EXERCISE 1.1

Problem 1 :

A person went to a restaurant for dinner. In the menu card, the person saw 10 Indian and 7 Chinese food items. In how many ways the person can select either an Indian or a Chinese food?

Solution :

Number of ways of selecting Chinese food items = 7

Number of ways of selecting Indian food items = 10

Here a person may choose any one food items, either an Indian or a Chinese food. So, we have to use "Addition" to find the total number of ways for selecting the food item.

Total number of selecting Indian or a Chinese food

$$= 10 + 7 = 17 \text{ ways}$$

Problem 2 :

There are 3 types of toy car and 2 types of toy train available in a shop. Find the number of ways a baby can buy a toy car and a toy train?

Solution :

According to the given question, a baby wants to buy both toy car and toy train. So we have to use the binary operation "Multiplication" to find the total number of ways.

Types of car available in the shop = 3

Types of train available in the shop = 2

Total number of ways of buying a car = 3 (2) = 6

Problem 3 :

How many two-digit numbers can be formed using 1,2,3,4,5 without repetition of digits?

Solution :

In order to form a two digit number, we have to select two numbers out of the given 5 numbers.

___ ___

To fill up the first dash, we have 5 options.

To fill up the second dash, we have 4 options.

Number of two digit numbers formed using the above numbers = 5 (4) = 20.

Problem 4 :

Three persons enter in to a conference hall in which there are 10 seats. In how many ways they can take their seats?

Solution :

1st person may choose 1 seat out of 10 seats

2nd person may choose 1 seat out of 9 seats

3rd person may choose 1 seat out of 8 seats

Total number of ways of selecting seat = 10 (9) (8) = 720 ways

Problem 5 :

In how many ways 5 persons can be seated in a row?

Solution :

5 persons may sit in 5 seats.

1st person may sit any one of the 5 seats

2nd person may sit any one of the 4 seats and so on.

Hence the total number of ways = 5 □ 4 □ 3 □ 2 □ 1 = 120 ways

2 Problem 1 :

A mobile phone has a passcode of 6 distinct digits. What is the maximum number of attempts one makes to retrieve the passcode?

Solution :

The passcode must be formed using the following digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

Each digits must be different.

Total number of ways = 10 □ 9 □ 8 □ 7 □ 6 □ 5 = 151,200

Since we have to use distinct digits, we cannot choose the repeated numbers.
Hence the total number of ways of retrieving the passcode is 151200.

Problem 2 :
Given four flags of different colors, how many different signals can be generated if each signal requires the use of three flags, one below the other?

Solution :

1st flag may be chosen out of 4 flags, 2nd flag may be chosen out of 3 flags and 3rd flag may be chosen out of 2 flags.

Total number of ways = 4 □ 3 □ 2 = 24 ways

Hence 24 different signals may be formed using 4 flags.

Problem 3 :
Four children are running a race.

(i) In how many ways can the first two places be filled?

(ii) In how many different ways could they finish the race?

Solution :

(i) Out of 4 children, any one may get first prize. Out of three children, any one may get the second prize.

Hence the total number of ways = 4 (3) = 12 ways.

(ii) Out of 4 ----> Any one may get 1st prize

Out of 3 ----> Any one may get 2nd prize

Out of 2 ----> Any one may get 3rd prize

Remaining 1 will get fourth place.

Hence total number of ways = 4 □ 3 □ 2 □ 1 = 24 ways

Problem 4 :
Count the number of three-digit numbers which can be formed from the digits 2, 4, 6, 8 if (i) repetitions of digits is allowed. (ii) repetitions of digits is not allowed

Solution :

Required three digit number

___ ___ ___

(i) repetitions of digits is allowed

Since repetition of digits is allowed, we have 4 options to fill each places.

Hence the numbers to be formed with the given digits are = 4 □ 4 □ 4 = 64

(ii) repetitions of digits is not allowed

Hundred place :

We may use any of the digits (2, 4, 6, 8), so we have 4 options.

Tens place :

Repetition is not allowed.So, we have 3 options.

Unit place :

By excluding the number used in the hundreds and tens place, we have 2 options.

Hence total numbers to be formed = 4 □ 3 □ 2 = 24

Question 5 :

How many three-digit numbers are there with 3 in the unit place? (i) with repetition (ii) without repetition.

Solution :

The following numbers are used to form any number.

0, 1, 2, 3, 4, 5, 6, 7, 8, 9

By using the above numbers, we have to form a three digit number. In which unit place must contain the number "3".

(i) with repetition

Unit place ----> must be 3 ---> 1 option

Ten's place :

Including 3, we have 10 options

Hundred's place :

It should not contain 0. So, we have 9 options

Total numbers can be formed = 9 □ 10 □ 1 = 90

(ii) without repetition. ______ ______ ______

Unit place ----> must be 3 ---> 1 option

Hundred's place :

By excluding 3 and 0, we have 8 options.

