Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Pages	Mark
3	
4 – 5	
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TOTAL	

AQA

Level 2 Certificate in Further Mathematics
June 2015

Further Mathematics 8360/1

Level 2

Paper 1 Non-Calculator

Monday 15 June 2015 9.00 am to 10.30 am

For this paper you must have:
- mathematical instruments.

You may **not** use a calculator.

Time allowed
- 1 hour 30 minutes

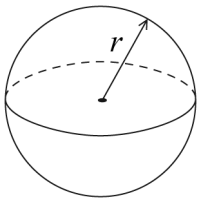
Instructions
- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- In all calculations, show clearly how you work out your answer.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 70.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer book.

Formulae Sheet

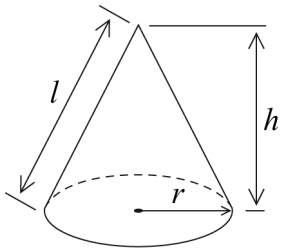
Volume of sphere $= \frac{4}{3}\pi r^3$

Surface area of sphere $= 4\pi r^2$



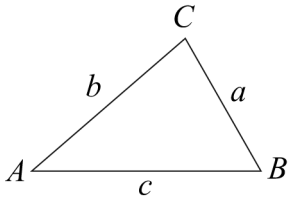
Volume of cone $= \frac{1}{3}\pi r^2 h$

Curved surface area of cone $= \pi r l$



In any triangle ABC

Area of triangle $= \frac{1}{2}ab\sin C$



Sine rule $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine rule $a^2 = b^2 + c^2 - 2bc\cos A$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$, where $a \neq 0$, are given by

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

Trigonometric Identities

$\tan\theta \equiv \dfrac{\sin\theta}{\cos\theta}$ $\qquad \sin^2\theta + \cos^2\theta \equiv 1$

Answer **all** questions in the spaces provided.

1 *GH* is a straight line.

The coordinates of *G* are (−2, 8)
The midpoint of *GH* is (5, −3)

Work out the coordinates of *H*.

[2 marks]

$= (5, -3) + (7, -11)$

$= (12, -19)$

Answer (12, −19)

Turn over for the next question

2

2 A straight line with equation $y = mx + c$ has gradient m and y-intercept c.

Here are the equations of four straight lines, P, Q, R and S.

P $2y - 4x = 5$ Q $5y = 2x - 4$

R $2y - 4 = 5x$ S $4y = 5 - 2x$

2 (a) Circle the line that passes through $(7, 2)$ **[1 mark]**

P Q R S

2 (b) Circle the line with gradient $2\frac{1}{2}$ **[1 mark]**

P Q R S

2 (c) Circle the line with y-intercept $2\frac{1}{2}$ **[1 mark]**

P Q R S

2 (d) Circle the line with a negative gradient. **[1 mark]**

P Q R S

2 (e) Circle a pair of perpendicular lines. **[1 mark]**

P Q R S

3 Solve $2(3x + 1) > 3 - 4x$ **[2 marks]**

$6x + 2 > 3 - 4x$

$10x > 1$

$x > 0.1$

Answer .....

Turn over for the next question

7

4 The equation of a curve is $y = x^2 - 5x$

4 (a) Work out $\frac{dy}{dx}$

[2 marks]

$2x - 5$

Answer .....

4 (b) *P* is a point on the curve.
The tangent to the curve at *P* has gradient 1

Work out the coordinates of *P*.

[2 marks]

$2x - 5 = 1$

$2x = 6$

$x = 3$

$y = x^2 - 5x$

$= 9 - 15$

$= -6$

Answer (3 , −6)

5 In the expansion of $(x+2)(x^2+kx-3)$ the coefficient of x^2 is zero.

5 (a) Work out the value of k.

[1 mark]

$x^3+2x^2+kx^2+2kx-6-3x$

$=x^3+(2+k)x^2+(2k-3)x-6$

$2-k=0$

$k=2$

Answer 2

5 (b) Work out the coefficient of x.

[2 marks]

$=2k-3$

$=4-3$ ECF = −7

$=1$

Answer .....

Turn over for the next question

6 A bag contains $5x$ red balls and $2x$ blue balls.

The number of red balls is **decreased** by 20% $= 4x$

The number of blue balls is **increased** by 30% $= \frac{13}{5}x$

There are now 35 **more** red balls than blue balls in the bag.

Work out the value of x.

[4 marks]

$4x - 35 = \frac{13}{5}x$

$1.4x = 35$

$x = 25$

Answer .....

7 $3x^3 - 2x^2 - 147x + 98 \equiv (ax - c)(bx + d)(bx - d)$

where a, b, c and d are positive integers.

