![AQA logo]

Level 2 Certificate
FURTHER MATHEMATICS

Paper 2   Calculator

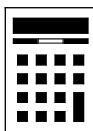Thursday 21 June 2018   Afternoon   Time allowed: 2 hours

Materials
For this paper you must have:
- a calculator
- mathematical instruments.

Instructions
- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- Do all rough work in this book. Cross through any work you do not want to be marked.
- In all calculations, show clearly how you work out your answer.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 105.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer book.
- The use of a calculator is expected but calculators with a facility for symbolic algebra must **not** be used.

For Examiner's Use	
Pages	Mark
3	
4–5	
6–7	
8–9	
10–11	
12–13	
14–15	
16–17	
18–19	
20–21	
22–23	
24–25	
26–27	
28–29	
TOTAL	

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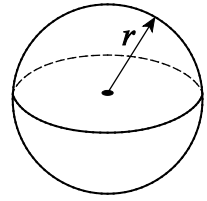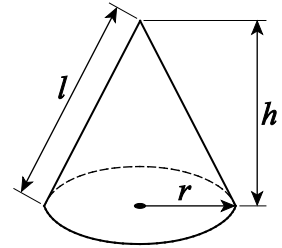**8360/2**

Formulae Sheet

Volume of sphere $= \dfrac{4}{3}\pi r^3$
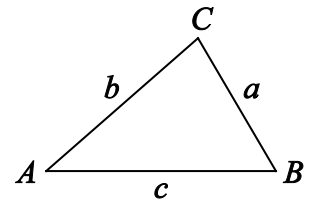
Surface area of sphere $= 4\pi r^2$

Volume of cone $= \dfrac{1}{3}\pi r^2 h$

Curved surface area of cone $= \pi r l$

In any triangle ABC

Area of triangle $= \dfrac{1}{2}ab \sin C$

Sine rule $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine rule $a^2 = b^2 + c^2 - 2bc \cos A$

$$\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$$

The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$, where $a \neq 0$, are given by $x = \dfrac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$

Trigonometric Identities

$\tan \theta \equiv \dfrac{\sin \theta}{\cos \theta} \qquad \sin^2 \theta + \cos^2 \theta \equiv 1$

Answer **all** questions in the spaces provided.

1 The nth term of a sequence is $\dfrac{1420 - 5n}{1420 + 5n}$

1 (a) Work out the **position** of the term that has the value zero.

[2 marks]

$$\frac{1420 - 5n}{1420 + 5n} = 0$$

$$1420 - 5n = 0$$

$$1420 = 5n$$

$$n = 284$$

Answer

1 (b) Write down the limiting value of the sequence as $n \to \infty$

[1 mark]

$$\lim n \to \infty = \frac{-5n}{+5n} = \frac{-5}{+5} = -1$$

Answer $x \to -1$

Turn over for the next question

Turn over ▶

2 P (−3, −10) and Q (a, b) are points on a straight line with gradient 12

Work out one possible pair of integer values for a and b.

[2 marks]

$-3 + 1 = -2$

$-10 + 12 = 2$

$a = \underline{\quad -2 \quad}$ $b = \underline{\quad 2 \quad}$

3 $p = \dfrac{m+2}{m^2+1}$

3 (a)   Work out the value of p when $m = -5.5$

[1 mark]

$$= \frac{-5.5+2}{(5.5)^2+1}$$

$$= \frac{-3.5}{31.25}$$

Answer $\qquad -\dfrac{18}{125} \qquad$

3 (b)   Work out the values of m when $p = 2$

[3 marks]

