

First, convert the equation

$$\hat{H}\psi = E\psi$$

where

$$\hat{H} = v\sigma \cdot \mathbf{p} + V(\mathbf{r})\sigma_z$$

into polar-coordinate form:

$$\begin{bmatrix} 0 & e^{-i\phi}(\partial_r - (i/r)\partial_\phi) \\ e^{i\phi}(\partial_r + (i/r)\partial_\phi) & 0 \end{bmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = k \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

with solution:

$$\psi_l = \begin{pmatrix} \psi_{1l} \\ \psi_{2l} \end{pmatrix} = e^{il\phi} \begin{pmatrix} J_l(kr) \\ ie^{i\phi} J_{l+1}(kr) \end{pmatrix}$$

boundary condition:

$$\frac{\psi_2(R_1)}{\psi_1(R_1)} = -ie^{i\phi}$$

and

$$\frac{\psi_2(R_2)}{\psi_1(R_2)} = ie^{i\phi}$$

note:

we get

$$j_l(E) = -j_{l+1}(E)$$

$$j_l(\eta E) = j_{l+1}(\eta E)$$