C3 X

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Question 1

a

$$y = \frac{x^2 - 6x + 12}{4x - 11}$$

$$v = 4x - 11$$
$$v' = 4$$

$$u = x^2 - 6x + 12$$
$$u' = 2x - 6$$

$$\begin{aligned}
\frac{\Delta y}{\Delta x} &= \frac{vu' - v'u}{v^2} \\
&= \frac{(4x-11)(2x-6) - 4(x^2 - 6x + 12)}{(4x-11)^2} \\
&= \frac{8x^2 - 46x + 66 - 4x^2 + 24x - 48}{(4x-11)^2} \\
&= \frac{4x^2 - 22x + 18}{(4x-11)^2} \\
&= \frac{4x^2 - 22x + 18}{16x^2 - 88x + 121}
\end{aligned}$$

b

y is decreasing $\therefore \frac{\Delta y}{\Delta x} < 0$

$$\frac{4x^2 - 22x + 18}{16x^2 - 88x + 121} < 0$$

Since $16x^2 - 88x + 121$ only has one repeated root, and is a positive curve, it will never go below zero, so we can ignore solutions from it.

$$\begin{aligned}
\frac{4x^2 - 22x + 18}{16x^2 - 88x + 121} &< 0 \\
4x^2 - 22x + 18 &< 0 \\
(2x-1)(4x-9) &< 0 \\
1 < x &< 4.5
\end{aligned}$$

Question 2

$$
\begin{aligned}
\sin 2\theta &= \cot\theta \\
2\sin\theta\cos\theta &= \frac{\cos\theta}{\sin\theta} \\
2\sin^2\theta\cos\theta &= \cos\theta \\
2\sin^2\theta\cos\theta - \cos\theta &= 0 \\
(\cos\theta)\left(2\sin^2\theta - 1\right) &= 0
\end{aligned}
$$

$$
\begin{aligned}
\cos\theta &= 0 \\
\theta &= 90^\circ
\end{aligned}
$$

$$
\begin{aligned}
2\sin^2\theta - 1 &= 0 \\
\sin^2\theta &= \frac{1}{2} \\
\sin\theta &= \pm\frac{\sqrt{2}}{2} \\
\theta &= 45^\circ, 135^\circ
\end{aligned}
$$

$$\theta = 45^\circ, 90^\circ, 135^\circ$$

Question 3

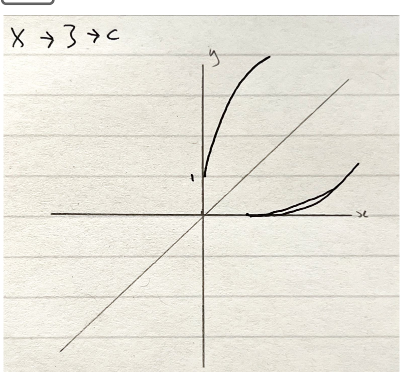
a

$$
\begin{aligned}
&= f\left(1+\sqrt{9}\right) \\
&= f(4) \\
&= 1+\sqrt{4} \\
&= 3
\end{aligned}
$$

b

$$
\begin{aligned}
y &= 1+\sqrt{x} \\
x &= 1+\sqrt{y} \\
x-1 &= \sqrt{y} \\
(x-1)^2 &= y \\
f(x) &= (x-1)^2
\end{aligned}
$$

c



d

$$
\begin{aligned}
1 + \sqrt{x} &= (x-1)^2 \\
\sqrt{x} &= x^2 - 2x \\
x &= \left(x^2 - 2x\right)^2 \\
0 &= x^4 - 4x^3 + 4x^2 - x
\end{aligned}
$$

We could say that $x = 0$, but we can see it is invalid on the graph and ignore it.

$$
\begin{aligned}
f(x) &= x^3 - 4x^2 + 4x - 1 \\
f(1) &= 1 - 4 + 4 - 1 = 0 \\
0 &= (x-1)(x^2 + bx + 1) \\
0 &= \ldots - x^2 + bx^2 + \ldots \\
-1 + b &= -4 \\
b &= -3 \\
f(x) &= (x-1)(x^2 - 3x + 1)
\end{aligned}
$$

We could say that $x = 1$, but we can see it is invalid on the graph and ignore it.

$$
\begin{aligned}
0 &= x^2 - 3x + 1 \\
x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
&= \frac{3 \pm \sqrt{9-4}}{2}
\end{aligned}
$$

We could say that $x = \frac{3-\sqrt{5}}{2}$, but we can see it is invalid on the graph and ignore it.

$$
x = \frac{3+\sqrt{5}}{2}
$$

Question 4

a

$$\frac{\Delta y}{\Delta x} = \frac{3y^2 - 4}{y^3 - 4y}$$

$$\frac{\Delta y}{\Delta x} = \frac{y^3 - 4y}{3y^2 - 4}$$

$$2 = \frac{3y^2 - 4}{y^3 - 4y}$$

$$1 = \frac{3y^2 - 4}{2y^3 - 8y}$$

$$2y^3 - 8y = 3y^2 - 4$$

$$y = \frac{3y^2 - 4}{2y^2 - 8}$$

???

b

???

