C3 Y

Jack Maguire

Question 1

$$v = e^{-x} \qquad u = \sin x$$
$$v' = -e^{-x} \qquad u' = \cos x$$

$$\begin{aligned}\frac{\Delta y}{\Delta x} &= vu' + v'u \\ &= e^{-x}\cos x + -e^{-x}\sin x \\ 0 &= e^{-x}\cos x - e^{-x}\sin x \\ &= (e^{-x})(\cos x - \sin x) \\ e^{-x} &\not\equiv \mathbb{R} \\ 0 &= \cos x - \sin x \\ \cos x &= \sin x \\ \tan x &= 1 \\ x &= \frac{1}{4}\pi\end{aligned}$$

$$\begin{aligned}e^{-x} &= e^{-x}\sin x \\ 1 &= \sin x \\ x &= \frac{1}{2}\pi\end{aligned}$$

$$\begin{aligned}&= \frac{1}{2}\pi - \frac{1}{4}\pi \\ &= \frac{1}{4}\pi\end{aligned}$$

Question 2

a

$$R\sin(x+\alpha) \equiv R\sin\alpha\cos x + R\sin\alpha\cos x$$

$$R^2 = 2^2 + 1^2$$
$$R = \sqrt{5}$$

$$\begin{aligned}2 &= \sin\alpha \\ 1 &= \cos\alpha \\ \tan\alpha &= 2 \\ \alpha &= 1.107\end{aligned}$$

$$2\cos x + \sin x \equiv \sqrt{5}\sin(x + 1.107)$$

b

$$-\sqrt{5} \leq f(x) \leq \sqrt{5}$$

$$g\left(-\sqrt{5}\right) = \frac{5}{5+5} = 2$$
$$g(0) = \frac{5}{0+5} = 1$$
$$g\left(\sqrt{5}\right) = \frac{5}{5+5} = 2$$

$$1 \leq g(f(x)) \leq 2$$

Question 3

a

$$10 = Ae^{-k*0}$$
$$A = 10$$

$$5 = Ae^{-5k}$$
$$5 = 10e^{-5k}$$
$$\frac{1}{2} = e^{-5k}$$
$$-5k = \ln\frac{1}{2}$$
$$5k = \ln 2$$
$$k = \frac{1}{5}\ln 2$$

b

$$M = 10e^{-\frac{1}{5}t}$$
$$\frac{\Delta M}{\Delta x} = \left(-\frac{1}{5}\right)\left(10e^{-\frac{1}{5}t}\right)$$
$$= -2e^{-\frac{1}{5}t}$$
$$\ln\left(\frac{1}{\sqrt{2}}\right) = -2e^{-\frac{1}{5}t}$$
$$\ln\sqrt{2} = 2e^{-\frac{1}{5}t}$$
$$\ln\sqrt[4]{2} = e^{-\frac{1}{5}t}$$
$$-\frac{1}{5}t = \ln\left(\ln\sqrt[4]{2}\right)$$
$$t = -5\ln\left(\ln\sqrt[4]{2}\right)$$
$$t = 8.764$$

Question 4

a

$$= \tan 2x$$
$$= \frac{\sin 2x}{\cos 2x}$$

$v = \cos 2x$
$v' = -2\sin 2x$

$u = \sin 2x$
$u' = 2\cos 2x$

$$\frac{\Delta}{\Delta x} = \frac{vu' - v'u}{v^2}$$
$$= \frac{2\cos^2 2x + 2\sin^2 2x}{\cos^2 2x}$$
$$= \frac{2}{\cos^2 2x}$$
$$= 2\sec^2 2x$$

b

$v = 6x$ $\qquad$ $u = \tan 2x$

$v' = 6$ $\qquad$ $u' = 2\sec^2 2x$

$$\begin{aligned}\frac{\Delta y}{\Delta x} &= vu' + v'u \\ &= 12x\sec^2 2x + 6\tan 2x \\ \frac{\Delta y}{\Delta x}_{\frac{1}{8}\pi} &= \frac{12}{8}\pi\sec^2\frac{1}{4}\pi + \tan\frac{1}{4}\pi \\ &= 3\pi + 1\end{aligned}$$

$$\begin{aligned}y &= 6x\tan 2x \\ &= \frac{6}{8}\pi\tan\frac{2}{8}\pi \\ &= \frac{3}{4}\pi\end{aligned}$$

$$\begin{aligned}y - y_1 &= m(x - x_1) \\ y - \frac{3}{4}\pi &= (3\pi + 1)\left(x - \frac{1}{8}\pi\right) \\ y &= (3\pi + 1)x - \frac{3\pi + 1}{8}\pi + \frac{3}{4}\pi \\ \therefore c &= -\frac{3\pi + 1}{8}\pi + \frac{3}{4}\pi \\ &= \frac{6\pi}{8} - \frac{3\pi^2 + \pi}{8} \\ &= \frac{-3\pi^2 + 5\pi}{8}\end{aligned}$$

