

Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										



Level 2 Certificate in Further Mathematics

Further Mathematics Level 2

8360/1

Practice Paper Set 4

Paper 1

Non-Calculator

For this paper you must have:

- mathematical instruments.

You may **not** use a calculator.



For Examiner's Use	
Examiner's Initials	
Pages	Mark
3	
4 – 5	
6 – 7	
8 – 9	
10 – 11	
12 – 13	
14 – 15	
16	
TOTAL	

Time allowed: 1 hour 30 minutes

Instructions

- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the space provided. Do not write outside the box around each page or on blank pages.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- In all calculations, show clearly how you work out your answer.

Information

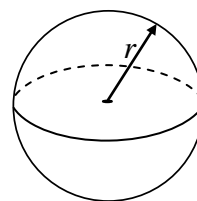
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 70.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer booklet.

PP4/8360/1

Formulae Sheet

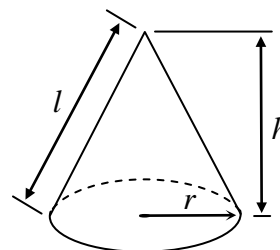
Volume of sphere $= \frac{4}{3} \pi r^3$

Surface area of sphere $= 4\pi r^2$



Volume of cone $= \frac{1}{3} \pi r^2 h$

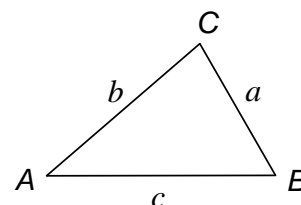
Curved surface area of cone $= \pi r l$



In any triangle ABC

Area of triangle $= \frac{1}{2} ab \sin C$

Sine rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$



Cosine rule $a^2 = b^2 + c^2 - 2bc \cos A$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

The Quadratic Equation

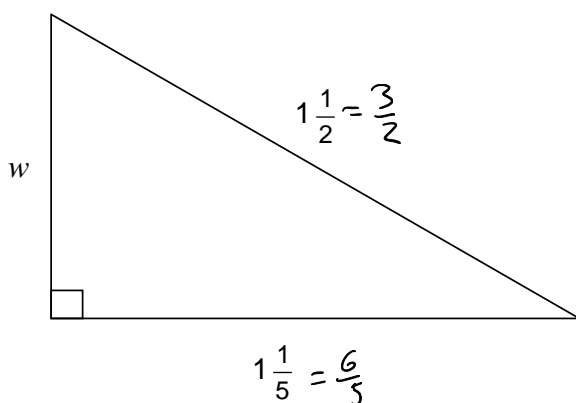
The solutions of $ax^2 + bx + c = 0$, where $a \neq 0$, are given by $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Trigonometric Identities

$$\tan \theta \equiv \frac{\sin \theta}{\cos \theta} \quad \sin^2 \theta + \cos^2 \theta \equiv 1$$

Answer **all** questions in the spaces provided.

1

Not drawn
accuratelyWork out the value of w .

$$\begin{aligned}
 a^2 + b^2 &= c^2 \\
 w &= \sqrt{\frac{3^2}{2} - \frac{6^2}{5}} \\
 &= \sqrt{\frac{9}{4} - \frac{36}{25}} \\
 &= \sqrt{\frac{225}{100} - \frac{144}{100}} \\
 &= \sqrt{\frac{81}{100}} \\
 &= \frac{9}{10}
 \end{aligned}$$

$$\begin{array}{r}
 36 \\
 \times 9 \\
 \hline
 144 \\
 \hline
 225 \\
 - 144 \\
 \hline
 081
 \end{array}$$

0.9
 $w = \dots\dots\dots$ (4 marks)

- 2 In this identity, h and k are integer constants.

$$4(hx - 1) - 3(x + h) \equiv 5(x + k)$$

Work out the values of h and k .

$$\begin{aligned} &\equiv 4hx - 4 - 3x - 3h \\ &= (4h-3)x - (4+3h) \end{aligned}$$

$$\begin{array}{l|l} 4h-3=5 & 4+3h=k \\ 4h=8 & 4+6=k \\ h=2 & k=10 \end{array}$$

$h = 2, k = 10$ (4 marks)

- 3 (a) $x : y = 3 : 2$

Write x in terms of y .

$$\frac{x}{y} = \frac{3}{2}$$

$$\begin{aligned} 3x &= 2y \\ x &= \frac{2}{3}y \end{aligned}$$

Or A

Answer $x = \frac{2}{3}y$ (2 marks)

