1) $x = \frac{3}{2} \ln(y^2+9)$

$\frac{dx}{dy} = \frac{3}{2} \times \frac{1}{y^2+9} \times 2y$

$= \frac{3y}{y^2+9}$

$\frac{dy}{dx} = \frac{y^2+9}{3y}$

$= \frac{y^2}{3y} + \frac{9}{3y}$

$= \frac{y}{3} + \frac{3}{y}$

C3 Z

Jack Maguire

Question 1

$$
\begin{aligned}
x &= \ln\left(y^2+9\right)^{\frac{3}{2}} \\
&= \left(\ln\left(y^2+9\right)\right)^{\frac{3}{2}} \\
u &= \ln\left(y^2+9\right) \\
\frac{\Delta u}{\Delta x} &= \frac{2y}{y^2+9} \\
\frac{\Delta y}{\Delta x} &= \frac{3}{2}\left(\ln\left(y^2+9\right)\right)^{\frac{1}{2}} \frac{2y}{y^2+9} \\
&= \frac{6y}{2y^2+18}\sqrt{\ln\left(y^2+9\right)}
\end{aligned}
$$

Log Laws

Question wrong? Cannot get to simplify and desmos disagrees

Question 2

$$\frac{\sin 2\phi}{\sin\phi} - \frac{\cos 2\phi}{\cos\phi} \equiv \sec\phi$$

$$
\begin{aligned}
\textbf{LHS} &\equiv \frac{\sin 2\phi}{\sin\phi} - \frac{\cos 2\phi}{\cos\phi} \\
&\equiv \frac{2\sin\cos\phi}{\sin\phi} - \frac{2\cos^2\phi - 1}{\cos\phi} \\
&\equiv 2\cos\phi - 2\cos\phi + \sec\phi \equiv \sec\phi \equiv \textbf{RHS} \quad \text{QED}
\end{aligned}
$$

✓

Question 3

a

$$v = 1 + 2\ln x \qquad u = x$$

$$v' = \frac{2}{x} \qquad u' = 1$$

$$\begin{aligned}
\frac{\Delta y}{\Delta x} &= \frac{vu' - v'u}{v^2} \\
&= \frac{1 + 2\ln x - \frac{2x}{x}}{(1 + 2\ln x)^2} \\
&= \frac{2\ln x - 1}{1 + 4\ln x + 4(\ln x)^2} \\
0 &= \frac{2\ln x - 1}{1 + 4\ln x + 4(\ln x)^2} \\
0 &= 2\ln x - 1 \\
\frac{1}{2} &= \ln x \\
x &= e^{\frac{1}{2}} = \sqrt{e} \\
y &= \frac{x}{1 + 2\ln x} \\
&= \frac{\sqrt{e}}{1+1} \\
&= \frac{1}{2}\sqrt{e} \\
\text{TP} &= (\sqrt{e}, \frac{1}{2}\sqrt{e})
\end{aligned}$$

b

$$v = (1 + 2\ln x)^2 \qquad u = 2\ln x - 1$$

$$v' = \frac{4 + 8\ln x}{x} \qquad u' = \frac{2}{x}$$

$$\frac{\Delta y}{\Delta x} = \frac{vu' - v'u}{v^2}$$

$$\frac{\Delta^2 y}{\Delta x^2} = \frac{\frac{2}{x}(1 + 2\ln x)^2 - \frac{4+8\ln x}{x}(2\ln x - 1)}{(1 + 2\ln x)^4}$$

$$\frac{\Delta^2 y}{\Delta x^2}_{\sqrt{e}} = \frac{\frac{2}{\sqrt{e}}(1+1)^2 - \frac{4+4}{\sqrt{e}}(1-1)}{(1+1)^4}$$

$$= \frac{\frac{2}{\sqrt{e}}}{(1+1)^2}$$

$$= \frac{2}{4\sqrt{e}}$$

$$= \frac{1}{2\sqrt{e}}$$

$\frac{1}{2\sqrt{e}} > 0 \therefore$ is a minimum point. ✓

Question 4

a

$$x = \tan y$$

$$\frac{\Delta x}{\Delta y} = \sec^2 y \;=\; 1 + \tan^2 y \;=\; 1 + x^2$$

$$\therefore \frac{dy}{dx} = \frac{1}{x^2}$$

$$\frac{\Delta y}{\Delta x} = \cos^2 y$$

$$= \cos^2\left(\tan^{-1} x\right)$$

???

