + 7$$

$$t = -8/2 + 7$$

$$= -2 + 7$$

$$= 5$$

$$t = 5$$

Answer ....

1 0

11 (a) Expand and simplify $(3w + 2y)(w - 4y)$

[3 marks]

$= 3w^2 + 2wy - 4wy - 8y^2$

-12

Answer $3w^2 - 2wy - 8y^2$

11 (b) Expand and simplify $\dfrac{3}{x^2}\left(\dfrac{x}{3} + 3x^2 - 1\right)$

[3 marks]

$= \dfrac{3\left(\frac{x}{3} + 3x^2 - 1\right)}{x^2} = \dfrac{9x^2 + x - 3}{x^2} = 9 + \dfrac{1}{x} - \dfrac{3}{x^2}$

Answer ....

10

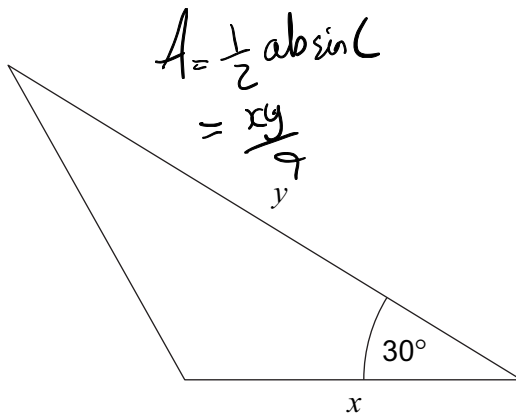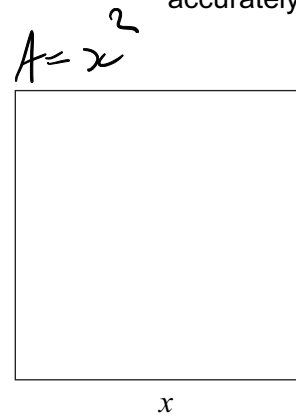**12** The area of the triangle is equal to the area of the square.
All dimensions are in centimetres.

Not drawn
accurately

$A = \frac{1}{2} ab \sin C$

$= \frac{xy}{4}$

y

$A = x^2$

30°

x

x

Write y in terms of x.

[2 marks]

$\frac{xy}{4} = x^2$

$xy = 4x^2$

$y = 4x$

Answer ....
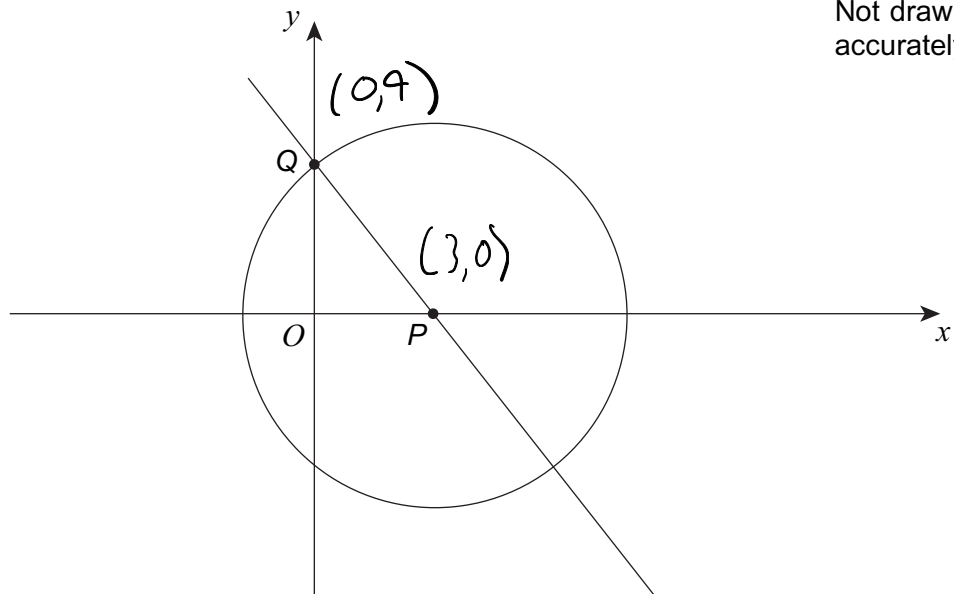
13 The diagram shows a circle, centre *P*, and a straight line passing through points *P* and *Q*.