$2 = \dfrac{m+2}{m^2+1}$

$2m^2 + 2 - m - 2 = 0$

$2m^2 - m = 0$

$m(2m-1)^2 = 0$

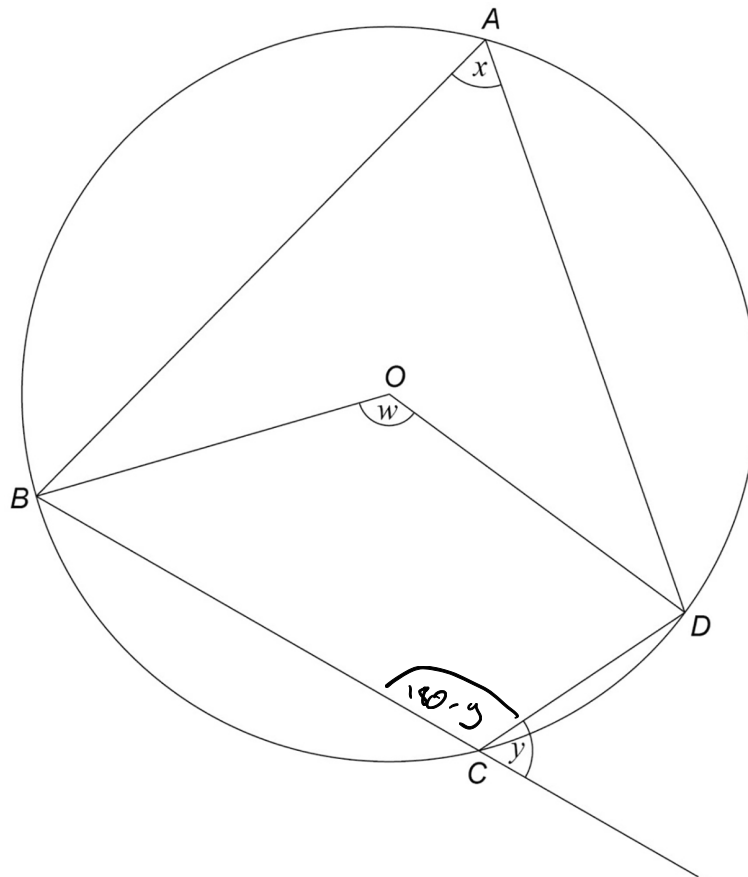$m = 0, \quad 2m-1 = 0$

$2m = 1$

$m = \frac{1}{2}$

Answer $\qquad 0, \frac{1}{2} \qquad$

Turn over for the next question

6

4   *A*, *B*, *C* and *D* are points on a circle, centre *O*.

$x + 180 - y = 180$

$x - y = 0$

$x = y$

Which statement is correct?

Tick **one** box.

[1 mark]

$x + y = 180°$ and $w = 2x$ ☐

$x + y = 180°$ and $x = 2w$ ☐

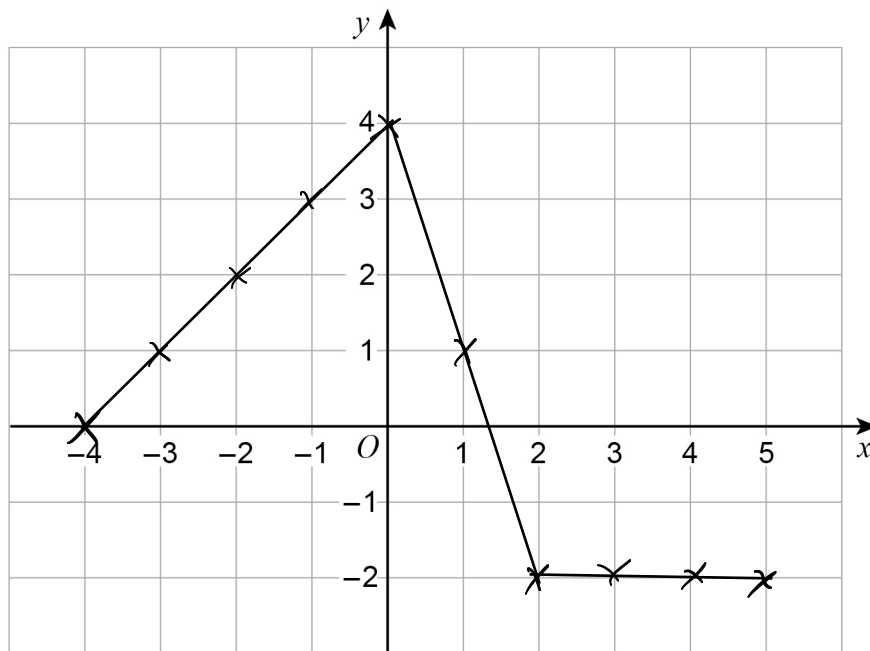$x = y$ and $w = 2x$ ✓

$x = y$ and $x = 2w$ ☐

5   On the grid, draw the graph of $y = f(x)$

$$f(x) = x + 4 \qquad -4 \leqslant x < 0$$

$$= 4 - 3x \qquad 0 \leqslant x < 2$$

$$= -2 \qquad\qquad 2 \leqslant x \leqslant 5$$

[4 marks]

Turn over for the next question

5

Turn over ▶

6 $f(x) = x^2 - 7$ for all values of x

$g(x) = 1 - 3x$ for $-4 \leqslant x \leqslant 4$

6 (a)   Work out the range of f(x).

