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Question 1

$$
\begin{aligned}
&= \sin 5x \\
&= \sin 2x \cos 3x + \sin 3x \cos 2x \\
&= 2 \sin x \cos x \cos 3x + \sin 3x \left(\cos^2 x - \sin^2 x\right) \\
&= 2 \sin x \cos x (\cos x \cos 2x - \sin x \sin 2x) + (\sin 2x \cos x + \sin x \cos 2x) \left(\cos^2 x - \sin^2 x\right) \\
&= 2 \sin x \cos x \left(\cos x \left(\cos^2 x - \sin^2 x\right) - 2 \sin^2 x \cos x\right) + \left(2 \sin x \cos^2 x + \sin x \left(\cos^2 x - \sin^2 x\right)\right) \left(\cos^2 x - \sin^2 x\right) \\
&= 2 \sin x \cos x \left(\cos^3 x - \sin^2 x \cos x - 2 \sin^2 x \cos x\right) + \left(2 \sin \cos^2 x + \sin x \cos^2 x - \sin^3 x\right) \left(\cos^2 x - \sin^2 x\right) \\
&= 2 \sin x \cos x \left(\cos^3 x - 3 \sin^2 x \cos x\right) + \left(3 \sin x \cos^2 x - \sin^3 x\right) \left(\cos^2 x - \sin^2 x\right) \\
&= 2 \sin x \cos^4 x - 6 \sin^3 x \cos^2 x + 3 \sin x \cos^4 x - \sin^3 x \cos^2 x - 3 \sin^3 \cos^2 x + \sin^5 x \\
&= 5 \sin x \cos^4 x - 10 \sin^3 x \cos^2 x + \sin^5 x
\end{aligned}
$$

$$
\begin{aligned}
&= \cos 5x \\
&= \cos 3x \cos 2x - \sin 3x \sin 2x \\
&= (\cos 2x \cos x - \sin 2x \sin x) \left(\cos^2 x - \sin^2 x\right) - 2 \sin x \cos x (\sin 2x \cos x + \sin x \cos 2x) \\
&= \left(\left(\cos^2 x - \sin^2 x\right) \cos x - 2 \sin^2 x \cos x\right) \left(\cos^2 x - \sin^2 x\right) - 2 \sin x \cos x \left(2 \sin x \cos^2 x + \left(\cos^2 x - \sin^2 x\right) \sin x\right) \\
&= \left(\cos^3 x - \sin^2 x \cos x - 2 \sin^2 x \cos x\right) \left(\cos^2 x - \sin^2 x\right) - 2 \sin x \cos x \left(2 \sin x \cos^2 x + \cos^2 x \sin x - \sin^3 x\right) \\
&= \left(\cos^3 x - 3 \sin^2 x \cos x\right) \left(\cos^2 x - \sin^2 x\right) - 2 \sin x \cos x \left(3 \sin x \cos^2 x - \sin^3 x\right) \\
&= \cos^5 x - \sin^2 x \cos^3 x - 3 \sin^2 x \cos^3 x + 3 \sin^4 x \cos x - 6 \sin^2 x \cos^3 x + 2 \sin^4 x \cos x \\
&= \cos^5 x - 10 \sin^2 x \cos^3 x + 5 \sin^4 x \cos x
\end{aligned}
$$

$$
\begin{aligned}
\sin x - \cos x &= \sin 5x - \cos 5x \\
\sin x - \cos x &= 5 \sin x \cos^4 x - 10 \sin^3 x \cos^2 x + \sin^5 x - \cos^5 x + 10 \sin^2 x \cos^3 x - 5 \sin^4 x \cos x
\end{aligned}
$$
???

Question 2

$$\binom{p}{2}(1)^{p-2}(-4)^2 = \binom{p}{4}(1)^{p-4}(-4)^4$$
$$4p(p-1) = \frac{32p(p-1)(p-2)(p-3)}{3}$$
$$3p(p-1) = 8p(p-1)(p-2)(p-3)$$
$$3 = (p-2)(p-3)$$
$$3 = p^2 - 5p + 6$$
$$0 = p^2 - 5p + 3$$
$$p = \frac{5 \pm \sqrt{13}}{2}$$

$$\binom{p}{3}(1)^{p-3}(-4)^3 \geq 0$$
$$\frac{32p(p-1)(p-2)}{3} \leq 0$$
$$\frac{32 * \frac{5+\sqrt{13}}{2} * (\frac{5+\sqrt{13}}{2} - 1)}{3} = 0$$
$$\frac{32 * \frac{5-\sqrt{13}}{2} * (\frac{5-\sqrt{13}}{2} - 1)}{3} = 0$$
$$p = \frac{5 \pm \sqrt{13}}{2}$$

