

Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										



Level 2 Certificate in Further Mathematics
June 2014

Further Mathematics

8360/2

Level 2

Paper 2 Calculator

Friday 20 June 2014 9.00 am to 11.00 am

<p>For this paper you must have:</p> <ul style="list-style-type: none"> • a calculator • mathematical instruments. 	
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Time allowed

- 2 hours

Instructions

- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- In all calculations, show clearly how you work out your answer.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 105.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer book.
- The use of a calculator is expected but calculators with a facility for symbolic algebra must **not** be used.

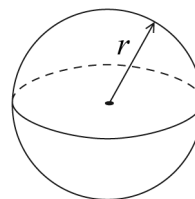
For Examiner's Use	
Examiner's Initials	
Pages	Mark
3	
4 – 5	
6 – 7	
8 – 9	
10 – 11	
12 – 13	
14 – 15	
16 – 17	
18 – 19	
20 – 21	
22 – 23	
24 – 25	
26 – 27	
28	
TOTAL	



Formulae Sheet

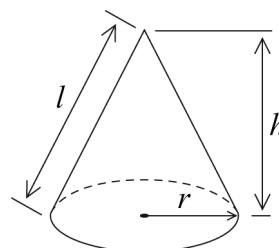
Volume of sphere $= \frac{4}{3}\pi r^3$

Surface area of sphere $= 4\pi r^2$



Volume of cone $= \frac{1}{3}\pi r^2 h$

Curved surface area of cone $= \pi r l$



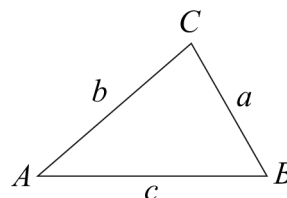
In any triangle ABC

Area of triangle $= \frac{1}{2}ab \sin C$

Sine rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule $a^2 = b^2 + c^2 - 2bc \cos A$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$



The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$, where $a \neq 0$, are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Trigonometric Identities

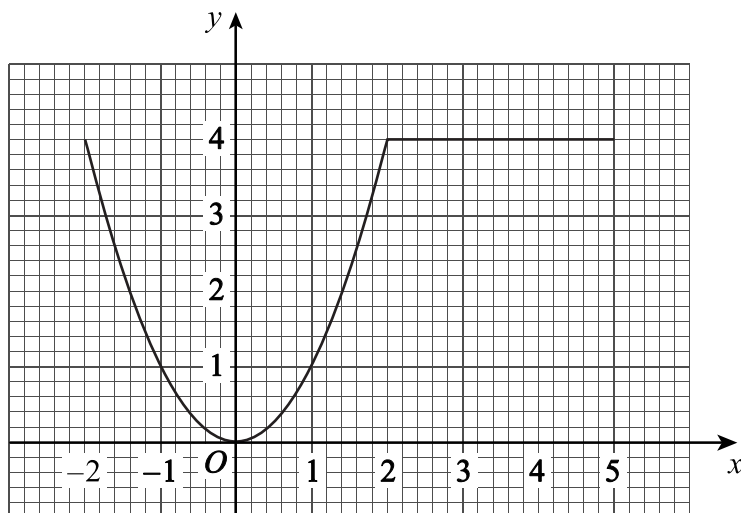
$$\tan \theta \equiv \frac{\sin \theta}{\cos \theta} \quad \sin^2 \theta + \cos^2 \theta \equiv 1$$



Answer **all** questions in the spaces provided.

1

The graph of $y = f(x)$ for the full domain is shown.
The graph consists of a quadratic curve and a straight line.



Complete the boxes to describe $f(x)$.

[3 marks]

$$f(x) = \boxed{x^2} \quad -2 \leq x \leq 2$$

$$= \boxed{4} \quad 2 < x \leq \boxed{5}$$

Turn over for the next question

Turn over ►

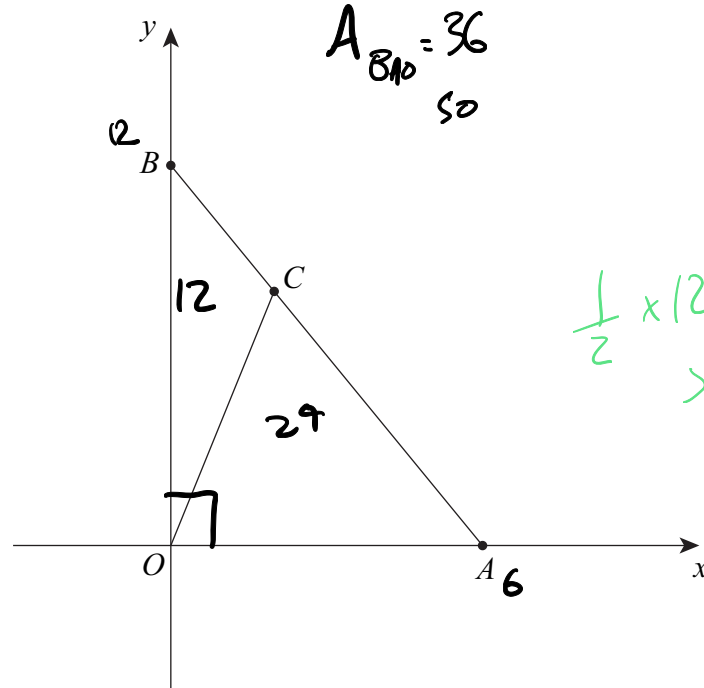


2

The equation of line AB is $y = 12 - 2x$ The area of triangle OCA is 24 square units.

$$y = 12 - 2x$$

$$(6, 0) \text{ and } (0, 12)$$

Not drawn
accuratelyWork out the coordinates of C .