Give your answer as an inequality.

[1 mark]

Answer $f(x) \geqslant -7$

6 (b)   Work out the range of g(x).

Give your answer as an inequality.

[2 marks]

$x =$ $1 - 3x$

-4 13

0 1

9 -11

Answer $-11 \leqslant g(x) \leqslant 13$

6 (c)   Solve $2f(x) = g(x)$

You **must** show your working.

Give your answers to 3 decimal places.

[4 marks]

$2x^2 - 19 = 1 - 3x$

$2x^2 + 3x - 15 = 0$

$2(x^2 + 1.5x) = 15$

$2((x + 0.75)^2 - \frac{9}{16}) = 15$

$2(x + 0.75)^2 - \frac{9}{8} = 15$

$2(x + 0.75)^2 = 16\frac{1}{8}$

$(x + 0.75)^2 = 8\frac{1}{16} = \frac{129}{16}$
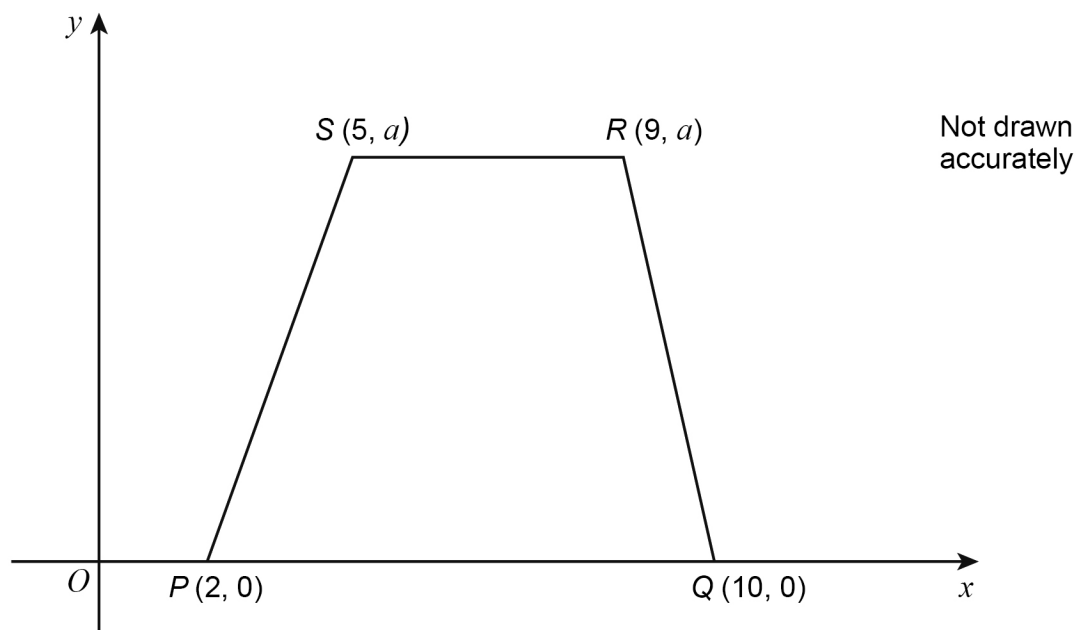
$x + 0.75 = \pm \frac{\sqrt{129}}{4}$

$x = \dfrac{+ - \sqrt{129} - 3}{4}$

Answer _____2. 089 , -3.589_____

Turn over for the next question

7

Turn over ▶

7 *PQRS* is a trapezium.

The area of the trapezium is 63 square units.

Work out the value of a.