Work out the values of a, b, c and d. i.e. Factorise that Cubic

[3 marks]

$F98 = 2 \times 7 \times 7$

$f(x) = 3x^3 - 2x^2 - 147x + 98$

~~$f(2) = 180$ $f(-2) = 360$~~

$f(7) = 0 \therefore$ Cor $d = 7$,

$= (3x - 7)(x + d)(x - d) = (3x^2 - 7x + 3dx - 7d)(x - d)$

$= 3x^3 - 7x^2 + 3dx^2 - 7dx - 3dx^2 - 7dx + 3dx^2 + 7d^2$ - d non-neg So $d = 7$

$= (3x - c)(x + 7)(x - 7)(3x + c)(x^2 - 99)$

$= 3x^3 - 147x - cx^2 + 99c$

$= 3x^3 - cx^2 - 147x + 99c$ $\quad 99 \times 2 = 98$

$-1 \times 2 = -2$

$a = 3$ $\quad b = 1$ $\quad c = 7$ $\quad d = 2$

DOUBLE CHECK BOXES

Turn over for the next question

7

8 Simplify fully $\dfrac{5x}{(x+4)(x-6)} - \dfrac{3}{(x-6)}$

[4 marks]

$$= \frac{5x(x-6) - 3(x+4)(x-6)}{(x+4)(x-6)(x-6)}$$

$$= \frac{5x - 3x - 12}{(x+4)(x-6)} \qquad = \frac{2x-12}{(x+4)(x+6)}$$

$$= \frac{2(x-6)}{(x+4)(x-6)}$$

$$= \frac{2}{x+4}$$

Answer .....

9 Given that $\begin{pmatrix} 3 & -1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} b \\ a+1 \end{pmatrix}$

work out the values of a and b.

[5 marks]

$= \begin{pmatrix} 3a-b \\ 2a+b \end{pmatrix} = \begin{pmatrix} b \\ a+1 \end{pmatrix}$

$3a - b = b$

(A) $3a = 2b$

$2a + b = a + 1$

(B) $a + b = 1$

(A) $3a - 2b = 0$

~~(B) $a + b = 1$~~

(2B) $2a + 2b = 2$

$5a = 2$

$a = \frac{2}{5}$

(B) $a + b = 1$

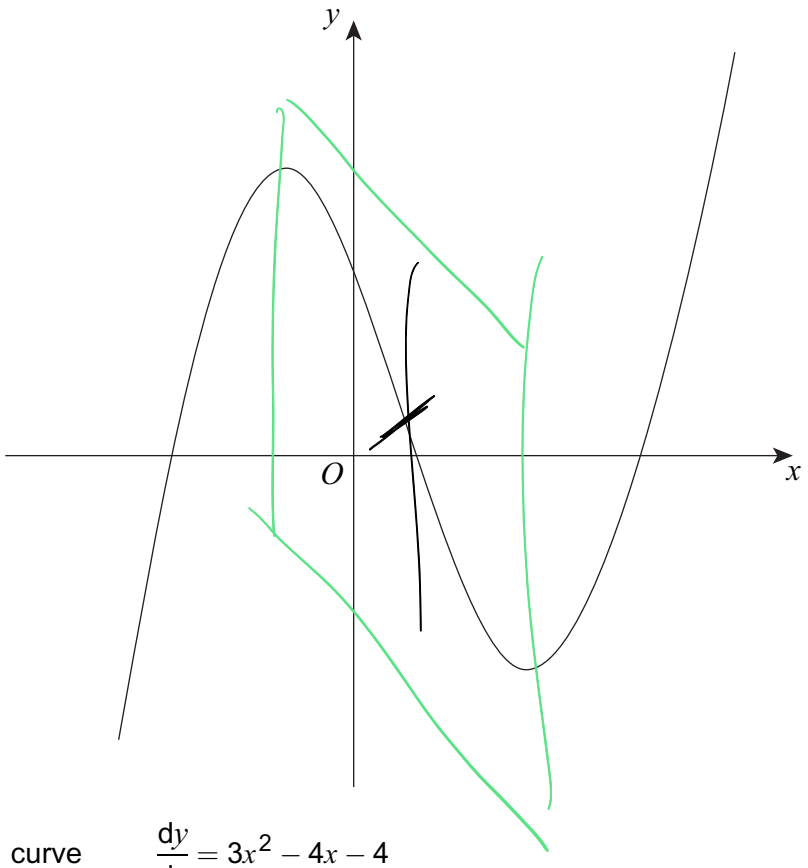
$\frac{2}{5} + b = 1$

$b = \frac{3}{5}$

$a =$ 2/5 , $b =$ 3/5

9

10 This is a sketch of the curve $y = f(x)$



10 (a) For this curve $\frac{dy}{dx} = 3x^2 - 4x - 4$

Work out the range of values of x for which $f(x)$ is a decreasing function.
Write your answer as an inequality.