???**Maybe 3 pie + 6**

Question 5

a

$$\begin{aligned}0 &= \sqrt{1 - (2x - 1)^2} \\ &= 1 - \left(4x^2 - 4x + 1\right) \\ &= -4x^2 + 4x \\ &= x^2 - x \\ &= x(x - 1) \\ x &= 0, 1 \\ a &= 1\end{aligned}$$

b

$$\begin{aligned} f(a \div 2) &= f(0.5) \\ &= \sqrt{1-(1-1)^2} \\ &= \sqrt{1} \\ &= 1 \\ 0 \leq f(x) &\leq 1 \end{aligned}$$

S

c

1. Stretch graph horizontally by a factor of 2
2. Stretch graph vertically by a factor of 2
3. Shift graph downwards by 2 units

NEED TO ACTUALLY DRAW THIS ON THE FIGURES BIT

d

Domain: $0 \leq x \leq 2$ Range: $-2 \leq f(x) \leq 0$

Question 6

a

$$\begin{aligned} \frac{2\cot\theta}{1+\cot^2\theta} &\equiv \sin 2\theta \\ \textbf{LHS} &\equiv \frac{2\cot\theta}{1+\cot^2\theta} \\ &\equiv \frac{2\cot\theta}{\csc^2\theta} \\ &\equiv 2\frac{\cos\theta}{\sin\theta}\sin^2\theta \\ &\equiv 2\cos\theta\sin\theta \equiv \sin 2\theta \equiv \textbf{RHS} \quad \text{QED} \end{aligned}$$

b

$$\begin{aligned} 4\cot^2\theta + 1 &= 2\sin 2\theta\left(1+\cot^2\theta\right) \\ 4\left(\cot^2\theta+1\right) - 3 &= (4\sin\theta\cos\theta)\left(\csc^2\theta\right) \\ 4\csc^2\theta - 3 &= 4\cot\theta \\ \frac{4}{\sin^2\theta} &= 4\frac{\cos\theta}{\sin\theta} + 3 \\ 4 &= 4\cos\theta\sin\theta + 3\sin^2\theta \\ 4 &= 2\sin 2\theta + 3\sin^2\theta \\ 0 &= 3\sin^2\theta + 2\sin 2\theta - 4 \\ 0 &= 3\sin^2\theta + 4\sin\theta\cos\theta - 4 \end{aligned}$$

???

Question 7

a

$$y = 3\sin^{-1}(x-1)$$

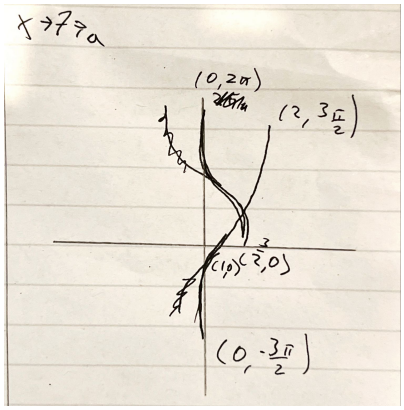
$$y = 3\sin^{-1}(0-1)$$
$$= 3 * -\frac{1}{2}\pi$$
$$= -\frac{3\pi}{2}\ //$$
$$y = 3\sin^{-1}(2-1)$$
$$= 3 * \frac{1}{2}\pi$$
$$= \frac{3\pi}{2}\ //$$

$$0 = 3\sin^{-1}(x-1)$$
$$\sin(0) = 3x - 3$$
$$0 = 3x - 3$$
$$3x = 3$$
$$x = 1$$

$$y = 2\cos^{-1}(x-1)$$

$$y = 2\cos^{-1}(0-1)$$
$$= 2\pi\ //$$
$$0 = 2\cos^{-1}(x-1)$$
$$\cos(0) = 2x - 2$$
$$1 = 2x - 2$$
$$3 = 2x$$
$$x = \frac{3}{2}\ //$$

$$3\sin^{-1}(x-1) = 2\cos^{-1}(x-1)$$
$$3x - 3 = \sin\left(2\cos^{-1}(x-1)\right)$$
???



b

???

Question 8

a

$$
\begin{aligned}
|f(x)| &= 12 \\
k\left(x^2 - 4x\right) &= \pm 12 \\
12/4 &= 3 \\
k &= 3
\end{aligned}
$$

Clarification?

b

$$
\begin{aligned}
3\left(x^2 - 4x\right) &= 12 \\
x^2 - 4x - 4 &= 0 \\
x &= 2 \pm 2\sqrt{2}
\end{aligned}
$$

$$
\begin{aligned}
-3\left(x^2 - 4x\right) &= 12 \\
x^2 - 4x + 4 &= 0 \\
(x-2)^2 &= 0 \\
x &= 2
\end{aligned}
$$

$$x = 2 - 2\sqrt{2} \quad 2 \quad 2 + 2\sqrt{2}$$