[2 marks]

$A = \frac{a+b}{2}h$
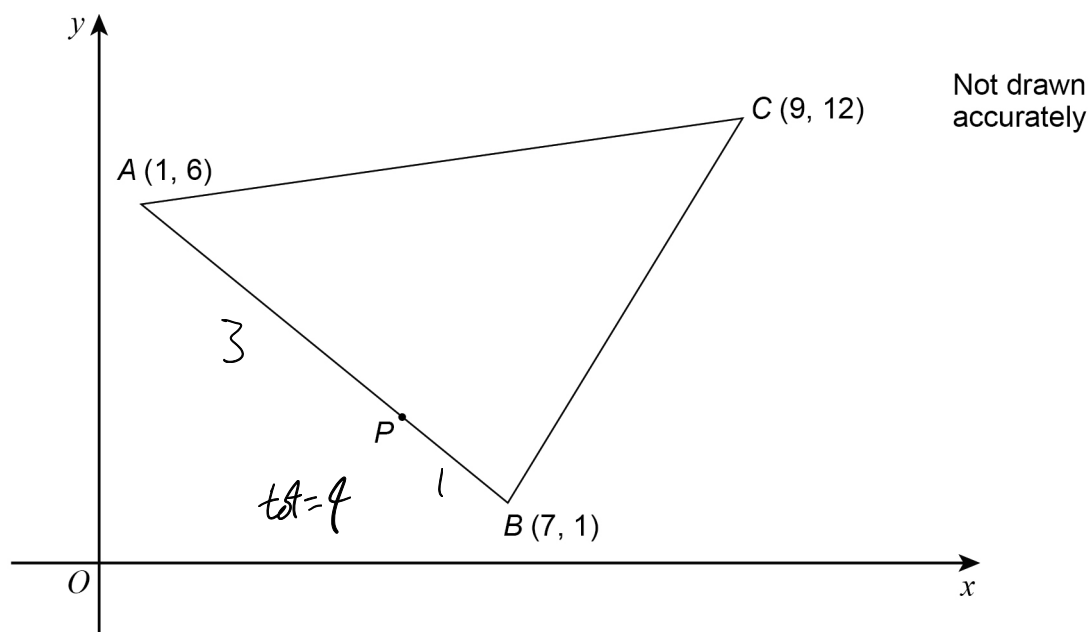
$63 = \frac{(9-5)+(10-2)}{2}a$

$63 = \frac{4+8}{2}a$

$63 = 6a$

$a = 10\frac{1}{2}$

Answer _____

8 Here is a sketch of triangle *ABC*.

P is a point on *AB*.

A (1, 6)

C (9, 12)

Not drawn accurately

3

P

$t\vec{A}=4$

1

B (7, 1)

O

x

y

AP : *PB*   is   3 : 1

Work out the length *PC*.

Give your answer to 4 significant figures.

[4 marks]

$\vec{AB} = (6, -5)$ $\frac{\vec{AB}}{4} = (\frac{3}{2}, -\frac{5}{4})$

$P = B - \frac{AB}{4} = (7 - \frac{3}{2}, 1 + \frac{5}{4}) = (5\frac{1}{2}, 2\frac{1}{4})$

$PC = \sqrt{(12 - 2\frac{1}{4})^2 + (9 - 5\frac{1}{2})^2}$

$= \sqrt{(9\frac{3}{4})^2 + (3\frac{1}{2})^2}$

$= 10.36$

Answer _____ units

6

Turn over ▶

9 $y = \dfrac{2x^7 + 15x^2}{3x} = \dfrac{2x^6 + 15x}{3} = \dfrac{2}{3}x^6 + 5x$

Work out the value of x when $\dfrac{dy}{dx} = 133$

[4 marks]

$\dfrac{dy}{dx} = 4x^5 + 5$

$4x^5 + 5 = 133$

$4x^5 = 128$

$x^5 = 32$

$x = 2$

Answer _____2_____

10 The transformation matrix $\begin{pmatrix} a & b \\ 2a & 3b \end{pmatrix}$ maps the point (1, −3) onto the point (1, 4)

Work out the values of a and b.

You **must** show your working.

[5 marks]

$$\begin{pmatrix} a & b \\ 2a & 3b \end{pmatrix} \begin{pmatrix} 1 \\ -3 \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$$

$$\begin{pmatrix} a - 3b \\ 2a - 9b \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$$

Ⓐ $a - 3b = 1$

Ⓑ $2a - 9b = 4$

②Ⓐ $2a - 6b = 2$

$-3b = 2$

$b = -\frac{2}{3}$

$a - 3b = 1$

$a + \frac{6}{3} = 1$

$a + 2 = 1$

$a = -1$

$a = $ -1

$b = $ $-\dfrac{2}{3}$

Turn over for the next question

9

Turn over ▶

11 Expand and simplify fully $(x + 2)(x + 3)(x + 4)$

[3 marks]

$$= (x^2 + 5x + 6)(x + 4)$$

$$= x^3 + 4x^2 + 5x^2 + 20x + 6x + 24$$

$$= x^3 + 9x^2 + 26x + 24$$

Answer $x^3 + 9x^2 + 26x + 24$

12 (a) Write $\dfrac{7}{9x} + \dfrac{2}{3x^2}$ as a single fraction in its simplest form.

