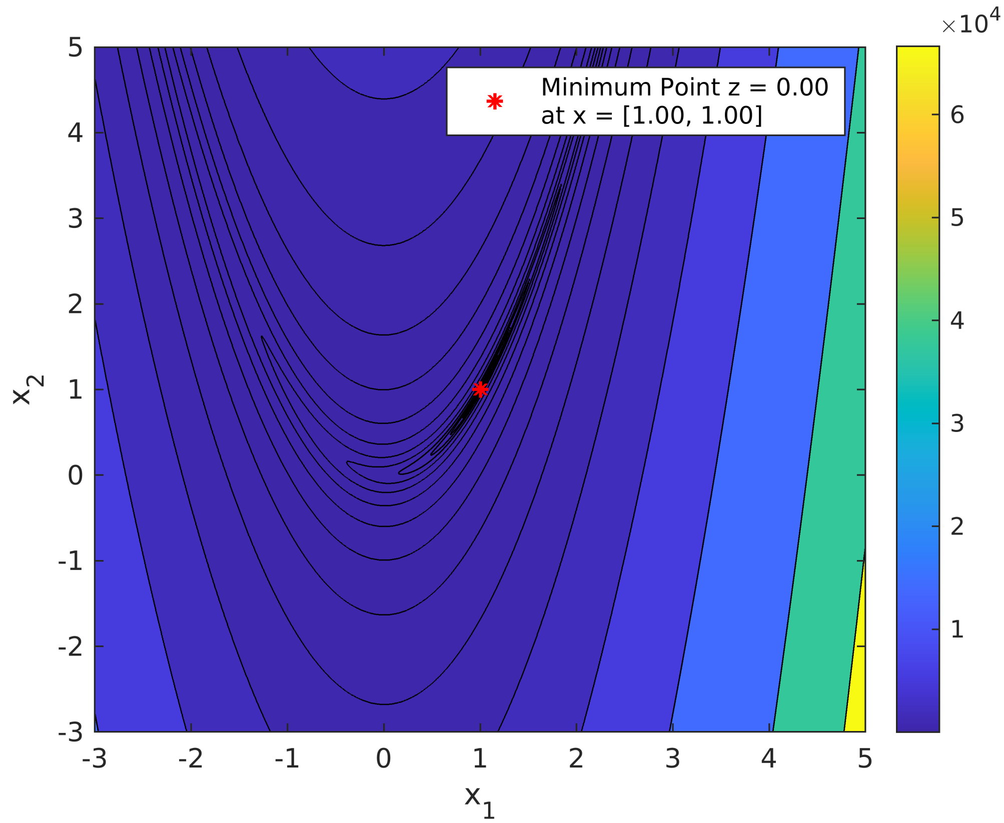
ASE387P Homework 3

Problem 1

```
clear; clc
```

Part a-b

```
f = @(x) (1-x(1))^2 + 100*(x(2)-x(1)^2)^2;
del = 0.01;
x1 = -3:del:5;
x2 = -3:del:5;
z = zeros(length(x1));
for i = 1:length(x1)
    for j = 1:length(x1)
        z(j, i) = f([x1(i), x2(j)]);
    end
end
contourf(x1, x2, z, [logspace(-4, 5, 22), max(max(z))*0.85], 'HandleVisibility',"off")
colorbar
[val, idx1] = min(z);
[val, idx2] = min(val);
idx1 = idx1(idx2);
hold on
scatter(x1(idx2), x2(idx1), 'r*', 'DisplayName', ...
    sprintf('Minimum Point z = %0.2f \nat x = [%0.2f, %0.2f]', val, x1(idx2), x2(idx1)
    'linewidth', 1.5)
legend
hold off
xlabel('x_1'); ylabel('x_2')
```




```
% exportgraphics(gcf, 'hw3p1a.png', 'Resolution', 200)
```

