```
clear; clc
```

Problem 2

Part 1

```
funcp = @(x, z) cos(-x(1)*x(2)*x(3) + z^2);
funcF = @(x, z) 2*z*sin(x(1)*x(3) + x(2)) + sin(z)*cos(x(1) +x(3) + x(2)) + 2;
cplxDiff(@(x) funcp(x, 0), [1; 2; 3], 1e-20)
```

```
ans = 1×3
    1.6765    0.8382    0.5588
```

Part 2

```
syms x_1 x_2 x_3 z
x = [x_1; x_2; x_3];
p = cos(-x(1)*x(2)*x(3) + z^2);
F = 2*z*sin(x(1)*x(3) + x(2)) + sin(z)*cos(x(1) + x(3) + x(2)) + 2
```

F = $2\,z\sin(x_2+x_1\,x_3)+\cos(x_1+x_2+x_3)\sin(z)+2$

```
Fx = [diff(F, x(1)) diff(F, x(2)) diff(F, x(3))];
Fz = diff(F, z);
dzdx = (-Fx/Fz)
```

dzdx =

$$\left(\frac{\sin(x_1+x_2+x_3)\sin(z)-2\,x_3\,z\cos(x_2+x_1\,x_3)}{\sigma_1}\quad -\frac{2\,z\cos(x_2+x_1\,x_3)-\sin(x_1+x_2+x_3)\sin(z)}{\sigma_1}\quad \frac{\sin(x_1+x}{}\right.$$

where

$$\sigma_1 = 2\sin(x_2+x_1\,x_3)+\cos(x_1+x_2+x_3)\cos(z)$$

```
px = [diff(p, x(1)) diff(p, x(2)) diff(p, x(3))]
```

px = $\left(x_2\,x_3\sin\left(z^2-x_1\,x_2\,x_3\right)\quad x_1\,x_3\sin\left(z^2-x_1\,x_2\,x_3\right)\quad x_1\,x_2\sin\left(z^2-x_1\,x_2\,x_3\right)\right)$

```
pz = diff(p, z)
```

pz = $-2\,z\sin\left(z^2-x_1\,x_2\,x_3\right)$

```
dpdx = px + pz*dzdx
```

dpdx =

$$\left(x_2\, x_3\, \sigma_2 - \frac{2\, z\, \sigma_2\ (\sigma_3 - 2\, x_3\, z \cos(x_2 + x_1\, x_3))}{\sigma_1} \quad x_1\, x_3\, \sigma_2 + \frac{2\, z\, \sigma_2\ (2\, z \cos(x_2 + x_1\, x_3) - \sigma_3)}{\sigma_1} \quad x_1\, x_2\, \sigma_2 - \frac{2\, z\, \sigma_2\ (} \right.$$

where

$$\sigma_1 = 2 \sin(x_2 + x_1\, x_3) + \cos(x_1 + x_2 + x_3) \cos(z)$$

$$\sigma_2 = \sin\left(z^2 - x_1\, x_2\, x_3\right)$$

$$\sigma_3 = \sin(x_1 + x_2 + x_3) \sin(z)$$

```
func = matlabFunction(dpdx);
func_test = matlabFunction(dzdx);
% func(1, 2, 3, 0)
```

```
z_test = 1.543295599106779
```

```
z_test = 1.5433
```

```
testa = func(1, 2, 3, z_test)
```

```
testa = 1×3
    0.5771    0.5118    0.0530
```

```
tic; func(1,2,3,z_test); toc
```

```
Elapsed time is 0.000588 seconds.
```

```
% cplxDiff(@(x) funcp(x, z_test), [1; 2; 3], 1e-20) % Wrong way to do it
testb = aaa(funcF, funcp, [1,2,3], z_test)
```

```
testb = 1×3
    0.5771    0.5118    0.0530
```

```
tic; aaa(funcF, funcp, [1,2,3], z_test); toc
```

```
Elapsed time is 0.004930 seconds.
```

```
testa-testb
```

```
ans = 1×3
10^-15 ×
   -0.4441   -0.2220   -0.2220
```

Part 3

```
cplxDiff(@(x) func(x(1), x(2), x(3), 0.0), [1;2;3], 1e-20)
```

```
ans = 3×3
  -34.5661  -16.4448  -10.9632
  -16.4448   -8.6415   -5.4816
  -10.9632   -5.4816   -3.8407
```