[3 marks]

$$= \frac{7(3x^2) + 2(9x)}{(9x)(3x^2)} = \frac{2(x^2 + 18x)}{27x^3} = \frac{3(7x+6)}{3(9x^2)} = \frac{7x+6}{9x^2}$$

Answer _____

12 (b) Show that $\quad \dfrac{x^{4\ \ 3}}{x+4} \times \dfrac{x+2}{x_1} \div \dfrac{x^2}{3x+12}$

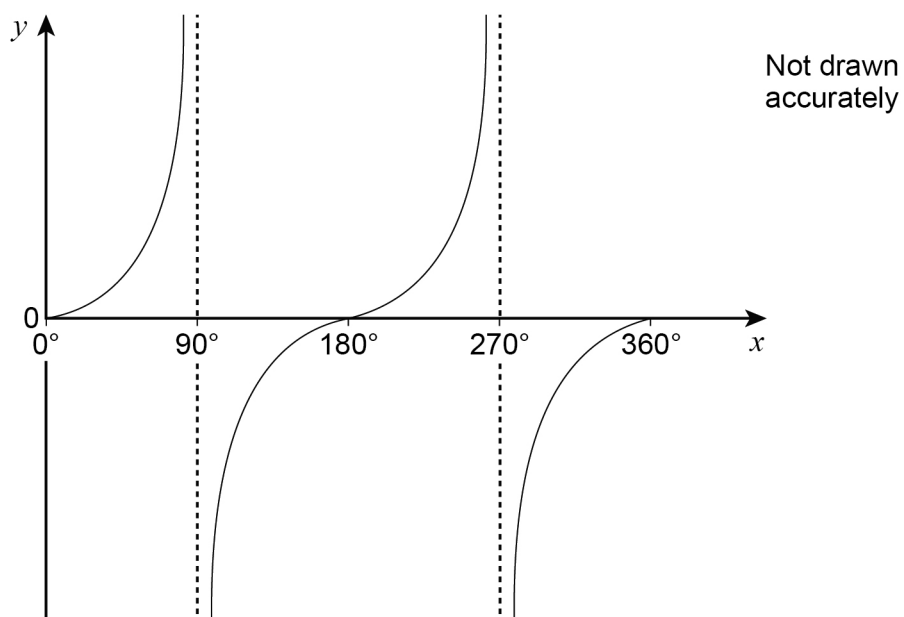simplifies to the form $ax^2 + bx$ where a and b are integers.

[4 marks]

$$= \frac{x^3(x+2)}{x+4} \times \frac{3x+12}{x} \quad \div \frac{x^2}{3x+12}$$

$$= \frac{x(x+2)(3x+12)}{x+4} = \frac{(x^2+2x)(3x+12)}{x+4}$$

$$= \frac{3x^3 + 18x^2 + 24x}{x+4} = \frac{3x(x^2+6x+8)}{x+4} = \frac{3x(x+4)(x+2)}{x+4}$$

$$= 3x(x+2) = 3x^2 + 6x$$

Turn over for the next question

10

Turn over ▶

13 (a) Here is a sketch of $y = \tan x$ for $0° \leqslant x \leqslant 360°$

Not drawn accurately

How many solutions of $\tan x = k$ where $k > 0$ are between 90° and 360° ?
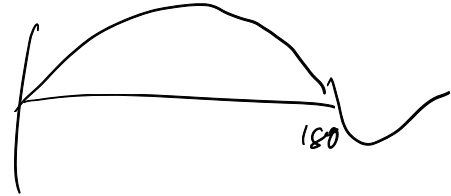
[1 mark]

$\tan x > 0$

Answer _____ 180 ? | _____

13 (b) $0 < p < 1$

How many solutions of $\sin x = p - 1$ are between 0° and 180° ?

You may use a sketch graph to help you.
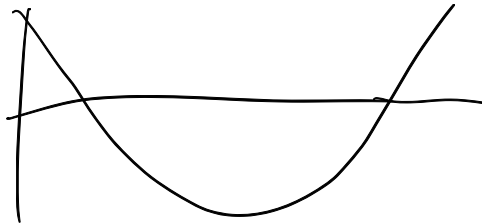
[1 mark]

$-1 < \sin x < 0$

180

Answer _____0_____

13 (c) State the coordinates of each point where the graph

$y = \cos x$ for $0° \leqslant x \leqslant 360°$

meets or intersects an axis.

