The Experimental Basis of Electromagnetism - Part III: The Magnetic Field

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The Experimental Basis of Electromagnetism—Part III: The Magnetic Field

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ABSTRACT. The principles outlined in previous parts, published in the *Proceedings of the Physical Society* in 1946 and 1948 respectively, which dealt with the direct current circuit and with electrostatics, are here applied to magnetism, with the object of showing how the basic concepts are defined in terms of the operations actually performed in measuring them. This part is confined to a discussion of the vector **B**, which is shown to be measurable independently of a knowledge of any other magnitude. It is solenoidal, and can therefore be determined at all points, even those within solid bodies.

§1. INTRODUCTION

The general purpose of this work, as stated in the abstract of Part I (Campbell and Hartshorn 1946) *, was "to show to what extent the working principles of electromagnetism can be soundly based on real experimental facts, as distinct from the imaginary experiments which are common to most expositions of the subject". The general principles by which magnitudes are established in terms of experimental operations were described in §2 of that paper, and at the outset we were optimistic enough to believe that by systematically applying these principles to operations that would be generally admitted as established practice, we should be able to derive the working laws of the experimentalist and a coherent outline of the whole subject in a form free from "mathematical fictions", that is to say, concepts like point charges that are so remote from real experiment that they must be classed as auxiliary devices invented by the human mind rather than anything encountered in nature.

We had some success in treating in this way the direct current circuit (Part I, 1946) and electrostatics (Part II, 1948), though it must be confessed that in Part II the jump from experiment to general law was not always so obvious as to leave no doubt in the mind of the reader about the wisdom of dispensing with the auxiliary devices. We have in the present part applied the same treatment to magnetism, and we have to admit that our early optimism has not been altogether justified. The laws that we have been able to base directly on real experiments can scarcely be regarded as forming a coherent structure, and it therefore seems

* For references seen end of Part IV, p. 000.

likely that theoretical magnitudes having no direct reference to real experiment form a necessary part of the subject as it is practised today. But though the magnitudes that arise directly from real experiments may not be sufficient, it is at least important that we shall know which they are, and their precise significance in terms of the operations that we actually perform. This part is therefore devoted to a consideration of the most characteristic experimental methods employed in the investigation of magnetic materials and of the magnitudes and laws which arise directly from them. These, in our view, constitute the experimental basis of the subject.

§ 2. THE FLUXMETER

In accordance with our general principles, our first step must be to seek among the instruments used in modern magnetic measurements one whose operations define a magnitude that can be measured independently of any other magnitude. Such an instrument is to be found in the fluxmeter. At first sight it may seem surprising that so complicated an instrument should be accepted as fundamental. The principle is, however, very similar to that which leads us to accept operations performed with balances and clocks, which are quite as complicated as fluxmeters, as the basis of our ideas of mass and time.

By a fluxmeter we mean the familiar combination of a search coil and ballistic galvanometer, the coil having its terminals close together, and being connected to the galvanometer, which is direction-sensitive, by long, flexible, twisted or concentric leads. (The term fluxmeter is often confined to an instrument in which the ballistic galvanometer is of a special kind, but we think it less objectionable to extend this meaning somewhat than to coin a new term.) If the search coil, after having been at rest for a long time in the neighbourhood of a magnet, is moved suddenly relative to it and is then brought to rest again, the galvanometer gives a characteristic response in the form of a transient deflection passing through one or more maximum values. We shall apply the term magnet to any system that produces such responses, so that it includes, not only permanent magnets and electromagnets in the conventional sense, but also current circuits not associated with any magnetic material. Similar responses can be produced by changes in the magnet, and in particular by changes in the current in a circuit forming part of the magnet without any relative motion of coil and magnet. The laws we are about to state in this section are true by whichever method the response is produced: it is convenient therefore to use terms that are equally applicable to either. We shall use the term "the system" to denote the whole combination of search coil, leads, galvanometer and magnet; and the "state of the system", that complex of conditions, sudden * change in any one of which may produce a response. Thus a change in the state of the system may include a change in the size or shape of the search coil, a displacement of the coil relative to the magnet, or a change in the state of the magnet, such as a change of current in a stationary coil or the displacement of a piece of magnetic material relative to neighbouring circuits.

