

# Volume of a sphere in 10 Dimensions

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## ABSTRACT

This study investigates the volume computation of a 10 dimensional sphere. The first step was to compute the volume by simulating the Monte Carlo Mean Value (MCMV) method. The last step was to ask Deepseek to solve the problem, and our results were compared. This example is a great introduction to Monte Carlo methods and its simulation.

## 1 Introduction

### 1.1 Monte Carlo integration

The MCMV method<sup>1,2</sup> arises when an integral is too hard to compute analytically. For example if we have an integral that has the form of:

$$I = \int_a^b f(x)dx \quad (1)$$

The mean value of  $f$  will become:

$$\langle f \rangle = \frac{1}{b-a} \int_a^b f(x)dx = E[f(x)]$$

So the integral becomes:

$$I = (b-a) \langle f \rangle \quad (2)$$

For the computation algorithm the integral that is going to be calculated is:

$$I \simeq \frac{b-a}{N} \sum_{i=1}^N f(x_i)$$

### 1.2 Multidimensional Integrals

The MCMV formula for the multidimensional integral of a  $f(\vec{r})$  in volume  $V$ :

$$I = \frac{V}{N} \sum_{i=1}^N f(\vec{r}_i) \quad (3)$$

To personalize the problem, the integral for the 10 dimensional sphere will become:

$$I = \frac{2^{10}}{N} \cdot \sum_{i=1}^N f(x_i, x_{i+1}, \dots, x_{i+9}) \quad (4)$$

where 10 in  $2^{10}$  is the number of dimensions and N is the number of samples for the range of [-1,1] for every x, where x are the dimensions. The function  $f(x_i, x_{i+1}, \dots, x_{i+9})$  is described by the equation of the sphere:

$$\sum_{i=1}^{10} x_i^2 = 1 \quad (5)$$

## 2 Solution

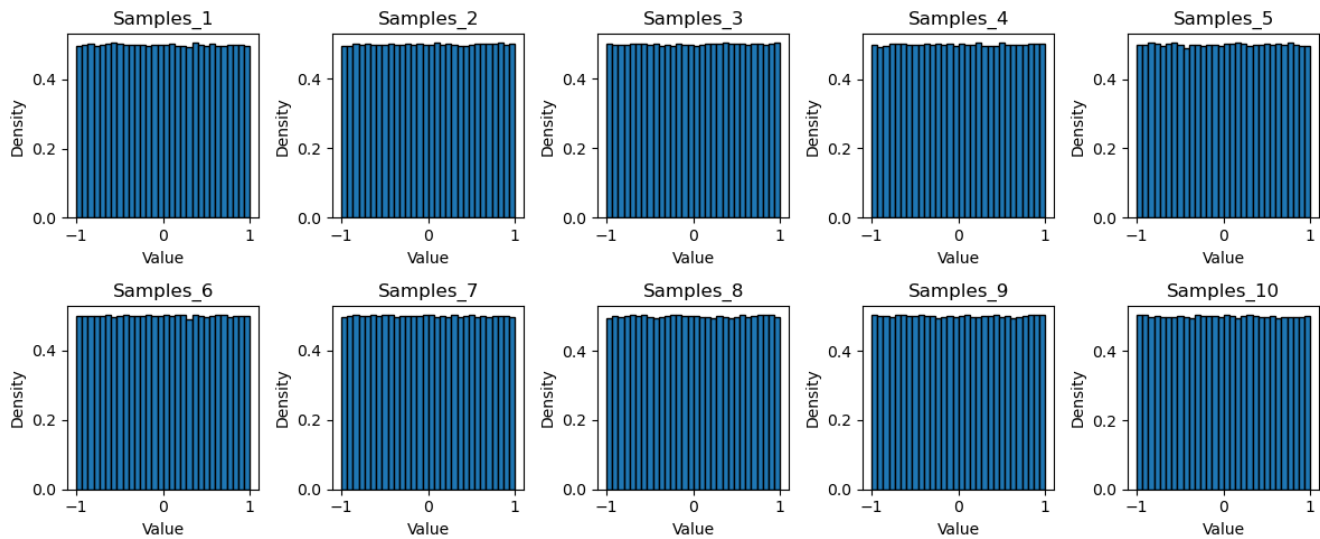
### 2.1 Personal code

The first step is to generate samples for every dimension of a uniform distribution that are is range of [-1,1] which is the radius of the sphere.

```
1 import numpy as np
2 import pandas as pd
3 import matplotlib.pyplot as plt
4 samples = pd.DataFrame()
5 sample_size = 1000000
6
7 for i in range(1,11):
8     exec(f"samples_{i} = np.random.uniform(-1, 1, {sample_size})")
9     exec(f"samples[f'samples_{i}'] = samples_{i}")
```

**Listing 1.** Samples generator

The samples if plotted appear:



**Figure 1.** Samples Distributions

The next step is to compute the integral is:

```

1 I = 0
2 samples = samples ** 2
3 samples['sum'] = samples.sum(axis=1)
4
5 for i in range(sample_size):
6     if samples['sum'].iloc[i] < 1.0:
7         I += 1
8
9 I = 2**10/sample_size * I

```

**Listing 2.** Integral calculation

and the result of the integral is equal to **2.564096**, which is close to the theoretical value of:

```

1 import math
2 theor = np.pi**(10 // 2) / math.factorial(10 // 2)

```

**Listing 3.** Theoretical calculation

```

1 2.550164

```

With the absolute error to be:

$$\varepsilon_a = 0.5465\%$$

## 2.2 DeepSeek's solution

### 2.2.1 Question

**User:** I want to compute the volume of a 10-dimensional sphere. Can you compute it for me computationally?

### 2.2.2 Solution

The volume of an  $n$ -dimensional sphere with radius  $R$  is given by:

$$V_n(R) = \frac{\pi^{n/2}}{\Gamma(\frac{n}{2} + 1)} R^n$$

where  $\Gamma$  is the gamma function.

For  $n = 10$  (10D sphere) and  $R = 1$  (unit sphere):

$$V_{10} = \frac{\pi^5}{\Gamma(6)} = \frac{\pi^5}{120}$$

### Numerical Computation (Python)

```
1 import math
2
3 # Volume of a 10-dimensional unit sphere
4 n = 10
5 volume = (math.pi ** (n / 2)) / math.gamma((n / 2) + 1)
6 print(f"Volume of a {n}-dimensional unit sphere: {volume}")
```

### Result

The program outputs:

$$V_{10} \approx 2.550164039877345$$

For a sphere with arbitrary radius  $R$ :

$$V_{10}(R) = \frac{\pi^5}{120} R^{10}$$

## References

1. Kosmidis, K. Computational mathematics ii notes.
2. Thompson, P. K. Node 19: Siesta. <https://web.ornl.gov/~kentpr/thesis/pkthnode19.html> (1994). Accessed: 2025-06-02.