- 3 (b) Use your answer to part (a) to simplify

$$2x + y : 3x - 2y$$

$$\frac{2}{3}y + y : \frac{3 \times 2}{3}y - 2y$$

$$\frac{5}{3}y : 2y - 2y$$

$$\frac{5}{3} : 0$$

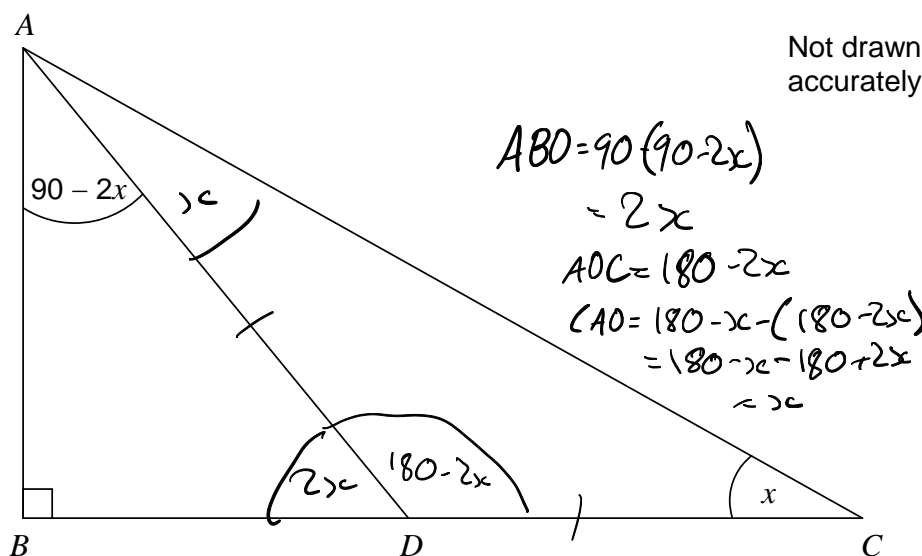
$$2x + y = 3y + y$$

$$3x - 2y = 3\left(\frac{3y}{2}\right) - 2y$$

$$8 : 5$$

Answer (2 marks)

4

 ABC is a right-angled triangle.Angle $ACB = x$ Angle $BAD = 90 - 2x$ Prove that ACD is an isosceles triangle. $\angle CAO = \angle ACO$, therefore $CA = CD$

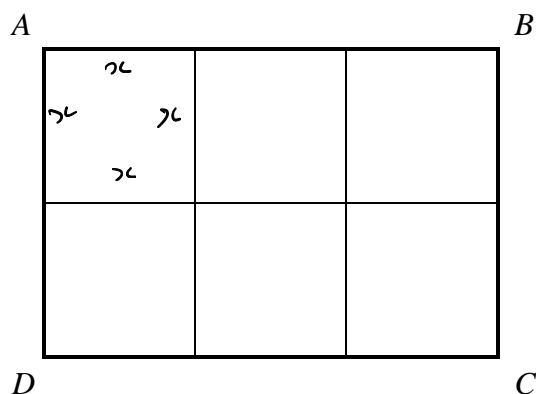
(3 marks)

Turn over for the next question

5

The rectangle $ABCD$ is divided into 6 identical squares.

The side of each square is x cm.



Not drawn
accurately

When the perimeter and the area of $ABCD$ are given in cm and cm^2 respectively, they have the same numerical value.

Work out x .

$$6x^2 = 10x \quad (\text{can just divide by } x, \text{ as } x > 0)$$

$$6x = 10$$

$$x = \frac{10}{6} = \frac{5}{3}$$

$$x = 1\frac{2}{3} \quad (4 \text{ marks})$$

6

$$y = \frac{3x(2x^4 - 5x)}{x^2}$$

$$y = \frac{6x^5 - 15x^2}{x^2}$$

Work out $\frac{dy}{dx}$

$$= 6x^3 - 15$$

$$\frac{dy}{dx} = 18x^2$$

$$\frac{dy}{dx} = 18x^2 \quad (3 \text{ marks})$$

7

Given that $\frac{2}{h} - \frac{3}{k} = 4$

show that $h = \frac{2k}{4k+3}$, AKA re-arrange to set h as subject

$$\frac{2-3h}{k} = 4$$

$$2k - 3h = 4hk$$

$$2k = 4hk + 3h$$

$$h(4k+3) = 2k$$

$$h = \frac{2k}{4k+3}$$

(3 marks)