Q lies on the *y*-axis and on the circumference of the circle.

The equation of the circle is $(x-3)^2 + y^2 = 25$

(0,4)

(3,0)

Not drawn accurately

Work out the equation of the straight line through *P* and *Q*.
Give your answer in the form $ax + by + c = 0$ where *a*, *b* and *c* are integers.

[4 marks]

$-3^2 + y^2 = 25$

$y = \sqrt{25-9} = \sqrt{16} = 4$

$m = -\frac{4}{3}$

$y = \frac{-4x}{3} + C$

$4 = 0 + C$

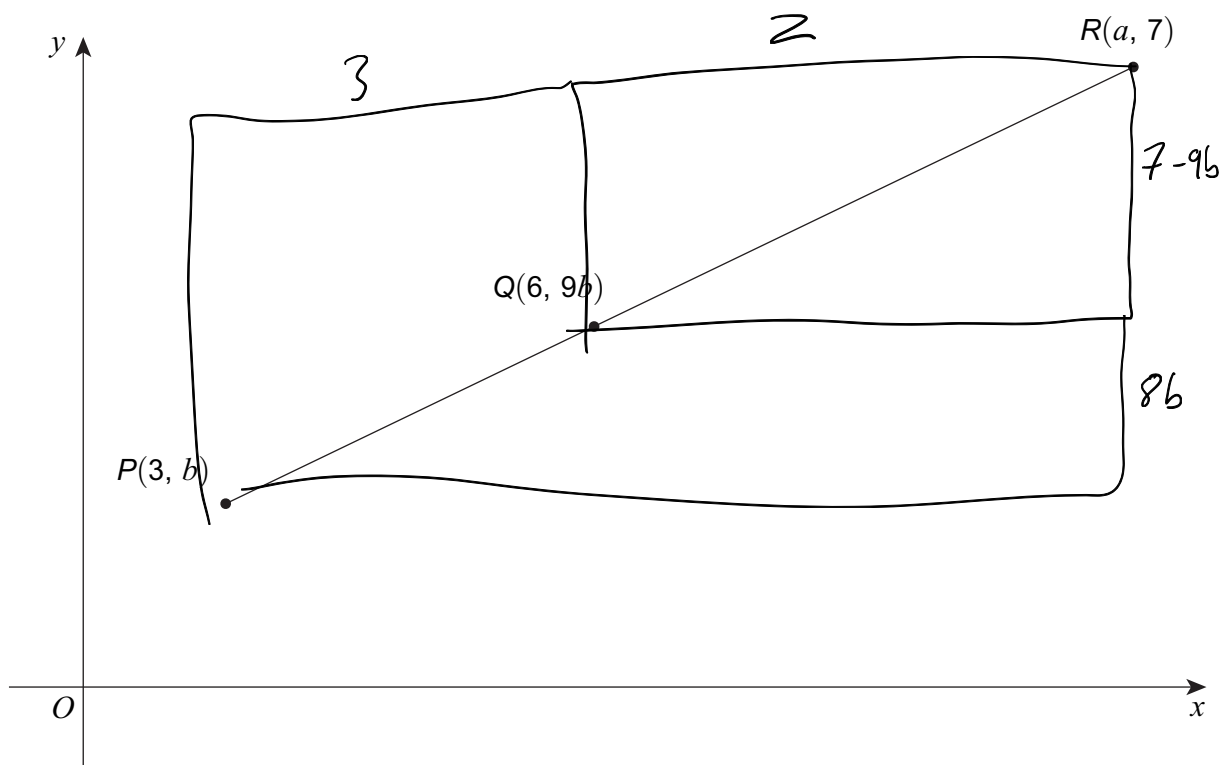$C = 4$

$4x + 3y - 12 = 0$

Answer $y = \frac{-4x}{3} + 4$

6

Turn over ▶

14   *PQR* is a straight line.
          PQ : *QR* is 2 : 3

Not drawn
accurately

3

2

$R(a, 7)$

$7 - 9b$

$Q(6, 9b)$

$8b$

$P(3, b)$

y

O

x

14 (a)   Show that $a = 10.5$

[2 marks]

$$3 \times 3 = 2 \times 2$$

$$2 = \frac{9}{2} = 4.5$$

$$R_x = Q_x + 2$$

$$= 6 + 4.5 = 10.5$$

14 (b)   Work out the value of b.

