

# AQA Level 2 Certificate in FURTHER MATHEMATICS (8365/1)

Paper 1

Specimen 2020

Time allowed: 1 hour 45 minutes

## Materials

For this paper you must have:

- mathematical instruments

You may **not** use a calculator



## Instructions

- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Fill in the boxes at the bottom of this page.
- Answer **all** questions.
- You must answer the questions in the space provided. Do not write outside the box around each page or on blank pages.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- In all calculations, show clearly how you work out your answer.

## Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer booklet.

Please write clearly, in block capitals, to allow character computer recognition.

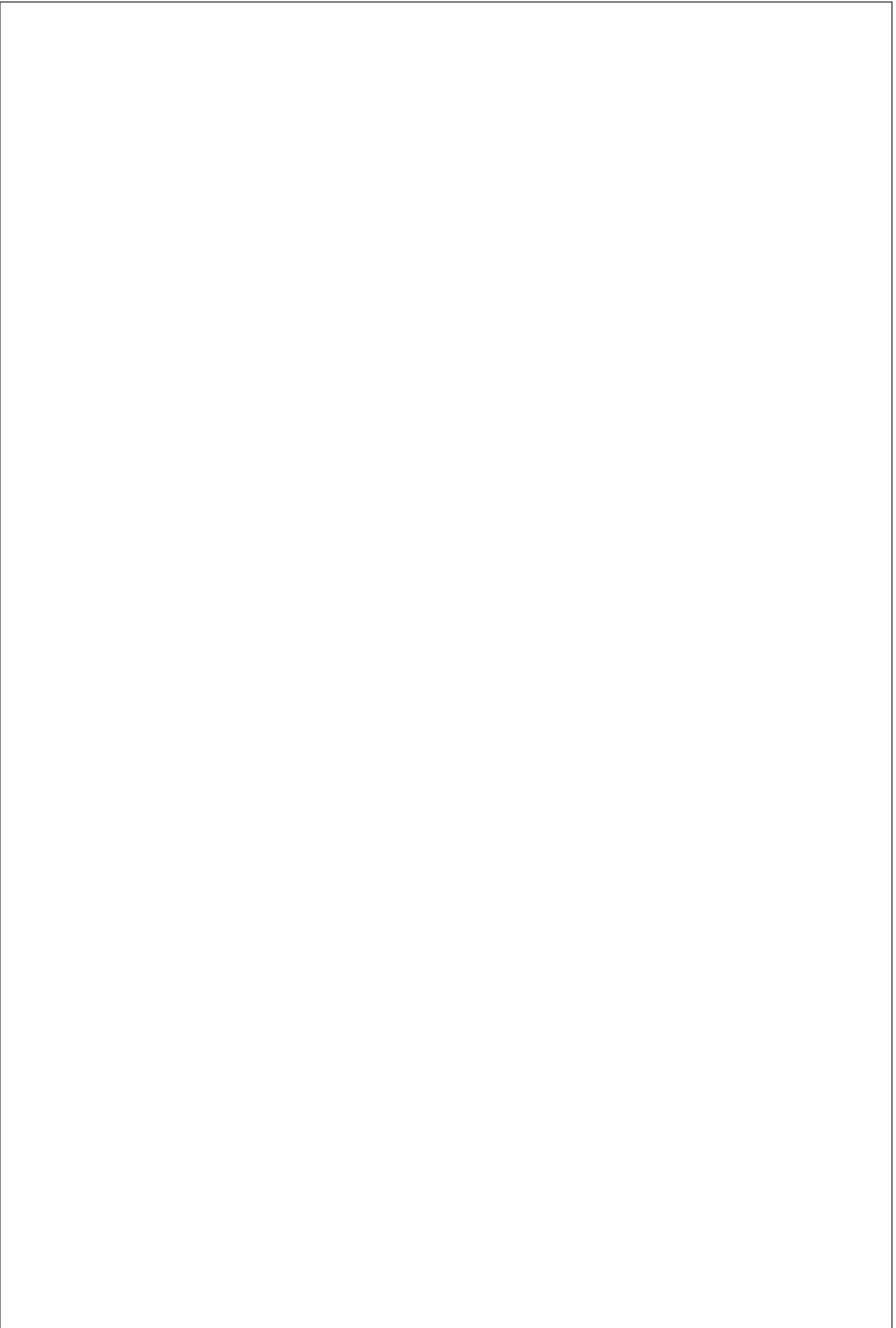
Centre number

Candidate number

Surname

Forename(s)