So long as the changes of the state of the system are always sufficiently sudden and the galvanometer always starts from the same zero, such a system obeys two qualitative laws: (*a*) If two responses agree in one feature, e.g. the first maximum,

* By terms such as "sudden", "close together", "long time", we mean *so* sudden, *so* close, *so* long, etc., that the statements in which the terms are used are true. The fact involved in any such statement is that it can be made true by making the motion sudden *enough*, or by placing the terminals close *enough*, and so on. In this, of course, we are only following the practice of many other writers.

they agree in all; accordingly, each response can be completely characterized by a single parameter, say this first maximum. (*b*) The response is determined wholly by the terminal states between which the change occurs; it is unaffected by the intermediate states traversed in the passage from one terminal state to the other and by the period occupied by the traverse, so long as it is short enough. The system is therefore characterized by pairs of states for which an unambiguous definition of equality can be given, namely that one pair of states is equal to another pair when a change between one pair produces the same response as the change between the other. If we can associate a definition of addition with this definition of equality, we shall arrive at a magnitude, say A_{xy}, characteristic of the pair of states, (x, y), and measurable independently of the measurement of any other magnitude, and which thus resembles direct current, direct voltage, resistance and capacitance, which are fundamental in the discussions of the previous Parts.

Let us adopt then, as the definition of addition,

$$A_{xz}=A_{xy}+A_{yz}, \qquad \ldots\ldots(2.1)$$

or, in words, the value of A to be associated with a change of state from x to z is the sum of the values to be associated with the changes from x to y and from y to z, where y is any third state, not necessarily intermediate between x and z. (It is just worth noting that (2.1) is *not* a consequence of the law that A_{xz} is determined only by the terminal states; for a change from x to y that halts at z long enough to observe, the deflection is not "sudden".) We can now calibrate the scale of the galvanometer for A_{xy} by the standard procedure for independently measurable magnitudes. We choose arbitrarily some pair of states (a, b), and assign to A_{ab} the value unity. We find by experiment other states $c, d, e \ldots$. such that, according to the definition of equality,

$$A_{ab}=A_{bc}=A_{cd}=A_{de}. \qquad \ldots\ldots(2.2)$$

Then, according to the definition of addition, the numerals 2, 3, 4 . . . must be assigned to A_{ac}, A_{ad}, A_{ae} etc. Tests must now be applied to establish that the rules of arithmetic are obeyed consistently; thus we must enquire whether in fact A_{bd}, A_{ce}, ... are all 2 (i.e. that changes of state from b to d, from c to e . . . . all give the value 2 on the calibrated scale), that A_{be} is 3, and so on. Since $A_{xx}=0$ (i.e. no change produces no deflection), we must have from (2.1) $A_{yx}=-A_{xy}$, and one of our tests must be to ascertain whether this is so. The necessity for this test is one reason why it was specified that the ballistic galvanometer should be direction-sensitive; of course it is not necessary that it should be such that the calibrated scale turns out to be symmetrical about the zero. A_{xy} in this general form, and without the limitations that will be imposed on it in the following sections, is not an important magnitude. It has no recognized name or symbol; no instrument is ever calibrated to read A_{xy} by this independent method, and no systematic tests are usually made to establish that (2.1) is actually true for fluxmeters calibrated in other ways. But it would be as wrong to dismiss as unimportant the fact that any satisfactory fluxmeter could be so calibrated as it would be to dismiss similarly the corresponding fact about D.C. ammeters, which also are never in practice calibrated by addition. The reasons are the same in both cases. The laws in virtue of which fluxmeters could be calibrated independently are often assumed to be true in using measurements made by fluxmeters calibrated in other ways; if a fluxmeter calibrated by some conventional method failed to obey those laws, then that calibration would have to be judged wrong and "an explanation" would be sought in some defect of the method or the instrument; the independent method provides a criterion whereby all other methods can be judged.

§ 3. FLAT SEARCH COILS: B_n AND B

We shall now proceed to derive from this magnitude A_{xy} characteristics of pairs of states of a system including a magnet and a fluxmeter, first a magnitude X_x characteristic of a single state of the system, magnet and search coil, and then a magnitude B dependent only on the state of the magnet and position relative to it.

Change of the ballistic galvanometer or of the leads (if they provide any appreciable part of the resistance of the fluxmeter circuit) will change the deflection associated with any given change of the state of the system; but if the same change of state is always taken as the unit, and the scale calibrated in terms of that unit, then the value to be associated with any given change of state is independent of both the ballistic galvanometer and the leads; that is an experimental fact. If it were similarly independent of the search coil, A_{xy} would depend only on the state of the magnet and of the positions of the search coil relative to it; but the mention of "position" shows at once that complete independence of the search coil is impossible. For the position of the search coil relative to the magnet, which enters into the definition of the unit of A_{xy}, has no meaning unless limits are placed on the form that the coil may take; apart from some geometrical convention, it would be meaningless to say that the position of a long helix relative to some system was the same as that of a flat coil.

