9I

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Question 1

a

$$\frac{\Delta y}{\Delta x} = 3x^2 - 6x + 1$$
$$\frac{\Delta^2 y}{\Delta x^2} = 6x - 6$$

Concave

$$6x - 6 < 0$$
$$x < 1$$

Convex

$$6x - 6 > 0$$
$$x > 1$$

b

$$\frac{\Delta y}{\Delta x} = 4x^3 - 9x^2 + 2$$
$$\frac{\Delta^2 y}{\Delta x^2} = 12x^2 - 18x$$

Concave

$$12x^2 - 18x < 0$$
$$2x^2 - 3x < 0$$
$$x(2x - 3) < 0$$
$$0 < x < \frac{3}{2}$$

Convex

$$12x^2 - 18x > 0$$
$$2x^2 - 3x > 0$$
$$x(2x - 3) > 0$$
$$x < 0 \cup x > \frac{3}{2}$$

c

$$\frac{\Delta y}{\Delta x} = \cos x$$
$$\frac{\Delta^2 y}{\Delta x^2} = -\sin x$$

Concave

$$-\sin x < 0$$
$$0 < x < \pi$$

Convex

$$-\sin x > 0$$
$$\pi < x > 2\pi$$

d

$$\frac{\Delta y}{\Delta x} = -2x + 3$$
$$\frac{\Delta^2 y}{\Delta x^2} = -2$$

Concave

$$-2 < 0$$

Always Concave

Convex

$$-2 > 0$$

Never Convex

e

$$\frac{\Delta y}{\Delta x} = e^x - 2x$$
$$\frac{\Delta^2 y}{\Delta x^2} = e^x - 2$$

Concave

$$e^x - 2 < 0$$
$$e^x < 2$$
$$x < \ln 2$$

Convex

$$e^x - 2 > 0$$
$$e^x > 2$$
$$x > \ln 2$$

f

$$\frac{\Delta y}{\Delta x} = \frac{1}{x}$$
$$\frac{\Delta^2 y}{\Delta x^2} = \frac{-1}{x^2}$$

Concave

$$\frac{-1}{x^2} < 0$$
$$-1 < 0$$

Always Concave

Convex

$$\frac{-1}{x^2} > 0$$
$$-1 > 0$$

Never Concave

Question 2

a

$$\sin^2 y + \cos^2 y = 1 \quad \therefore \quad \cos y = \sqrt{1 - \sin^2 y}$$

$$y = \arcsin x \quad \therefore \quad x = \sin y$$

$$\begin{aligned} 1 &= \cos y \frac{\Delta y}{\Delta x} \\ \frac{\Delta y}{\Delta x} &= \frac{1}{\cos y} \\ &= \frac{1}{\sqrt{1 - \sin^2 y}} \\ &= \frac{1}{\sqrt{1 - x^2}} \end{aligned}$$

b

$$\begin{aligned} \frac{\Delta y}{\Delta x} &= \left(1 - x^2\right)^{-\frac{1}{2}} \\ \frac{\Delta^2 y}{\Delta x^2} &= -\frac{1}{2}\left(1 - x^2\right)^{-\frac{3}{2}}(-2x) \\ &= \frac{x}{2\sqrt{1 - x^2}^3} \end{aligned}$$

$$\begin{aligned} \frac{x}{2\sqrt{1 - x^2}^3} &< 0 \\ x &< 0 \\ -1 < x &< 0 \end{aligned}$$

c

$$\begin{aligned} \frac{x}{2\sqrt{1 - x^2}^3} &> 0 \\ x &> 0 \\ 0 < x &< 1 \end{aligned}$$

d

$$\begin{aligned} x &= 0 \\ y &= \arcsin x = \arcsin 0 \\ &= 0 \end{aligned}$$

$$= (0, 0)$$

Question 3

a

$$
\begin{aligned}
\frac{\Delta y}{\Delta x} &= -2\cos x \sin x - 2\cos x \\
\frac{\Delta^2 y}{\Delta x^2} &= -2\left(-\sin^2 x + \cos^2 x\right) + 2\sin x \\
&= -2\left(\cos^2 x - \sin^2 x\right) + 2\sin x
\end{aligned}
$$

$$
\begin{aligned}
0 &= -2\left(\cos^2 x - \sin^2 x\right) + 2\sin x \\
0 &= \cos^2 x - \sin^2 x - \sin x \\
0 &= 1 - \sin^2 x - \sin^2 x - \sin x \\
0 &= 2\sin^2 x + \sin x - 1 \\
0 &= (\sin x + 1)(2\sin x - 1) \\
\sin x &= -1, \frac{1}{2} \\
x &= \frac{3}{2}\pi, \frac{1}{6}\pi, \frac{5}{6}\pi
\end{aligned}
$$

- $(\frac{3}{2}\pi, 2)$
- $(\frac{1}{6}\pi, -\frac{1}{4})$
- $(\frac{5}{6}\pi, -\frac{1}{4})$

b

$$
\begin{aligned}
\frac{\Delta y}{\Delta x} &= -\frac{\left(3x^2-4x+1\right)(x-2)-\left(x^3-2x^2+x-1\right)}{(x-2)^2} \\
&= -\frac{3x^3-6x^2-4x^2+8x+x-2-x^3+2x^2-x+1}{(x-2)^2} \\
&= \frac{-2x^3+8x^2-8x+1}{x^2-4x+4} \\
&= \frac{-2x^3+8x^2-8x}{x^2-4x+4}+\frac{1}{x^2-4x+4} \\
&= \frac{-2x(x^2-4x+4)}{x^2+4x+4}+\frac{1}{x^2-4x+4} \\
&= -2x+\frac{1}{x^2-4x+4} \\
\frac{\Delta^2 y}{\Delta x^2} &= -2-\left(x^2-4x+4\right)^{-2}(2x-4) \\
&= \frac{4-2x}{(x-2)^4}-2
\end{aligned}
$$

