

Vectors Exam Qs

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Question 1

i

$$\begin{aligned}p(2i + 6j) + q(2i - 4j) &= 6i - 7j \\2pi + 6pj + 2qi - 4qj &= 6i - 7j\end{aligned}$$

$$\begin{aligned}2p + 2q &= 6 & 6p - 4q &= -7 \\4p + 4q &= 12 & 6p - 4q &= -7\end{aligned}$$

$$10p = 5$$

$$p = \frac{1}{2}$$

$$2p + 2q = 6$$

$$1 + 2q = 6$$

$$2q = 5$$

$$q = \frac{5}{2}$$

$$p = \frac{1}{2} \quad q = \frac{5}{2}$$

ii

??? Will use algebra.

$$(2 + 2k)^2 + (6 - 4k)^2 = 25$$

$$4 + 8k + 4k^2 + 36 - 48k + 16k^2 = 25$$

$$20k^2 - 40k + 15 = 0$$

$$4k^2 - 8k + 3 = 0$$

$$k = \frac{1}{2}, \frac{3}{2}$$

Question 2

i

$$6i + 3j - 5i - 2j = ai + bj + 2i - j$$

$$-i + 2j = ai + bj$$

$$D = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$$

ii

$$M = \frac{\begin{pmatrix} 2 \\ 5 \end{pmatrix} + \begin{pmatrix} 6 \\ 3 \end{pmatrix}}{2}$$

$$= \begin{pmatrix} 4 \\ 4 \end{pmatrix}$$

$$AM = \begin{pmatrix} 4 \\ 4 \end{pmatrix} - \begin{pmatrix} -2 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 6 \\ 3 \end{pmatrix}$$

$$|AM| = \sqrt{6^2 + 3^2} = 6.71$$

Question 3

i

a

$$= 2i + j$$

b

$$\text{dir}(OA) = \arctan \frac{-2}{1} = -63.4^\circ$$

$$\text{dir}(OB) = 26.6^\circ, -153^\circ$$

+180 not mins

ii

C points form a circle around A, radius 2. You need to find the unit vector of A, then scale it by 2. Then, either add/subtract that from A to get the max/min.

$$|OA| = \sqrt{1^2 + 2^2} = \sqrt{5}$$

$$OA_{\text{Unit}} = \frac{OA}{\sqrt{5}}$$

$$= \frac{1}{\sqrt{5}}i - \frac{2}{\sqrt{5}}j$$

$$= \frac{\sqrt{5}}{5}i - \frac{2\sqrt{5}}{5}j$$

$$|AC| = \frac{2\sqrt{5}}{5}i - \frac{4\sqrt{5}}{5}j$$

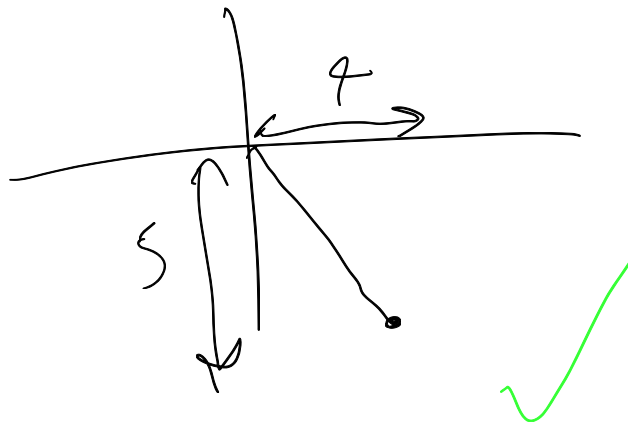
$$\text{Max} = i - 2j + \frac{2\sqrt{5}}{5}i - \frac{4\sqrt{5}}{5}j$$

$$\text{Min} = i - 2j - \frac{2\sqrt{5}}{5}i + \frac{4\sqrt{5}}{5}j$$

??

Question 4

i



ii

$$|r| = \sqrt{4^2 + (-5)^2}$$

$$= 6.40$$

$$\theta = \arctan \frac{-5}{4}$$

$$= -51.3^\circ$$

4 180

iii

$$= \begin{pmatrix} 12 \\ -15 \end{pmatrix} \quad \checkmark$$

Question 5

i

$$|p| = \sqrt{8^2 + 1^2} = \sqrt{65}$$

$$|q| = \sqrt{4^2 + (-7)^2} = \sqrt{65} \quad \checkmark$$

ii

$$p + q = 8i + j + 4i - 7j$$

$$= 12i - 6j$$

$$\theta_{p+q} = \arctan \frac{-6}{12} = \arctan -\frac{1}{2}$$

$$\theta_{2i-j} = \arctan \frac{-1}{2} = \arctan -\frac{1}{2} \quad \checkmark$$

Angles are the same \therefore they are parallel.

iii