10

Turn over ►

8

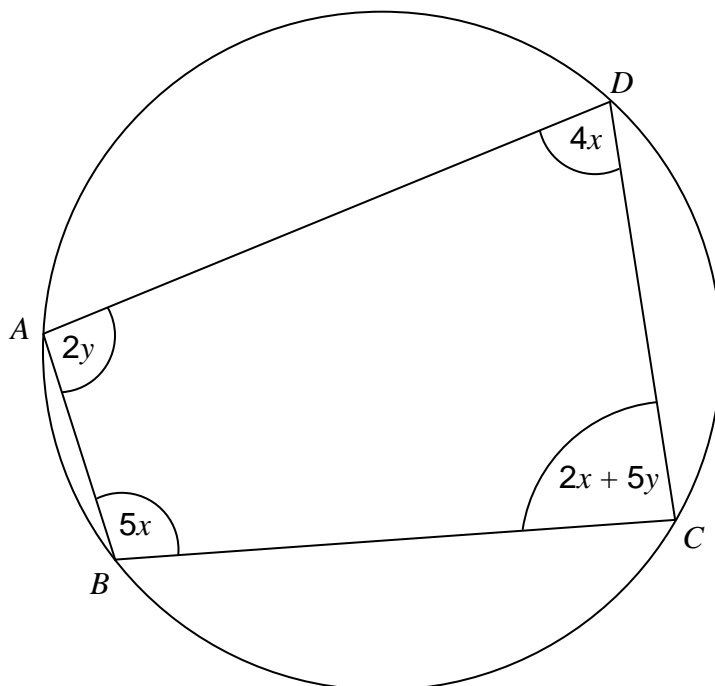
Work out the gradient of the curve $y = (3x - 4)(x + 2)$ at the point $(2, 8)$

$$y = 3x^2 + 2x - 8$$

$$\frac{dy}{dx} = 6x + 2, \quad x = 2, \quad \frac{dy}{dx} = 12 + 2 = 14$$

Answer 14 (3 marks)

9

 $ABCD$ is a cyclic quadrilateral.Not drawn
accuratelyProve that $x = y$

$$4x + 5y = 180$$

$$4x = 180 - 5y$$

$$x = \frac{180 - 5y}{4}$$

$$2y + 2x + 5y = 180$$

$$2x + 7y = 180$$

$$\frac{180 - 5y}{2} + 7y = 180$$

$$180 - 5y + 14y = 360$$

$$9y = 180$$

$$y = 20^\circ$$

$$x = \frac{180 - 5y}{4}$$

$$= \frac{180 - 100}{4}$$

$$= \frac{80}{4}$$

$$x = 20^\circ$$

$$x = y$$

(4 marks)

10

$$y = 10 - 8x - x^3 \quad \text{for all values of } x.$$

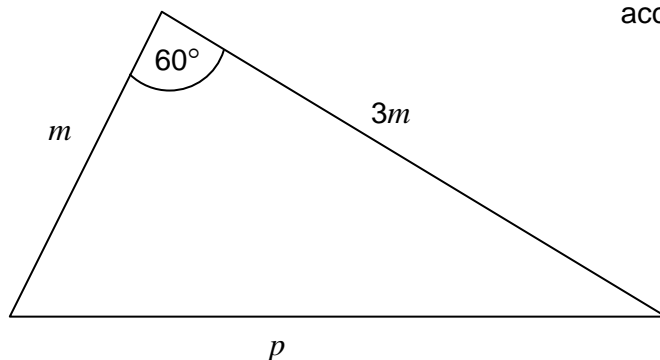
Show that y is a decreasing function for all values x .

$$\begin{aligned} \frac{dy}{dx} &= -8 - 3x^2 & -8 - 3x^2 &> 0 \\ & & -3x^2 &> 8 \\ & & 3x^2 &< -8 \\ & & x^2 &= -\frac{8}{3}, \text{ cannot square root } \therefore \text{ no solutions} \\ & & & \therefore \text{ decreasing} \end{aligned}$$

(3 marks)