[2 marks]

Answer $(0, 1) \, (90, 0) \, (270, 0)$

4

Turn over ▶

14 (a) Factorise fully $12pq^3r - 18pq^2r^2 + 24pq^2r$

[2 marks]

$6pq^2r(2q^3 - 3r^2 + 4)$

Answer _____

14 (b) Factorise fully $6(y+3)^5 + 4(y+3)^4$

Give your answer in its simplest form.

Do **not** attempt to expand $(y+3)^5$ or $(y+3)^4$

[3 marks]

$(y+3)^4(6(y+3)+4)$

$(y+3)^4(6y+18+4)$

$(y+3)^4(6y+22)$

$2(y+3)^4(3y+11)$

Answer _____

14 (c) Factorise fully $48 - 75x^2$

[2 marks]

$= -3(25x^2 - 16)$

$= -3(5x+4)(5x-4)$

Answer _____

15 Work out the rate of change of y with respect to x at the point on the curve

$y = x^2(x^2 - 9)$ where $x = -2$

You **must** show your working.

[4 marks]

$y = x^4 - 9x^2$

$\frac{dy}{dx} = 4x^3 - 18x$, $x = -2$, $= 4 \times -8 - 18 \times -2$

$= -32 + 36$

$= 4$

Answer $\qquad 4 \qquad$

Turn over for the next question

Turn over ▶

16 $A = 2 - 5x$ $B = 3x - 1$ $C = x^2$

Show that $(2A + 3B)^2 \equiv A + B + C$

[4 marks]

$(2A+3B)^2 \equiv \left(2(2-5x) + 3(3x-1)\right)^2$

$\equiv (4-10x+9x-3)^2$

$\equiv (1-x)^2$

$\equiv x^2-2x+1$ ⎤

$A+B+C \equiv x^2+3x-1+2-5x$ ⎥ — SAME

$\equiv x^2-2x+1$ ⎦

17   A circle has equation $x^2 + y^2 = 29$

P is the point (–5, 2)

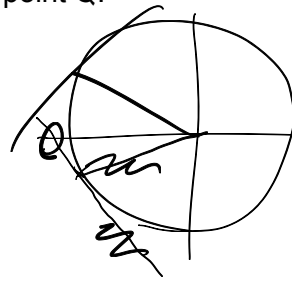**17 (a)**   Show that *P* is on the circle.

[1 mark]

$= 2^2 + (-5)^2$

$= 9 + 25 = 29$

17 (b) The tangent to the circle at *P* intersects the *x*-axis at point *Q*.

Work out the *x*-coordinate of *Q*.

You **must** show your working.

[4 marks]

$\frac{dy}{dx} = \frac{-2}{5}$, neg inv $= \frac{5}{2}$

$y = mx + c$

$y = \frac{5x}{2} + c$

$2 = \frac{5 \times -5}{2} + c$

$2 = \frac{-25}{2} + c$

$c = \frac{29}{2} = 14\frac{1}{2}$

$y = \frac{5}{2}x + \frac{29}{2}$

$2y = 5x + 29$

$0 = 5x + 29$

$-29 = 5x$

$x = -5\frac{4}{5}$

Answer _____

9

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18 (a) Work out all the **integer** values of x for which

$$-5 < 4x + 3 \leqslant 13$$

[3 marks]

$-8 < 4x \leqslant 10$

$-2 < x \leqslant 2.5$

Answer $\quad -1, 0, 1, 2$

18 (b) Work out the range of values of x for which

$$x^2 - 11x + 28 > 0$$

You **must** show your working.

[3 marks]

$(x-7)(x-4) > 0$

$x < 4$ and $x > 7$

Answer

19   Use **matrix multiplication** to show that, in the x–y plane,

- a reflection in the line $y = -x$, followed by
- a rotation, 90° anticlockwise about the origin, followed by
- a reflection in the x-axis

is equivalent to a transformation by the identity matrix.

