Topic assessment

Take $g = 9.8$ ms^{-2} unless otherwise instructed.

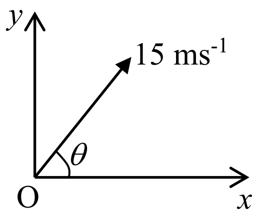
1. A particle is projected from a point on horizontal ground with a velocity of 25 ms^{-1} at an angle of 35° to the ground.
   (a) Calculate the maximum height reached by the particle. [3]
   (b) Calculate the distance from the point of projection to the point at which it lands on the ground. [4]

2. A pebble is thrown horizontally with a velocity of u ms^{-1} from the top of a vertical cliff 28 m above the sea below. The pebble lands 45 m from the foot of the cliff. Calculate the value of u. [4]

3. **Take $g = 10$ ms^{-2} in this question**. Air resistance should be neglected.

   A ball is thrown from a point O with a velocity of 15 ms^{-1} at an angle of θ to the horizontal, where $\cos\theta = 0.6$ and $\sin\theta = 0.8$.

   The diagram below shows x- and y-axes drawn through O, the point of projection. The units of the axes are metres.



   (a) Show that, after time t seconds, the position of the ball is given by
   $$x = 9t, \qquad y = 12t - 5t^2.$$ [3]

   (b) Show that the equation of the ball is
   $$y = \tfrac{4}{3}x - \tfrac{5}{81}x^2.$$ [3]

   (c) Hence calculate how far from the origin the ball lands. [3]

4. A stone is projected over horizontal ground from a point O on the ground. The velocity of projection is 30 ms^{-1} at 40° to the horizontal. The effects of air resistance should be neglected.
   (a) State the modelling assumptions used in the standard projectile model. [2]
   (b) The stone is at a horizontal distance x m from the point of projection and y m above the ground t seconds after projection. Write down an expressions for x and y in terms of t. [2]

   The stone passes directly over a wall which is at a horizontal distance of 34 m from O.

   (c) Find the time taken to reach the wall. [2]

(d) Determine the speed of the stone as it passes over the wall. Calculate also the angle between the direction of motion of the ball and the horizontal at that time, making it clear whether the ball is rising or falling. [8]

5. A tennis player serves the ball with a velocity of U ms^{-1} at an angle of $\alpha°$ above horizontal from a point which is 2.5 m above the ground. The ball reaches a maximum height of 2.8 m above the ground.

(a) Show that $U^2 = \dfrac{3g}{5\sin^2\alpha}$ [2]

(b) The ball just passes over the net which is 90cm high and 12 m from the server.

Show that $120\tan^2\alpha - 12\tan\alpha - \dfrac{8}{5} = 0$. [4]

(c) Hence calculate the values of α and U. [4]

(d) To be a legal serve the ball must land less than 18.4 m from the server. Determine whether the serve is legal. [4]

(e) Explain why a particle model may not be good enough to predict the flight of a tennis ball. [2]

Total 50 marks

OCR A level Maths Projectiles Assessment solutions

Solutions to topic assessment

1. (a) Vertical motion $u_y = 25\sin 35°$

At the highest point $v_y = 0$

Using $v^2 = u^2 + 2as$

$0 = \left(25\sin 35°\right)^2 - 2\times 9.8s$

$s = \dfrac{(25\sin 35°)^2}{19.6} = 10.5$ m (3sf) [3]

(b) Vertical motion $u_y = 25\sin 35°$

When the projectile reaches the ground $y = 0$

Using $s = ut + \dfrac{1}{2}at^2$

$0 = \left(25\sin 35°\right)t - 4.9t^2$

$t\left(25\sin 35° - 4.9t\right) = 0$

$t = 0,\ \ 2.9264...$

Looking at horizontal motion $u_x = 25\cos 35°$ constant speed

When $t = 2.9264...$ $x = \left(25\cos 35°\right)\times 2.9264... = 59.9$

Range is 59.9 m (3sf) [4]

2. Vertical motion $y = -28, u = 0, a = -g$

Using $s = ut + \frac{1}{2}at^2$ $-28 = -\frac{1}{2}gt^2 = -4.9t^2$

Which gives $t = \sqrt{\dfrac{28}{4.9}}$ s

Horizontal motion speed is constant so $x = ut$

Giving $45 = 2.39u$ so $u = 18.8$ ms^{-1} [4]

3. (a) Horizontally, speed $= 15\cos\theta = 15\times 0.6 = 9$

As horizontal speed is constant, $x = 9t$.