Question 5

$$
\begin{aligned}
\frac{\Delta y}{\Delta x} &= (-1)\left(4e^{2-x}\right) - (-2)\left(e^{4-2x}\right) \\
&= -4e^{2-x} + 2e^{4-2x} \\
0 &= -4e^{2-x} + 2e^{4-2x} \\
2e^{2-x} &= e^{4-2x} \\
2e^{2-x} &= \left(e^{4-2x}\right)^2
\end{aligned}
$$

$$
\begin{aligned}
\text{Let } y &= e^{2-x} \\
2y &= y^2 \\
0 &= y^2 - 2y \\
y &= 0, 2 \\
\mathbb{R} \not\subseteq \ln 0 \\
e^{2-x} &= 2 \\
x &= 2 - \ln 2 \\
&= 1.307
\end{aligned}
$$

$$
\begin{aligned}
y &= 4e^{2-x} - e^{4-2x} \\
&= 4
\end{aligned}
$$

$$
\begin{aligned}
\frac{\Delta y}{\Delta x}_{1.25} &= 0.49 \\
\frac{\Delta y}{\Delta x}_{1.307} &= 0 \\
\frac{\Delta y}{\Delta x}_{1.35} &= -0.32
\end{aligned}
$$

- Stationary Point = $(1.307, 4)$
- Kind = Maximum

Question 6

a

???

b

$$R\cos(\theta - \alpha) \equiv R\cos\theta\cos\alpha + R\sin\theta\sin\alpha$$

$$\begin{aligned} R\sin\alpha &= 2 \\ R\cos\alpha &= 5 \\ \tan\alpha &= \frac{2}{5} \\ \alpha &= 21.8^\circ \end{aligned}$$

$$\begin{aligned} R^2 &= 5^2 + 2^2 \\ R &= \sqrt{25+4} \\ R &= \sqrt{29} \end{aligned}$$

$$5\cos\theta + 2\sin\theta \equiv \sqrt{29}\cos(\theta - 21.8^\circ)$$

c

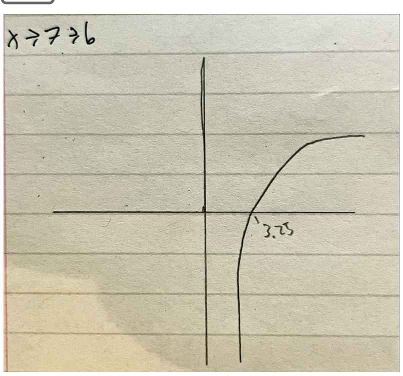
???

Question 7

a

1. Take the natural logarithm of x value.
2. Shift the graph right by 12 units.
3. Shrink the graph horizontally by a factor of 4.

b



Question 8

a

$$v = x \qquad u = \sqrt{x+1}$$
$$v' = 1 \qquad u' = \frac{1}{2}(x+1)^{-\frac{1}{2}}$$

$$\begin{aligned}
\frac{\Delta y}{\Delta x} &= vu' + v'u \\
&= x\frac{1}{2}(x+1)^{-\frac{1}{2}} + (x+1)^{\frac{1}{2}} \\
&= (x+1)^{-\frac{1}{2}}\left(x\frac{1}{2} + (x+1)\right) \\
&= (x+1)^{-\frac{1}{2}}\left(\frac{3}{2}x + 1\right) \\
&= \frac{1}{2}(3x+2)(x+1)^{-\frac{1}{2}}
\end{aligned}$$

b

$$v = x\sqrt{x+1} \qquad u = \sin 2x$$
$$v' = \frac{1}{2}(3x+2)(x+1)^{-\frac{1}{2}} \qquad u' = 2\cos 2x$$

$$\begin{aligned}
\frac{\Delta y}{\Delta x} &= vu' + v'u \\
&= \left(x\sqrt{x+1}\right)(2\cos 2x) + \left(\frac{1}{2}(3x+2)(x+1)^{-\frac{1}{2}}\right)(\sin 2x) \\
&= \left(\frac{\pi}{2}\sqrt{\frac{\pi}{2}+1}\right)(2\cos \pi) + \left(\frac{1}{2}(3\frac{\pi}{2}+2)(\frac{\pi}{2}+1)^{-\frac{1}{2}}\right)(\sin \pi) \\
&= -2\left(\frac{\pi}{2}\sqrt{\frac{\pi}{2}+1}\right) \\
&= -\pi\sqrt{\frac{\pi}{2}+1}
\end{aligned}$$