[3 marks]

$$3 \times 8b = 2(7-9b)$$

$$24b = 14 - 18b$$

$$42b = 14$$

$$b = \frac{1}{3}$$

Answer $\frac{1}{3}$

15   Use algebra to prove that the value of $\dfrac{8c^2 + 16}{3c^2 + 6} + \dfrac{1}{3}$ is an integer for all values of c.
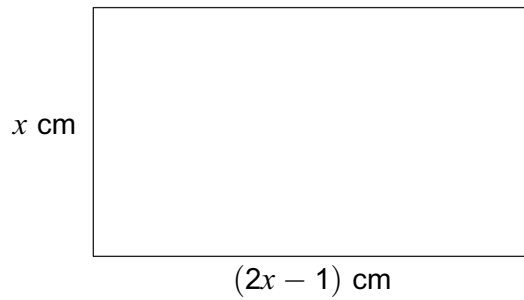
[3 marks]

$$= \frac{8(c^2+2)}{3(c^2+2)} + \frac{1}{3} = \frac{8}{3} + \frac{1}{3} = \frac{9}{3} = 3$$

8

16 The diagram shows a rectangle with area 9 cm^2

x cm

$(2x - 1)$ cm

Not drawn accurately

Set up and solve an equation to work out the value of x.
Give your answer to 3 significant figures.

[5 marks]

$(x)(2x-1) = 9$

$2x^2 - x - 9 = 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{1 \pm \sqrt{1 - 4 \times 2 \times -9}}{2 \times 2} = \dfrac{1 \pm \sqrt{73}}{4}$
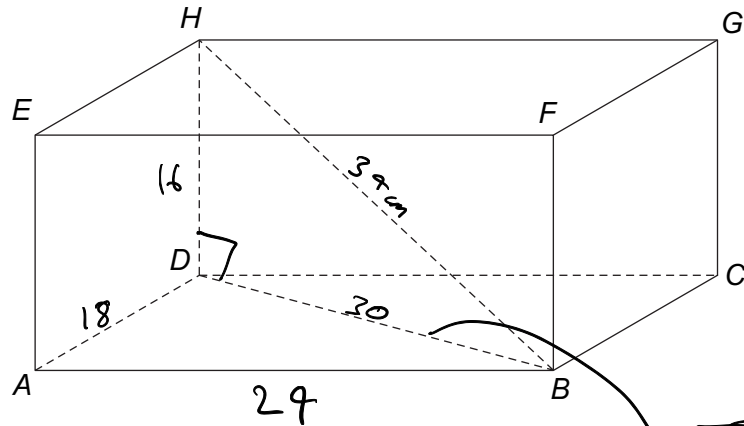
len not neg

$= 2.39$

$x = $ ....

17 *ABCDEFGH* is a cuboid.

H   G

E   F

34 cm

16

D   C

18   30

A   24   B

$\approx \sqrt{24^2 + 18^2}$

$= 30$

HB = 34 cm
HD = 16 cm
AD = 18 cm

17 (a) Work out the length of *AB*.

[3 marks]

$a^2 + b^2 + c^2 = d^2$

$AB = \sqrt{34^2 - 16^2 - 18^2}$

$= \sqrt{576}$

$= 24 \, cm$

Answer... cm

17 (b) Work out the angle between *HB* and *ABCD*.

[2 marks]

