AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname _____

Forename(s) _____

Candidate signature _____

Level 2 Certificate in Further Mathematics
FURTHER MATHEMATICS

Level 2 Paper 1 Non-Calculator

Monday 20 June 2016 Morning Time allowed: 1 hour 30 minutes

Materials

For this paper you must have:
- mathematical instruments.

You must **not** use a calculator.

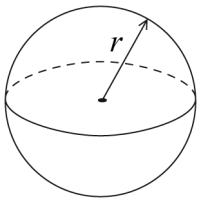
Instructions
- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- Do all rough work in this book. Cross through any work you do not want to be marked.
- In all calculations, show clearly how you work out your answer.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 70.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer book.

Formulae Sheet

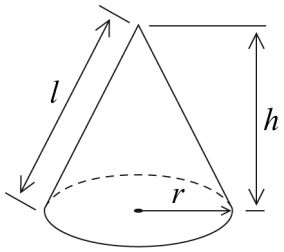
Volume of sphere $= \frac{4}{3}\pi r^3$

Surface area of sphere $= 4\pi r^2$



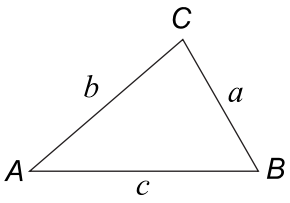
Volume of cone $= \frac{1}{3}\pi r^2 h$

Curved surface area of cone $= \pi r l$



In any triangle *ABC*

Area of triangle $= \frac{1}{2}ab\sin C$



Sine rule $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine rule $a^2 = b^2 + c^2 - 2bc\cos A$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$, where $a \neq 0$, are given by

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

Trigonometric Identities

$\tan\theta \equiv \dfrac{\sin\theta}{\cos\theta}$ $\qquad \sin^2\theta + \cos^2\theta \equiv 1$

Answer **all** questions in the spaces provided.

1 $y = x^2(x - 10)$

Work out $\frac{dy}{dx}$

$y = x^3 - 10x^2$

$\frac{dy}{dx} = 3x^2 - 20x$

[3 marks]

Answer _____

2 $4\begin{pmatrix} 1 - 2a \\ a \end{pmatrix} = \begin{pmatrix} b \\ 12 \end{pmatrix}$

Work out the values of a and b.

[3 marks]

$\begin{pmatrix} 4 - 8a \\ 4a \end{pmatrix} = \begin{pmatrix} b \\ 12 \end{pmatrix}$

$4a = 12$

$a = 3$

$4 - 8a = b$

$4 - 24 = b$

$b = -20$

$a =$ 3

$b =$ -20

6

3 The nth term of a sequence is $\dfrac{3n}{5n+12}$

3 (a) Work out the position of the term that has a value of $\frac{1}{2}$ **[2 marks]**

$\frac{3n}{5n+12} = \frac{1}{2}$

$\frac{6n}{5n+12} = 1$

$6n = 5n + 12$

$n = 12$

Answer 12

3 (b) Write down the limiting value of $\dfrac{3n}{5n+12}$ as $n \to \infty$ **[1 mark]**

Answer

$= \frac{3n}{5n} = \frac{3}{5}$

4 The equation of a circle is $(x+5)^2+(y-8)^2=10$

4 (a) What are the coordinates of the centre of the circle?
Circle your answer.

[1 mark]

$(-5, -8)$ $(-5, 8)$ $(5, 8)$ $(5, -8)$

4 (b) Write down the radius of the circle.