??

[5 marks]

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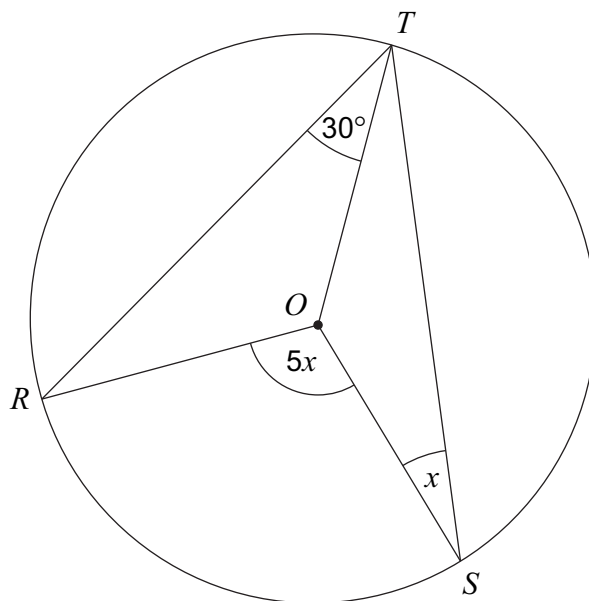
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Answer (..... ,)



- 3 R , S and T are on the circumference of a circle, centre O .



Not drawn
accurately

- 3 (a) Give a reason why angle $OTS = x$

? $OT = OS$

[1 mark]

so isosceles

- 3 (b) Work out the value of x .

? $5x = 2(x + 30)$

[3 marks]

30

Answer..... degrees

Turn over for the next question

Turn over ►



4 (a) Expand $x^2(x - 2)$

[2 marks]

Answer $x^3 - 2x^2$

4 (b) A curve has equation $y = x^2(x - 2)$

Work out the gradient of the curve at the point (3, 9).

[3 marks]

$\frac{dy}{dx} = 3x^2 - 4x$
 $x=3, m=15$

Answer

4 (c) Line L is the tangent to the curve $y = x^2(x - 2)$ at the point (3, 9).

Work out the equation of L.

Give your answer in the form $y = mx + c$

[2 marks]

$m=15$, $y = 15x + c$
 $9 = 45 + c$
 $c = -36$

Answer $y = 15x - 36$



5

Solve

$$\frac{4c+3}{2} + \frac{c-8}{5} = 1$$

[4 marks]

$$\frac{5(4c+3)}{2} + c - 8 = 5$$

$$5(4c+3) + 2(c-8) = 10$$

$$20c + 15 + 2c - 16 = 10$$

$$22c = 11$$

$$c = 0.5$$

$c = \dots\dots\dots$

Turn over for the next question



6

Two circles, each with centre O , are shown.
The equations of the circles are

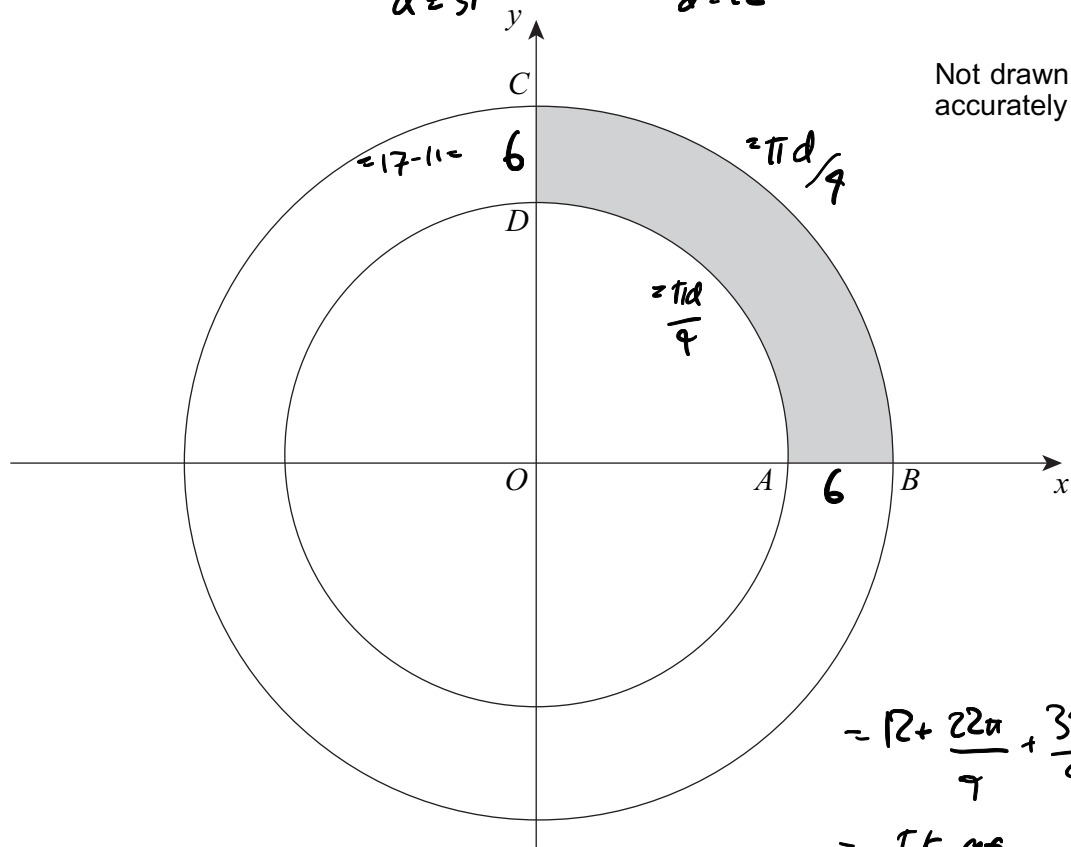
$$x^2 + y^2 = 289 \quad \text{and} \quad x^2 + y^2 = 121$$