Part c

```
syms x_1 x_2
f_sym = (1-x_1)^2 + 100*(x_2-x_1^2)^2
```

f_sym = $(x_1 - 1)^2 + 100\,(x_2 - x_1^2)^2$

```
eqs = [diff(f_sym, x_1) == 0; diff(f_sym, x_2) == 0]
```

eqs =

$$\begin{pmatrix} 2\,x_1 - 400\,x_1\,(x_2 - x_1^2) - 2 = 0 \\ 200\,x_2 - 200\,x_1^2 = 0 \end{pmatrix}$$

```
sol = solve(eqs); fprintf('Derivatives = 0 \nat x = [%0.2f, %0.2f]', sol.x_1, sol.x_2)
```

```
Derivatives = 0
at x = [1.00, 1.00]
```

2nd Order Necessary and Sufficient Condition

Necessary: $\nabla f(x_*) = \vec{0}$

Sufficient: $\nabla^2 f(x_*)$ is positive definite

```
grad = [diff(f_sym, x_1); diff(f_sym, x_2)]
```

grad =

$$\begin{pmatrix} 2\,x_1 - 400\,x_1\,\left(x_2 - {x_1}^2\right) - 2 \\ 200\,x_2 - 200\,{x_1}^2 \end{pmatrix}$$

```
f_grad_ = matlabFunction(grad);
fprintf('Norm of the first gradient of f is %0.2f,\n thus the necessary condition is sa
```

```
Norm of the first gradient of f is 0.00,
 thus the necessary condition is satisfied
```

```
grad = [diff(grad(1), x_1) diff(grad(1), x_2);
    diff(grad(2), x_1) diff(grad(2), x_2)]
```

grad =

$$\begin{pmatrix} 1200\,{x_1}^2 - 400\,x_2 + 2 & -400\,x_1 \\ -400\,x_1 & 200 \end{pmatrix}$$

```
f_grad2_ = matlabFunction(grad);
[A, flag] = chol(f_grad2_(sol.x_1, sol.x_2))
```

A =

$$\begin{pmatrix} \sqrt{802} & -\frac{200\,\sqrt{802}}{401} \\ 0 & \frac{10\,\sqrt{2}\,\sqrt{401}}{401} \end{pmatrix}$$

flag = 0

```
fprintf('As the Cholesky factorization was successful, and returned a flag of %i\n the
```

```
As the Cholesky factorization was successful, and returned a flag of 0
 the sufficient condition was satisfied
```

Problem 2

```
% SETUP
s = [-1 1].';
x0 = [4 -1].';
t0 = 0.1;
f_grad = @(x) f_grad_(x(1), x(2));
f_grad2 = @(x) f_grad2_(x(1), x(2));

% BRACKETING MINIMUM
[pts, line, iters] = lineSearch(x0, s, f, t0, 4, 2)
```

pts = 1×3 struct

Fields	x	z
1	[1.7000;...	253.3000
2	[1.3000;...	0.1000
3	[0.5000;...	506.5000

line = 2×13

```
    4.0000    3.9000    3.8000    3.7000    3.5000    3.3000    3.1000    2.9000 ···
   -1.0000   -0.9000   -0.8000   -0.7000   -0.5000   -0.3000   -0.1000    0.1000
iters = 13
```