[1 mark]

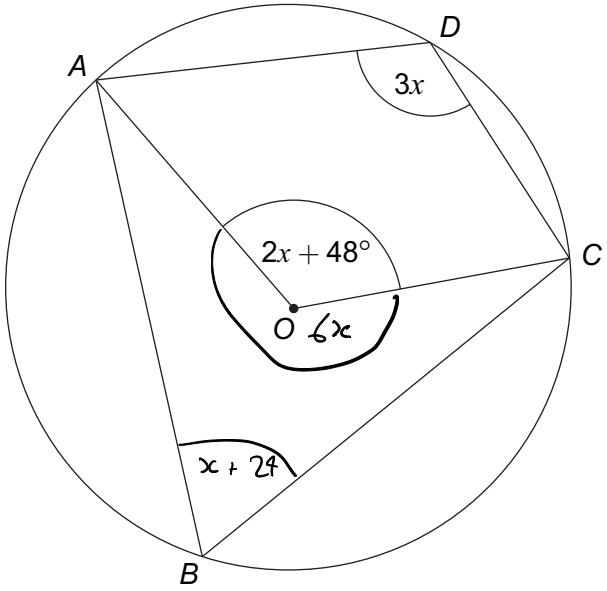
Answer $\sqrt{10}$

Turn over for the next question

5

5 *A*, *B*, *C* and *D* are points on a circle, centre *O*.

Not drawn accurately



Work out the value of x.

[3 marks]

$3x + x + 24 = 180$

$4x = 156$

$x = 39°$

$x =$ _____ degrees

6 $mx + 4 - 2(x + p) \equiv 6(x + 1)$ where m and p are integers.

Work out the values of m and p.

[4 marks]

$= mx + 4 - 2x - 2p$

$= (-2+m)x + (4 - 2p) = (-2+m)x + 2(2-p)$

$= 6\left(\frac{(-2+m)x}{6} + \frac{2-p}{3}\right)$

$\frac{-2+m}{6} = 1$ $\quad$ $\frac{2-p}{3} = 1$

$-2+m = 6$ $\quad$ $2-p = 3$

$m = 8$ $\quad$ $2 = 3+p$

$p = -1$

$m =$ 8 2w

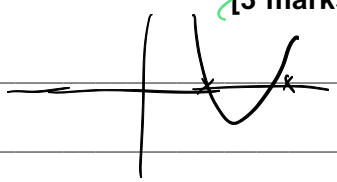
$p =$ -1

7 Work out the integer values of x for which $x^2 - 20x + 96 < 0$

[3 marks]

2

$(x - 12)(x - 8) < 0$



$8 < x < 12$

Answer 9, 10, 11

10

8 Solve $(3-\sqrt{x})^{\frac{1}{3}} = -2$ **[3 marks]**

$\sqrt[3]{3-x^{1/2}} = -2$

$3 - x^{1/2} = -8$

$3 = -8 + x^{1/2}$

$x^{1/2} = 11$

$x = 121$

$x =$ 121

9 Expand and simplify $(x-5)^3$ **[3 marks]**

$= \binom{3}{0}(x)^3(-5)^0 + \binom{3}{1}(x)^2(-5)^1 + \binom{3}{2}(x)^1(-5)^2 + \binom{3}{3}(x)^0(-5)^3$

$= 1 \times x^3 + 3 \times x^2 \times -5 + 3 \times x \times (-5)^2 + 1 \times (-5)^3$

$= x^3 + -15x^2 + 75x + -125$

$= x^3 - 15x^2 + 75x - 125$

Answer $x^3 - 15x^2 + 75x - 125$

10 $\sqrt[4]{x} = 2$ and $y^{-2} = 25$

$x > 0$ and $y < 0$

Work out the value of $\frac{x}{y}$

[4 marks]

$\sqrt[4]{x} = 2$

$x = 16$

$y^{-2} = 25$

$y = 25^{-\frac{1}{2}}$

$y = \frac{1}{5}$

$= 16 \div -\frac{1}{5}$

$= 16 \times -5$

$= -80$

Answer -80

Turn over for the next question

11 $A(1\frac{1}{5}, 3\frac{4}{5})$, $B(2, 1\frac{4}{5})$ and $C(5, 3)$ are points on a coordinate grid.

Show that the line segments AB and BC are perpendicular.