$$\sqrt{} = 17$$

$$d = 34$$

$$\sqrt{} = 11$$

$$d = 22$$



Work out the perimeter of the shaded section $ABCD$.

[5 marks]

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Answer **56.0**



7 (a) Simplify $\sqrt{x^5 \times x^9}$

Give your answer in the form x^p where p is an integer.

[2 marks]

$$\begin{aligned}
 &= \sqrt{x^{14}} \\
 &= (x^{14})^{1/2} = x^{14/2} \\
 &= x^7
 \end{aligned}$$

$$x^7$$

Answer

7 (b) Solve $y^{-3} = 125$

[2 marks]

$$\begin{aligned}
 \frac{1}{y^3} &= 125 \\
 y^3 &= \frac{1}{125} \\
 y &= \sqrt[3]{\frac{1}{125}} \\
 &= \frac{1}{5}
 \end{aligned}$$

$y =$

Turn over for the next question

Turn over ►



8

$$M = \begin{pmatrix} -2 & -1 \\ 3 & 1 \end{pmatrix}$$

Show that $M^3 = I$ 2 [4 marks]

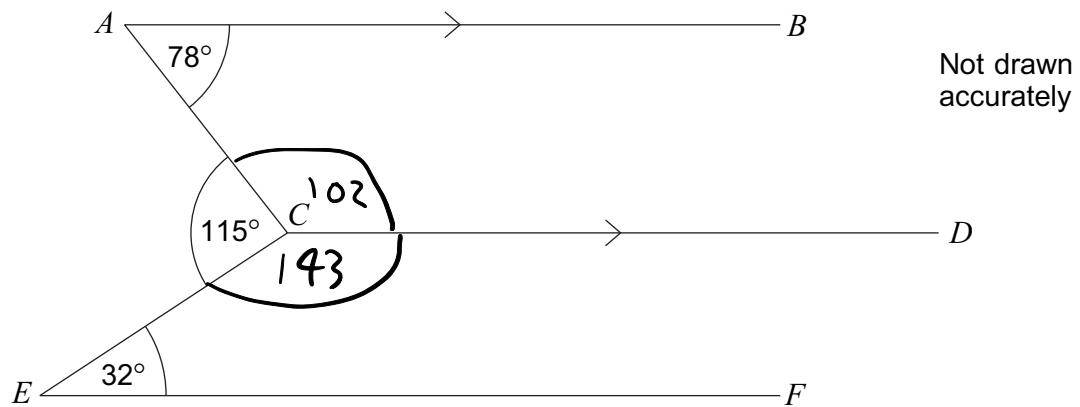
$$= \begin{pmatrix} -2 & -1 \\ 3 & 1 \end{pmatrix} \begin{pmatrix} -2 & -1 \\ 3 & 1 \end{pmatrix} \begin{pmatrix} -2 & -1 \\ 3 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 1 \\ -3 & -2 \end{pmatrix} \begin{pmatrix} -2 & -1 \\ 3 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$



9



AB is parallel to CD .

Is EF parallel to CD ?