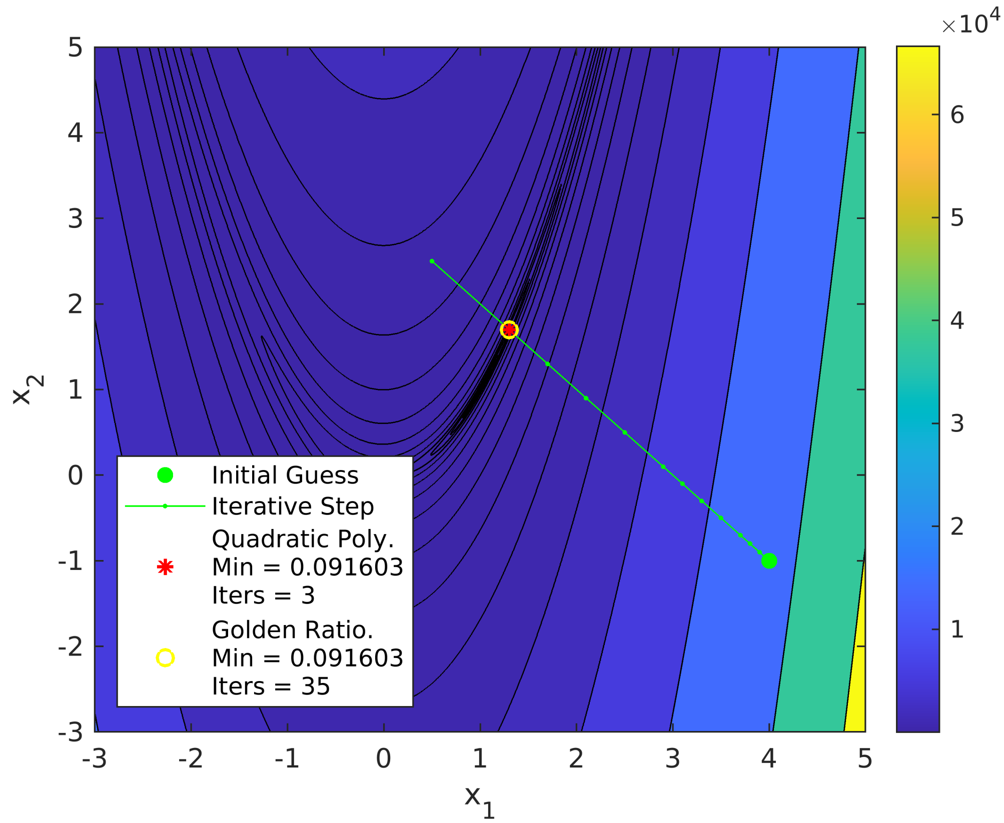
```
% QUADRATIC POLYNOMIAL LINE SEARCH
[x_min_quad, z_min_quad, iter_quad] = quadMin([pts(:).x], [pts(:).z], s, f, f_grad, f_g
```

```
x_min_quad = 2×1
    1.3025
    1.6975
z_min_quad = 0.0916
iter_quad = 3
```

```
% GOLDEN RATIO LINE SEARCH
[x_min_gr, z_min_gr, iter_gr, allX] = grMin([pts(:).x], [pts(:).z], s, f)
```