[3 marks]

$m_{AB} = \frac{dy}{dx} = \frac{3\frac{4}{5} - 1\frac{4}{5}}{1\frac{1}{5} - 2} = \frac{2}{-\frac{9}{5}} = -\frac{5}{2}$

$m_{BC} = \frac{dy}{dx} = \frac{1\frac{4}{5} - 3}{2 - 5} = \frac{-\frac{6}{5}}{-3} = \frac{2}{5}$

If line A is perp to line B, gradient of $B = -\frac{1}{A}$

$-\left(\frac{1}{-\frac{5}{2}}\right) = \frac{2}{5}$ ∴ perpendicular

12 You are given that $x^2 + 6x + 2 \equiv (x + h)^2 + k$

12 (a) Work out the values of h and k.

[2 marks]

$= (x+3)^2 - 9 + 2$

$= (x+3)^2 - 7$

$h =$ 3

$k =$ −7

12 (b) Write down the coordinates of the minimum point on the curve $y = x^2 + 6x + 2$

[1 mark]

Answer (−3 , −7)

$\frac{dy}{dx} = 2x + 6$

$2x + 6 = 0$

$x = -3$

$y = x^2 + 6x + 2, \; x = -3$

$= -7$

12 (c) Solve the equation $x^2 + 6x + 2 = 0$

Give your answers in the form $a \pm \sqrt{b}$

[1 mark]

$(x+3)^2 = 7$

$(x+3) = \pm\sqrt{7}$

$x = -3 \pm \sqrt{7}$

Answer _____

7

13 Solve $\sqrt{125} + \sqrt{20} = \sqrt{80} + \sqrt{x}$ **[3 marks]**

$5\sqrt{5} + 2\sqrt{5} - 4\sqrt{5} = \sqrt{x}$

$3\sqrt{5} = \sqrt{x}$

$\sqrt{45} = \sqrt{x}$

$x = 45$

$x =$ 45

14 (a) $(x - 3)$ is a factor of $x^3 - 8x^2 + ax + 42$ where a is an integer.

Show that the value of a is 1

[2 marks]

$F(3) = 0$, 3 is a factor

$3^3 - 8 \times 3^2 + 3a + 42 = 0$

$27 - 72 + 42 = -3a$

$-3 = 3a$

$a = 1$

14 (b) Hence, factorise fully $x^3 - 8x^2 + x + 42$

$-42 = 2 \times 3 \times 7$

other factors are $2, -7$ or $7, -2$

[3 marks]

$(x-3)(x+2)(x-7) = (x^2 - x - 6)(x-7)$

$= (x^3 - x^2 - 6x - 7x^2 + 7x + 42)$

$= x^3 - 8x^2 + x + 42$ YAY!

Answer $(x-3)(x+2)(x-7)$

8

15 Rationalise the denominator and simplify fully $\dfrac{6}{\sqrt{7}+2}$ **[3 marks]**

$= \dfrac{6}{\sqrt{7}+2} \times \dfrac{\sqrt{7}-2}{\sqrt{7}-2}$

$= \dfrac{6\sqrt{7}-12}{7-4} = \dfrac{6\sqrt{7}-12}{3} = 2\sqrt{7}-4$

Answer _____

16 Angle θ is obtuse and $\sin\theta = \frac{\sqrt{11}}{6}$ ≈ 0.55

Work out the value of $\cos\theta$ $\sin^{-1}(\ ) = 33.5°$

[4 marks]

true $\theta = 180 - 33.5$

$= 146.5°$

$\cos\theta = \cos(146.5)$

$= -5/6$

Answer _____

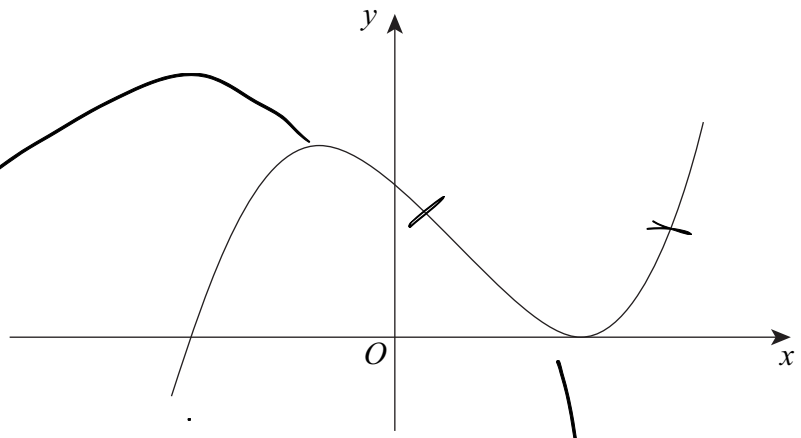
Turn over for the next question

17 The diagram shows a sketch of the cubic curve $y = \frac{1}{3}x^3 - x^2 - 3x + k$

where k is a constant.