You **must** show your working.

$$143 + 32 = 175 \neq 180. \text{ Not parallel.}$$

[3 marks]

Turn over for the next question

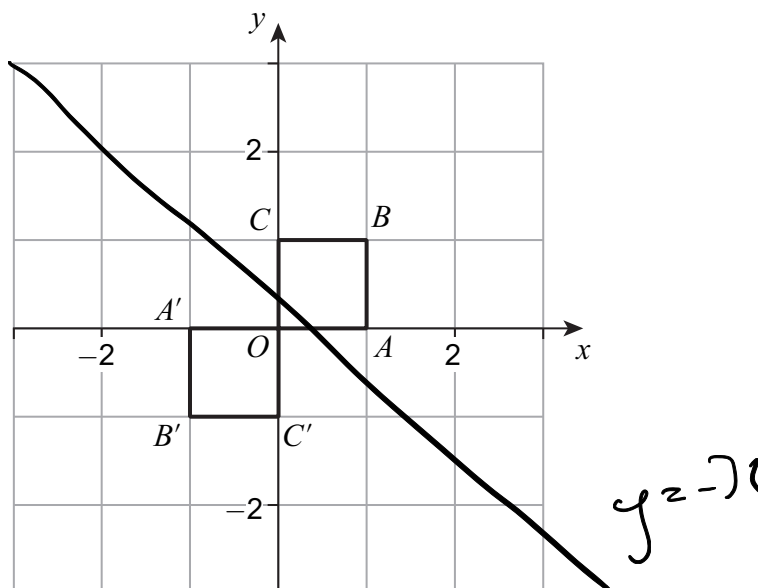
Turn over ►



10 The unit square $OABC$ has vertices

$$O(0, 0) \quad A(1, 0) \quad B(1, 1) \quad C(0, 1)$$

10 (a) $OABC$ is mapped to $OA'B'C'$ under transformation matrix \mathbf{M} .



Work out matrix \mathbf{M} .

[2 marks]

$$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

Answer

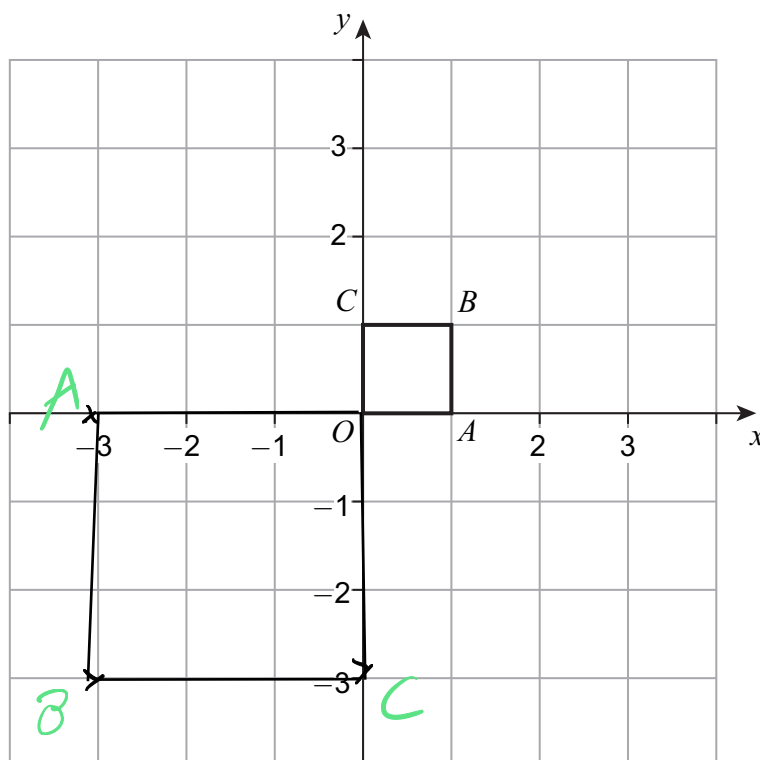


10 (b) $OABC$ is mapped to $OA''B''C''$ under transformation matrix $\begin{pmatrix} -3 & 0 \\ 0 & -3 \end{pmatrix}$

Draw **and** label $OA''B''C''$ on the diagram below.

$y = -x$ mult by 3 [3 marks]

②



Turn over for the next question



11 (a) Simplify fully

$$\frac{8c^7}{15d^6} \div \frac{6c^2}{5d^3}$$

[3 marks]

$$\begin{aligned} &= \frac{8c^7}{15d^6} \times \frac{5d^3}{6c^2} \\ &= \frac{\cancel{8}^2 \cancel{c}^5 \cancel{d}^3}{\cancel{15}^3 \cancel{c}^2 \cancel{d}^3} = \frac{4c^5}{9d^3} \end{aligned}$$

Answer

11 (b) Write as a single fraction

$$\frac{5}{m+1} + \frac{6}{m-4}$$

Give your answer in its simplest form.

[4 marks]