$a^2 = b^2 + c^2 - 2bc \cos A$

$A = \cos^{-1} \left(\frac{34^2 + 30^2 - 16^2}{2 \times 34 \times 30} \right)$

$= \cos^{-1} \left(\frac{15}{17} \right) = 28.1$

Answer... degrees
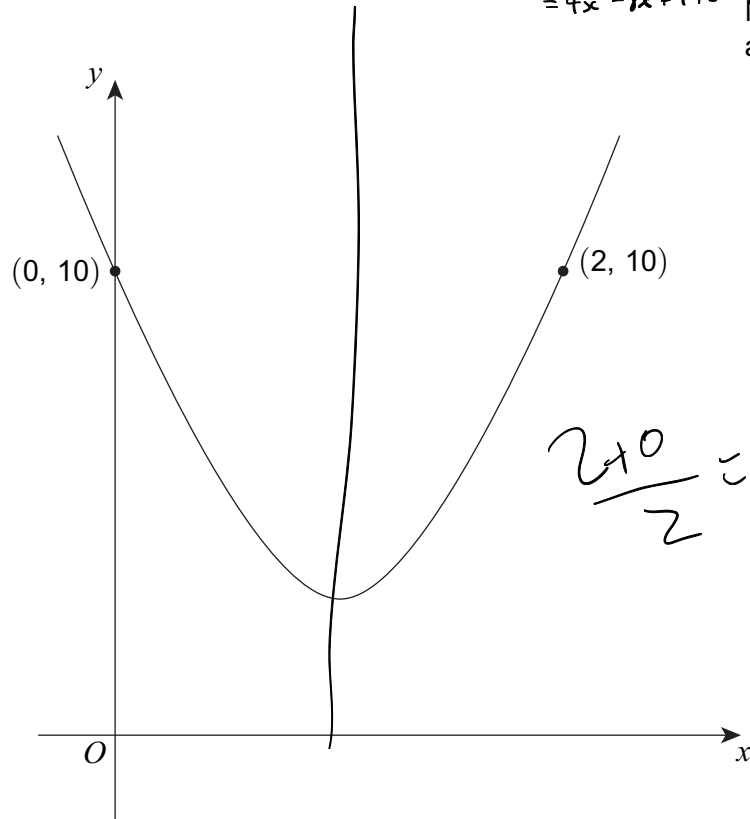
10

Turn over ▶

18 The sketch shows the quadratic curve $y = 4(x - a)^2 + b$
The curve passes through (0, 10) and (2, 10)

$y = 4(x^2 - 2x + 1) + b$
$= 4x^2 - 8x + 4 + b$

Not drawn accurately

(0, 10)

(2, 10)

$\dfrac{2 + 0}{2} = 1$

18 (a) Give reasons why the value of a is 1.

[2 marks]

Because the midpoint (at the minimum point) has an x of 1.

18 (b) Work out the value of b.

[2 marks]

$y = 4x^2 - 8x + 9 + b$

$10 = 0 - 0 + 9 + b$

$10 - 9 = b$

$b = 6$

Answer ....

18 (c) Write the equation of the curve in the form $y = px^2 + qx + r$

[2 marks]

$y = 4x^2 - 8x + 9 + 6$

Answer $y = 4x^2 - 8x + 10$

19 Use the factor theorem to show that $(x - 3)$ is **not** a factor of $x^3 - 10x - 3$

[2 marks]

$(x-3), 3$ is a factor, when $x = 3$ $y = 0$

$f(x) = 4x^2 - 8x + 10$

$f(3) = 36 - 24 + 10 = 22 \neq 0$

8

Turn over ▶

20 (a) The transformation matrix **P** represents a 90° anti-clockwise rotation about the origin.

Describe fully the **single** transformation represented by the matrix P^3

[2 marks]

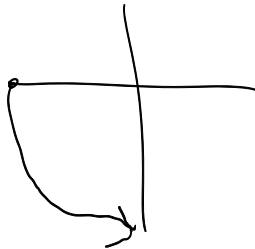a 90° clockwise turn around the origin.

20 (b) The transformation matrix **Q** is $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

The transformation matrix **R** is $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

Describe fully the **single** transformation represented by the matrix **QR**.

[2 marks]