The x-axis is a tangent to the curve at its minimum point.

Not drawn accurately



Work out the value of k.

[5 marks]

3

$\frac{dy}{dx} = x^2 - 2x - 3$

$(x+1)(x-3)$

$x = 1, 3$

3 is a factor of k

???

plug into eq

$k = 9$

$k = -3$

18 Factorise fully $x^4 - 81$ **[2 marks]**

$= (x^2 + 9)(x^2 - 9)$

$= (x+3)(x-3)(x+3)(x-3)$

$= (x+3)^2(x-3)^2$

Answer $(+3 \ -3)\ (x^2+9)$

cannot factorise

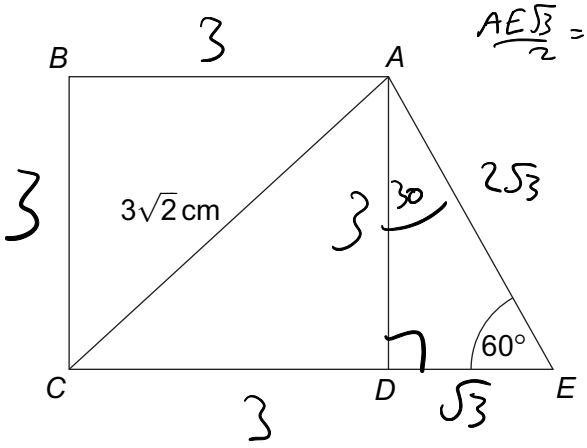
Turn over for the next question

7

Do not write outside the box

19 *ABCD* is a square.
CDE is a straight line.

AC is $3\sqrt{2}$ cm and angle $DEA = 60°$



Not drawn accurately

19 (a) Show that the side of the square is 3 cm

[2 marks]

$2a^2 = 18$

$a^2 = 9$

$a = 3$

19 (b) Show that the perimeter of trapezium *ABCE* is $3(3 + \sqrt{3})$ cm

[4 marks]

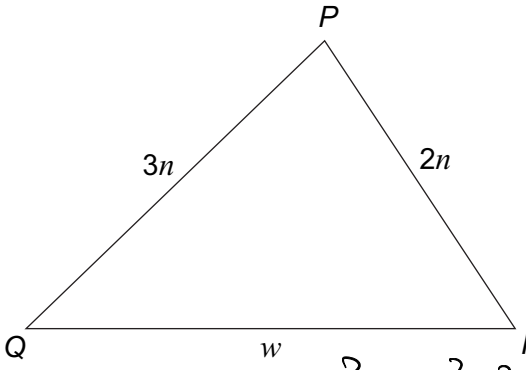
$= 3 \times 3 + \sqrt{3} + 2\sqrt{3}$

$= 3 \times 3 + 3\sqrt{3}$

$= 3(3 + \sqrt{3})$

20 In triangle PQR, $\cos P = \frac{1}{3}$

Not drawn accurately



$w = 3n$ or $3n = 2n$ or $w = 2n$

Show that triangle PQR is isosceles.

[4 marks]

$a^2 = b^2 + c^2 - 2bc\cos A$

$w^2 = (3n)^2 + (2n)^2 - 2 \times 3n \times 2n \times \frac{1}{3}$

$w^2 = 13n^2 - 4n^2$

$w = \sqrt{13n^2 - 4n^2}$

$w^2 = 9n^2$

$w = 3n$

???

END OF QUESTIONS

10

There are no questions printed on this page

DO NOT WRITE ON THIS PAGE
ANSWER IN THE SPACES PROVIDED