Ten's place :

Excluding 3 and the number filled in the hundreds place, we have 8 options.

Total numbers can be formed = 8 □ 8 □ 1 = 64

Question 6 :

How many numbers are there between 100 and 500 with the digits 0, 1, 2, 3, 4, 5 ? if (i) repetition of digits allowed (ii) the repetition of digits is not allowed.

Solution :

We must form a three digit number.

(i) repetition of digits allowed

Unit place --> It may contain any of 6 numbers --> we have 6 options

Tens place --> It may contain any of 6 numbers --> we have 6 options

Hundreds place --> excluding 0 and 5 --> we have 4 options

Note : Any three digit number starts with 0. It should not 5, because we have to form a three digit number less than 500.

Total numbers can be formed = 4 □ 6 □ 6 = 144

(ii) the repetition of digits is not allowed.

Hundreds place :

Excluding 0 and 5 --> we have 4 options

Tens place :

Since repetition in not allowed including 0, we have 5 options.

Unit place :

Excluding the number filled in the tens and hundreds place, we have 4 options.

Total numbers can be formed = 4 □ 5 □ 4 = 80

Question 7:

Question 8:

Count the numbers between 999 and 10000 subject to the condition that there are (i) no restriction. (ii) no digit is repeated. (iii) at least one of the digits is repeated

Solution :

The numbers lie between 999 to 10000 must be a 4 digit number.

— — — —

(i) no restriction.

Thousands place :

Other than 0, we have 9 options to fill up the thousands place.

Hundreds place :

Including 0, we have 10 options.

Tens place :

Including 0, we have 10 options.

Unit place :

Including 0, we have 10 options.

Hence total numbers lie between 999 and 10000 is 9 □ 10 □ 10 □ 10 = 9000

(ii) no digit is repeated.

Thousands place :

Other than 0, we have 9 options to fill up the thousands place.

Hundreds place :

Here we may accept 0, but we should not use the number that we have already used in the thousands place. So, we have 9 options.

Tens place :

We have 8 options.

Unit place :

We have 7 options.

So, the total numbers are 9 □ 9 □ 8 □ 7 = 4536

(iii) at least one of the digits is repeated

At least one of digits is repeated means, we may have one repeated digit, two repeated digits or more.
In order to find the answer for this question, we have to subtract no digits repeated from total numbers.
Hence the answer is 9000 - 4536 = 4464.

Question 9 :

How many three-digit numbers, which are divisible by 5, can be formed using the digits 0, 1, 2, 3, 4, 5 if (i) repetition of digits are not allowed? (ii) repetition of digits are allowed?

Solution :

— — —

(i) repetition of digits are not allowed?

Case 1 :

If we use 0 in the unit place, we will have only one option.
In tens place, we will have 5 options.
In hundreds place, we will have 4 options.

Case 2 :

If we use 5 in the unit place, we will have only one option.
In hundreds place other than 0 and 5, we will have 4 options.
In tens place, we will have 4 options including 0.
Hence total numbers formed = 20 + 16 = 36 numbers

(ii) repetition of digits are allowed?

Unit place :

Since the required numbers are divisible by 5, it ends with 0 or 5. So, we have 2 options.

Hundreds place :

Other than 0, we will have 5 options.

Tens place :

Sine the repetition is allowed , we have 6 options.
Hence total numbers to be formed = 5 □ 6 □ 2 = 60 numbers

Question 10 :

How many numbers are there between 1 and 1000 (both inclusive) which are divisible neither by 2 nor by 5?

Solution :

We may form one digit, two digit and three digit numbers from 1 to 1000.

One digit numbers not divisible by 2 and 5

The numbers 1, 3, 7 and 9 are not divisible by both 2 and 5.= 4 numbers

Two digit numbers not divisible by 2 and 5

Since the required numbers are not divisible by bot 2 and 5, it ends with (1, 3, 7, 9)

Unit digit :

We have 4 options

Tens digit :

Other than 0, we have 9 options = 9 □ 4 = 36 numbers

Three digit numbers not divisible by 2 and 5

Since the required numbers are not divisible by bot 2 and 5, it ends with (1, 3, 7, 9)

Unit digit :

We have 4 options

Hundreds digit :

Other than 0, we have 9 options

Tens digit :

Including 0, we have 10 options = 10 □ 9 □ 4 = 360 numbers
Hence total number to be formed = 4 + 36 + 360 = 400 numbers

Question 11:

How many strings can be formed using the letters of the word LOTUS if the word (i) either starts with L or ends with S? (ii) neither starts with L nor ends with S?

Solution :

(i) either starts with L or ends with S?

