C3 X

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Question 1

a

$$y = \frac{x^2 - 6x + 12}{4x - 11}$$

$$v = 4x - 11$$
$$v' = 4$$

$$u = x^2 - 6x + 12$$
$$u' = 2x - 6$$

$$\frac{\Delta y}{\Delta x} = \frac{vu' - v'u}{v^2}$$

$$= \frac{(4x - 11)(2x - 6) - 4(x^2 - 6x + 12)}{(4x - 11)^2}$$

$$= \frac{8x^2 - 46x + 66 - 4x^2 + 24x - 48}{(4x - 11)^2}$$

$$= \frac{4x^2 - 22x + 18}{(4x - 11)^2}$$

$$= \frac{4x^2 - 22x + 18}{16x^2 - 88x + 121}$$

b

y is decreasing $\therefore \frac{\Delta y}{\Delta x} < 0$

$$\frac{4x^2 - 22x + 18}{16x^2 - 88x + 121} < 0$$

Since $16x^2 - 88x + 121$ only has one repeated root, and is a positive curve, it will never go below zero, so we can ignore solutions from it.

$$\frac{4x^2 - 22x + 18}{16x^2 - 88x + 121} < 0$$
$$4x^2 - 22x + 18 < 0$$
$$(2x - 1)(4x - 9) < 0$$
$$1 < x < 4.5$$

$$\sin 2\theta = \cot \theta$$
$$2 \sin \theta \cos \theta = \frac{\cos \theta}{\sin \theta}$$
$$2 \sin^2 \theta \cos \theta = \cos \theta$$
$$2 \sin^2 \theta \cos \theta - \cos \theta = 0$$
$$(\cos \theta)\left(2 \sin^2 \theta - 1\right) = 0$$

$$\cos \theta = 0$$
$$\theta = 90°$$

$$2 \sin^2 \theta - 1 = 0$$
$$\sin^2 \theta = \frac{1}{2}$$
$$\sin \theta = \pm \frac{\sqrt{2}}{2}$$
$$\theta = 45°, 135°$$

$$\theta = 45°, 90°, 135°$$

a

$$= f\left(1 + \sqrt{9}\right)$$
$$= f(4)$$
$$= 1 + \sqrt{4}$$
$$= 3$$

b

$$y = 1 + \sqrt{x}$$
$$x = 1 + \sqrt{y}$$
$$x - 1 = \sqrt{y}$$
$$(x - 1)^2 = y$$
$$f(x) = (x - 1)^2$$

c

X → 3 → c
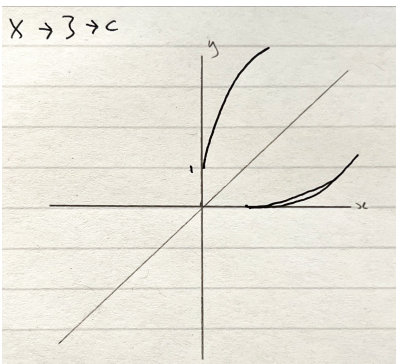
$$1 + \sqrt{x} = (x - 1)^2$$
$$\sqrt{x} = x^2 - 2x$$
$$x = \left(x^2 - 2x\right)^2$$
$$0 = x^4 - 4x^3 + 4x^2 - x$$

We could say that $x = 0$, but we can see it is invalid on the graph and ignore it.

$$f(x) = x^3 - 4x^2 + 4x - 1$$
$$f(1) = 1 - 4 + 4 - 1 = 0$$
$$0 = (x - 1)(x^2 + bx + 1)$$
$$0 = \ldots - x^2 + bx^2 + \ldots$$
$$-1 + b = -4$$
$$b = -3$$
$$f(x) = (x - 1)(x^2 - 3x + 1)$$

We could say that $x = 1$, but we can see it is invalid on the graph and ignore it.

$$0 = x^2 - 3x + 1$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{3 \pm \sqrt{9 - 4}}{2}$$

We could say that $x = \frac{3 - \sqrt{5}}{2}$, but we can see it is invalid on the graph and ignore it.

$$x = \frac{3 + \sqrt{5}}{2}$$

Question 4

a

$$\frac{\Delta y}{\Delta x} = \frac{3y^2 - 4}{y^3 - 4y}$$

$$\frac{\Delta y}{\Delta x} = \frac{y^3 - 4y}{3y^2 - 4}$$

$$2 = \frac{3y^2 - 4}{y^3 - 4y}$$

$$1 = \frac{3y^2 - 4}{2y^3 - 8y}$$

$$2y^3 - 8y = 3y^2 - 4$$

$$y = \frac{3y^2 - 4}{2y^2 - 8}$$

???

b

???