[4 marks]

$\frac{d^2y}{dx} = 6x - 4$ just $\frac{dy}{dx}$ then solve

$6x - 4 < 0$

$6x < 4$

$x < \frac{2}{3}$

Answer .....

10 (b) Work out the equation of the normal to the curve at the point $(1, -2)$
Give your answer in the form $y = mx + c$

[5 marks]

haven't done integration, but guess the curve is: $y = x^3 - 2x^2 - 4x + c$

$-2 = 1 - 2 - 4 + c$

$c = 3$

$\tan m = 3x^2 - 4x - 4$ where $x = 1$

$= 3 - 4 - 4 = -5$

$\text{region} = \frac{1}{5}$

$y = \frac{x}{5} + c$

$-2 = \frac{1}{5} + c$

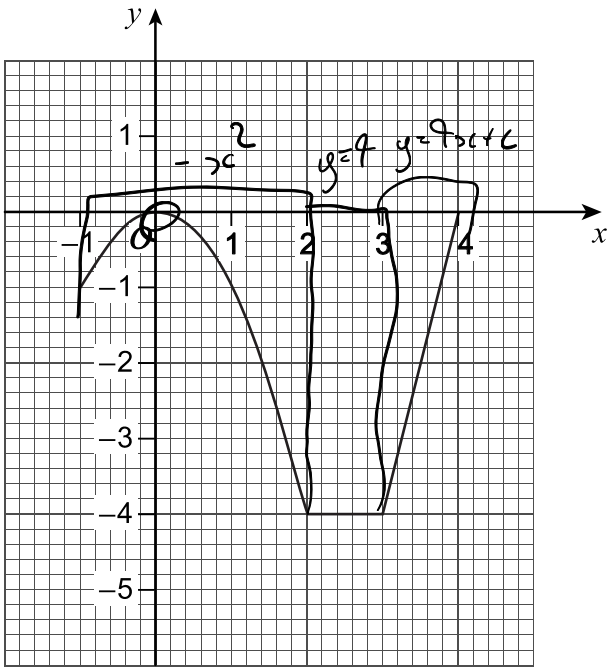
$c = -2.2$

Answer $y = \frac{x - 11}{5}$

Turn over for the next question

11 Here is the graph of $y = f(x)$

It consists of a quadratic curve and two straight lines.



Define $f(x)$, stating clearly the domain for each part.

[4 marks]

$y = 9x + c$

$0 = 9 \times 9 + c$

$c = -16$

$f(x) = -1 < x \leqslant 2 : y = -x^2$ f(x) = not y =

$= 2 < x \leqslant 3 : y = 4$

$= 3 < x \leqslant 4 : y = 9x - 16$

12 Make y the subject of $\sqrt{\frac{3xy}{x+y}} = 4$

[4 marks]

$\frac{3xy}{x+y} = 16$

$3xy = 16x + 16y$

$3xy - 16y = 16x$

$y(3x - 16) = 16x$

$y = \frac{16x}{3x - 16}$

Answer .....

Turn over for the next question

13 $x^2 + 2ax + b \equiv (x - 5)^2 - a$

Work out the values of a and b.

[3 marks]

$(x-5)^2 - a = x^2 - 10x + 25 - a$

$x^2 - 10x + 25 - a \equiv x^2 + 2ax + b$

$a = -5$

$x^2 - 10x + 30 \equiv x^2 - 10x + b$

$a =$ -5, $b =$ 30

14 Write $\dfrac{5\sqrt{2}}{3\sqrt{6}-7}$ in the form $\sqrt{w}+\sqrt{k}$ where w and k are integers.

[5 marks]

$= \dfrac{5\sqrt{2}}{-7+3\sqrt{6}} \times \dfrac{-7-3\sqrt{6}}{-7-3\sqrt{6}} = \dfrac{-35\sqrt{2}-30\sqrt{3}}{49-54}$

$= \dfrac{-35\sqrt{2}-30\sqrt{3}}{-5} = 7\sqrt{2}+6\sqrt{3}$! $= \sqrt{19}+\sqrt{18}$

$\sqrt{18}+\sqrt{98}$

Answer .....