Let us therefore place limits on the form of the search coil and adopt the convention that all are to be "flat" and rigid, though they may differ in area and in the number of turns, and are to have one "marked" face. Then the position of any search coil relative to any coordinate frame fixed in the magnet is perfectly determinate when the position of its centre and the direction of the normal to the marked face are given; and we can inquire whether, if some particular state of the magnet and positions of the search coil are always used to define the unit, the value of A_{xy} is independent of the search coil used in measuring it and characteristic of the states of the magnet and positions relative to it. We find that it is so characteristic so long as the area of each coil is small enough, and that the permissible upper limit depends upon the magnet (i.e. on the uniformity of its field, in ordinary terms); so we adopt the further convention that, in measurements on any particular magnet, the area of the search coil must not exceed this limit. We thus arrive at a magnitude (in spite of the added conventions, it will still be denoted by A_{xy}), measurable independently of the measurement of any other magnitude, characteristic (for any given magnet) of pairs of positions around it, each position being defined by the situation of a point and the direction of a line drawn from that point.

The form of the law (2.1) on which the measurement of A_{xy} depends shows that, given any distribution of A_{xy}, it must be possible to find a set of magnitudes X_x, X_y each characteristic of a single position (not, like A_{xy}, of a pair of positions) such that

$$A_{xy}=k(X_x-X_y). \qquad \ldots\ldots(3.1)$$

But, unless something is added, the X's, unlike normal magnitudes, would involve an arbitrary additive constant, as well as the arbitrary scale factor k. This difference would vanish if we could find facts or laws that distinguish one choice of the additive constant from all others, and thus make it no longer arbitrary, and especially if we could find a reason for attributing the value 0 to X_0, characteristic of some position or set of positions denoted by suffix 0.

Such a reason can be found. Suppose that the flat search coil is symmetrical about an axis lying in its own plane, so that on rotation through 180° about the axis the winding of the coil occupies the same space as before, but the normal to the unmarked face replaces that to the marked face; such a rotation will be called "reversal". Then experiment shows that, in general, reversal produces a response of the galvanometer, as does any other motion of the coil; but that for each point near the magnet there is a plane such that, if the marked normal originally lies in that plane, reversal produces no response; when this condition is fulfilled, the coil will be said to be in a "zero" position characterized by X_0. The plane and the zero position are, in general, different for different points, but if the coil is suddenly transferred from one zero position to another, no response occurs, and none occurs if it is suddenly transferred from one of these zero positions near the magnet to a point very distant from it, where any magnitude determined by the magnet would be expected to be zero.* All these facts support the idea that the value 0 should be assigned to every X_0, so that X_x characteristic of any non-zero position x will be A_{x0}/k, where A_{x0} is the reading on the calibrated scale when the search coil is suddenly transferred from x to a zero position.

The reader will no doubt have realized already that X_x, apart from a scale factor, is the familiar quantity that might be written $(B_n)_x$, i.e. the component of the flux-density normal to the small flat search coil when its centre is located at the point and its marked normal is turned in the direction characteristic of the position denoted by x; when B_n has been measured for all positions x, the fact that it is the normal component of a vector **B** will appear from a study of the measurements, and the complete distribution of **B** will be determined. Accordingly the final result of this discussion is to show that **B**, the flux-density (or magnetic induction), can be measured by methods that do not involve any measurement of any other magnitude (and, in particular, of any other magnetic magnitude), methods that involve only laws of equality and of addition together with conventions (namely the limitation to small flat coils) that are based upon real and definite experiments.

§4. FLUX-LINKAGE

The distribution of **B** round magnets of various kinds can therefore be determined by direct measurement, and it can be established that **B**, wherever it can be measured, is solenoidal, i.e. that the surface integral of its normal component over any closed surface is zero. It cannot be measured at all inside solid bodies, for small flat coils cannot be introduced into, or rotated within, solid bodies. It is tempting to assume that **B** is solenoidal, even where it cannot be directly measured, and if that assumption is made, **B** can be determined everywhere; for the distribution, within a closed surface, of a vector known to be everywhere solenoidal can be determined by calculation from values of the vector measured at appropriate points outside the surface. However, in Part II, §16, we had an example of a vector that is solenoidal in some regions but not in others; the general assumption must therefore not be made without adequate reason. Good reason for making the assumption so far as non-magnetic bodies are concerned

* The neglect of the earth's field here and elsewhere (e.g. § 6) is justifiable because the experiments described could really be conducted in a space where the earth's field is neutralized.