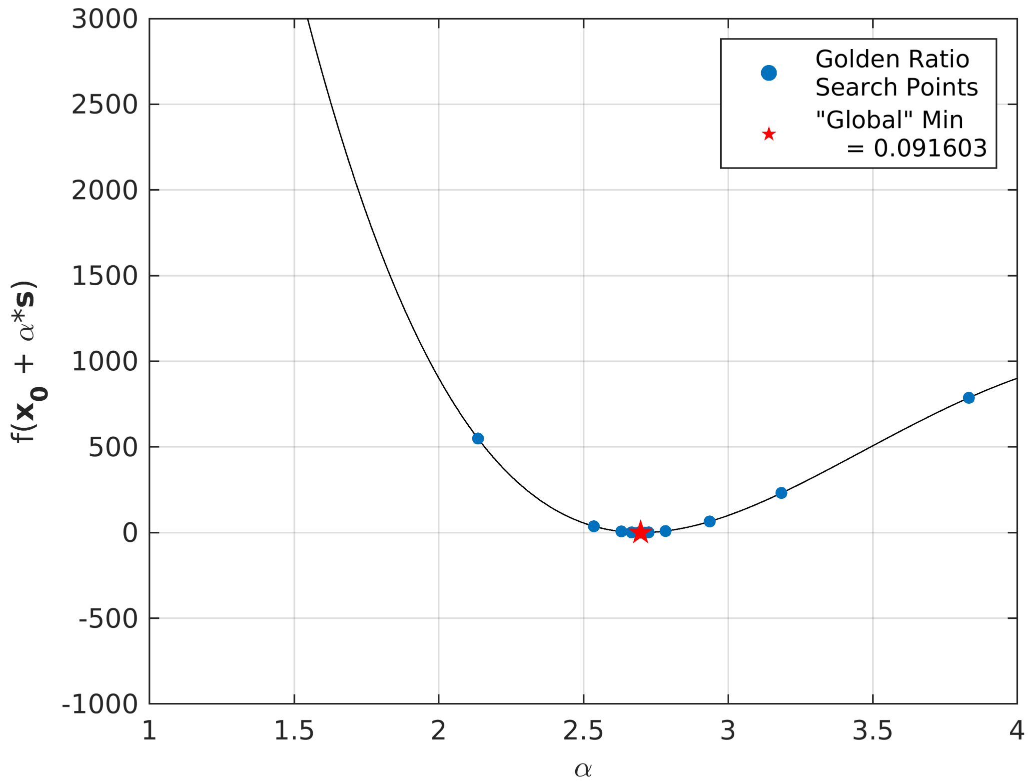
```
x_min_gr = 2×1
    1.3025
    1.6975
z_min_gr = 0.0916
iter_gr = 35
allX = 2×39
    1.3025    1.3025    1.3025    1.3025    1.3025    1.3025    1.3025    1.3025 ···
    1.6975    1.6975    1.6975    1.6975    1.6975    1.6975    1.6975    1.6975
```

```
% PLOTTING
contourf(x1, x2, z, [logspace(-4, 5, 22), max(max(z))*0.85], 'HandleVisibility',"off")
colorbar
[val, idx1] = min(z);
[val, idx2] = min(val);
idx1 = idx1(idx2);
hold on
% scatter(x1(idx2), x2(idx1), 'r*', 'DisplayName', ...
%     sprintf('Minimum Point z = %0.2f \nat x = [%0.2f, %0.2f]', val, x1(idx2), x2(idx1
%     'linewidth', 1.5)
scatter(line(1, 1), line(2, 1), 'go', 'filled', 'DisplayName', 'Initial Guess')
plot(line(1, :), line(2, :), 'g.-', 'DisplayName', 'Iterative Step')
scatter(x_min_quad(1), x_min_quad(2), 'r*', 'DisplayName', ...
    sprintf('Quadratic Poly.\nMin = %0.6f\nIters = %i', z_min_quad, iter_quad), ...
    'linewidth', 1.5)
scatter(x_min_gr(1), x_min_gr(2), 'yo', 'DisplayName', ...
    sprintf('Golden Ratio.\nMin = %0.6f\nIters = %i', z_min_gr, iter_gr), ...
    'linewidth', 1.5)
lgn = legend; lgn.Location = 'southwest';
% axis equal
hold off
xlabel('x_1'); ylabel('x_2')
```




```
% exportgraphics(gcf, 'hw3p2.png', 'Resolution', 200)

% ADDITIONAL PLOT
line = linspace(0, 7, 50000);
y = zeros(length(line), 1);
for i = 1:length(y); y(i) = f(x0 + line(i).*s); end
plot(line(1:50:end), y(1:50:end), 'k-', 'HandleVisibility', "off")
hold on
[val, idx] = min(y);
ylim([-1000 3000])
legend
ylabel('f({\bfx_0} + \alpha*{\bfs})', "Interpreter","tex")
xlabel('\alpha')
dist = zeros(size(allX, 2), 1); ydist = dist;
for i = 1:size(allX, 2)
    dist(i) = norm(allX(:, i)-x0)-1.1173;
    ydist(i) = f(x0 + dist(i).*s);%*1.85;
end
scatter(dist, ydist, 20, 'o', 'filled', 'MarkerFaceColor', [0 0.4470  0.7410],...
    'DisplayName', sprintf('Golden Ratio \nSearch Points'))
scatter(line(idx), y(idx), 100, 'rp', 'filled',...
    'DisplayName', sprintf('"Global" Min\n    = %0.6f', val))
xlim([1 4])
grid on
```




```
% CREATING TABLE OF RESULTS
% tMult = [1.1 1.25 1.5 2 2.5 3 5];
% tSteps = [2 4 8 16];
% C = cell(length(tSteps), length(tMult));
% for i = 1:length(tSteps)
%     for j = 1:length(tMult)
%         % FINDING ITERATIONS
%         [pts, ~, iters_line] = lineSearch(x0, s, f, t0, tSteps(i), tMult(j));
%         [~, ~, iter_quad] = quadMin([pts(:).x], [pts(:).z], s, f, f_grad, f_grad2);
%         [~, ~, iter_gr] = grMin([pts(:).x], [pts(:).z], s, f);
%
%         % COLLECTING
%         C{i, j} = [iters_line, iter_quad, iter_gr];
%     end
% end
% C = cat(2, num2cell(tSteps.'), C);
% colNames = {'NumSteps', '1.1 Multiplier', '1.25 Multiplier', '1.5 Multiplier', '2.0 M
% rowNames = {'2 Steps', '4 Steps', '8 Steps', '16 Steps'};
% dimNames = ["Num Steps","Multiplier"];
% T = cell2table(C(:, 2:end), 'DimensionNames', dimNames, 'VariableNames', colNames(2:e