$$= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$
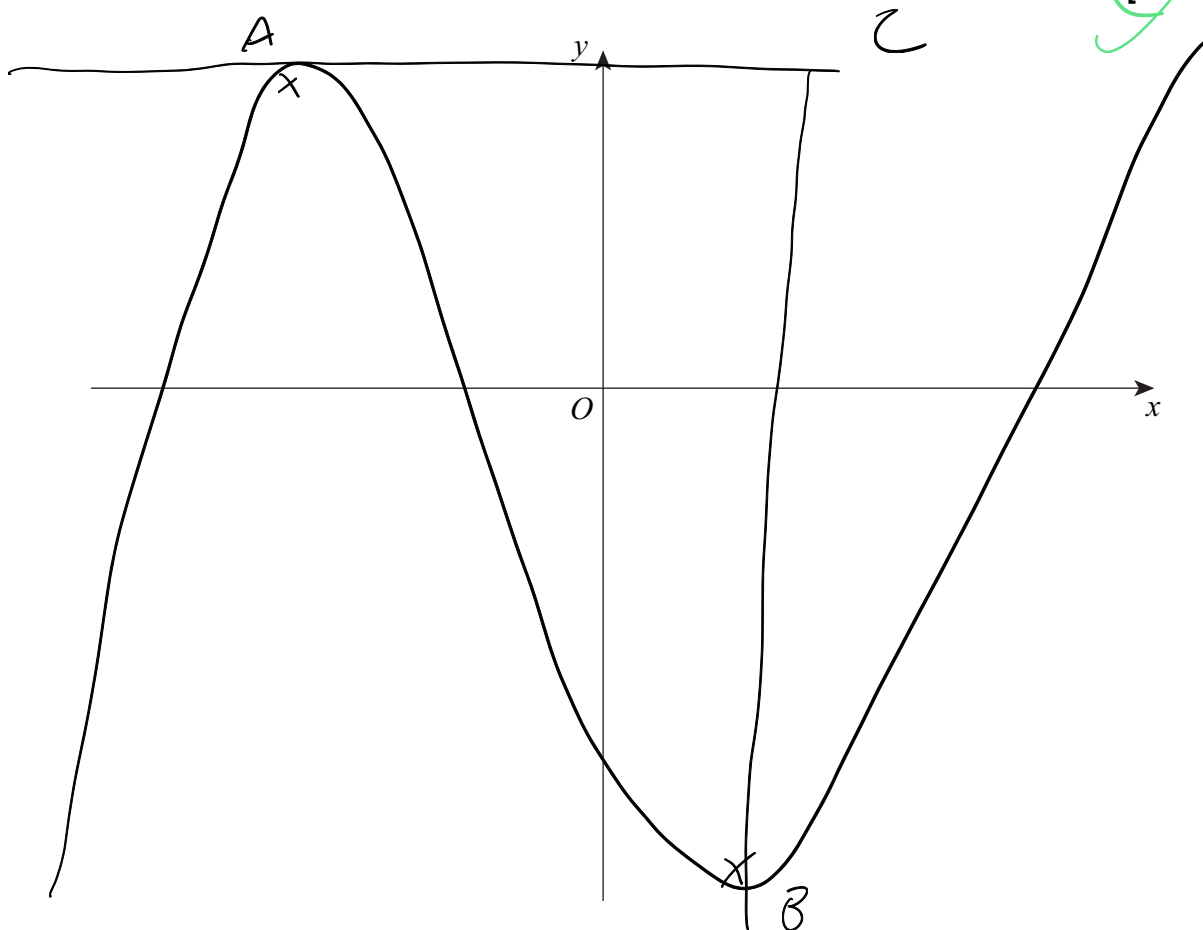
Rotate 90° AC around the origin.   ✗ SF = -1 enlargement

21 A cubic curve has

a maximum point at A $(-4, 10)$

a minimum point at B $(2, -26)$

The tangent to the curve at A and the normal to the curve at B intersect at point C.

Work out the area of triangle ABC.
You may sketch a diagram to help you.

[3 marks]

$$A = \frac{bh}{2} = \frac{\left|(-4 - 2)\right| \times \left|(10 - 26)\right|}{2} = \frac{6 \times 36}{2} = 108$$

Answer..square units

7

Turn over ▶

22 A quadratic sequence starts

$$302 \xrightarrow{+298} 600 \xrightarrow{+294} 894 \xrightarrow{+290} 1184 \quad \ldots\ldots$$

(with -4 above the $+298 \to +294$ and -4 above $+294 \to +290$)

22 (a) Work out an expression for the nth term.

[3 marks]

$$-2n^2: \quad -2 \quad -8 \quad -18 \quad -32$$

$$\text{seq}: \quad 302 \quad 600 \quad 894 \quad 1184$$

$$\text{rem}: \quad 304 \quad 608 \quad 912 \quad 1216$$

$$+304 \qquad +304 \qquad +304 \qquad +304$$

$$= -2n^2 + 304n$$

Answer ....

22 (b) A term in the sequence has value 0

Find the position of this term.

[2 marks]

$$-2n^2 + 304n = 0 \qquad n = -0 = 0$$

$$(-n)(2n - 304) = 0 \qquad 2n - 304 = 0$$

$$2n = 304$$

$$n = 152$$

Answer .... 0 or 152

0 not valid. Only provide 1 ans

23 The continuous curve $y = f(x)$ has exactly **two** stationary points.

P is a maximum point when $x = a$

Q is a stationary point of inflection when $x = b$

$a < b$

Which of these is correct?