Candidate signature \_\_\_\_\_



Answer **all** questions in the spaces provided.

1 (a)

$$\frac{y^6 \times y}{y^m} = y^4$$

$$\frac{y^7}{y^m} = y^4$$

Circle the value of  $m$ .

– 2

1.5

2

3

[1 mark]

1 (b)

$$a^n \times a^5 = a^5$$

Work out the value of  $n$ .

[1 mark]

Answer

0

1 (c)

$$(c^5)^p = (c^2)^6$$

$$= \frac{12}{5}$$

Work out the value of  $p$ .

[2 marks]

Answer

$\frac{12}{5}$

2 Solve  $\sqrt[3]{7x-13} = 2$

[2 marks]

$$7x - 13 = 8$$

$$7x = 21$$

$$x = 3$$

$$x = 3$$

3  $3a(2x - 1) + 4(ax + 5) \equiv 60x + b$

Work out the values of  $a$  and  $b$ .

[4 marks]

$$= 6ax - 3a + 4ax + 20$$

$$= (10ax) + (20 - 3a)$$

$$10ax = 60x$$

$$a = 6$$

$$20 - 3a = b$$

$$20 - 18 = b$$

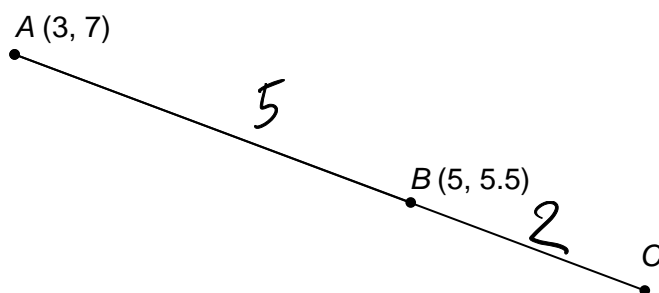
$$b = 2$$

$$a = 6$$

$$b = 2$$

4

ABC is a straight line with  $AB : BC = 5 : 2$



Not drawn  
accurately

Work out the coordinates of C.

$$C_x = 5 + \frac{(5-3) \times 2}{5} = 5 + \frac{4}{5} = 5.8$$

$$C_y = 5.5 + \frac{(7-5.5) \times 2}{5} = 5.5 + \frac{6}{10} = 6.1$$

[4 marks]

Answer ( 5.8 , 6.1 )

5

$$y = 2x^{10} - \frac{3}{x^2}$$

Work out  $\frac{dy}{dx}$

9

[3 marks]

$$= 2x^{10} - 3 \times x^{-2}$$

$$= 3 \times 2x^{-3}$$

$$\frac{dy}{dx} = 20x^9 + (3 \times 2x^{-3})$$

$$= \frac{6}{2x^3}$$

$$(3 \times 2)x^{-3} \text{ so } 6x^{-3}$$

Answer  $20x^9 + \frac{6}{2x^3}$

6

Simplify fully  $\frac{15x^2y - 5xy^2}{12x - 4y}$

[3 marks]