b

$$\frac{dy}{dx} = \frac{1}{x^2+1} - 4x\,\frac{1}{1+x^2} \times 2x - 6x$$

$$= \frac{1}{1+x^2} - \frac{8x}{1+x^2} - 6x$$

$$0 = \frac{1-8x}{1+x^2} - 6x$$

$$6x + 6x^3 = 1 - 8x$$

$$6x^3 + 14x - 1 = 0$$

$$\frac{\Delta y}{\Delta x} = 1 + x^2 - \frac{8}{1+x^2} - 6x$$

$$= x^2 - 6x - \frac{8}{1+x^2} + 1$$

$$0 = x^2 - 6x - \frac{8}{1+x^2} + 1$$

$$= \left(1+x^2\right)x^2 - \left(1+x^2\right)6x - 8 + \left(1+x^2\right)$$

$$= x^2 + x^4 - 6x - 6x^3 - 8 + 1 + x^2$$

$$= x^4 - 6x^3 + 2x^2 - 6x - 7$$

???

c

$$f(x) = 6x^3 + 14x - \cancel{1}f(0) = -1$$

realise

$$f(1) = 19$$

Between positive and negative ∴ root inbetween.

d

???

NC

e

???

"

Question 5

a

$$\begin{aligned}
H &= k\left(1 - e^{-\frac{1}{12}t}\right) \\
41 &= k\left(1 - e^{-\frac{5}{12}}\right) \\
\frac{41}{k} &= 1 - e^{-\frac{5}{12}} \\
k &= \frac{41}{1 - e^{-\frac{5}{12}}} \\
k &= 120.319\ldots \\
&= 120
\end{aligned}$$

b

$$\begin{aligned}
H &= 120\left(1 - e^{-\frac{1}{12}t}\right) \\
\frac{90}{120} &= 1 - e^{-\frac{1}{12}t} \\
\frac{1}{4} &= e^{-\frac{1}{12}t} \\
\ln\frac{1}{4} &= -\frac{1}{12}t \\
-12t &= -\ln 4 \\
t &= 12\ln 4 \\
&= 24\ln 2
\end{aligned}$$

c

$$H = 120\left(1 - e^{-\frac{1}{12}t}\right)$$
$$\frac{H}{120} = 1 - e^{-\frac{1}{12}t}$$
$$1 - \frac{H}{120} = e^{-\frac{1}{12}t}$$
$$-\frac{1}{12}t = \ln\left(1 - \frac{H}{120}\right)$$
$$t = -12\ln\left(1 - \frac{H}{120}\right)$$

$$H = 120\left(1 - e^{-\frac{1}{12}t}\right)$$
$$a = 1 - e^{-\frac{1}{12}t}$$
$$\frac{\Delta a}{\Delta t} = \frac{1}{12}e^{-\frac{1}{12}t}$$
$$\frac{\Delta H}{\Delta t} = 120\frac{1}{12}e^{-\frac{1}{12}t}$$
$$= 10e^{-\frac{1}{12}t}$$
$$\cancel{= 10e^{-12\ln\left(1-\frac{H}{120}\right)}}$$
$$\cancel{= \frac{-10}{12}H}$$

$$\frac{H}{120} = 1 - e^{-\frac{1}{12}t}$$
$$e^{-\frac{1}{12}t} = 1 - \frac{H}{120}$$
$$10e^{-\frac{1}{12}t} = 10 - \frac{H}{12}$$

d

rewline

$$\frac{\Delta H}{\Delta t} = \frac{-10}{12}\cancel{H}\left.\frac{\Delta H}{\Delta t}\right._{7.5} \qquad = -\frac{-10}{12}7.5$$
$$= \frac{-25}{4}$$

