

**QUESTION:**

Over all real numbers, find the minimum value of a positive real number,  $y$  such that  
 $y = \sqrt{(x+6)^2 + 25} + \sqrt{(x-6)^2 + 121}$

**SOLUTION:**

$$y = \sqrt{(x+6)^2 + 25} + \sqrt{(x-6)^2 + 121}$$

$$y = [(x+6)^2 + 25]^{1/2} + [(x-6)^2 + 121]^{1/2}$$

$$\frac{dy}{dx} = \frac{x+6}{\sqrt{(x+6)^2 + 25}} + \frac{x-6}{\sqrt{(x-6)^2 + 121}} = 0$$

There would always be a value of  $y$  for all values of  $x$  over all integer planes.

However, the minimum value of  $y$  exists at  $x = 0$  which is the smallest positive real number.

$$@ x = 0;$$

$$y = \sqrt{(0+6)^2 + 25} + \sqrt{(0-6)^2 + 121}$$

$$y = \sqrt{61} + \sqrt{157}$$

$$y = 20.34$$