11

Not drawn
accurately



Use the cosine rule to show that $p = m\sqrt{7}$

$$\begin{aligned} a^2 &= b^2 + c^2 - 2bc \cos A \\ p^2 &= m^2 + 9m^2 - 2 \times m \times 3m \times \cos 60 \\ &= \sqrt{10m^2 + 3m^2} \\ &= \sqrt{13m^2} \\ &= m\sqrt{13} \end{aligned}$$

(3 marks)

12

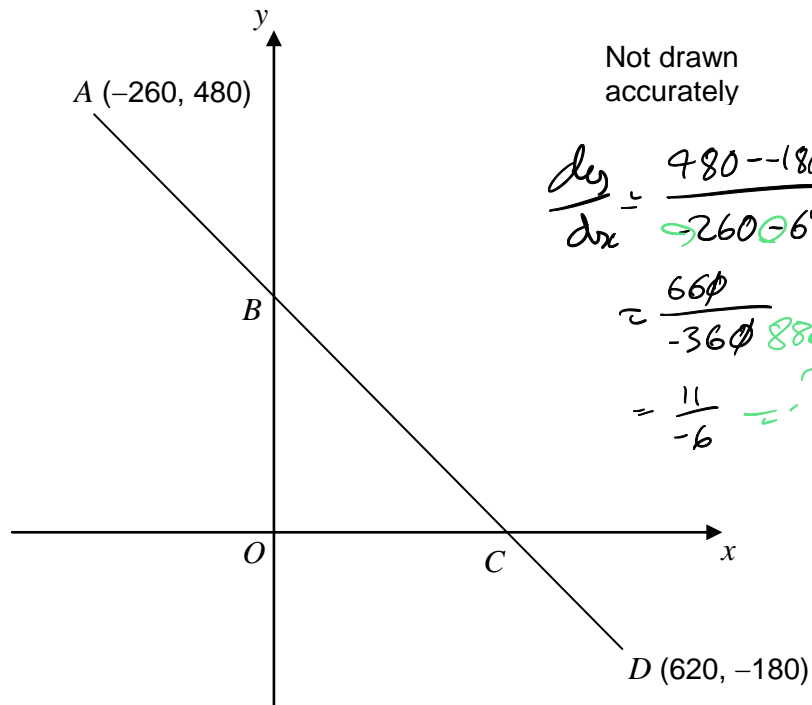
The diagram shows a straight line $ABCD$.

A is the point $(-260, 480)$

D is the point $(620, -180)$

The line cuts the y -axis at B and the x -axis at C .

$$\begin{array}{r} 11 \\ x \ 260 \\ \hline 260 \\ + \ 2600 \\ \hline 2860 \end{array}$$



Work out the coordinates of B and C .

$$y = \frac{11x}{-6} + C$$

$$480 = \frac{11x - 260}{-6} + C$$

$$480 = 2860 + C$$

$$C = ?$$

$B = (\dots, \dots) \quad C = (\dots, \dots)$ (6 marks)