[5 marks]

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$
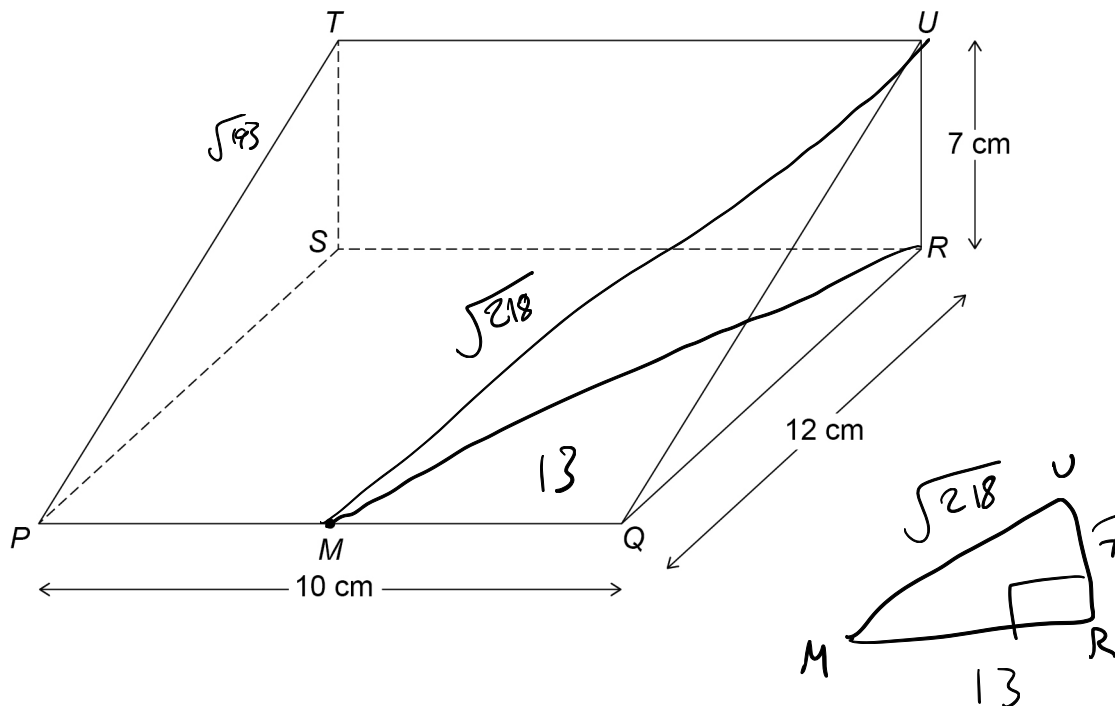
Turn over for the next question

11

Turn over ▶

20   *PQRSTU* is a triangular prism.

PQRS is a rectangle and angle *QRU* = 90°

PQ = 10 cm QR = 12 cm UR = 7 cm

M is the midpoint of *PQ*.

T   U

√193

7 cm

S   R

√218

12 cm

13

√218   U

7

M   R

13

P   M   Q

10 cm

20 (a)   Calculate the size of the angle between the line *UM* and the plane *PQRS*.

[4 marks]

$$a^2 + b^2 = c^2$$

$$UQ = \sqrt{7^2 + 12^2} = \sqrt{193}$$

$$UM = \sqrt{193 + 5^2} = \sqrt{218}$$

$$RM = \sqrt{12^2 + 5^2} = 13$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$UMR = \cos^{-1}\left(\frac{218 + 13^2 - 7^2}{2 \times \sqrt{218} \times 13}\right)$$

$$= \cos^{-1}(0.88\ldots$$

$$= 28.3°$$

Answer _____28.3°_____ degrees

20 (b) Calculate the size of the angle between the planes *UMR* and *UQR*. AKA $M\hat{Q}R$

[2 marks]

$\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

$M\hat{Q}R = \cos^{-1}\left(\dfrac{13^2 + 12^2 - 5^2}{2 \times 13 \times 12}\right)$

$= \cos^{-1}\left(\dfrac{12}{13}\right)$

$= 22.6$

Answer _____ degrees

Turn over for the next question

Turn over ▶

21 The continuous curve $y = f(x)$ has exactly two stationary points.

Here is some information about the curve.

$x < -1$	$x = -1$	$-1 < x < 2$	$x = 2$	$x > 2$
$\dfrac{dy}{dx}$ is positive	$\dfrac{dy}{dx}$ is zero	$\dfrac{dy}{dx}$ is negative	$\dfrac{dy}{dx}$ is zero	$\dfrac{dy}{dx}$ is negative

$f(-1) = 3$ and $f(2) = 1$

State the coordinates **and** the nature of each of the stationary points.

