C3 Y

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Question 1

$$v = e^{-x} \qquad u = \sin x$$
$$v' = -e^{-x} \qquad u' = \cos x$$

$$\begin{aligned} \frac{\Delta y}{\Delta x} &= vu' + v'u \\ &= e^{-x}\cos x + -e^{-x}\sin x \\ 0 &= e^{-x}\cos x - e^{-x}\sin x \\ &= (e^{-x})(\cos x - \sin x) \end{aligned}$$

$e^{-x} \not\equiv \mathbb{R}$

$$\begin{aligned} 0 &= \cos x - \sin x \\ \cos x &= \sin x \\ \tan x &= 1 \\ x &= \frac{1}{4}\pi \end{aligned}$$

$$\begin{aligned} e^{-x} &= e^{-x}\sin x \\ 1 &= \sin x \\ x &= \frac{1}{2}\pi \end{aligned}$$

$$\begin{aligned} &= \frac{1}{2}\pi - \frac{1}{4}\pi \\ &= \frac{1}{4}\pi \end{aligned}$$

Question 2

a

$$R\sin(x+\alpha) \equiv R\sin\alpha\cos x + R\sin\alpha\cos x$$

$$R^2 = 2^2 + 1^2$$
$$R = \sqrt{5}$$

$$\begin{aligned} 2 &= \sin\alpha \\ 1 &= \cos\alpha \\ \tan\alpha &= 2 \\ \alpha &= 1.107 \end{aligned}$$

$$2\cos x + \sin x \equiv \sqrt{5}\sin(x + 1.107)$$

b

$$-\sqrt{5} \leq f(x) \leq \sqrt{5}$$

$$g\left(-\sqrt{5}\right) = \frac{5}{5+5} = 2$$
$$g(0) = \frac{5}{0+5} = 1$$
$$g\left(\sqrt{5}\right) = \frac{5}{5+5} = 2$$

$$1 \leq g(f(x)) \leq 2$$

Question 3

a

$$10 = Ae^{-k*0}$$
$$A = 10$$

$$5 = Ae^{-5k}$$
$$5 = 10e^{-5k}$$
$$\frac{1}{2} = e^{-5k}$$
$$-5k = \ln\frac{1}{2}$$
$$5k = \ln 2$$
$$k = \frac{1}{5}\ln 2$$

b

$$M = 10e^{-\frac{1}{5}t}$$
$$\frac{\Delta M}{\Delta x} = \left(-\frac{1}{5}\right)\left(10e^{-\frac{1}{5}t}\right)$$
$$= -2e^{-\frac{1}{5}t}$$
$$\ln\left(\frac{1}{\sqrt{2}}\right) = -2e^{-\frac{1}{5}t}$$
$$\ln\sqrt{2} = 2e^{-\frac{1}{5}t}$$
$$\ln\sqrt[4]{2} = e^{-\frac{1}{5}t}$$
$$-\frac{1}{5}t = \ln\left(\ln\sqrt[4]{2}\right)$$
$$t = -5\ln\left(\ln\sqrt[4]{2}\right)$$
$$t = 8.764$$

Question 4

a

$$= \tan 2x$$
$$= \frac{\sin 2x}{\cos 2x}$$

$v = \cos 2x$
$v' = -2\sin 2x$

$u = \sin 2x$
$u' = 2\cos 2x$

$$\frac{\Delta}{\Delta x} = \frac{vu' - v'u}{v^2}$$
$$= \frac{2\cos^2 2x + 2\sin^2 2x}{\cos^2 2x}$$
$$= \frac{2}{\cos^2 2x}$$
$$= 2\sec^2 2x$$

b

$v = 6x$ $\qquad$ $u = \tan 2x$

$v' = 6$ $\qquad$ $u' = 2\sec^2 2x$

$$\begin{aligned}
\frac{\Delta y}{\Delta x} &= vu' + v'u \\
&= 12x\sec^2 2x + 6\tan 2x \\
\frac{\Delta y}{\Delta x}_{\frac{1}{8}\pi} &= \frac{12}{8}\pi\sec^2\frac{1}{4}\pi + \tan\frac{1}{4}\pi \\
&= 3\pi + 1
\end{aligned}$$

$$\begin{aligned}
y &= 6x\tan 2x \\
&= \frac{6}{8}\pi\tan\frac{2}{8}\pi \\
&= \frac{3}{4}\pi
\end{aligned}$$

$$\begin{aligned}
y - y_1 &= m\,(x - x_1) \\
y - \frac{3}{4}\pi &= (3\pi + 1)\left(x - \frac{1}{8}\pi\right) \\
y &= (3\pi + 1)x - \frac{3\pi + 1}{8}\pi + \frac{3}{4}\pi \\
\therefore c &= -\frac{3\pi + 1}{8}\pi + \frac{3}{4}\pi \\
&=
\end{aligned}$$