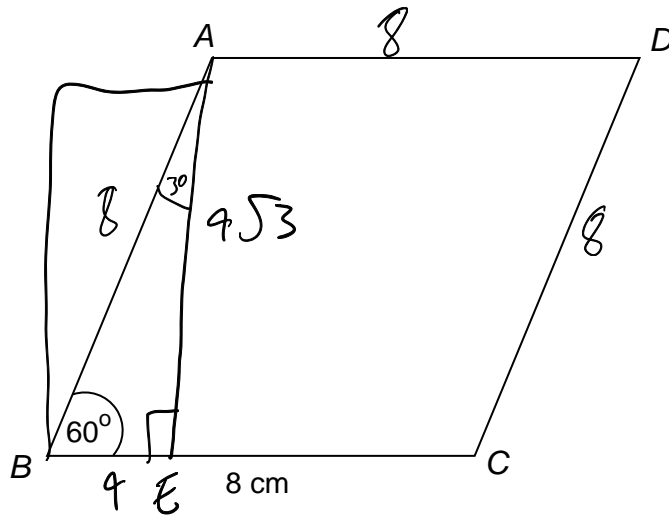
$$= \frac{5xy(3x - y)}{4(3x - y)}$$

Answer  $\frac{5xy}{4}$

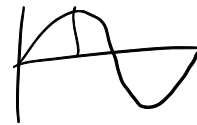
7

$ABCD$  is a rhombus with side length 8 cm

Angle  $ABC = 60^\circ$



Not drawn  
accurately



Work out the area of the rhombus.

Give your answer in the form  $a\sqrt{b}$  cm<sup>2</sup> where  $a$  and  $b$  are integers.

[3 marks]

$$\frac{a}{\sin A} = \frac{b}{\sin B} \quad AE = \frac{8}{\sin 90} \times \sin 60 = 8 \times \frac{\sqrt{3}}{2} = 4\sqrt{3}$$

$$BE = \frac{8}{\sin 90} \times \sin 30 = 8 \times \frac{1}{2} = 4$$

$$A = (4\sqrt{3} \times 4) + (0 \times 4\sqrt{3})$$

Answer  $16\sqrt{3}$  cm<sup>2</sup>

8

The curve  $y = 2x^3 - 3x^2 - 12x + 6$

has a maximum point at  $L(-1, 13)$

has a minimum point at  $M(2, -14)$

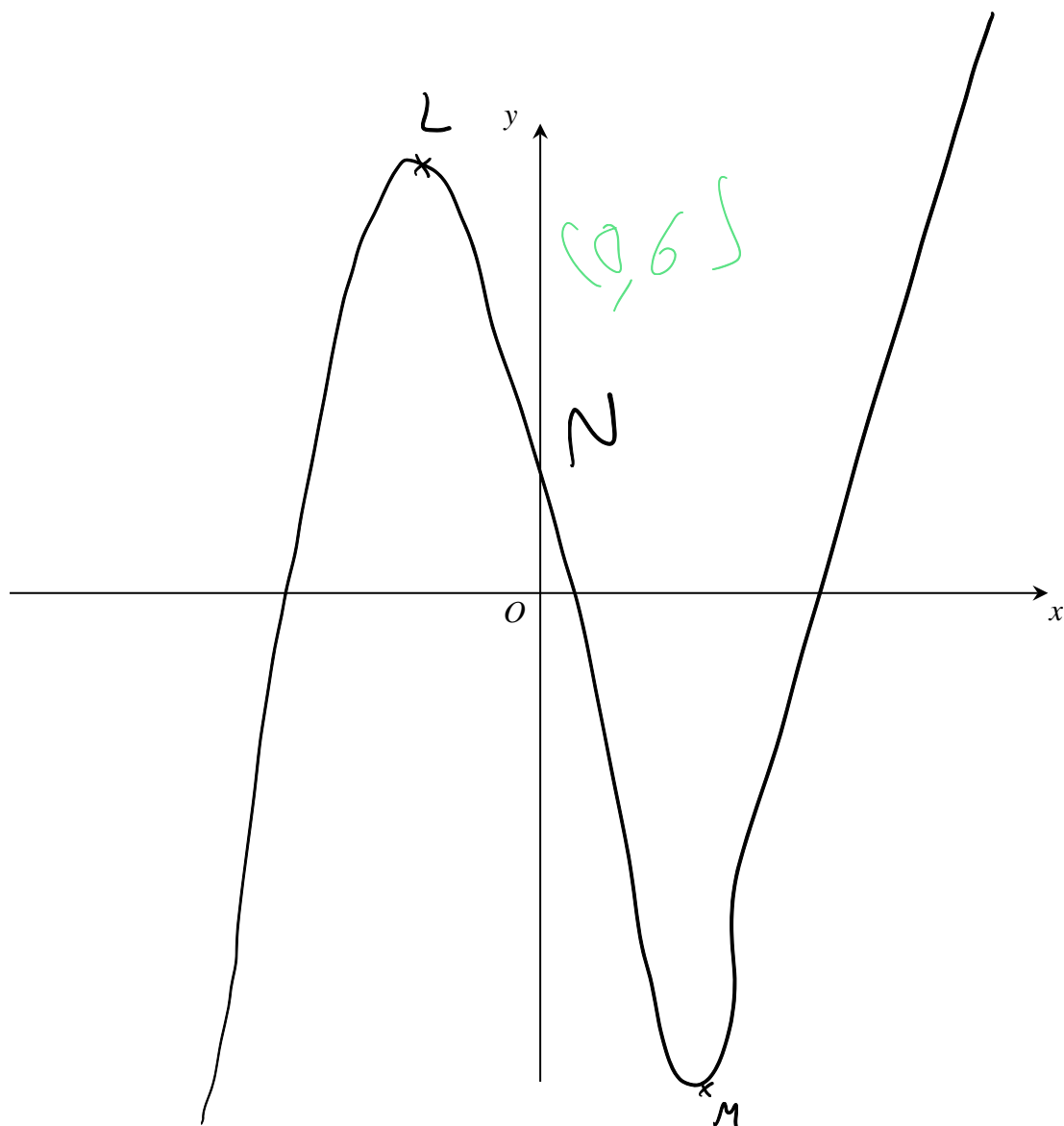
intersects the  $y$ -axis at  $N$ .

