C3 Z

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Question 1

$$
\begin{aligned}
x &= \ln\left(y^2+9\right)^{\frac{3}{2}} \\
&= \left(\ln\left(y^2+9\right)\right)^{\frac{3}{2}} \\
u &= \ln\left(y^2+9\right) \\
\frac{\Delta u}{\Delta x} &= \frac{2y}{y^2+9} \\
\frac{\Delta y}{\Delta x} &= \frac{3}{2}\left(\ln\left(y^2+9\right)\right)^{\frac{1}{2}}\frac{2y}{y^2+9} \\
&= \frac{6y}{2y^2+18}\sqrt{\ln\left(y^2+9\right)}
\end{aligned}
$$

Question wrong? Cannot get to simplify and desmos disagrees

Question 2

$$\frac{\sin 2\phi}{\sin\phi} - \frac{\cos 2\phi}{\cos\phi} \equiv \sec\phi$$

$$
\begin{aligned}
\textbf{LHS} &\equiv \frac{\sin 2\phi}{\sin\phi} - \frac{\cos 2\phi}{\cos\phi} \\
&\equiv \frac{2\sin\cos\phi}{\sin\phi} - \frac{2\cos^2\phi - 1}{\cos\phi} \\
&\equiv 2\cos\phi - 2\cos\phi + \sec\phi \equiv \sec\phi \equiv \textbf{RHS} \quad \text{QED}
\end{aligned}
$$

Question 3

a

$$v = 1 + 2\ln x \qquad\qquad u = x$$

$$v' = \frac{2}{x} \qquad\qquad u' = 1$$

$$\begin{aligned}
\frac{\Delta y}{\Delta x} &= \frac{vu' - v'u}{v^2} \\
&= \frac{1 + 2\ln x - \frac{2x}{x}}{(1 + 2\ln x)^2} \\
&= \frac{2\ln x - 1}{1 + 4\ln x + 4(\ln x)^2} \\
0 &= \frac{2\ln x - 1}{1 + 4\ln x + 4(\ln x)^2} \\
0 &= 2\ln x - 1 \\
\frac{1}{2} &= \ln x \\
x &= e^{\frac{1}{2}} = \sqrt{e} \\
y &= \frac{x}{1 + 2\ln x} \\
&= \frac{\sqrt{e}}{1 + 1} \\
&= \frac{1}{2}\sqrt{e} \\
\text{TP} &= (\sqrt{e}, \frac{1}{2}\sqrt{e})
\end{aligned}$$

b

$$v = (1+2\ln x)^2$$
$$v' = \frac{4+8\ln x}{x}$$

$$u = 2\ln x - 1$$
$$u' = \frac{2}{x}$$

$$\begin{aligned}
\frac{\Delta y}{\Delta x} &= \frac{vu' - v'u}{v^2} \\
\frac{\Delta^2 y}{\Delta x^2} &= \frac{\frac{2}{x}(1+2\ln x)^2 - \frac{4+8\ln x}{x}(2\ln x - 1)}{(1+2\ln x)^4} \\
\frac{\Delta^2 y}{\Delta x^2}_{\sqrt{e}} &= \frac{\frac{2}{\sqrt{e}}(1+1)^2 - \frac{4+4}{\sqrt{e}}(1-1)}{(1+1)^4} \\
&= \frac{\frac{2}{\sqrt{e}}}{(1+1)^2} \\
&= \frac{2}{4\sqrt{e}} \\
&= \frac{1}{2\sqrt{e}}
\end{aligned}$$

$\frac{1}{2\sqrt{e}} > 0 \therefore$ is a minimum point.

Question 4

a

$$\begin{aligned}
x &= \tan y \\
\frac{\Delta x}{\Delta y} &= \sec^2 y \\
\frac{\Delta y}{\Delta x} &= \cos^2 x
\end{aligned}$$

???

b

$$\begin{aligned}
\frac{\Delta y}{\Delta x} &= 1 + x^2 - \frac{8}{1+x^2} - 6x \\
&= x^2 - 6x - \frac{8}{1+x^2} + 1 \\
0 &= x^2 - 6x - \frac{8}{1+x^2} + 1 \\
&= \left(1+x^2\right)x^2 - \left(1+x^2\right)6x - 8 + \left(1+x^2\right) \\
&= x^2 + x^4 - 6x - 6x^3 - 8 + 1 + x^2 \\
&= x^4 - 6x^3 + 2x^2 - 6x - 7
\end{aligned}$$

???

c

$$f(x) = 6x^3 + 14x - 1 f(0) \qquad = -1$$
$$f(1) = 19$$

Between positive and negative ∴ root inbetween.

d

???

e

???

