Binomial Hypothesis Testing

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Question 1

i

$$x \sim B(20, 0.78)$$

- A: $P(x = 19) = 3.91\%$
- B: $P(x \leq 18) = 95.38\%$
- C: ???

ii

Let p be the proportion of patients who are cured.

- $H_0 : p = 0.78$
- $H_1 : p > 0.78$

Let x be the number of patients cured out of 20. Under H_0, $x \sim B(20, 0.78)$

$$p = P(x \geq 19) = 0.0461$$

$0.0461 > 0.01$ $\therefore$ there is insufficient evidence to reject H_0 in favour of H_1. This is sufficient evidence that the new treatment is no different to the old one.

ii

$0.0461 < 0.05$ $\therefore$ there is sufficient evidence to reject H_0 in favour of H_1. This is sufficient evidence that the new treatment is better than the old one.

Question 2

i

$$x \sim B(15, 0.85)$$

- A: $P(x = 12) = 21.84\%$
- B: $P(x \leq 11) = 17.73\%$

ii

Let p be the proportion of seeds that germinate out of 15.

- $H_0 : p = 0.85$
- $H_1 : p < 0.85$

The alternate hypothesis uses <, as we are checking if fewer seeds germinated than before as our hypothesis.

iii

Let x be the number of seeds that germinate out of 20. Under H_0, $x \sim B\left(20, 0.85\right)$

$$p = P\left(x \leq 13\right) = 0.0219$$

$0.0219 > 0.01$ $\therefore$ there is insufficient evidence to reject H_0 in favour of H_1. This is sufficient evidence that the old seeds show no change in germination rate.

Question 3

i

$$x \sim B\left(10, 0.35\right)$$

- A: $P\left(x = 5\right) = 15.35\%$
- B: $P\left(x \geq 5\right) = 24.85\%$
- C: ???

ii

Let p be the proportion of the customers who use the internet.

- $H_0 : p = 0.35$
- $H_1 : p \neq 0.35$

Let x be the customers who use the internet out of 20. Under H_0, $x \sim B\left(10, 0.35\right)$

$$p = P\left(x \geq 10\right) = 0.1217$$

$0.1217 < \frac{0.5}{2}$ $\therefore$ there is insufficient evidence to reject H_0 in favour of H_1. This is sufficient evidence that the proportion of internet users hasn't changed.