The curve crosses the  $x$ -axis at three distinct points.

On the axes below, sketch the curve.

Label the points  $L$ ,  $M$  and  $N$  on your sketch.

[3 marks]





9

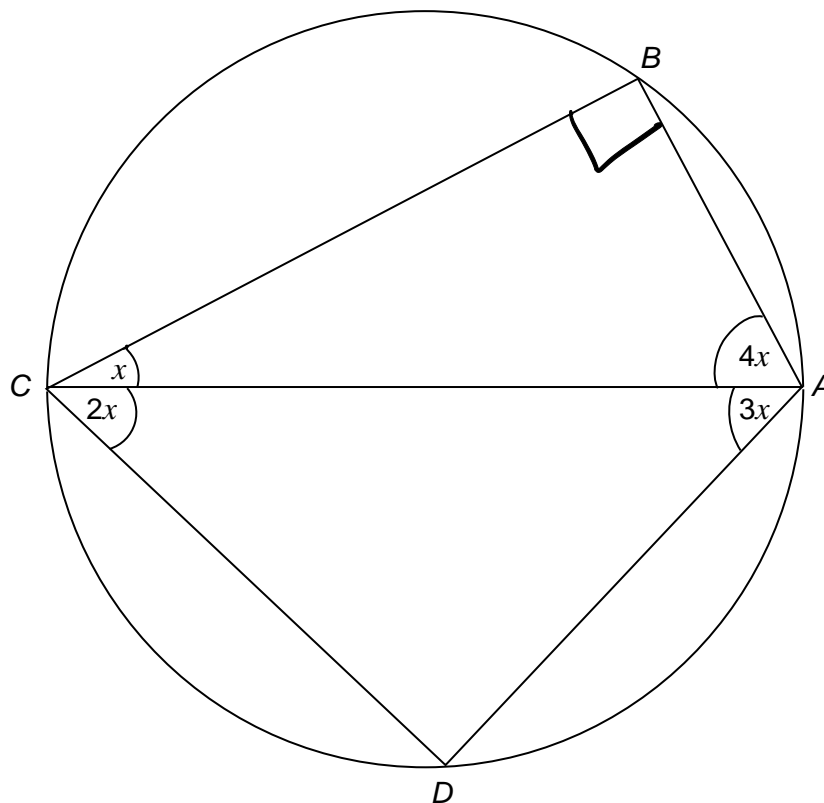
$A, B, C$  and  $D$  are points on a circle.