Question 5

a

$$
\begin{aligned}
H &= k\left(1 - e^{-\frac{1}{12}t}\right) \\
41 &= k\left(1 - e^{-\frac{5}{12}}\right) \\
\frac{41}{k} &= 1 - e^{-\frac{5}{12}} \\
k &= \frac{41}{1 - e^{-\frac{5}{12}}} \\
k &= 120.319\ldots \\
&= 120
\end{aligned}
$$

b

$$
\begin{aligned}
H &= 120\left(1 - e^{-\frac{1}{12}t}\right) \\
\frac{90}{120} &= 1 - e^{-\frac{1}{12}t} \\
\frac{1}{4} &= e^{-\frac{1}{12}t} \\
\ln\frac{1}{4} &= -\frac{1}{12}t \\
-12t &= -\ln 4 \\
t &= 12\ln 4 \\
&= 24\ln 2
\end{aligned}
$$

c

$$
\begin{aligned}
H &= 120\left(1 - e^{-\frac{1}{12}t}\right)\\
\frac{H}{120} &= 1 - e^{-\frac{1}{12}t}\\
1 - \frac{H}{120} &= e^{-\frac{1}{12}t}\\
-\frac{1}{12}t &= \ln\left(1 - \frac{H}{120}\right)\\
t &= -12\ln\left(1 - \frac{H}{120}\right)
\end{aligned}
$$

$$
\begin{aligned}
H &= 120\left(1 - e^{-\frac{1}{12}t}\right)\\
a &= 1 - e^{-\frac{1}{12}t}\\
\frac{\Delta t}{\Delta t} &= \frac{1}{12}e^{-\frac{1}{12}t}\\
\frac{\Delta H}{\Delta t} &= 120\frac{1}{12}e^{-\frac{1}{12}t}\\
&= 10e^{-\frac{1}{12}t}\\
&= 10e^{-12\ln\left(1-\frac{H}{120}\right)}\\
&= \frac{-10}{12}H
\end{aligned}
$$

d

$$
\begin{aligned}
\frac{\Delta H}{\Delta t} &= \frac{-10}{12}H\frac{\Delta H}{\Delta t}_7.5 \qquad = -\frac{-10}{12}7.5\\
&= \frac{-25}{4}
\end{aligned}
$$

e

$$
\begin{aligned}
\frac{\Delta H}{\Delta t} &= 120\frac{1}{12}e^{-\frac{1}{12}t}\\
0 &= 10e^{-\frac{1}{12}t}\\
e^{-\frac{1}{12}t} &= 0
\end{aligned}
$$

Since $\mathbb{R} \not\subseteq \ln 0$, we can see that this model exponentially grows, that is, there is no maximum height.

Question 6

a

$$2 \le f(x) \le 4$$
$$a = 3$$
$$\pi \longrightarrow 2\pi \therefore b = 0.5$$

b

$$\begin{aligned} y &= 3 + \cos\frac{x}{2} \\ x &= 3 + \cos\frac{y}{2} \\ \cos\frac{y}{2} &= x - 3 \\ y &= 2\cos^{-1}(x-3) \end{aligned}$$

c

$$2 \le x \le 4$$
$$0 \le f(x) \le 2\pi$$

d

$$\frac{\Delta y}{\Delta x} = -\frac{1}{2}\sin\frac{x}{2}\frac{\Delta y}{\Delta x}_{\frac{4}{3}\pi} \qquad = -\frac{1}{2}\sin\frac{2\pi}{3}$$
$$= -\frac{1}{2}\frac{\sqrt{3}}{2}$$
$$= -\frac{\sqrt{3}}{4}$$

e

$$-\frac{\sqrt{3}}{4}$$

Question 7

a

$$\sin P + \sin Q \equiv 2\sin\left(\frac{P+Q}{2}\right)\cos\left(\frac{P-Q}{2}\right)$$

$$\textbf{RHS} \equiv 2\sin\left(\frac{P+Q}{2}\right)\cos\left(\frac{P-Q}{2}\right)$$

$$\equiv 2\sin\left(\frac{P}{2}+\frac{Q}{2}\right)\cos\left(\frac{P}{2}-\frac{Q}{2}\right)$$

$$\equiv 2\left(\sin\frac{P}{2}\cos\frac{Q}{2}+\sin\frac{Q}{2}\cos\frac{P}{2}\right)\left(\cos\frac{P}{2}\cos\frac{Q}{2}+\sin\frac{P}{2}\sin\frac{Q}{2}\right)$$

$$\equiv 2\left(\sin\frac{P}{2}\cos\frac{P}{2}\cos^2\frac{Q}{2}+\sin^2\frac{P}{2}\sin\frac{Q}{2}\cos\frac{Q}{2}+\sin\frac{Q}{2}\cos^2\frac{P}{2}\cos\frac{Q}{2}+\sin\frac{P}{2}\sin^2\frac{Q}{2}\cos\frac{P}{2}\right)$$

???

$$\equiv \sin P + \sin Q \equiv \text{RHS} \quad \text{QED}$$

b

$$\sin\theta - \sin 3\theta + \sin 5\theta = 0\textbf{???}$$

Question 8