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Question 1

$$= \sin 5x$$

$$= \sin 2x \cos 3x + \sin 3x \cos 2x$$

$$= 2 \sin x \cos x \cos 3x + \sin 3x (\cos^2 x - \sin^2 x)$$

$$= 2 \sin x \cos x (\cos x \cos 2x - \sin x \sin 2x) + (\sin 2x \cos x + \sin x \cos 2x) (\cos^2 x - \sin^2 x)$$

$$= 2 \sin x \cos x (\cos x (\cos^2 x - \sin^2 x) - 2 \sin^2 x \cos x) + (2 \sin x \cos^2 x + \sin x (\cos^2 x - \sin^2 x)) (\cos^2 x - \sin^2 x)$$

$$= 2 \sin x \cos x (\cos^3 x - \sin^2 x \cos x - 2 \sin^2 x \cos x) + (2 \sin \cos^2 x + \sin x \cos^2 x - \sin^3 x) (\cos^2 x - \sin^2 x)$$

$$= 2 \sin x \cos x (\cos^3 x - 3 \sin^2 x \cos x) + (3 \sin x \cos^2 x - \sin^3 x) (\cos^2 x - \sin^2 x)$$

$$= 2 \sin x \cos^4 x - 6 \sin^3 x \cos^2 x + 3 \sin x \cos^4 x - \sin^3 x \cos^2 x - 3 \sin^3 \cos^2 x + \sin^5 x$$

$$= 5 \sin x \cos^4 x - 10 \sin^3 x \cos^2 x + \sin^5 x$$

$$= \cos 5x$$

$$= \cos 3x \cos 2x - \sin 3x \sin 2x$$

$$= (\cos 2x \cos x - \sin 2x \sin x) (\cos^2 x - \sin^2 x) - 2 \sin x \cos x (\sin 2x \cos x + \sin x \cos 2x)$$

$$= ((\cos^2 x - \sin^2 x) \cos x - 2 \sin^2 x \cos x) (\cos^2 x - \sin^2 x) - 2 \sin x \cos x (2 \sin x \cos^2 x + (\cos^2 x - \sin^2 x) \sin x)$$

$$= (\cos^3 x - \sin^2 x \cos x - 2 \sin^2 x \cos x) (\cos^2 x - \sin^2 x) - 2 \sin x \cos x (2 \sin x \cos^2 x + \cos^2 x \sin x - \sin^3 x)$$

$$= (\cos^3 x - 3 \sin^2 x \cos x) (\cos^2 x - \sin^2 x) - 2 \sin x \cos x (3 \sin x \cos^2 x - \sin^3 x)$$

$$= \cos^5 x - \sin^2 x \cos^3 x - 3 \sin^2 x \cos^3 x + 3 \sin^4 x \cos x - 6 \sin^2 x \cos^3 x + 2 \sin^4 x \cos x$$

$$= \cos^5 x - 10 \sin^2 x \cos^3 x + 5 \sin^4 x \cos x$$

$$\sin x - \cos x = \sin 5x - \cos 5x$$

$$\sin x - \cos x = 5 \sin x \cos^4 x - 10 \sin^3 x \cos^2 x + \sin^5 x - \cos^5 x + 10 \sin^2 x \cos^3 x - 5 \sin^4 x \cos x ???$$

Question 2

$$\binom{p}{2}(1)^{p-2}(-4)^2 = \binom{p}{4}(1)^{p-4}(-4)^4$$

$$4p(p-1) = \frac{32p(p-1)(p-2)(p-3)}{3}$$

$$3p(p-1) = 8p(p-1)(p-2)(p-3)$$

$$3 = (p-2)(p-3)$$

$$3 = p^2 - 5p + 6$$

$$0 = p^2 - 5p + 3$$

$$p = \frac{5 \pm \sqrt{13}}{2}$$

$$\binom{p}{3}(1)^{p-3}(-4)^3 \geq 0$$

$$\frac{32p(p-1)(p-2)}{3} \leq 0$$

$$\frac{32 * \frac{5+\sqrt{13}}{2} * (\frac{5+\sqrt{13}}{2} - 1)}{3} = 0$$

$$\frac{32 * \frac{5-\sqrt{13}}{2} * (\frac{5-\sqrt{13}}{2} - 1)}{3} = 0$$

$$p = \frac{5 \pm \sqrt{13}}{2}$$

Question 3

$$\frac{\Delta y}{\Delta x} = \frac{\Delta y \div \Delta t}{\Delta x \div \Delta t} = \frac{y'(t)}{x'(t)}$$

$$x = 15t - t^3$$

$$x' = 15 - 3t^2$$

$$y = 3 - 2t^2$$

$$y' = -4t$$

$$\frac{\Delta y}{\Delta x} = \frac{-4t}{15 - 3t^2}$$

$$x = 15t - t^3$$

$$14 = 15t - t^3$$

$$0 = -t^3 + 15t - 14$$

$$t = 1, \dots$$

$$y = 3 - 2t^2$$

$$1 = 3 - 2t^2$$

$$0 = -2t^2 + 3t - 1$$

$$t = 1, 0.5$$

$$t = 1$$

$$\frac{\Delta y}{\Delta x_1} = \frac{-4}{15-3}$$

$$= -\frac{1}{3}$$

Normal = 3

$$3 = \frac{-4t}{15-3t^2}$$

$$45 - 9t^2 = -4t$$

$$0 = 9t^2 - 4t - 45$$

Question 4

$$x^3 + y^3 - 3xy = 48$$

$$y^3 - 3xy = 48 - x^3$$

$$y(y^2 - 3x) = 48 - x^3$$