C3 Y

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Question 1

$$v = e^{-x} \qquad\qquad u = \sin x$$
$$v' = -e^{-x} \qquad\qquad u' = \cos x$$

$$e^{-x} = e^{-x} \sin x$$
$$1 = \sin x$$
$$x = \frac{1}{2}\pi$$

$$\frac{\Delta y}{\Delta x} = vu' + v'u$$
$$= e^{-x} \cos x + -e^{-x} \sin x$$
$$0 = e^{-x} \cos x - e^{-x} \sin x$$
$$= (e^{-x})(\cos x - \sin x)$$
$$\mathbb{R} \nsubseteq e^{-x}$$
$$0 = \cos x - \sin x$$
$$\cos x = \sin x$$
$$\tan x = 1$$
$$x = \frac{1}{4}\pi$$

$$= \frac{1}{2}\pi - \frac{1}{4}\pi$$
$$= \frac{1}{4}\pi$$

Question 2

a

$$R \sin(x + \alpha) \equiv R \sin\alpha \cos x + R \sin\alpha \cos x$$

$$R^2 = 2^2 + 1^2 \qquad\qquad 2 = \sin\alpha$$
$$R = \sqrt{5} \qquad\qquad 1 = \cos\alpha$$
$$\tan\alpha = 2$$
$$\alpha = 1.107$$

$$2\cos x + \sin x \equiv \sqrt{5} \sin(x + 1.107)$$

b

$$-\sqrt{5} \leq f(x) \leq \sqrt{5}$$

$$g\left(-\sqrt{5}\right) = \frac{5}{5+5} = 2$$
$$g(0) = \frac{5}{0+5} = 1$$
$$g\left(\sqrt{5}\right) = \frac{5}{5+5} = 2$$

$$1 \leq g(f(x)) \leq 2$$

Question 3

a

$$10 = Ae^{-k*0}$$
$$A = 10$$

$$5 = Ae^{-5k}$$
$$5 = 10e^{-5k}$$
$$\frac{1}{2} = e^{-5k}$$
$$-5k = \ln\frac{1}{2}$$
$$5k = \ln 2$$
$$k = \frac{1}{5}\ln 2$$

b

$$M = 10e^{-\frac{1}{5}t}$$
$$\frac{\Delta M}{\Delta x} = \left(-\frac{1}{5}\right)\left(10e^{-\frac{1}{5}t}\right)$$
$$= -2e^{-\frac{1}{5}t}$$
$$\ln\left(\frac{1}{\sqrt{2}}\right) = -2e^{-\frac{1}{5}t}$$
$$\ln\sqrt{2} = 2e^{-\frac{1}{5}t}$$
$$\ln\sqrt[4]{2} = e^{-\frac{1}{5}t}$$
$$-\frac{1}{5}t = \ln\left(\ln\sqrt[4]{2}\right)$$
$$t = -5\ln\left(\ln\sqrt[4]{2}\right)$$
$$t = 8.764$$

a

$$= \tan 2x$$

$$= \frac{\sin 2x}{\cos 2x}$$

$v = \cos 2x$ $\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ $u = \sin 2x$

$v' = -2\sin 2x$ $\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ $u' = 2\cos 2x$

$$\frac{\Delta}{\Delta x} = \frac{vu' - v'u}{v^2}$$

$$= \frac{2\cos^2 2x + 2\sin^2 2x}{\cos^2 2x}$$

$$= \frac{2}{\cos^2 2x}$$

$$= 2\sec^2 2x$$

b

$$v = 6x$$
$$v' = 6$$

$$u = \tan 2x$$
$$u' = 2\sec^2 2x$$

$$\frac{\Delta y}{\Delta x} = vu' + v'u$$
$$= 12x \sec^2 2x + 6\tan 2x$$
$$\frac{\Delta y}{\Delta x}_{\frac{1}{8}\pi} = \frac{12}{8}\pi \sec^2 \frac{1}{4}\pi + \tan \frac{1}{4}\pi$$
$$= 3\pi + 1$$

$$y = 6x\tan 2x$$
$$= \frac{6}{8}\pi \tan \frac{2}{8}\pi$$
$$= \frac{3}{4}\pi$$

$$y - y_1 = m(x - x_1)$$
$$y - \frac{3}{4}\pi = (3\pi + 1)\left(x - \frac{1}{8}\pi\right)$$
$$y = (3\pi + 1)x - \frac{3\pi + 1}{8}\pi + \frac{3}{4}\pi$$
$$\therefore c = -\frac{3\pi + 1}{8}\pi + \frac{3}{4}\pi$$
$$= \frac{6\pi}{8} - \frac{3\pi^2 + \pi}{8}$$
$$= \frac{-3\pi^2 + 5\pi}{8}$$