Question 3

$$\frac{\Delta y}{\Delta x} = \frac{\Delta y \div \Delta t}{\Delta x \div \Delta t} = \frac{y'(t)}{x'(t)}$$

$$x = 15t - t^3$$
$$x' = 15 - 3t^2$$

$$y = 3 - 2t^2$$
$$y' = -4t$$

$$\frac{\Delta y}{\Delta x} = \frac{-4t}{15 - 3t^2}$$

$$x = 15t - t^3$$
$$14 = 15t - t^3$$
$$0 = -t^3 + 15t - 14$$
$$t = 1, \ldots$$

$$y = 3 - 2t^2$$
$$1 = 3 - 2t^2$$
$$0 = -2t^2 + 3t - 1$$
$$t = 1, 0.5$$

$$\begin{aligned} t &= 1 \\ \frac{\Delta y}{\Delta x}_1 &= \frac{-4}{15-3} \\ &= -\frac{1}{3} \\ \therefore \text{Normal} &= 3 \end{aligned}$$

$$\begin{aligned} 3 &= \frac{-4t}{15-3t^2} \\ 45-9t^2 &= -4t \\ 0 &= 9t^2-4t-45 \\ t &= \frac{2 \pm \sqrt{409}}{9} \end{aligned}$$

Question 4

$$\begin{aligned} x^3+y^3-3xy &= 48 \\ y^3-3xy &= 48-x^3 \\ y\left(y^2-3x\right) &= 48-x^3\textbf{???} \end{aligned}$$

Question 5

a

$$y = \sin(\cos x)$$

$$\begin{aligned} 0 &= \sin(\cos x) \\ 0 &= \cos x \\ x &= \pm\frac{\pi}{2} \end{aligned}$$

$$\begin{aligned} y &= \sin(\cos 0) \\ y &= \sin 1 \\ y &= 0.841 \end{aligned}$$

A $(-\frac{\pi}{2}, 0)$
B $(\frac{\pi}{2}, 0)$
C $(0, 0.841)$

b

$$\begin{aligned} y &= \sin(\cos x) \\ u &= \cos x \\ \frac{\Delta x}{\Delta u} &= \sin x \\ \frac{\Delta y}{\Delta x} &= -\cos(\cos x)\sin x \\ \frac{\Delta y}{\Delta x}_0 &= 0 \end{aligned}$$

$\frac{\Delta y}{\Delta x}_0 = 0 \therefore B$ is a stationary point.

c, d

I know how to show this using a graph, but I'm not sure how to do this using algebra.

Question 6

a

$$\begin{aligned} m_2 - x^{n_2} &= m_1 - x^{n_1} \\ m_2 - x^{12-n_1} &= m_1 - x^{n_1} \end{aligned}$$

$$\begin{aligned} x &= 3 \\ m_2 - 3^{12-n_1} &= m_1 - 3^{n_1} \\ m_2 - \frac{3^{12}}{3^{n_1}} &= m_1 - 3^{n_1} \end{aligned}$$

$$\begin{aligned} x &= -3 \\ m_2 - (-3)^{12-n_1} &= m_1 - (-3)^{n_1} \\ m_2 - \frac{-3^{12}}{-3^{n_1}} &= m_1 - (-3)^{n_1} \\ m_2 - \frac{3^12}{3^{n_1}} &= m_1 - (-3)^{n_1} \end{aligned}$$

???

Question 7

a

You can only separate brackets if you fully factorise, with factors on one side and a zero on the other - you separate brackets by dividing by each set of brackets. This only works with zero - because $\frac{0}{x} \equiv 0$.

b

$$x^3 + \frac{3}{4}x - \frac{1}{2} = 0$$
$$f\left(\frac{1}{2}\right) = 0$$
$$(x - \frac{1}{2})(x^2 + bx + 1) = 0$$
$$\ldots - \frac{1}{2}x^2 + bx^2 + \ldots = 0$$
$$-\frac{1}{2} + b = 0$$
$$b = \frac{1}{2}$$
$$x^3 + \frac{3}{4}x - \frac{1}{2} \equiv (x - \frac{1}{2})(x^2 + \frac{1}{2}x + 1)$$

$$x^2 + \frac{1}{2}x + 1 = 0\Delta \qquad = b^2 - 4ac$$
$$= \frac{1}{2}^2 - 4 * 1 * 1$$
$$= -\frac{15}{4}$$

$\Delta < 0 \therefore$ no real solutions.

c, d

???