Tick **one** box only.

[1 mark]

When $a < x < b$, $\dfrac{dy}{dx}$ is positive

and

when $x > b$, $\dfrac{dy}{dx}$ is positive

When $a < x < b$, $\dfrac{dy}{dx}$ is positive

and

when $x > b$, $\dfrac{dy}{dx}$ is negative

When $a < x < b$, $\dfrac{dy}{dx}$ is negative

and

when $x > b$, $\dfrac{dy}{dx}$ is positive

When $a < x < b$, $\dfrac{dy}{dx}$ is negative
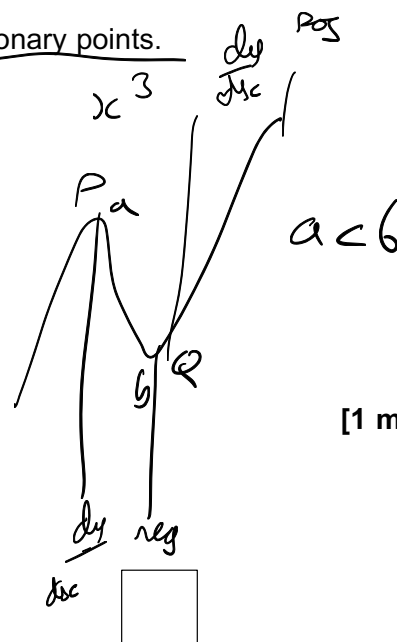
and

when $x > b$, $\dfrac{dy}{dx}$ is negative

6

Turn over ▶

24 $a^2 < 4$ and $a + 2b = 8$

Work out the range of possible values of b.
Give your answer as an inequality.

[4 marks]

$-2 < a < 2$

$-2 + 2b = 8$ $2 + 2b = 8$
$2b = 10$ $2b = 6$
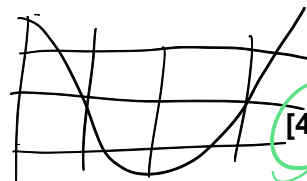$b = 5$ $b = 3$

Answer $3 < b < 5$

25 Work out the values of x between $0°$ and $360°$ for which

$$25\cos^2 x = 9$$

Give your answers to 1 decimal place.

[4 marks]

$\cos^2 x = \frac{9}{25}$

$\cos x = \frac{3}{5}$ | $\cos x = -\frac{3}{5}$

$x = 53.1$ | $x_3 = 127$

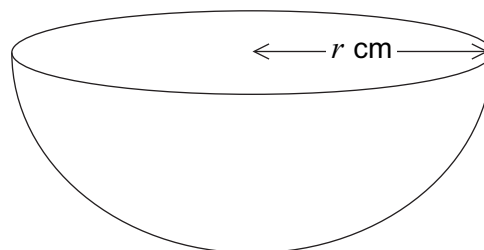$360 - x = x_2$ | $360 - x_3 = x_4$
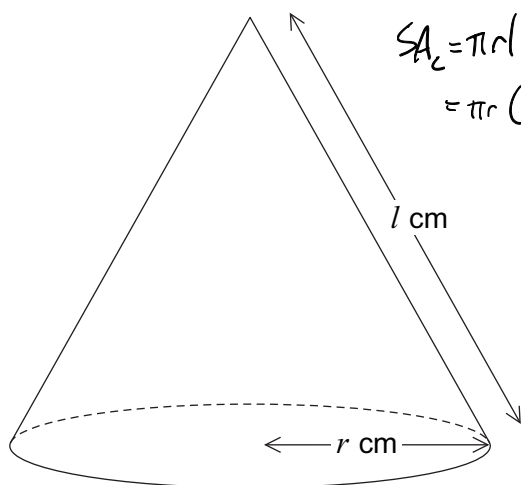
$x_2 = 307$ | $x_4 = 233$

Answer $53.1, 127, 307, 233$

8

26 A cone has base radius r cm and slant height l cm

A hemisphere has radius r cm

$SA_S = \dfrac{4\pi r^2}{2} + \pi r^2$

$= 3\pi r^2$

$= \pi r (3r)$

$SA_c = \pi r l + \pi r^2$

$= \pi r (l + r)$

l cm

r cm

r cm

26 (a) The curved surface area of the cone equals the curved surface area of the hemisphere.

