9G and 9H

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9G

Question: 3

a

$$\frac{\Delta x}{\Delta t} = e^t \qquad \frac{\Delta y}{\Delta t} = e^t - e^{-t}$$

$$\begin{aligned}\frac{\Delta y}{\Delta x} &= \frac{e^t - e^{-t}}{e^t} \\ &= \frac{e^t}{e^t} - \frac{e^{-t}}{e^t} \\ &= 1 - e^{-2t}\end{aligned}$$

$$\begin{aligned}t &= 0 \\ \frac{\Delta y}{\Delta x} &= 1 - e^0 \\ &= 0\end{aligned}$$

$$x = e^0 = 1 \qquad y = e^0 + e^0 = 2$$

$$\begin{aligned}y - y_1 &= m\,(x - x_1) \\ y - 2 &= 0 * (\ldots) \\ y &= 2\end{aligned}$$

b

$$\frac{\Delta x}{\Delta t} = 2\sin 2t \qquad \frac{\Delta y}{\Delta t} = 2\cos 2t$$

$$\begin{aligned}\frac{\Delta y}{\Delta x} &= \frac{2\cos 2t}{2\sin 2t} \\ &= \cot 2t\end{aligned}$$

$$\begin{aligned}t &= \frac{\pi}{6} \\ \frac{\Delta y}{\Delta x} &= \cot\frac{\pi}{3} \\ &= \frac{\sqrt{3}}{3}\end{aligned}$$

$$x = 1 - \cos\frac{\pi}{3} = \frac{1}{2} \qquad y = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$$

$$\begin{aligned}y - y_1 &= m\,(x - x_1) \\ y - \frac{\sqrt{3}}{2} &= \frac{\sqrt{3}}{3}\left(x - \frac{1}{2}\right) \\ y - \frac{\sqrt{3}}{2} &= \frac{\sqrt{3}}{3}x - \frac{\sqrt{3}}{6} \\ y &= \frac{\sqrt{3}}{3}x + \frac{\sqrt{3}}{3}\end{aligned}$$

Question: 4

$$\begin{aligned}x &= \frac{t}{1-t} \\ \frac{\Delta x}{\Delta t} &= \frac{vu' - v'u}{v^2} \\ &= \frac{1 - t + t}{1 - 2t + t^2} \\ &= \frac{1}{t^2 - 2t + 1}\end{aligned}$$

$$\begin{aligned}y &= \frac{t^2}{1-t} \\ \frac{\Delta y}{\Delta t} &= \frac{vu' - v'u}{v^2} \\ &= \frac{2t\,(1-t) + t^2}{1 - 2t + t^2} \\ &= \frac{2t - t^2}{t^2 - 2t + 1}\end{aligned}$$

$$
\begin{aligned}
\frac{\Delta y}{\Delta x} &= \frac{2t - t^2}{1 - 2t + t^2} \div \frac{1}{t^2 - 2t + 1} \\
&= \frac{\left(2t - t^2\right)\left(t^2 - 2t + 1\right)}{t^2 - 2t + 1} \\
&= 2t - t^2 \\
0 &= 2t - t^2 \\
t &= 0, 2
\end{aligned}
$$

$t = 0$

$$
\begin{aligned}
x &= \frac{t}{1 - t} \\
&= 0 \\
y &= \frac{t^2}{1 - t} \\
&= 0
\end{aligned}
$$

$t = 2$

$$
\begin{aligned}
x &= \frac{t}{1 - t} \\
&= -2 \\
y &= \frac{t^2}{1 - t} \\
&= -4
\end{aligned}
$$

$$= (0, 0), (-2, -4)$$

9H

Question: 4

$$
\begin{aligned}
0 &= 2x + 6xy\frac{\Delta y}{\Delta x} + 3y^2 - 3y^2\frac{\Delta y}{\Delta x} \\
-2x - 3y^2 &= 6xy\frac{\Delta y}{\Delta x} - 3y^2\frac{\Delta y}{\Delta x} \\
\frac{\Delta y}{\Delta x} &= \frac{-2x - 3y^2}{6xy - 3y^2} \\
&= -\frac{3y^2 + 2x}{3y^2 - 6xy}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{3 * 1^2 + 2 * 2}{3 * 1^2 - 6 * 2 * 1} \\
&= -\frac{7}{9}
\end{aligned}
$$

Question: 5

$$
\begin{aligned}
2x + \frac{\Delta y}{\Delta x} &= 3(x + y)^2\left(1 + \frac{\Delta y}{\Delta x}\right) \\
2x + \frac{\Delta y}{\Delta x} &= 3(x + y)^2 + 3\frac{\Delta y}{\Delta x}(x + y)^2 \\
3(x + y)^2 - 2x &= \frac{\Delta y}{\Delta x} - 3\frac{\Delta y}{\Delta x}(x + y)^2 \\
\frac{\Delta y}{\Delta x} &= \frac{3(x + y)^2 - 2x}{1 - 3(x + y)^2}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{3(1 + 0)^2 - 2 * 1}{1 - 3(1 + 0)^2} \\
&= -\frac{1}{2}
\end{aligned}
$$