$$
\begin{aligned}
\frac{4-2x}{(x-2)^4}-2 &= 0 \\
\frac{4-2x}{(x-2)^4} &= 2 \\
2-x &= (x-2)^4 \\
-(x-2) &= (x-2)^4 \\
-1 &= (x-2)^3 \\
x-2 &= -1 \\
x &= 1
\end{aligned}
$$

$$= (1,1)$$

C

$$
\begin{aligned}
\frac{\Delta y}{\Delta x} &= \frac{2x^2\left(x^2-4\right)-x^3(2x)}{\left(x^2-4\right)^2} \\
&= \frac{3x^4-8x^2-2x^4}{\left(x^2-4\right)^2} \\
&= \frac{x^4-12x^2}{\left(x^2-4\right)^2} \\
\frac{\Delta^2 y}{\Delta x^2} &= \frac{\left(4x^3-24x\right)\left(x^2-4\right)^2-2\left(x^4-12x^2\right)\left(x^2-4\right)(2x)}{\left(x^2-4\right)^4} \\
&= \frac{\left(4x^3-24x\right)\left(x^2-4\right)-4x\left(x^4-12x^2\right)}{\left(x^2-4\right)^3} \\
&= \frac{4x^5-16x^3-24x^3+96x-4x^5+48x^3}{\left(x^2-4\right)^3} \\
&= \frac{8x^3+96x}{\left(x^2-4\right)^3} \\
&= \frac{8x\left(x^2+12\right)}{\left(x^2-4\right)^3}
\end{aligned}
$$

$$0 = \frac{8x\left(x^2+12\right)}{\left(x^2-4\right)^3} 0 = 8x\left(x^2+12\right)$$

$$x = 0$$

$$0 = x^2 + 12$$

Discard

$$= (0, 0)$$

Question 4

$$
\begin{aligned}
\frac{\Delta y}{\Delta x} &= \frac{2x^2}{x} + 4x\ln x \\
&= 2x + 4x\ln x \\
&= 2x(1+2\ln x) \\
\frac{\Delta^2 y}{\Delta x^2} &= 2(1+2\ln x) + 2x\frac{2}{x} \\
&= 2 + 4\ln x + 4 \\
&= 6 + 4\ln x
\end{aligned}
$$

$$0 = 6 + 4\ln x$$
$$-\frac{3}{2} = \ln x$$
$$x = e^{-\frac{3}{2}}$$
$$x0.223$$

$$= (0.223, -0.149)$$

Question 5

a

$$\frac{\Delta y}{\Delta x} = e^x\left(x^2 - 2x + 2\right) + e^x\left(2x - 2\right) \qquad = e^x x^2$$

$$0 = e^x x^2$$

$$0 = e^x$$
$$x = \ln 0$$
Discard

$$0 = x^2$$
$$x = 0$$

$$= (0, 2)$$

$f'(-0.1)$	0.09
$f'(0.1)$	0.011

Point of Inflection?

b

$$\frac{\Delta^2 y}{\Delta x^2} = e^x\left(x^2 + 2x\right) = e^x x(x + 2)$$

$$0 = e^x$$
$$x = \ln 0$$
Discard

$$x = 0$$

$$0 = x + 2$$
$$x = -2$$

Since $x = 0$ is a stationary point, I only need $x = 2$.

$$= (2, 2e^2)$$

Question 6

a

$$\frac{\Delta y}{\Delta x} = e^x(x+1)$$

$$0 = e^x(x+1)$$

$0 = e^x$

$x = \ln 0$

Discard

$0 = x + 1$

$x = -1$

$$= (-1, \frac{-1}{e})$$

$f'(-0.1)$	-0.09
$f'(0.1)$	0.11

Minimum Point

b

$$\frac{\Delta^2 y}{\Delta x^2} = e^x\,(x+1+1) = e^x(x+2)$$

$$0 = e^x(x+2)$$

$0 = e^x$

$x = \ln 0$

Discard

$0 = x + 2$

$x = -2$

$$= (-2, \frac{-2}{e^2}))$$

c

Question 7

	f'	f''
A	Negative	Positive
B	Zero	Positive
C	Positive	Negative
D	Zero	Negative

Question 8

$$\frac{\Delta y}{\Delta x} = \sec^2 x$$

$$\frac{\Delta^2 y}{\Delta x^2} = 2\tan x \sec^2 x$$

$$0 = 2\tan x \sec^2 x$$

$$0 = \frac{2\sin x}{\cos^3 x}$$

$$0 = 2\sin x x$$

$$x = 0$$

$$= \arcsin 0$$

$$= (0, 0)$$

Question 9

a

$$\frac{\Delta y}{\Delta x} = 15x(3x-1)^4 + (3x-1)^5 \frac{\Delta^2 y}{\Delta x^2} \qquad = 180x^2(3x-1)^3 + 15(3x-1)^4 + 15(3x-1)^4$$

$$= (3x-1)^3\,(180x + 30(3x-1))$$

$$= (3x-1)^3(270x-30)$$

b

$$(3x-1)^3(270x-30)$$

$$0 = (3x-1)^3$$

$$0 = 3x - 1$$

$$x = \frac{1}{3}$$

$$0 = 270x - 30$$

$$30 = 270x$$

$$x = \frac{1}{9}$$

$$= \left(\frac{1}{3}, 0\right), \left(\frac{1}{9}, -\frac{32}{2187}\right)$$