$$\angle BCA = x$$

$$\angle ACD = 2x$$

$$\angle CAD = 3x$$

$$\angle CAB = 4x$$



Not drawn  
accurately

Prove that  $AC$  is a diameter.

[4 marks]

$$x + 2x + 4x + 3x = 180^\circ$$

$$10x = 180^\circ$$

$$x = 18^\circ$$

$$\angle BDA = 180 - 4x - x$$

$$= 180 - 5 \times 18$$

$$= 180 - 90$$

$= 90^\circ$ , so must be diameter

10

$$f(x) = \left(\frac{9x}{2}\right)^{-1}$$

$$g(x) = \sqrt{1 - px^3} \quad \text{where } p \text{ is a constant}$$

Given that  $f\left(\frac{1}{3}\right) = g\left(\frac{1}{3}\right)$  work out the value of  $p$ .

[5 marks]

$$\frac{2}{9 \times \frac{1}{3}} = \sqrt{1 - p\left(\frac{1}{3}\right)^3}$$

$$\frac{2}{3} = \sqrt{1 - \frac{p}{27}}$$

$$\frac{4}{9} = 1 - \frac{p}{27}$$

$$\frac{-5}{9} = -\frac{p}{27}$$

$$\frac{15}{27} = \frac{p}{27}$$

$$p = 15$$

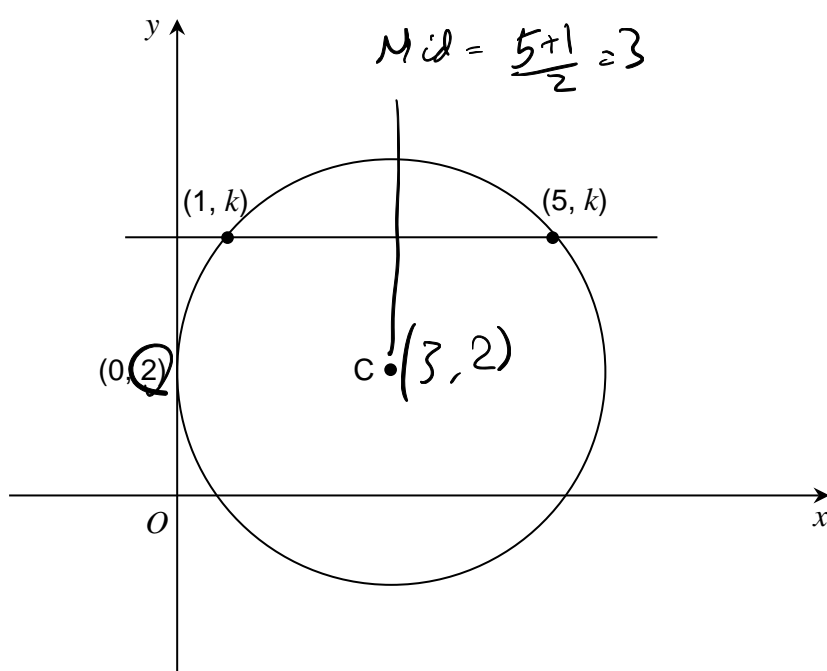
Answer

15

11

A circle, centre  $C$ , touches the  $y$ -axis at the point  $(0, 2)$

The line  $y = k$  intersects the circle at the points  $(1, k)$  and  $(5, k)$



Not drawn  
accurately

Work out the equation of the circle.