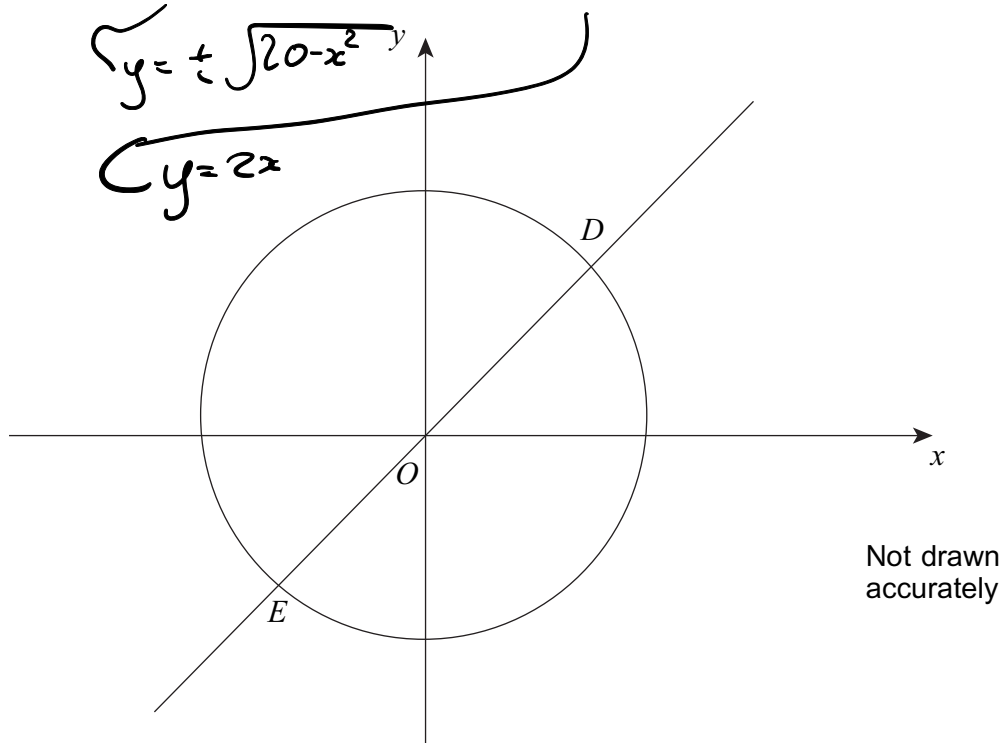
$$\begin{aligned} &\frac{5(m-4) + 6(m+1)}{(m+1)(m-4)} \\ &= \frac{5m - 20 + 6m + 6}{m^2 - 3m - 4} \\ &= \frac{11m - 14}{m^2 - 3m - 4} \end{aligned}$$

Answer



12

The circle $x^2 + y^2 = 20$ and the line $y = 2x$ intersect at points D and E .



Work out the coordinates of D and E .

Do **not** use trial and improvement.

You **must** show your working.

$$2x = \sqrt{20 - x^2}$$

[5 marks]

$$x^2 + y^2 = 20$$

$$4x^2 = 20 - x^2$$

$$2^2 + 4^2 = 4 + 16 = 20$$

$$5x^2 - 20 = 0$$

$$-2^2 + -4^2 = 4 + 16 = 20$$

$$(5x + 10)(x - 2) = 0$$

$$x = 2, -2$$

$$y = 2x, y = 4, -4$$

$$D(2, 4) \quad E(-2, -4)$$

Turn over ►

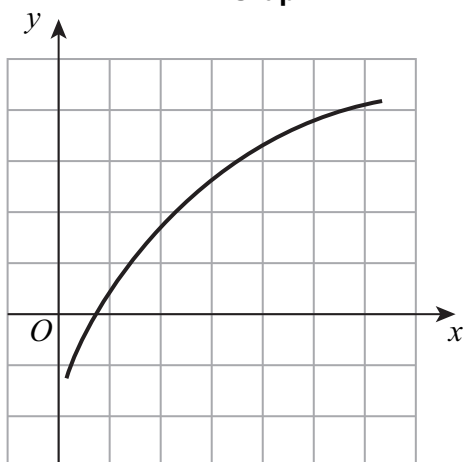
12



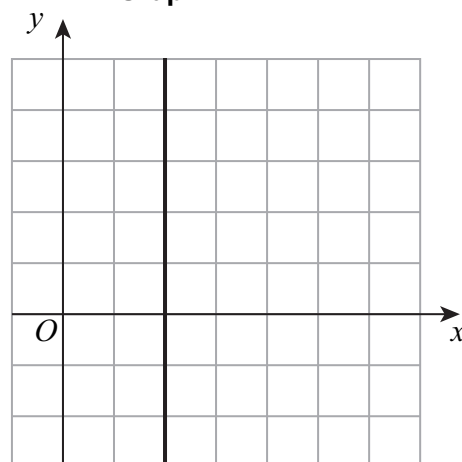
13

Here are five graphs.