Part 4

```
Fxx = [diff(Fx, x(1)); diff(Fx, x(2)); diff(Fx, x(3))]
```

Fxx =

$$\begin{pmatrix} -\sigma_2 - 2\,x_3^2\,z\,\sigma_3 & -\sigma_2 - 2\,x_3\,z\,\sigma_3 & \sigma_1 \\ -\sigma_2 - 2\,x_3\,z\,\sigma_3 & -2\,z\,\sigma_3 - \sigma_2 & -\sigma_2 - 2\,x_1\,z\,\sigma_3 \\ \sigma_1 & -\sigma_2 - 2\,x_1\,z\,\sigma_3 & -\sigma_2 - 2\,x_1^2\,z\,\sigma_3 \end{pmatrix}$$

where

$$\sigma_1 = 2\,z\cos(x_2 + x_1\,x_3) - \sigma_2 - 2\,x_1\,x_3\,z\,\sigma_3$$

$$\sigma_2 = \cos(x_1 + x_2 + x_3)\sin(z)$$

$$\sigma_3 = \sin(x_2 + x_1\,x_3)$$

```
Fxz = diff(Fx, z).'
```

Fxz =

$$\begin{pmatrix} 2\,x_3\cos(x_2 + x_1\,x_3) - \sin(x_1 + x_2 + x_3)\cos(z) \\ 2\cos(x_2 + x_1\,x_3) - \sin(x_1 + x_2 + x_3)\cos(z) \\ 2\,x_1\cos(x_2 + x_1\,x_3) - \sin(x_1 + x_2 + x_3)\cos(z) \end{pmatrix}$$

```
Fzx = [diff(Fz, x(1)), diff(Fz, x(2)), diff(Fz, x(3))]
```

Fzx = $\left(2\,x_3\cos(x_2 + x_1\,x_3) - \sin(x_1 + x_2 + x_3)\cos(z) \quad 2\cos(x_2 + x_1\,x_3) - \sin(x_1 + x_2 + x_3)\cos(z) \quad 2\,x_1\cos(x_2 + x\right.$

```
Fzz = diff(Fz, z)
```

Fzz = $-\cos(x_1 + x_2 + x_3)\sin(z)$

```
d2zdx2 = Fz^-2 * Fx.' * (Fzx + Fzz*dzdx) - Fz^-1 * (Fxx + Fxz*dzdx);
pxx = [diff(px, x(1)); diff(px, x(2)); diff(px, x(3))];
pxz = diff(px, z).';
pzx = [diff(pz, x(1)), diff(pz, x(2)), diff(pz, x(3))];
pzz = diff(pz, z);
d2pdy2 = pxx + (pxz*dzdx + dzdx.'*pzx) + pzz*(dzdx.')*dzdx + pz*d2zdx2
```

d2pdy2 =

$$\begin{pmatrix} \frac{4\,x_2\,x_3\,z\cos(\sigma_{25})\,\sigma_{24}}{\sigma_{23}} - \frac{{\sigma_{24}}^2\,\sigma_{20}}{{\sigma_{23}}^2} - 2\,z\sin(\sigma_{25})\left(\frac{\sigma_{27}\sin(z) + 2\,{x_3}^2\,z\,\sigma_{26} - \frac{\sigma_4\,\sigma_{24}}{\sigma_{23}}}{\sigma_{23}} + \frac{\sigma_{24}\,\sigma_1}{{\sigma_{23}}^2}\right) - {x_2}^2\,{x_3}^2\cos(\sigma_{25}) \\ x_3\sin(\sigma_{25}) - 2\,z\sin(\sigma_{25})\left(\frac{\sigma_{27}\sin(z) - \frac{\sigma_{18}\,\sigma_{24}}{\sigma_{23}} + 2\,x_3\,z\,\sigma_{26}}{\sigma_{23}} - \frac{\sigma_{22}\,\sigma_1}{{\sigma_{23}}^2}\right) + \sigma_7 - \sigma_{17} + \sigma_{13} - \sigma_9 \\ x_2\sin(\sigma_{25}) + 2\,z\sin(\sigma_{25})\left(\frac{2\,z\,\sigma_{28} - \sigma_{27}\sin(z) + \frac{\sigma_5\,\sigma_{24}}{\sigma_{23}} - \sigma_{19}}{\sigma_{23}} - \frac{\sigma_{21}\,\sigma_1}{{\sigma_{23}}^2}\right) - \sigma_{16} - \sigma_8 + \sigma_{14} + \sigma_{10} \end{pmatrix}$$

