

C3 Y

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Question 1

$$v = e^{-x}$$

$$v' = -e^{-x}$$

$$u = \sin x$$

$$u' = \cos x$$

$$e^{-x} = e^{-x} \sin x$$

$$1 = \sin x$$

$$x = \frac{1}{2}\pi$$

$$\frac{\Delta y}{\Delta x} = vu' + v'u$$

$$= e^{-x} \cos x + -e^{-x} \sin x$$

$$0 = e^{-x} \cos x - e^{-x} \sin x$$

$$= (e^{-x})(\cos x - \sin x)$$

$$e^{-x} \neq \mathbb{R}$$

$$0 = \cos x - \sin x$$

$$\cos x = \sin x$$

$$\tan x = 1$$

$$x = \frac{1}{4}\pi$$

$$= \frac{1}{2}\pi - \frac{1}{4}\pi$$

$$= \frac{1}{4}\pi$$

Question 2

a

$$R \sin(x + \alpha) \equiv R \sin \alpha \cos x + R \cos \alpha \sin x$$

$$R^2 = 2^2 + 1^2$$

$$R = \sqrt{5}$$

$$2 = \sin \alpha$$

$$1 = \cos \alpha$$

$$\tan \alpha = 2$$

$$\alpha = 1.107$$

$$2 \cos x + \sin x \equiv \sqrt{5} \sin(x + 1.107)$$

b

$$-\sqrt{5} \leq f(x) \leq \sqrt{5}$$

$$g(-\sqrt{5}) = \frac{5}{5+5} = 2$$

$$g(0) = \frac{5}{0+5} = 1$$

$$g(\sqrt{5}) = \frac{5}{5+5} = 2$$

$$1 \leq g(f(x)) \leq 2$$

Question 3

a

$$10 = Ae^{-k \cdot 0}$$

$$A = 10$$

$$5 = Ae^{-5k}$$

$$5 = 10e^{-5k}$$

$$\frac{1}{2} = e^{-5k}$$

$$-5k = \ln \frac{1}{2}$$

$$5k = \ln 2$$

$$k = \frac{1}{5} \ln 2$$

b

$$M = 10e^{-\frac{1}{5}t}$$

$$\begin{aligned} \frac{\Delta M}{\Delta x} &= \left(-\frac{1}{5}\right) \left(10e^{-\frac{1}{5}t}\right) \\ &= -2e^{-\frac{1}{5}t} \end{aligned}$$

$$\ln\left(\frac{1}{\sqrt{2}}\right) = -2e^{-\frac{1}{5}t}$$

$$\ln \sqrt{2} = 2e^{-\frac{1}{5}t}$$

$$\ln \sqrt[4]{2} = e^{-\frac{1}{5}t}$$

$$-\frac{1}{5}t = \ln\left(\ln \sqrt[4]{2}\right)$$

$$t = -5 \ln\left(\ln \sqrt[4]{2}\right)$$

$$t = 8.764$$

Question 4

α

$$\begin{aligned} &= \tan 2x \\ &= \frac{\sin 2x}{\cos 2x} \end{aligned}$$

$$\begin{aligned} v &= \cos 2x \\ v' &= -2 \sin 2x \end{aligned}$$

$$\begin{aligned} u &= \sin 2x \\ u' &= 2 \cos 2x \end{aligned}$$

$$\begin{aligned} \frac{\Delta}{\Delta x} &= \frac{vu' - v'u}{v^2} \\ &= \frac{2 \cos^2 2x + 2 \sin^2 2x}{\cos^2 2x} \\ &= \frac{2}{\cos^2 2x} \\ &= 2 \sec^2 2x \end{aligned}$$

b

$$v = 6x$$

$$v' = 6$$

$$u = \tan 2x$$

$$u' = 2 \sec^2 2x$$

$$\begin{aligned}\frac{\Delta y}{\Delta x} &= vu' + v'u \\ &= 12x \sec^2 2x + 6 \tan 2x \\ \frac{\Delta y}{\Delta x} \frac{1}{8}\pi &= \frac{12}{8}\pi \sec^2 \frac{1}{4}\pi + \tan \frac{1}{4}\pi \\ &= 3\pi + 1\end{aligned}$$

$$\begin{aligned}y &= 6x \tan 2x \\ &= \frac{6}{8}\pi \tan \frac{2}{8}\pi \\ &= \frac{3}{4}\pi\end{aligned}$$

$$\begin{aligned}y - y_1 &= m(x - x_1) \\ y - \frac{3}{4}\pi &= (3\pi + 1)\left(x - \frac{1}{8}\pi\right) \\ y &= (3\pi + 1)x - \frac{3\pi + 1}{8}\pi + \frac{3}{4}\pi \\ \therefore c &= -\frac{3\pi + 1}{8}\pi + \frac{3}{4}\pi \\ &= \frac{6\pi}{8} - \frac{3\pi^2 + \pi}{8} \\ &= \frac{-3\pi^2 + 5\pi}{8}\end{aligned}$$

???Maybe 3 pie + 6

Question 5

a

$$\begin{aligned}0 &= \sqrt{1 - (2x - 1)^2} \\ &= 1 - (4x^2 - 4x + 1) \\ &= -4x^2 + 4x \\ &= x^2 - x \\ &= x(x - 1) \\ x &= 0, 1 \\ a &= 1\end{aligned}$$

b

$$\begin{aligned}f(a \div 2) &= f(0.5) \\&= \sqrt{1 - (1 - 1)^2} \\&= \sqrt{1} \\&= 1 \\0 &\leq f(x) \leq 1\end{aligned}$$

c

1. Stretch graph horizontally by a factor of 2
2. Stretch graph vertically by a factor of 2
3. Shift graph downwards by 2 units

NEED TO ACTUALLY DRAW THIS ON THE FIGURES BIT

d

Domain: $0 \leq x \leq 2$ Range: $-2 \leq f(x) \leq 0$

Question 6

a

$$\begin{aligned}\frac{2 \cot \theta}{1 + \cot^2 \theta} &\equiv \sin 2\theta \\ \text{LHS} &\equiv \frac{2 \cot \theta}{1 + \cot^2 \theta} \\ &\equiv \frac{2 \cot \theta}{\csc^2 \theta} \\ &\equiv 2 \frac{\cos \theta}{\sin \theta} \sin^2 \theta \\ &\equiv 2 \cos \theta \sin \theta \equiv \sin 2\theta \equiv \text{RHS} \quad \text{QED}\end{aligned}$$

b

$$4 \cot^2 \theta + 1 = 2 \sin 2\theta (1 + \cot^2 \theta)$$
$$4 (\cot^2 \theta + 1) - 3 = (4 \sin \theta \cos \theta) (\csc^2 \theta)$$

$$4 \csc^2 \theta - 3 = 4 \cot \theta$$

$$\frac{4}{\sin^2 \theta} = 4 \frac{\cos \theta}{\sin \theta} + 3$$

$$4 = 4 \cos \theta \sin \theta + 3$$

$$\frac{1}{4} = \cos \theta \sin \theta$$

$$\frac{1}{4} = \frac{1}{2} \sin 2\theta$$

$$\frac{1}{2} = \sin 2\theta$$

$$0^\circ \leq 2\theta \leq 720^\circ$$

$$2\theta = \sin^{-1} \frac{1}{2} = 30^\circ, 150^\circ, 390^\circ, 510^\circ$$

$$\theta = 15^\circ, 75^\circ, 195^\circ, 255^\circ$$