8

15 The diagram shows two circles touching externally at T.
Points X, Y and W lie on the larger circle.

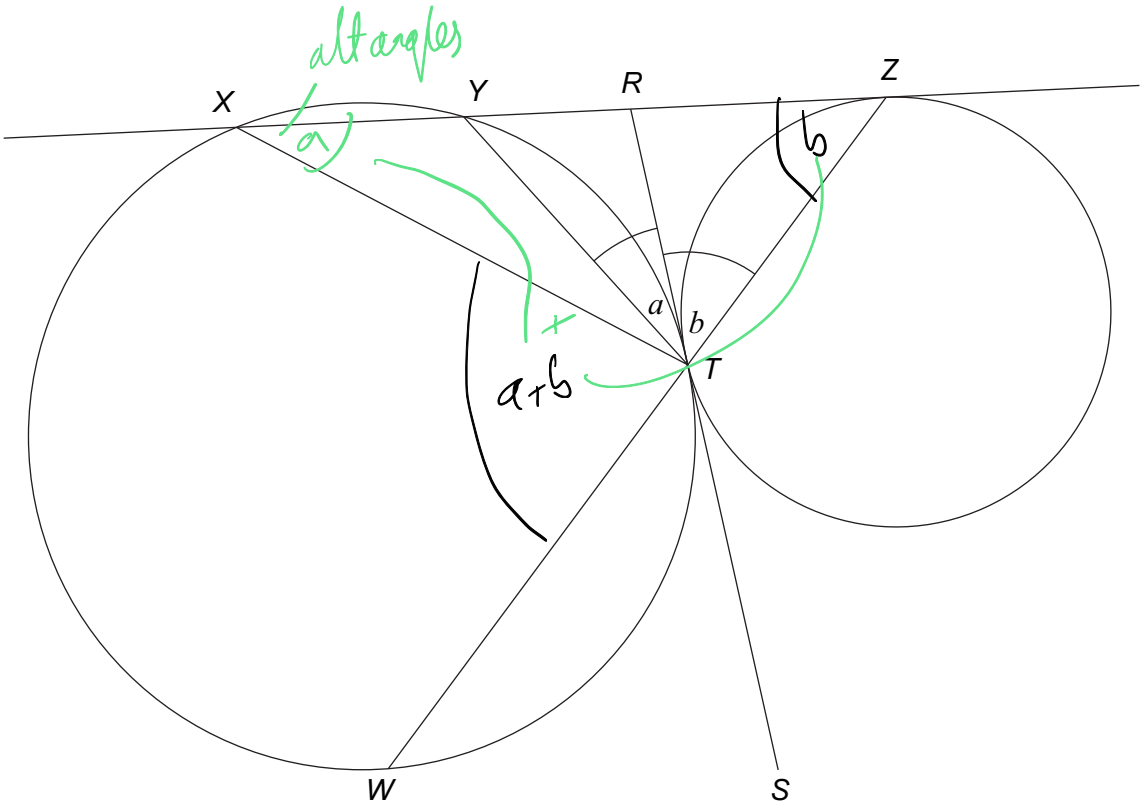
RTS is a tangent to both circles.

$XYRZ$ is a tangent to the smaller circle at Z.

ZTW is a straight line.

Angle $YTR = a$ and angle $ZTR = b$

Not drawn accurately



15 (a) Give reasons why angle $RZT = b$

[2 marks]

? tangents equal length
isos tri

15 (b) Angle $RZT = b$

Prove that angle XTW = angle YTZ

[3 marks]

?

5

16 By factorising fully, simplify $\dfrac{x^4 - x^3 - 2x^2}{x^4 - 5x^2 + 4}$ **[5 marks]**

$x^2(x^2 - x - 2)$ ✓ → $x^2(x+1)(x-2)$

$x^2(x^2-3)+4$ $(x^2-1)(x^2-4)$ .....

?

Answer .....

17 Prove that $2\tan^2\theta + 1 \equiv \dfrac{1+\sin^2\theta}{1-\sin^2\theta}$ where $\sin^2\theta \neq 1$ to avoid /key 0

[3 marks]

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ $1-\sin^2\theta = \sin^2\theta$

$1-\cos^2\theta = \cos^2\theta$

$\equiv \dfrac{1+\sin^2\theta}{1-\sin^2\theta} \equiv \dfrac{1+\sin^2\theta}{\cos^2\theta}$ ✓ $\equiv \dfrac{1}{\cos^2\theta} + \dfrac{\sin^2\theta}{\cos^2\theta}$

$\equiv \dfrac{1}{\cos^2\theta} + \tan^2\theta$

???

$\dfrac{\cos^2+\sin^2+\tan^2}{\cos^2} = \dfrac{\cos^2}{\cos^2} + \dfrac{2\sin^2}{\cos^2}$

. . .

END OF QUESTIONS

8

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