4

Question 5

$$\frac{\Delta y}{\Delta x} = (-1)\left(4e^{2-x}\right) - (-2)\left(e^{4-2x}\right)$$
$$= -4e^{2-x} + 2e^{4-2x}$$
$$0 = -4e^{2-x} + 2e^{4-2x}$$
$$2e^{2-x} = e^{4-2x}$$
$$2e^{2-x} = \left(e^{4-2x}\right)^2$$

Let $y = e^{2-x}$
$$2y = y^2$$
$$0 = y^2 - 2y$$
$$y = 0, 2$$
$$\ln 0 \neq \mathbb{R}$$
$$e^{2-x} = 2$$
$$x = 2 - \ln 2$$
$$= 1.307$$

$$y = 4e^{2-x} - e^{4-2x}$$
$$= 4$$

$$\frac{\Delta y}{\Delta x}_{1.25} = 0.49$$
$$\frac{\Delta y}{\Delta x}_{1.307} = 0$$
$$\frac{\Delta y}{\Delta x}_{1.35} = -0.32$$

- Stationary Point = $(1.307, 4)$
- Kind = Maximum

Question 6

a

???

b

$$R\cos(\theta - \alpha) \equiv R\cos\theta\cos\alpha + R\sin\theta\sin\alpha$$

$$R\sin\alpha = 2 \qquad\qquad\qquad R^2 = 5^2 + 2^2$$
$$R\cos\alpha = 5 \qquad\qquad\qquad R = \sqrt{25 + 4}$$
$$\tan\alpha = \frac{2}{5} \qquad\qquad\qquad R = \sqrt{29}$$
$$\alpha = 21.8°$$

$$5\cos\theta + 2\sin\theta \equiv \sqrt{29}\cos(\theta - 21.8°)$$
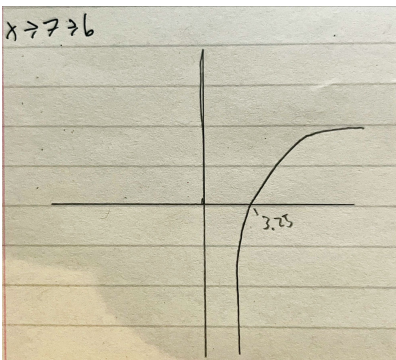
c

???

Question 7

a

1. Take the natural logarithm of x value.

2. Shift the graph right by 12 units.

3. Shrink the graph horizontally by a factor of 4.

b

a

$$v = x$$
$$v' = 1$$

$$u = \sqrt{x + 1}$$
$$u' = \frac{1}{2}(x + 1)^{-\frac{1}{2}}$$

$$\frac{\Delta y}{\Delta x} = vu' + v'u$$

$$= x\frac{1}{2}(x + 1)^{-\frac{1}{2}} + (x + 1)^{\frac{1}{2}}$$

$$= (x + 1)^{-\frac{1}{2}}\left(x\frac{1}{2} + (x + 1)\right)$$

$$= (x + 1)^{-\frac{1}{2}}\left(\frac{3}{2}x + 1\right)$$

$$= \frac{1}{2}(3x + 2)(x + 1)^{-\frac{1}{2}}$$

b

$$v = x\sqrt{x + 1}$$
$$v' = \frac{1}{2}(3x + 2)(x + 1)^{-\frac{1}{2}}$$

$$u = \sin 2x$$
$$u' = 2\cos 2x$$

$$\frac{\Delta y}{\Delta x} = vu' + v'u$$

$$= \left(x\sqrt{x + 1}\right)(2\cos 2x) + \left(\frac{1}{2}(3x + 2)(x + 1)^{-\frac{1}{2}}\right)(\sin 2x)$$

$$= \left(\frac{\pi}{2}\sqrt{\frac{\pi}{2} + 1}\right)(2\cos \pi) + \left(\frac{1}{2}(3\frac{\pi}{2} + 2)(\frac{\pi}{2} + 1)^{-\frac{1}{2}}\right)(\sin \pi)$$

$$= -2\left(\frac{\pi}{2}\sqrt{\frac{\pi}{2} + 1}\right)$$

$$= -\pi\sqrt{\frac{\pi}{2} + 1}$$