(bodies that are air-equivalent in the sense of Part I, § 15) is immediately found; for direct experiment shows that if we take any system in air, for which **B** has been shown to be solenoidal, the mere insertion of non-magnetic bodies into any region of the system makes no difference to any response of the fluxmeter; in other words, bodies that are air-equivalent with respect to compass needles are also air-equivalent with respect to fluxmeters. Thus **B** is established as solenoidal and measurable throughout any system which is composed of non-magnetic materials only. In systems which include magnetic material, it is measurable and solenoidal in the air spaces, but we have as yet no evidence concerning points inside magnetic solids.

The further consideration of this matter is facilitated by the removal of the limitation to small flat search coils that was imposed in order to establish **B** as a measurable property. We therefore examine search coils of various shapes and sizes, and seek a law connecting the response of the fluxmeter with the magnitude **B** and the shape and size of the search coil. Experiment shows that a simple law can be found provided we limit it to search coils that satisfy the following conditions: (*a*) the wire must be so thin and the terminals so close together that the geometrical properties of the coil are completely represented by a single closed curve; (*b*) the resistance of the whole fluxmeter circuit shall remain unaltered when any change of coil is made. It is scarcely necessary to add that the coil must be constructed entirely of non-magnetic material, for this condition has been assumed throughout, any magnetic material included in the system always being regarded as part of the magnet. (This condition is of course always satisfied in practice.) We have already seen that the responses of a fluxmeter which includes such a search coil serve to determine a magnitude X characteristic of the state x of the system under examination. We can now establish a relation between X, the field of **B**, and the geometrical properties of the search coil. In practice, systems which are found to possess a uniform field of **B** throughout some particular region are of outstanding importance. It is found that the value of X for a search coil that lies wholly within such a region is determined only by the magnitude of **B** and the total projected area A of the search coil on a plane perpendicular to **B** (i.e., A is the area enclosed by the curve traced out on the specified plane by the projection of a point P on the coil as this point travels once completely round the winding from one terminal to the other). The law may be written

$$X = kBA, \qquad \ldots\ldots (4.1)$$

where the proportionality factor k is independent of the size, shape and orientation of the coil and of the field of **B** so long as the stipulated conditions are satisfied. This simple relation suggests the more general law, which is implied by calling **B** the flux-density at a specified point and X the flux-linkage of the search coil for the particular state of the system, that X is proportional to the surface integral Φ of the normal component of **B** over any continuous surface bounded by the closed curve along which the winding of the search coil lies. It must be noted that this surface integral only has a unique value for the closed curve if the distribution of **B** is solenoidal throughout the whole field. The application of the law to systems containing magnetic material therefore involves the assumption that **B** is solenoidal within magnetic materials. Now X can certainly be measured for many systems that include magnetic material, and the values are always uniquely associated with particular distributions of such values of **B** as can be measured, and they are, moreover, consistent in every way with a solenoidal distribution of

B throughout the system. The conception of **B** as a magnitude that is solenoidal throughout all magnetic systems whatever their composition is, therefore, fully justified by experiment, and since values of **B** inside a magnetic solid can, with this conception, be determined from measured values of X, we may legitimately regard **B** as measurable everywhere. The general law may be written

$$X = \Phi = \int B_n dS, \qquad \ldots\ldots(4.2)$$

the scale factor being omitted since the units of **B** and X are always chosen so as to make it unity. We shall therefore henceforward regard X and Φ as the same magnitude and use the same symbol Φ for both.

We may note here that Φ can be measured for search coils of any size and shape, but that for the measurement of **B** the coil must be small enough to lie wholly within a region throughout which **B** can be shown to be sensibly uniform; it must also possess a definite axis rigidly fixed with respect to the winding, and a determinate „ effective area" A, which is its projected area on a plane perpendicular to this axis. Measurements of **B** can then be made in accordance with the law

$$\Phi = B_a A, \qquad \ldots\ldots(4.3)$$

where B_a denotes the component of **B** in the direction of the axis of the search coil.