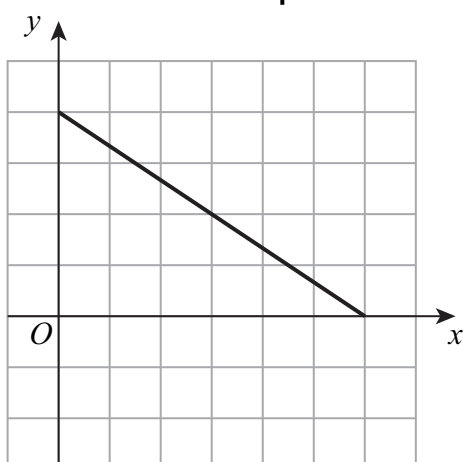
Graph A



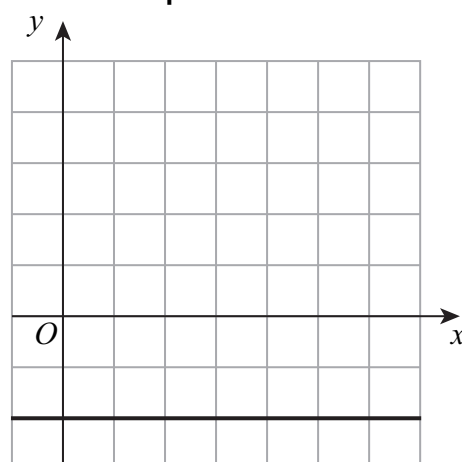
Graph B



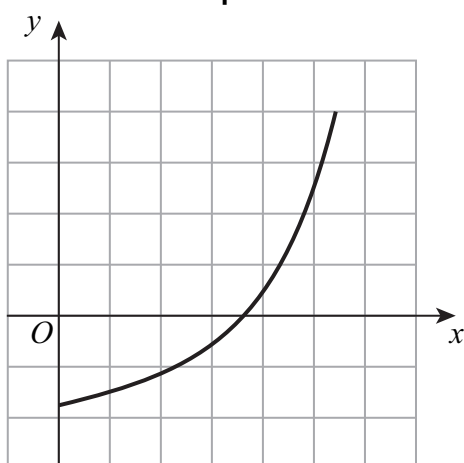
Graph C



Graph D



Graph E



For each of the following statements, decide which graph is being described.
Circle your answer each time.

- 13 (a) The rate of change of y with respect to x is always negative.

[1 mark]

A B C D E

- 13 (b) The rate of change of y with respect to x is always zero.

[1 mark]

A B C D E

- 13 (c) As x increases, the rate of change of y with respect to x decreases.

[1 mark]

A B C D E

Turn over for the next question



14

Rearrange

$$x = \frac{2w+1}{5-3w} \quad \text{to make } w \text{ the subject.}$$

[4 marks]

$$5x - 3wx = 2w + 1$$

$$5x - 1 = 2w + 3wx$$

$$5x - 1 = w(2 + 3x)$$

$$w = \frac{5x-1}{2+3x}$$

Answer



- 15 (a) The n th term of a sequence is $n^2 + 12n + 27$

By factorising, or otherwise, show that the 20th term can be written as the product of two prime numbers.

$$n=20 \Rightarrow 20^2 + 12 \times 20 + 27 = 667 = 23 \times 29$$

[2 marks]

- 15 (b) The n th term of a different sequence is $n^2 - 6n + 14$

By completing the square, or otherwise, show that every term is positive.

$$\frac{dy}{dx} = 2n - 6, \text{ MIN} \Rightarrow 2n - 6 = 0$$

$$2n = 6$$

$$n = 3$$

$$n^2 - 6n + 14, n=3 \Rightarrow 5$$

Min Point is positive, so all terms positive.

\therefore is positive quadratic.

[3 marks]

