Vectors Exam Qs

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Question 1

i

$$p(2i + 6j) + q(2i - 4j) = 6i - 7j$$
$$2pi + 6pj + 2qi - 4qj = 6i - 7j$$

$$2p + 2q = 6 \qquad 6p - 4q = -7$$
$$4p + 4q = 12 \qquad 6p - 4q = -7$$

$$\begin{aligned} 10p &= 5 \\ p &= \frac{1}{2} \\ 2p + 2q &= 6 \\ 1 + 2q &= 6 \\ 2q &= 5 \\ q &= \frac{5}{2} \end{aligned}$$

$$p = \frac{1}{2} \quad q = \frac{5}{2}$$

ii

??? Will use algebra.

$$\begin{aligned} (2 + 2k)^2 + (6 - 4k)^2 &= 25 \\ 4 + 8k + 4k^2 + 36 - 48k + 16k^2 &= 25 \\ 20k^2 - 40k + 15 &= 0 \\ 4k^2 - 8k + 3 &= 0 \end{aligned}$$

$$k = \frac{1}{2}, \frac{3}{2}$$

Question 2

i

$$6i + 3j - 5i - 2j = ai + bj + 2i - j$$
$$-i + 2j = ai + bj$$

$$D = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$$

ii

$$M = \frac{\begin{pmatrix} 2 \\ 5 \end{pmatrix} + \begin{pmatrix} 6 \\ 3 \end{pmatrix}}{2}$$
$$= \begin{pmatrix} 4 \\ 4 \end{pmatrix}$$
$$AM = \begin{pmatrix} 4 \\ 4 \end{pmatrix} - \begin{pmatrix} -2 \\ 1 \end{pmatrix}$$
$$= \begin{pmatrix} 6 \\ 3 \end{pmatrix}$$
$$|AM| = \sqrt{6^2 + 3^2} = 6.71$$

Question 3

i

a

$$= 2i + j$$

b

$$\text{dir}\,(OA) = \arctan \frac{-2}{1} = -63.4^\circ$$
$$\text{dir}\,(OB) = 26.6^\circ, -153^\circ$$

ii

C points form a circle around A, radius 2. You need to find the unit vector of A, then scale it by 2. Then, either add/subtract that from A to get the max/min.

$$
\begin{aligned}
|OA| &= \sqrt{1^2 + 2^2} = \sqrt{5} \\
OA_{\text{Unit}} &= \frac{OA}{\sqrt{5}} \\
&= \frac{1}{\sqrt{5}}i - \frac{2}{\sqrt{5}}j \\
&= \frac{\sqrt{5}}{5}i - \frac{2\sqrt{5}}{5}j \\
|AC| &= \frac{2\sqrt{5}}{5}i - \frac{4\sqrt{5}}{5}j
\end{aligned}
$$

$$
\begin{aligned}
\text{Max} &= i - 2j + \frac{2\sqrt{5}}{5}i - \frac{4\sqrt{5}}{5}j \\
\text{Min} &= i - 2j - \frac{2\sqrt{5}}{5}i + \frac{4\sqrt{5}}{5}j
\end{aligned}
$$

Question 4

i

ii

$$
\begin{aligned}
|r| &= \sqrt{4^2 + (-5)^2} \\
&= 6.40 \\
\theta &= \arctan\frac{-5}{4} \\
&= -51.3^\circ
\end{aligned}
$$

iii

$$= \begin{pmatrix} 12 \\ -15 \end{pmatrix}$$

Question 5

i

$$|p| = \sqrt{8^2 + 1^2} = \sqrt{65}$$
$$|q| = \sqrt{4^2 + (-7)^2} = \sqrt{65}$$

ii

$$\begin{aligned} p + q &= 8i + j + 4i - 7j \\ &= 12i - 6j \\ \theta_{p+q} &= \arctan \frac{-6}{12} = \arctan -\frac{1}{2} \\ \theta_{2i-j} &= \arctan \frac{-1}{2} = \arctan -\frac{1}{2} \end{aligned}$$

Angles are the same ∴ they are parallel.

iii