Vertically, initial speed $= 15\sin\theta = 15\times 0.8 = 12$

$u = 12$	$s = ut + \frac{1}{2}at^2$
$a = -10$	$y = 12t - 5t^2$
$t = t$	
$s = y$	

[3]

(b) $x = 9t \Rightarrow t = \dfrac{x}{9}$

Substituting into $y = 12t - 5t^2$,

$$y = \frac{12x}{9} - \frac{5x^2}{81}$$

$$y = \tfrac{4}{3}x - \tfrac{5}{81}x^2$$

[3]

(c) When y = 0, $0 = \tfrac{4}{3}x - \tfrac{5}{81}x^2$

$$0 = 108x - 5x^2$$

$$0 = x(108 - 5x)$$

$$x = 0 \text{ or } x = \tfrac{108}{5} = 21.6$$

(x = 0 is the point where the ball is projected)

This ball lands 21.6 m from O. [3]

4. (a) The acceleration due to gravity is constant.

Air resistance is negligible.

The particle does not spin. [2]

(b) For vertical motion:

$u = 30\sin 40°$ $\quad s = ut + \tfrac{1}{2}at^2$

$s = y$ $\quad y = 30t\sin 40° - 4.9t^2$

$t = t$

$g = -9.8$

For horizontal motion speed is constant so $x = 30t\cos 40°$ [2]

(c) The stone passes over the wall when $x = 34$

When it passes over the wall, $34 = 30t\cos 40$

$$t = \frac{34}{30\cos 40} = 1.4794...$$

So the stone passes over the wall after 1.48 s [2]

(d) Vertically:

$u = 30\sin 40°$ $\quad v = u + at$

$v = v$ $\quad = 30\sin 40° - \dfrac{9.8 \times 34}{30\cos 40°}$

$t = \dfrac{34}{30\cos 40°}$ $\quad = 4.7849$

$a = -9.8$

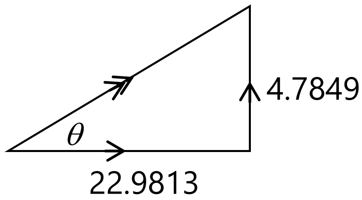
Speed $= \sqrt{22.9813^2 + 4.7849^2}$

$= 23.5$ (3 s.f.)

The speed of the stone as it passes over the wall is 23.5 ms^{-1} (3 s.f.)

$\tan\theta = \dfrac{4.7849}{22.9813}$

$\theta = 11.8°$ (1 d.p.)



The direction of motion of the stone is 11.8° above the horizontal.

The stone is rising, since its vertical velocity is positive. [8]

5. (a) Vertically, the ball reaches maximum height when $v_y = 0$

The vertical displacement at this time is $2.8 - 2.5 = 0.3$ m

Using $u_y = U\sin\alpha$ and the equation $v^2 = u^2 + 2as$

$0^2 = (U\sin\alpha)^2 - 2g \times 0.3$

Giving $U^2 = \dfrac{3g}{5\sin^2\alpha}$ [2]

(b) The ball passes through the point where $x = 12,\ y = 0.9$

Horizontally speed is constant so $x = 12 = U\cos\alpha t$ giving $t = \dfrac{12}{U\cos\alpha}$

Vertically the displacement is $0.9 - 2.5 = -1.6$ m

Using $s = ut + \frac{1}{2}at^2$

$-1.6 = (U\sin\alpha)\left(\dfrac{12}{U\cos\alpha}\right) - \dfrac{1}{2}g\left(\dfrac{12}{U\cos\alpha}\right)^2$

$-1.6 = 12\tan\alpha - \dfrac{72g}{U^2\cos^2\alpha}$

Substituting $U^2 = \dfrac{3g}{5\sin^2\alpha}$ gives $-1.6 = 12\tan\alpha - \dfrac{72g}{\left(\dfrac{3g}{5\sin^2\alpha}\right)\cos^2\alpha}$

So $-1.6 = 12\tan\alpha - \dfrac{72g \times 5\sin^2\alpha}{3g\cos^2\alpha}$

Using $\tan^2\alpha = \dfrac{\sin^2\alpha}{\cos^2\alpha}$ this gives $-1.6 = 12\tan\alpha - 120\tan^2\alpha$

So $120\tan^2\alpha - 12\tan\alpha - \dfrac{8}{5} = 0$. [4]

(c) Solving the quadratic equation and taking the positive root,

$\tan\alpha = 0.1758...$

So $\alpha = \tan^{-1} 0.1758... = 10.0°$

Substituting $\alpha. = 10.0°$ gives $U = \sqrt{\dfrac{3g}{5\sin^2 10.0°}} = 14.0 \text{ ms}^{-1}$ [4]

(d) To find x when the ball lands, use $y = -2.5$ in the equation $s = ut + \frac{1}{2}at^2$

$-2.5 = 14.0\sin 10.0° t - \frac{1}{2}gt^2$

Gives $4.9t^2 - 14.0\sin 10.0° t - 2.5 = 0$ with positive root $t = 1.004...$

Horizontally $x = 14.0\cos 10.0° \times 1.004 = 13.8$ m

This is less than 18.4 m so the serve is legal (if the direction is correct) [4]

(e) The modelling assumptions treat the ball as a particle so the spin on the ball is ignored. In reality, spin is an important factor in the trajectory of a tennis ball. [2]