where

$$\sigma_1 = \sigma_{29}\cos(z) - 2\,x_3\,\sigma_{28} + \frac{\sigma_{27}\sin(z)\,\sigma_{24}}{\sigma_{23}}$$

$$\sigma_2 = \sigma_{29}\cos(z) - 2\,x_1\,\sigma_{28} + \frac{\sigma_{27}\sin(z)\,\sigma_{21}}{\sigma_{23}}$$

$$\sigma_3 = 2\,\sigma_{28} - \sigma_{29}\cos(z) + \frac{\sigma_{27}\sin(z)\,\sigma_{22}}{\sigma_{23}}$$

$$\sigma_4 = 2\,x_3\,\sigma_{28} - \sigma_{29}\cos(z)$$

$$\sigma_5 = 2\,x_1\,\sigma_{28} - \sigma_{29}\cos(z)$$

$$\sigma_6 = \frac{\sigma_{22}\,\sigma_{21}\,\sigma_{20}}{{\sigma_{23}}^2}$$

$$\sigma_7 = \frac{\sigma_{22}\,\sigma_{24}\,\sigma_{20}}{{\sigma_{23}}^2}$$

$$\sigma_8 = \frac{\sigma_{21}\,\sigma_{24}\,\sigma_{20}}{{\sigma_{23}}^2}$$

$$\sigma_9 = \frac{2\,x_2\,x_3\,z\cos(\sigma_{25})\,\sigma_{22}}{\sigma_{23}}$$

$$\sigma_{10} = \frac{2\,x_2\,x_3\,z\cos(\sigma_{25})\,\sigma_{21}}{\sigma_{23}}$$

$$\sigma_{11} = \frac{2\,x_1\,x_3\,z\cos(\sigma_{25})\,\sigma_{21}}{\sigma_{23}}$$

$$\sigma_{12} = \frac{2\,x_1\,x_2\,z\cos(\sigma_{25})\,\sigma_{22}}{\sigma_{23}}$$

```
func2 = matlabFunction(d2pdy2)
```

```
func2 = function_handle with value:
    @(x_1,x_2,x_3,z)reshape([-x_2.^2.*x_3.^2.*cos(z.^2-x_1.*x_2.*x_3)-1.0./(sin(x_2+x_1.*x_3).*2.0+cos(x_1
```

```
z_test = 1.543295599106779
```

```
z_test = 1.5433
```

```
func2(1, 2, 3, z_test)
```

```
ans = 3×3
  -22.2226   -4.9360   -7.1680
   -4.9360   -0.9918   -1.5241
   -7.1680   -1.5241   -2.0855
```

```
% tic; func(1,2,3,z_test); toc
cplxDiff(@(x) func(x(1), x(2), x(3), z_test), [1;2;3], 1e-20)
```

```
ans = 3×3
  -14.4862   -1.8615   -4.0935
    0.6120    1.2130    0.6807
   -5.8336   -0.9938   -1.5551
```

```
% aaa(funcF, funcp, [1,2,3], z_test)
norm(func2(1, 2, 3, z_test), 'fro')
```

```
ans = 25.5990
```

```
norm(cplxDiff(@(x) func(x(1), x(2), x(3), z_test), [1;2;3], 1e-20), 'fro')
```

```
ans = 16.4261
```

```
% tic; aaa(funcF, funcp, [1,2,3], z_test); toc
```

```
function fp =  cplxDiff(f, x0, h)
    L = length(x0);
    sz_func = size(f(x0));
    sz_inpt = size(x0);
    for i = 1:L
        H = zeros(size(x0));
        H(i) = h*1i;
        fp(:, i) = imag(f(x0 + H))/h;
    end
end

function out = aaa(F, p, x0, z0)
    h = 1e-20;
    Fx = cplxDiff(@(x) F(x, z0), x0, h);
    Fz = cplxDiff(@(z) F(x0, z), z0, h);
    px = cplxDiff(@(x) p(x, z0), x0, h);
    pz = cplxDiff(@(z) p(x0, z), z0, h);

    out = px - pz*(Fx/Fz);
```

```
end
```