ECF

e

$$\frac{\Delta H}{\Delta t} = 120\frac{1}{12}e^{-\frac{1}{12}t}$$
$$0 = 10e^{-\frac{1}{12}t}$$
$$e^{-\frac{1}{12}t} = 0$$

$t \to \infty$, $e^{-\frac{1}{12}t} \to 0$

$\therefore H \to 120$

$t = 120$

Since $\mathbb{R} \not\subseteq \ln 0$, we can see that this model exponentially grows, that is, there is no maximum height.

Question 6

a

$$2 \le f(x) \le 4$$
$$a = 3$$
$$\pi \longrightarrow 2\pi \therefore b = 0.5$$

b

$$y = 3 + \cos\frac{x}{2}$$
$$x = 3 + \cos\frac{y}{2}$$
$$\cos\frac{y}{2} = x - 3$$
$$f^{-1}(x) = \cancel{y} = 2\cos^{-1}(x-3)$$

c

$$2 \le x \le 4$$
$$0 \le f(x) \le 2\pi$$

d

$$\frac{\Delta y}{\Delta x} = -\frac{1}{2}\sin\frac{x}{2}\frac{\Delta y}{\Delta x}_{\frac{4}{3}\pi} \qquad = -\frac{1}{2}\sin\frac{2\pi}{3}$$
$$= -\frac{1}{2}\frac{\sqrt{3}}{2}$$
$$= -\frac{\sqrt{3}}{4}$$

e

$$-\frac{\sqrt{3}}{4}$$ X reciprocal

7a)

$\sin(A+B) + \sin(A-B)$

$= 2\sin A\cos B$

$A+B = P$

$A-B = Q$

$A = \frac{P+Q}{2}$, $B = \frac{P-Q}{2}$

$\therefore \sin P + \sin Q = 2\sin\frac{P+Q}{2}\cos\frac{P-Q}{2}$

7b)

$2\sin\left(\frac{5\theta+\theta}{2}\right)\cos\left(\frac{5\theta-\theta}{2}\right) = \sin 3\theta$

$2\sin 3\theta\cos 2\theta = \sin 3\theta$

$\sin 3\theta(2\cos 2\theta - 1) = 0$

solve.

$= 0, 30, 60, 120, 150, 180$

Question 7

a

$$\sin P + \sin Q \equiv 2\sin\left(\frac{P+Q}{2}\right)\cos\left(\frac{P-Q}{2}\right)$$

$$\textbf{RHS} \equiv 2\sin\left(\frac{P+Q}{2}\right)\cos\left(\frac{P-Q}{2}\right)$$

$$\equiv 2\sin\left(\frac{P}{2}+\frac{Q}{2}\right)\cos\left(\frac{P}{2}-\frac{Q}{2}\right)$$

$$\equiv 2\left(\sin\frac{P}{2}\cos\frac{Q}{2}+\sin\frac{Q}{2}\cos\frac{P}{2}\right)\left(\cos\frac{P}{2}\cos\frac{Q}{2}+\sin\frac{P}{2}\sin\frac{Q}{2}\right)$$

$$\equiv 2\left(\sin\frac{P}{2}\cos\frac{P}{2}\cos^2\frac{Q}{2}+\sin^2\frac{P}{2}\sin\frac{Q}{2}\cos\frac{Q}{2}+\sin\frac{Q}{2}\cos^2\frac{P}{2}\cos\frac{Q}{2}+\sin\frac{P}{2}\sin^2\frac{Q}{2}\cos\frac{P}{2}\right)$$

???

$\equiv \sin P + \sin Q \equiv$ RHS QED

b

$\sin\theta - \sin 3\theta + \sin 5\theta = 0$**???**

Question 8