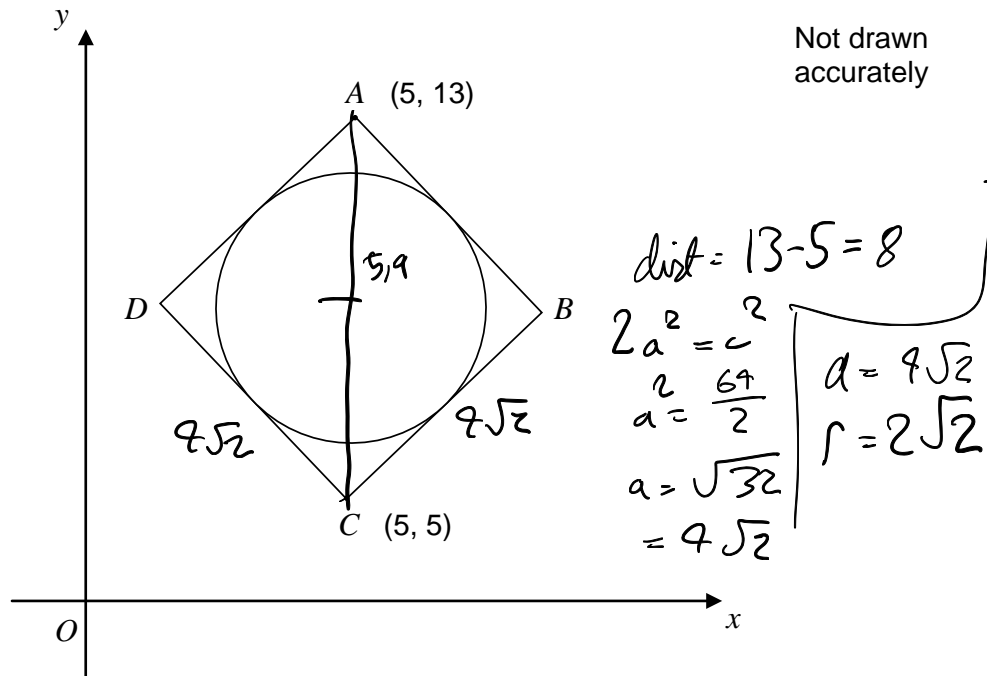
13

$ABCD$ is a square.

A is the point $(5, 13)$

C is the point $(5, 5)$

The circle touches the sides of the square.



Work out the equation of the circle.

.....

.....

.....

.....

.....

.....

.....

Answer $(x-5)^2 + (y-9)^2 = 8$

(5 marks)

- 14 (a) Show that $(x-2)$ is a factor of $x^3 + 8x^2 + x - 42$

$$f(x) = x^3 + 8x^2 + x - 42$$

$$f(2) = 8 + 32 + 2 - 42$$

$$= 42 - 42$$

$$= 0$$

(2 marks)

- 14 (b) Hence, or otherwise, work out **all** solutions of $x^3 + 8x^2 + x - 42 = 0$

$$= (x-2)(x^2 + kx + 21)$$

$$= x^3 + kx^2 + 21x - 2x^2 - 2kx - 42$$

$$= x^3 + (k-2)x^2 + (21-2k)x - 42$$

$$k-2=8$$

$$k=10$$

OR

$$21 \div 2 = 21$$

$$x^3 \div x = x^2$$

$$21 - 2k = 1$$

$$-2k = -20$$

$$k=10$$

$$= (x-2)(x^2 + 10x + 21)$$

$$= (x-2)(x+7)(x+3)$$

Answer

2, -7, -3

(4 marks)

15

Rationalise the denominator and simplify $\frac{5\sqrt{5} - 2}{2\sqrt{5} - 3}$

$$= \frac{5\sqrt{5} - 2}{-3 + 2\sqrt{5}} \times \frac{-3 - 2\sqrt{5}}{-3 - 2\sqrt{5}} = \frac{-15\sqrt{5} - 50 + 6 + 4\sqrt{5}}{9 - 20}$$

$$= \frac{-11\sqrt{5} - 44}{-11}$$

$$= \frac{11\sqrt{5} + 44}{11} = \sqrt{5} + 4 = 4 + \sqrt{5}$$

Answer $4 + \sqrt{5}$ (4 marks)

16

Prove that, for all values of x , $2x^2 - 8x + 9 > 0$ 

$$2x^2 - 8x + 9 < 0$$

$$2(x^2 - 4x) < -9$$

$$2(x-2)^2 - 9 < -9$$

$$2(x-2)^2 - 8 < -9$$

$$2(x-2)^2 < -1$$

no real roots

$(x-2)^2 < -\frac{1}{2}$, is negative, so no possible solutions where $x \geq 0$,
so all solutions are > 0

(5 marks)

Turn over for the next question

17

$$\begin{pmatrix} 2 & a \\ 1 & -3 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$$

Work out **all** possible pairs of values of a and b .

$$\begin{pmatrix} 2a+ab \\ a-3b \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$$

$$b = \frac{a-2}{3}$$

$$a = -3$$

$$b = \frac{-3-2}{3}$$

$$= \frac{-5}{3}$$

$$= -1\frac{2}{3}$$

$$a = -1$$

$$b = \frac{-1-2}{3}$$

$$= \frac{-3}{3}$$

$$= -1$$

$$a-3b=2$$

$$-3b=2-a$$

$$3b=a-2$$

$$b = \frac{a-2}{3}$$

$$2a+ab=-1$$

$$2a + a\left(\frac{a-2}{3}\right) = -1$$

$$6a + a^2 - 2a = -3$$

$$a^2 + 4a + 3 = 0$$

$$(a+3)(a+1) = 0$$

$$a = -3, -1$$

Answer $a = -3, b = -1\frac{2}{3}$ OR $a = -1, b = -1$

(6 marks)

END OF QUESTIONS