[3 marks]

Answer  $(x-3)^2 + (y-2)^2 = 9$

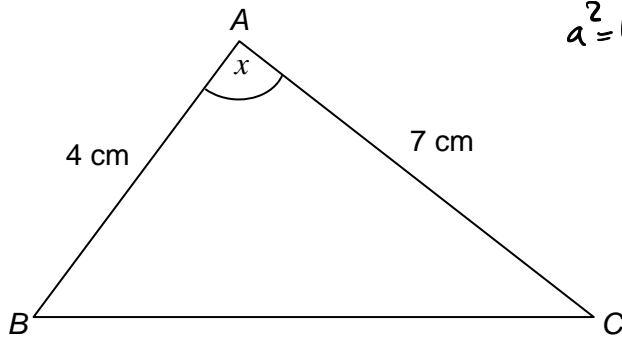
12

$AB = 4 \text{ cm}$

$AC = 7 \text{ cm}$

$\cos x = -\frac{2}{7}$

$a^2 = b^2 + c^2 - 2bc \cos A$

Work out the length of BC.

[3 marks]

$$\begin{aligned}
 BC &= \sqrt{16 + 49 - 2 \times 4 \times 7 \times -\frac{2}{7}} \\
 &= \sqrt{65 + 16} \\
 &= \sqrt{81} \\
 &= 9
 \end{aligned}$$

Answer 9 cm

13

Rearrange  $t = \frac{3w^3 + a}{w^3 - 2}$  to make  $w$  the subject.

[5 marks]

$t w^3 - 2t = 3w^3 + a$

$t w^3 - 3w^3 = a + 2t$

$w^3(t - 3) = a + 2t$

$w^3 = \frac{a + 2t}{t - 3}$

$w = \sqrt[3]{\frac{a + 2t}{t - 3}}$

Answer  $\sqrt[3]{\frac{a + 2t}{t - 3}}$

14 Rationalise and simplify  $\frac{\sqrt{3}-7}{\sqrt{3}+1}$

Give your answer in the form  $a + b\sqrt{3}$  where  $a$  and  $b$  are integers

[4 marks]

$$= \frac{-7+\sqrt{3}}{1+\sqrt{3}} \times \frac{1-\sqrt{3}}{1-\sqrt{3}} = \frac{-7-6\sqrt{3}-3}{1-3} = \frac{-10-6\sqrt{3}}{-2} = 5+3\sqrt{3}$$

Answer

$$5-4\sqrt{3}$$

15 Point A lies on the curve  $y = x^2 + 5x + 8$

The  $x$ -coordinate of A is -4

15 (a) Show that the equation of the normal to the curve at A is  $3y = x + 16$

[5 marks]

$$\frac{dy}{dx} = 2x + 5$$

$$M: 2x + 5 \text{ at } x = -4 = -3, \text{ gradient} = \frac{1}{3}$$

$$y = x^2 + 5x + 8 \text{ at } x = -4 = 4$$

$$y = mx + c, \quad 4 = \frac{-4}{3} + c$$

$$12 = -4 + 3c$$

$$16 = 3c$$

$$c = \frac{16}{3}$$

$$y = \frac{2x}{3} + \frac{16}{3}$$

$$3y = x + 16 //$$

- 15 (b) The normal at A also intersects the curve at B.

Work out the x-coordinate of B.

[4 marks]

$$y = \frac{x}{3} + \frac{16}{3}, \quad y = x^2 + 5x + 8$$

$$\frac{x}{3} + \frac{16}{3} = x^2 + 5x + 8$$

$$x + 16 = 3x^2 + 15x + 24$$

$$3x^2 + 14x + 8 = 0$$

$$(3x + 2)(x + 4) = 0$$

$$x = -\frac{2}{3} \quad x = -4, \text{ Also discard}$$

Answer

$$-\frac{2}{3}$$

- 16 The coefficient of the  $x^4$  term in the expansion of  $(2x + a)^6$  is 60

Work out the possible values of  $a$ .