???**Maybe 3 pie + 6**

Question 5

a

$$0 = \sqrt{1 - (2x - 1)^2}$$
$$= 1 - \left(4x^2 - 4x + 1\right)$$
$$= -4x^2 + 4x$$
$$= x^2 - x$$
$$= x(x - 1)$$
$$x = 0, 1$$
$$a = 1$$

b

$$f(a \div 2) = f(0.5)$$
$$= \sqrt{1 - (1-1)^2}$$
$$= \sqrt{1}$$
$$= 1$$
$$0 \le f(x) \le 1$$

S

c

1. Stretch graph horizontally by a factor of 2

2. Stretch graph vertically by a factor of 2

3. Shift graph downwards by 2 units

NEED TO ACTUALLY DRAW THIS ON THE FIGURES BIT

d

Domain: $0 \le x \le 2$ Range: $-2 \le f(x) \le 0$

Question 6

a

$$\frac{2\cot\theta}{1 + \cot^2\theta} \equiv \sin 2\theta$$
$$\mathbf{LHS} \equiv \frac{2\cot\theta}{1 + \cot^2\theta}$$
$$\equiv \frac{2\cot\theta}{\csc^2\theta}$$
$$\equiv 2\frac{\cos\theta}{\sin\theta}\sin^2\theta$$
$$\equiv 2\cos\theta\sin\theta \equiv \sin 2\theta \equiv \mathbf{RHS} \quad \text{QED}$$

b

$$4\cot^2\theta + 1 = 2\sin 2\theta\left(1 + \cot^2\theta\right)$$
$$4\left(\cot^2\theta + 1\right) - 3 = (4\sin\theta\cos\theta)\left(\csc^2\theta\right)$$
$$4\csc^2\theta - 3 = 4\cot\theta$$
$$\frac{4}{\sin^2\theta} = 4\frac{\cos\theta}{\sin\theta} + 3$$
$$4 = 4\cos\theta\sin\theta + 3\sin^2\theta$$
$$4 = 2\sin 2\theta + 3\sin^2\theta$$
$$0 = 3\sin^2\theta + 2\sin 2\theta - 4$$
$$0 = 3\sin^2\theta + 4\sin\theta\cos\theta - 4$$
???

a

$$y = 3\sin^{-1}(x - 1)$$

$$y = 3\sin^{-1}(0 - 1)$$
$$= 3 * -\frac{1}{2}\pi$$
$$= -\frac{3\pi}{2} \; //$$

$$y = 3\sin^{-1}(2 - 1)$$
$$= 3 * \frac{1}{2}\pi$$
$$= \frac{3\pi}{2} \; //$$

$$0 = 3\sin^{-1}(x - 1)$$
$$\sin(0) = 3x - 3$$
$$0 = 3x - 3$$
$$3x = 3$$
$$x = 1$$

$$y = 2\cos^{-1}(x - 1)$$

$$y = 2\cos^{-1}(0 - 1)$$
$$= 2\pi \; //$$
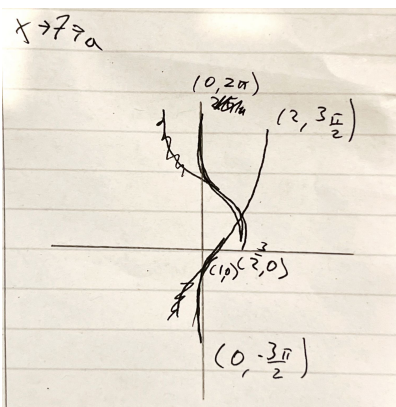$$0 = 2\cos^{-1}(x - 1)$$
$$\cos(0) = 2x - 2$$
$$1 = 2x - 2$$
$$3 = 2x$$
$$x = \frac{3}{2} \; //$$

$$3\sin^{-1}(x - 1) = 2\cos^{-1}(x - 1)$$
$$3x - 3 = \sin\left(2\cos^{-1}(x - 1)\right)$$
???

b

???

Question 8

a

$$|f(x)| = 12$$
$$k(x^2 - 4x) = \pm 12$$
$$12/4 = 3$$
$$k = 3$$

Clarification?

b

$$3(x^2 - 4x) = 12 \qquad\qquad -3(x^2 - 4x) = 12$$
$$x^2 - 4x - 4 = 0 \qquad\qquad x^2 - 4x + 4 = 0$$
$$x = 2 \pm 2\sqrt{2} \qquad\qquad (x - 2)^2 = 0$$
$$\qquad\qquad\qquad\qquad x = 2$$

$$x = 2 - 2\sqrt{2} \quad 2 \quad 2 + 2\sqrt{2}$$