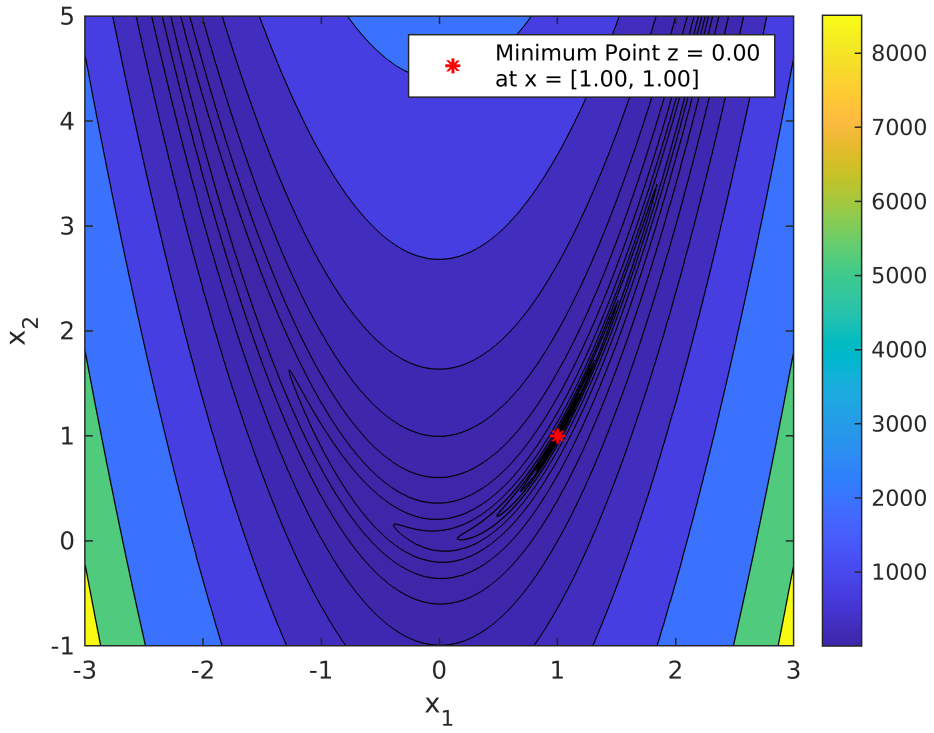
ASE387P Homework 3

Problem 1

```
clear; clc
```

Part a-b

```
f = @(x) (1-x(1))^2 + 100*(x(2)-x(1)^2)^2;
del = 0.01;
x1 = -3:del:3;
x2 = -1:del:5;
z = zeros(length(x1));
for i = 1:length(x1)
    for j = 1:length(x1)
        z(j, i) = f([x1(i), x2(j)]);
    end
end
contourf(x1, x2, z, [logspace(-4, 5, 22), max(max(z))*0.85], 'HandleVisibility',"off")
colorbar
[val, idx1] = min(z);
[val, idx2] = min(val);
idx1 = idx1(idx2);
hold on
scatter(x1(idx2), x2(idx1), 'r*', 'DisplayName', ...
    sprintf('Minimum Point z = %0.2f \nat x = [%0.2f, %0.2f]', val, x1(idx2), x2(idx1))
    'linewidth', 1.5)
legend
hold off
xlabel('x_1'); ylabel('x_2')
exportgraphics(gcf, 'hw3p1a.png', 'Resolution', 200)
```



Part c

```
syms x_1 x_2
f_sym = (1-x_1)^2 + 100*(x_2-x_1^2)^2
```

f_sym = $(x_1 - 1)^2 + 100\ \left(x_2 - x_1^2\right)^2$

```
eqs = [diff(f_sym, x_1) == 0; diff(f_sym, x_2) == 0]
```

eqs =

$$\begin{pmatrix} 2\,x_1 - 400\,x_1\ \left(x_2 - x_1^2\right) - 2 = 0 \\ 200\,x_2 - 200\,x_1^2 = 0 \end{pmatrix}$$

```
sol = solve(eqs); fprintf('Derivatives = 0 \nat x = [%0.2f, %0.2f]', sol.x_1, sol.x_2)
```

```
Derivatives = 0
at x = [1.00, 1.00]
```

2nd Order Necessary and Sufficient Condition

Necessary: $\nabla f(x_*) = \vec{0}$

Sufficient: $\nabla^2 f(x_*)$ is positive definite

```
grad = [diff(f_sym, x_1); diff(f_sym, x_2)]
```

grad =

$$\begin{pmatrix} 2x_1 - 400x_1\left(x_2 - x_1^2\right) - 2 \\ 200x_2 - 200x_1^2 \end{pmatrix}$$

```
f_grad = matlabFunction(grad);
fprintf('Norm of the first gradient of f is %0.2f,\n thus the necessary condition is sa
```

```
Norm of the first gradient of f is 0.00,
 thus the necessary condition is satisfied
```

```
grad = [diff(grad(1), x_1) diff(grad(1), x_2);
    diff(grad(2), x_1) diff(grad(2), x_2)]
```

```
grad =
```

$$\begin{pmatrix} 1200x_1^2 - 400x_2 + 2 & -400x_1 \\ -400x_1 & 200 \end{pmatrix}$$

```
f_grad2 = matlabFunction(grad);
[A, flag] = chol(f_grad2(sol.x_1, sol.x_2))
```

```
A =
```

$$\begin{pmatrix} \sqrt{802} & -\frac{200\sqrt{802}}{401} \\ 0 & \frac{10\sqrt{2}\sqrt{401}}{401} \end{pmatrix}$$

```
flag = 0
```

```
fprintf('As the Cholesky factorization was successful, and returned a flag of %i\n the
```

```
As the Cholesky factorization was successful, and returned a flag of 0
 the sufficient condition was satisfied
```