Turn over for the next question

Turn over ►



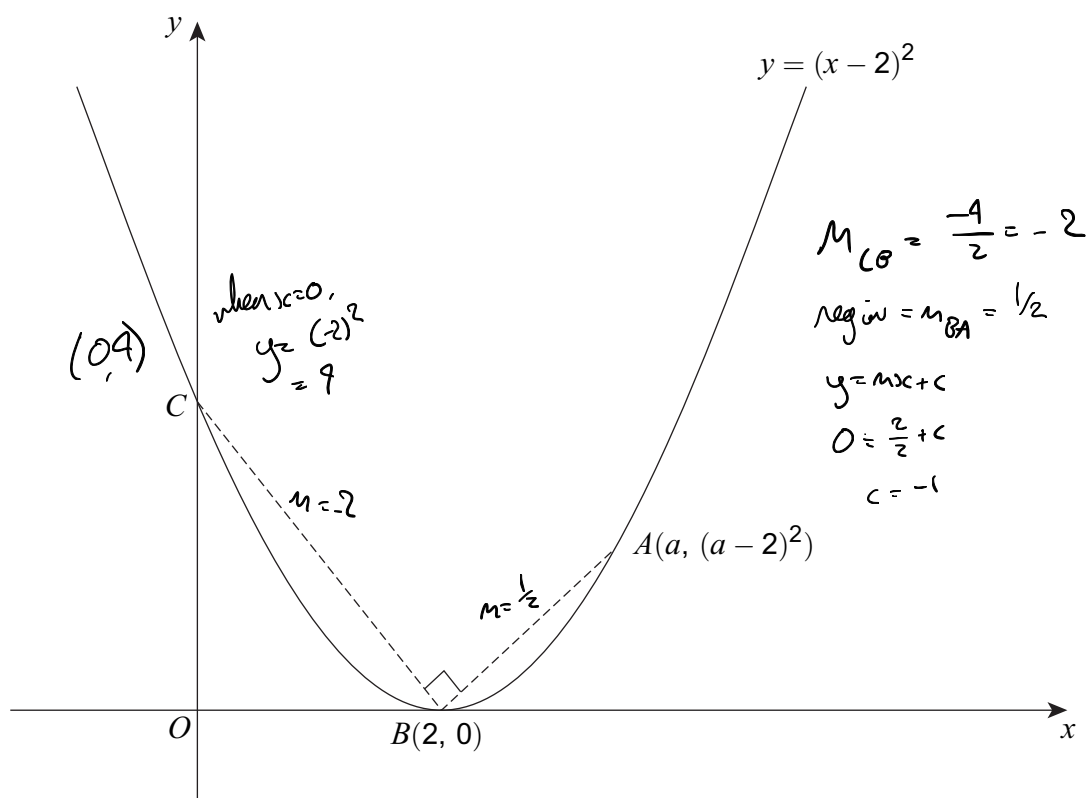
16 (a) Simplify $\frac{(a-2)^2}{a-2}$

$$\frac{(a-2)(a-2)}{(a-2)}$$

[1 mark]

Answer $a-2$

16 (b) Here is a sketch of the curve $y = (x-2)^2$



- The curve touches the x -axis at B and intersects the y -axis at C .
- Angle ABC is 90° .
- The curve passes through $A(a, (a-2)^2)$

$$\begin{aligned} BA \Rightarrow y &= \frac{x}{2} - 1 \\ (a-2)^2 &= \frac{a}{2} - 1 \\ a^2 - 4a + 4 &= \frac{a}{2} - 1 \\ 2a^2 - 8a + 8 &= a - 2 \\ 2a^2 - 7a + 10 &= 0 \\ (2a-5)(a-2) & \end{aligned}$$

$$a = \frac{5}{2}, 2$$



Work out the value of a .

9 [5 marks]

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Answer ~~2~~, $5/2$

Turn over for the next question

Turn over ►



17 (a) Factorise fully $12c^2d - 9d^2$

[2 marks]

$$3d(4c^2 - 3d)$$

Answer

17 (b) Factorise fully $(w+4)^3 - (w+4)^2(w+1)$

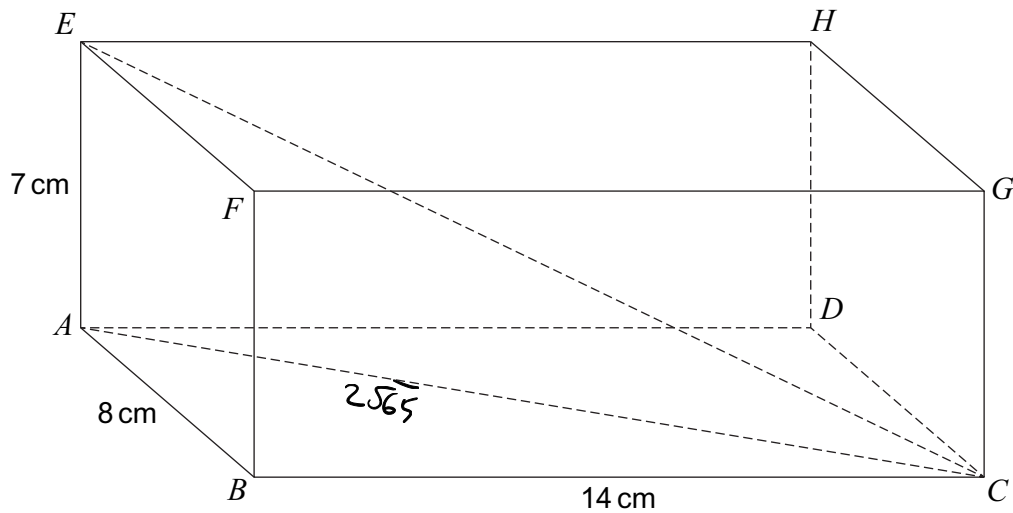
[3 marks]

$$\begin{aligned}
 &= \binom{3}{0} w^3 + \binom{3}{1} w^2 4 + \binom{3}{2} w 4^2 + \binom{3}{3} 4^3 - (w^2 + 8w + 16)(w+1) \\
 &= w^3 + 12w^2 + 48w + 64 - (w^3 + 8w^2 + 16w + w^2 + 8w + 16) \\
 &= \cancel{w^3} + 12w^2 + 48w + 64 - (\cancel{w^3} + 9w^2 + 24w + 16) \\
 &= -3w^2 + 24w + 48 \\
 &= -3(w^2 + 8w + 16) \\
 &= -3(w+4)^2
 \end{aligned}$$

Answer



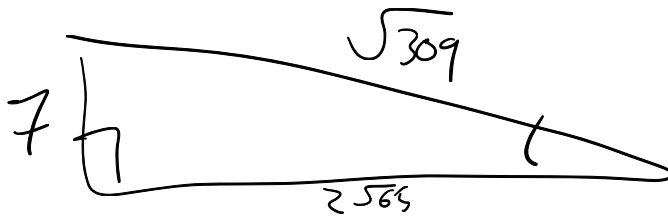
18

 $ABCDEFGH$ is a cuboid.Work out the angle between EC and $ABCD$.