$$(2x+a)^6 = \binom{6}{1}(2x)^6 + \binom{6}{2}(2x)^5(a) + \boxed{\binom{6}{3}(2x)^4(a)^2}$$

$$\binom{6}{3}(2x)^4(a)^2 = \cancel{20} \times \overset{16}{16}x^4 \times a^2$$

$$320a^2 = 60$$

$$32a^2 = 6$$

$$a^2 = \frac{\overset{60}{60}}{32} = \frac{3}{16}$$

$$a = \pm \frac{\sqrt{3}}{4}$$

$$\pm \frac{1}{2}$$

①  
[4 marks]

Answer  $\pm \frac{\sqrt{3}}{4}$



17

Solve the simultaneous equations

$$\textcircled{3A} \quad 6a + 3b - 3c = 24$$

$$\textcircled{4} \quad 2a + b - c = 8$$

$$\textcircled{3} \quad 4a - 3b - 2c = -9$$

$$\textcircled{C} \quad 6a + 3b + c = 0$$

$$C+B = \textcircled{D} \quad 10a - c = -9$$

$$3A+B = \textcircled{E} \quad 10a - 5c = 15$$

$$10a - c = -9$$

$$10a + 6 = -9$$

$$10a = -15$$

$$a = -1.5$$

OR

$$10a - 5c = 15$$

$$10a - 5 \times 6 = 15$$

$$10a = -15$$

$$a = -1.5$$

[5 marks]

$$D-E = \quad 9c = -24$$

$$c = -6$$

$$2a + b - c = 8$$

$$-3 + b + 6 = 8$$

$$b + 3 = 8$$

$$b = 5$$

OR

$$4a - 3b - 2c = -9$$

$$-6 - 15 + 12 = -9$$

$$-9 = -9$$

$$a = -1.5 \quad b = 5 \quad c = -6$$

18

Solve  $x^{\frac{2}{3}} = 12\frac{1}{4}$

$$\sqrt[3]{x^2} = \frac{49}{4}$$

$$\begin{array}{r} 99 \\ \times 7 \\ \hline 343 \end{array}$$

[3 marks]

$$\sqrt[3]{x^2} = \frac{49}{4}$$

$$\sqrt[3]{x} = \frac{7}{2}$$

$$x = \frac{8}{343}$$

x =

19

$$f(x) = 2x^3 - 12x^2 + 25x - 11$$

Use differentiation to show that  $f(x)$  is an increasing function for all values of  $x$ . $x > 0$ 

[4 marks]

$$\frac{dy}{dx} = 6x^2 - 24x + 25$$

??????

$$6(x^2 - 4x) \dots$$

$$6(x-2)^2 \dots$$

$$6(x-2)^2 + 1 > 0$$

+arg

20 (a) Show that  $2 \cos^2 \theta \equiv 2 - 2 \sin^2 \theta$

$$\cos^2 \theta = 1 - \sin^2 \theta$$

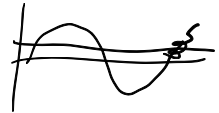
[1 mark]

$$= 2 \cos^2 \theta$$

$$= 2(\cos^2 \theta)$$

$$= 2(1 - \sin^2 \theta)$$

$$= 2 - 2 \sin^2 \theta$$



20 (b) Hence, solve  $2 \cos^2 \theta + 3 \sin \theta = 3$  for  $0 < \theta < 180^\circ$

[4 marks]

$$2 - 2 \sin^2 \theta + 3 \sin \theta - 3 = 0$$

$$\text{Let } x = \sin \theta$$

$$-2x^2 + 3x - 1 = 0$$

$$2x^2 - 3x + 1 = 0$$

$$(2x - 1)(x - 1)$$

$$2 \sin \theta = 1, \sin \theta = \frac{1}{2}$$

$$\theta = 30^\circ, (180 - 30) = 150$$

$$\sin \theta = 1$$

$$\theta = 90^\circ$$

Answer

30, 90, 150

END OF QUESTIONS

**There are no questions printed on this page**

**DO NOT WRITE ON THIS PAGE  
ANSWER IN THE SPACES PROVIDED**