```

```
function [pts, x, iterTot] = lineSearch(x0, s, f, t0, steps, stepMod)
    % SETUP
    z = f(x0);
    x = zeros(length(x0), 1);
    x(:, end) = x0;            % Enforcing column vectors
```

```
    t = t0;
    iter = 1; iterTot = 1;

    % ITERATING UNTIL MINIMUM BRACKETED
    while length(z) < 2 || z(end) < z(end-1)
        % INCREASING STEP SIZE IF NOT BRACKETING
        if iter == steps
            t = stepMod*t;
            iter = 0;
        end

        % STEPPING AND EVALUATING
        x = cat(2, x, x(:, end) + t*s);
        z(end+1) = f(x(:, end));
        iter = iter+1;
        iterTot = iterTot+1;

        if iter == 31
            pts = [];
            warning("lineSearch() ran out of iterations")
            return
        end
    end

    % OUTPUTTING POINTS
    pts = struct('x', x(:, end-2), 'z', z(end-2));
    pts(end+1) = struct('x', x(:, end-1), 'z', z(end-1));
    pts(end+1) = struct('x', x(:, end), 'z', z(end));
end

function [xmin, zmin, iters] = quadMin(x, z, s, f, f_grad, f_grad2)
    % SETUP
    alpha = inf;
    iters = 0;

    % ITERATING
    while abs(alpha) > 1e-8
        [~, idx] = min(z);

        % FINDING STEP SIZE
        alpha = -(f_grad(x(:, idx)).'*s) / (s.' * f_grad2(x(:, idx)) * s);

        % STEPPING
        x = cat(2, x, x(:, idx) + alpha*s);
        z(end+1) = f(x(:, end));
        iters = iters+1;
        if iters > 51; break; end
    end

    % OUTPUTTING
    [~, idx] = min(z);
    xmin = x(:, idx);
    zmin = z(idx);
end
```

```matlab
function [xmin, zmin, iters, allX] = grMin(x0, z0, s, f)
    % SETUP
    alpha = (3 - sqrt(5))/2;   % Step Size

    xL = x0(:, 1);             % Creating Points
    xR = x0(:, 3);             % |
    x1 = xL + alpha*(xR - xL); % |
    x2 = xR - alpha*(xR - xL); % #

    fL = z0(1);                % Evaluating at Points
    f1 = f(x1);                % |
    f2 = f(x2);                % |
    fR = z0(3);                % #

    X = [xL x1 x2 xR];         % Creating Test Matrix
    dx = inf;                  % Initializing Error
    iters = 0;

    % ITERATING
    allX = X;
    while abs(dx) > 1e-8
        if f1 > f2
            % MOVING BOUNDS
            xL = x1; fL = f1;
            x1 = x2; f1 = f2;

            % RE-EVALUTING POINT
            x2 = xR - alpha*(xR - xL);
            allX = cat(2, x2, allX);
            f2 = f(x2);
        else
            % MOVING BOUINDS
            xR = x2; fR = f2;
            x2 = x1; f2 = f1;

            % RE-EVALUTING POINT
            x1 = xL + alpha*(xR - xL);
            allX = cat(2, x1, allX);
            f1 = f(x1);
        end

        Xnew = [xL, x1, x2, xR];
        dx = norm(Xnew, 'fro') - norm(X, 'fro');
        X = Xnew;
        iters = iters+1;
        if iters > 51; break; end
    end

    % FINDING RESULTS
    [zmin, idx] = min([fL, f1, f2, fR]);
    xmin = X(:, idx);
end
```