Show that $l = 2r$
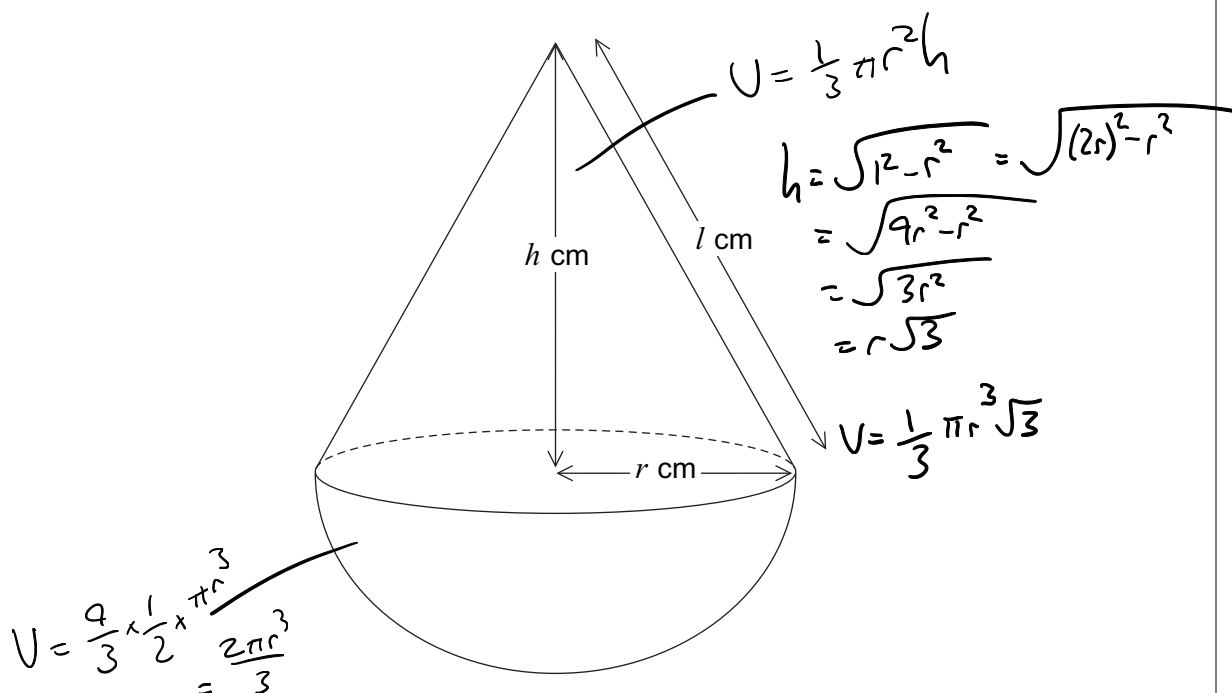
[1 mark]

$\pi r (l+r) = \pi r (3r)$

$l + r = 3r$

$l = 2r$

26 (b) The cone has vertical height h cm

The cone and hemisphere are joined to make the shape shown below.

$V = \frac{1}{3}\pi r^2 h$

$h = \sqrt{l^2 - r^2} = \sqrt{(2r)^2 - r^2}$

$= \sqrt{9r^2 - r^2}$

$= \sqrt{3r^2}$

$= r\sqrt{3}$

$V = \frac{1}{3}\pi r^3 \sqrt{3}$

h cm l cm

r cm

$V = \frac{4}{3} \times \frac{1}{2} \times \pi r^3$

$= \frac{2\pi r^3}{3}$

Show that the volume of the shape can be written as

$\frac{1}{3}\pi r^3 (a + \sqrt{b})$ cm³   where a and b are integers.

[4 marks]

$V = \frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^3 \sqrt{3}$

$= \frac{1}{3}\pi r^3 (2 + \sqrt{3})$

5

Turn over ▶

27 Work out the values of a when

$$2^{a^2} = 8^a \times 16$$

Do **not** use trial and improvement.
You **must** show your working.

[4 marks]

$$2^{(a^2)} = 2^{3a} \times 2^4$$

$$a^2 = 3a + 4$$

$$a^2 - 3a - 4 = 0$$

$$(a - 4)(a + 1) = 0$$

$$a = 4, -1$$

Answer ....

END OF QUESTIONS

4