§ 5. INDUCED E.M.F.: FARADAY'S LAW

At this stage it becomes practicable to establish Faraday's law of induction in the familiar form

$$e = k\, d\Phi/dt, \qquad \ldots\ldots(5.1)$$

i.e. the instantaneous E.M.F. generated in any search coil by a change of state of the system is proportional to the rate of change of flux-linkage of the coil. Instantaneous E.M.F. is of course a generalization of the magnitude E established in Part I, consistent with the instantaneous voltage and current of Part II. Thus e is defined as instantaneous open-circuit voltage and (5.1) can be considered as an experimental law that can be established by means of experiments with, say, a cathode-ray oscillograph calibrated as a voltmeter.

It is natural at this point to see whether e, like E, satisfies Kirchhoff's second law, and therefore accounts for the transient currents indicated by the galvanometer of the fluxmeter. However, it is sufficient to note here that Kirchhoff's law can only be applied to circuits of varying current if terms involving self-inductance and capacitance are included in addition to the ΣIR of the D.C. circuit. The detailed consideration of these matters is more appropriate to the laws of alternating current circuits than to magnetism, and we shall therefore not pursue them here.

Sometimes (5.1) is quoted as a definition of Φ, and not a true experimental law: we would therefore emphasize that we have some knowledge of Φ independent of this relation. It must also be remembered that (5.1) is not a complete statement of the relation between e and Φ for any circuit. If the circuit is not wholly filamentary, but passes through an extended conductor which is in motion, as in the Faraday wheel and Lorentz apparatus, an E.M.F. can be measured even when $d\Phi/dt = 0$. The complete law must be written

$$e = -\left\{\frac{\partial\Phi}{\partial t} + \int_c (\mathbf{B} \times \mathbf{v})\, dl\right\}, \qquad \ldots\ldots(5.2)$$

where $\partial\Phi/\partial t$ represents the rate of change in the absence of motion, and $\mathbf{v}$ is the velocity of the element dl of the material in the circuit relative to the field $\mathbf{B}$: that is to say, the complete law must include *both* "change of linkage" and "flux-cutting". The Lorenz method for the absolute determination of the ohm constitutes an accurate verification of (5.2), which, therefore, we also regard as an established experimental law. In (5.2) the proportionality factor k of (5.1) has been made -1, which is of course the usual choice, and serves the very important practical purpose of defining the unit of E by reference to the units of Φ and $\mathbf{B}$. It would be tedious to trace in detail the reason for the negative sign, but it may be worth noting that it is determined solely by conventions laid down in connection with other laws, e.g. that fixing the sense in which circulations are linked positively, and that prescribing that in any circuit an E.M.F. and the current it drives shall have the same sign.

The next problem is to decide by what the field of $\mathbf{B}$ is determined. Here we meet the conception of "magnetic materials", and some difficulties rather different in kind from those we have discussed so far. These are dealt with in the following paper.

The Experimental Basis of Electromagnetism—Part IV: Magnetic Materials

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ABSTRACT. This paper concludes the inquiry which was undertaken by N. R. Campbell and the author with the object of elucidating the connection between the concepts and principles of electromagnetism and the experimental operations actually performed in the laboratory that constitute their factual basis. In Part III the vector $\mathbf{B}$ was established as measurable everywhere, even within solid bodies. The vector $\mathbf{H}$ and the scalar $\mu=\mathbf{B}/\mathbf{H}$ are now shown to be measurable in special circumstances by means of the magnetometer and permeameter, but in general their values depend on a hypothesis, which is stated. The significant facts concerning the magnetic properties of real materials are briefly reviewed, and the two-fold aspect of the science is emphasized: the self-consistent mathematical theories based on postulates, on the one hand, and the complex of experimental laws on the other. It is important that the relation between the two shall be clearly established, and not merely assumed as self-evident.

§ 6. NON-MAGNETIC MEDIA*

In Part III of this inquiry we reached the stage at which $\mathbf{B}$ was established as a vector magnitude measurable independently of any other magnitude and characteristic of the states of certain systems, which we called magnets, and of position in and around them. We now proceed to consider what determines the distribution of $\mathbf{B}$, and for simplicity we shall restrict ourselves in the first instance to systems in which all the materials are non-magnetic. Since $\mathbf{B}$ is everywhere solenoidal, it can be determined experimentally at every point of the system. We can also readily show that $\mathbf{B}_0$ at any point (we shall use the suffix 0

* Section headings continue in sequence from Part III.