[3 marks]
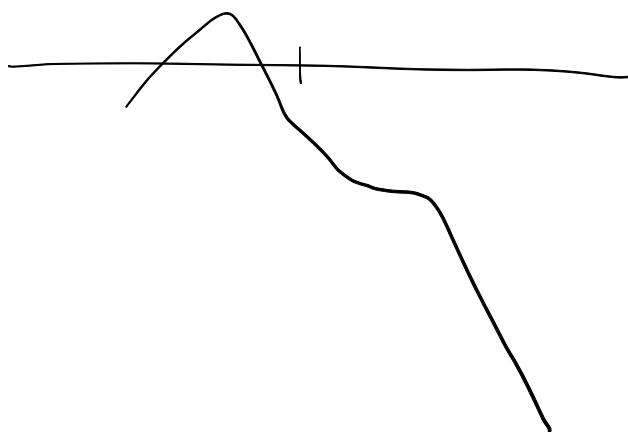
stationary point (__−1__ , __3__)   nature __Max__

stationary point (__2__ , __1__)   nature __? inflection__

22 (a) $8 \cos x + 5 \sin x = 0$ where $90° < x < 180°$

Work out the size of angle x.

[3 marks]

?

$\div \cos x$

$8 + 5 \frac{\sin x}{\cos x} = 0$

$5 \tan x = -8$

$\tan = 1.6$

$\theta = -57.99, + 180 = 122$

Answer _____122_____ degrees

22 (b) $6 \sin^2 x + 4 \cos^2 x \equiv A + B \cos^2 x$ where A and B are integers.

Work out the values of A and B.

You **must** show your working.

[2 marks]

$\sin^2 x + \cos^2 x = 1$

$= 6 \sin^2 x + 4 \cos^2 x$

$= 6 (1 - \cos^2 x) + 4 \cos^2 x$

$= 6 - 6 \cos^2 x + 4 \cos^2 x$

A = _____6_____   B = _____-2_____

8

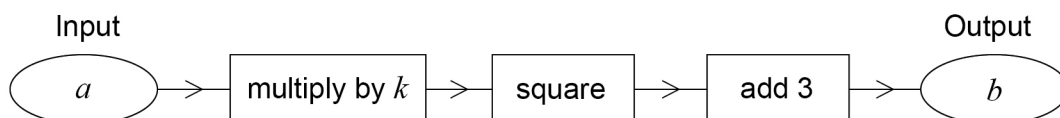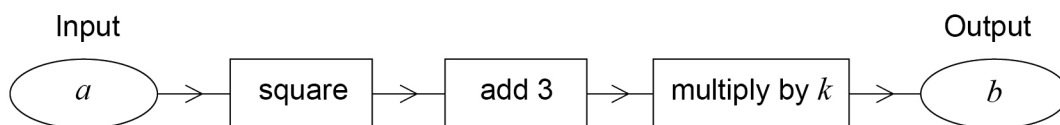**23** For each of these two function machines, when the input is a the output is b.

$k > 0$ and $k \neq 1$ and $a > 0$

Input   Output

a → square → add 3 → multiply by k → b

Input   Output

a → multiply by k → square → add 3 → b

Work out an expression for a in terms of k.

Give your answer in its simplest form.

⑤

[6 marks]

$$k\left(a^2 + 3\right) = b, \quad ka^2 + 3k = b$$

$$(ak)^2 + 3 = b, \quad a^2k^2 + 3 = b$$

$$ka^2 + 3k = a^2k^2 + 3$$

$$a^2k - a^2k^2 = 3 - 3k$$

$$a^2(k - k^2) = 3 - 3k$$

$$a^2 = \frac{3 - 3k}{k - k^2}$$

Cancel out $(1-k)$

$$a = \sqrt{\frac{3 - 3k}{k - k^2}}$$

$$a = \sqrt{\frac{3}{k}}$$

Answer _____

24 Work out the value of p when

$$9^{0.5p} \times 81 = 27^{2p-1}$$

[4 marks]

$$3^{2^{0.5p}} \times 3^4 = 3^{3^{2p-1}}$$

$$3^{p+4} = 3^{6p-3}$$

$$p + 4 = 6p - 3$$

$$7 = 5p$$

$$p = \frac{7}{5}$$

Answer _____

END OF QUESTIONS

10

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