[3 marks]

$$AC = \sqrt{8^2 + 14^2} = 2565$$

$$EC = \sqrt{8^2 + 7^2 + 14^2} = \sqrt{309}$$



$$\sin \theta = \frac{7}{\sqrt{309}}$$

$$\theta = \sin^{-1}\left(\frac{7}{\sqrt{309}}\right)$$

$$= 23.96^\circ$$

23.5

Answer degrees

Turn over ►

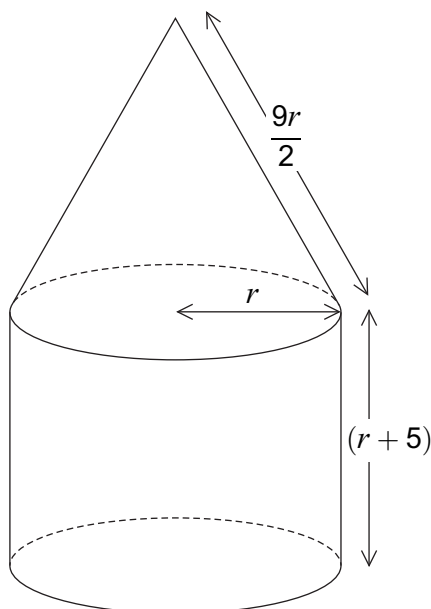


19

On this diagram all lengths are given in centimetres.
A cylinder and cone are joined together to make a solid.

The cylinder has radius r and height $(r + 5)$

The cone has radius r and slant height $\frac{9r}{2}$



19 (a)

Show that the **total** surface area of the solid, in cm^2 , is

$$\frac{5\pi r}{2}(3r + 4)$$

[4 marks]

$$CSA_{\text{cone}} = \pi r l = \frac{9r}{2} r \pi = \frac{9r^2 \pi}{2}$$

$$CSA_{\text{cylinder}} = \pi r h = \pi r (r + 5) = (r^2 + 5r) \pi$$

$$TSA = \frac{9r^2 \pi}{2} + (r^2 + 5r) \pi$$

$$= \frac{9r^2 \pi + 2r^2 \pi + 10r \pi}{2}$$

$$= \frac{11r^2 \pi + 10r \pi}{2}$$

$$= \frac{r \pi (11r + 10)}{2}$$

UHH?



- 19 (b) The total surface area of the solid is $1200\pi \text{ cm}^2$

Work out the value of r .

[5 marks]

$$= \frac{5\pi r (3r+4)}{2} = 1200\pi$$

$$= 5\pi r (3r+4) = 2400\pi$$

$$= 5r (3r+4) = 2400$$

$$= 15r^2 + 20r = 2400$$

$$15r = 2380$$

$$r = 158 \frac{2}{3} \text{ cm}$$

NO! - 12

Answer

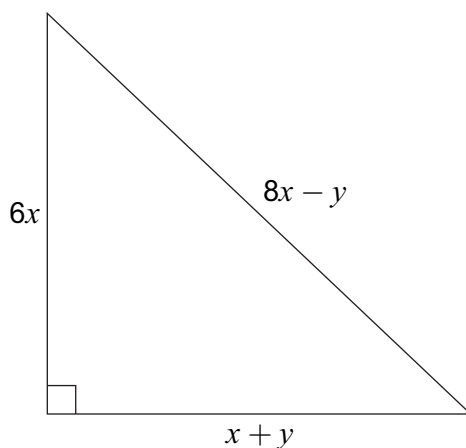
Turn over for the next question

Turn over ►



20

The diagram shows a right-angled triangle.

Not drawn
accurately

Prove algebraically that

$x : y = 2 : 3$

$$(8x - y)^2 = (6x)^2 + (x + y)^2$$

$$64x^2 - 16xy + y^2 = 36x^2 + x^2 + 2xy + y^2$$

$$27x^2 - 18xy = 0$$

$$27x^2 = 18xy$$

$$27x = 18y$$

$$3x = 2y$$

$$x : y = 2 : 3$$

[6 marks]



21

Solve

$16 \sin^2 x = 1$

for

$0^\circ \leq x \leq 270^\circ$

[5 marks]

???

$$\sin x = \sqrt{\frac{1}{16}} = \frac{1}{4}$$

$$\sin x = -\sqrt{\frac{1}{16}} = -\frac{1}{4}$$
$$= 14.5$$

$$= 14.5, 198.5$$

Answer

Turn over for the next question

Turn over ►



22

The curve $y = f(x)$ has $\frac{dy}{dx} = kx(x-3)^3$ where k is a **negative** constant.

There is a stationary point at $x = 3$

Determine the nature of this stationary point.
You **must** show your working.

[3 marks]

$$\frac{dy}{dx} = kx(x^3 - 9x^2 + 27x - 27)$$

$$\frac{dy}{dx} = kx^4 - 9kx^3 + 27kx^2 - 27kx$$

$$\frac{d^2y}{dx^2} = 4kx^3 - 27kx^2 + 54kx - 27k$$

$x = 3$:

$$\text{all } k = -1$$

$$k = -100$$

$$= 0$$

$$= 0$$

Answer MIN

END OF QUESTIONS