Case 1 :

Number of words starts with L,

L ___ ___ ___ ___

1st 2nd 3rd 4th 5th

1st dash --> We have 1 option (L)
2nd dash --> We have 4 options

3^{rd} dash --> We have 3 options

4^{rd} dash --> We have 2 options

5^{rd} dash --> We have 1 option

= 1 □ 4 □ 3 □ 2 □ 1

= 24 words

Case 2 :

Number of words ends with S,

___ ___ ___ ___ S

1^{st} 2^{nd} 3^{rd} 4^{th} 5^{th}

5^{rd} dash --> We have 1 option (S)

1^{st} dash --> We have 4 options

2^{nd} dash --> We have 3 options

3^{rd} dash --> We have 2 options

4^{rd} dash --> We have 1 options

= 4 □ 3 □ 2 □ 1 □ 1

= 24 words

Case 3 :

Number of words starts with L and ends with S.

L □ 3 □ 2 □ 1 □ S

= 6

Either starts with L or ends with S = 24 + 24 - 6 = 42 words

(ii) neither starts with L nor ends with S?

Total number of words formed without restriction = 5!

= 120 words

Number of words starts with L nor S

= Total number of words - Number of words either starts with L or ends with S

= 120 - 42

= 78 words

Question 12:

(i) Count the total number of ways of answering 6 objective type questions, each question having 4 choices.

(ii) In how many ways 10 pigeons can be placed in 3 different pigeon holes ?

(iii) Find the number of ways of distributing 12 distinct prizes to 10 students?

Solution :

(i) Each question is having 4 options.There are 4 ways to answer each question.

1^{st} question = 4 ways

2^{nd} question = 4 ways ....

Total number of ways = 4^6

(ii) In how many ways 10 pigeons can be placed in 3 different pigeon holes ?

Note : If n different objects are to be placed in m places, then the number of ways of placing is m

10 - Number of pigeons (different things)

3 - Number of places (pigeon holes)

Applying the formula m^n,

Total number of ways = 3^{10}

(iii) 12 - Number of prizes (different things) 10 - Number of place (students)

Applying the formula m^n,

Total number of ways = 10^{12}

Problem 13:

Find the value of

(i) 6!

(ii) 4! + 5!

(iii) 3! – 2!

(iv) 3! × 4!

(v) 12! / (9! × 3!)

(vi) (n + 3)! / (n + 1)!

Solution :

(i) 6! = 6□ 5 □ 4 □ 3 □ 2 □ 1

= 720

(iii) 3! – 2!

= 3 □2! - 2!

= 2! (3 - 1)

= 2 (2)

= 4

(v) 12! / (9! × 3!)

= (12□11□10□9!) /(9! □ 3!)

= (12□11□10)/(3□2□1)

= 2 □ 11 □ 10

= 220

(ii) 4! + 5!

= 4! + 5 □ 4!

= 4! (1 + 5)

= 24 (6)

= 144

(iv) 3! × 4!

= (3 □ 2 □ 1) (4 □ 3 □ 2 □ 1)

= 6 □ 24

= 144

(vi) (n + 3)! / (n + 1)!

= (n+3)(n+2)(n+1)!/(n+1)!

= (n+3)(n+2)

= n^2 + 3n + 2n + 6

= n^2 + 5n + 6

Problem 14

Evaluate n! / r!(n – r)! when

(i) n = 6, r = 2 (ii) n = 10, r = 3 (iii) For any n with r = 2.

Solution :

(i) n! / r!(n – r)! = 6!/(2! □ 4!) = (6 □ 5 □ 4!)/(2! □ 4!) = (6 □ 5) / 2 = 15

(ii) n = 10, r = 3

n! / r!(n – r)! = 10!/(3! □ 7!) = 120

(iii) For any n with r = 2.

n! / r!(n – r)!

= n!/2!(n - 2)! = [n(n - 1)(n-2)!] / [2!(n - 2)!] = n (n - 1) / 2

Problem 15:

Find the value of n if

(i) (n + 1)! = 20(n – 1)! (ii) (1/8!) + (1/9!) = (n/10!)

Solution :

(i) (n + 1)! = 20(n – 1)!

(n + 1) n (n - 1)! = 20 (n - 1)!

n(n + 1) = 20

n^2 + n - 20 = 0

(n - 4)(n + 5) = 0

n = 4 and n = -5

Hence the answer is 4.

(ii) (1/8!) + (1/9!) = (n/10!)

(1/8!) + (1/9)(1/8!) = (n/90)(1/8!)

1 + (1/9) = n/90

10/9 = n/90

n = 100

Ex 1.1B

Question

1.a If ${}^{(n-1)}P_3 : {}^{n}P_4$, find n:

Solution:

$$\frac{(n-1)!}{(n-4)!} \times \frac{(n-4)!}{n!} = \frac{1}{10}$$ i.e. n = 10

1. b $\frac{x}{10!} = \frac{1}{8!} + \frac{1}{9!}$,

SOLUTION On multiplying both sides by 10 !, we get

$$n = \frac{10!}{8!} + \frac{10!}{9!} \Rightarrow n = \frac{10 \times 9 \times (8!)}{(8!)} + n\frac{10 \times (9!)}{(9!)}$$

$$\Rightarrow n = (10 \times 9) + 10 = (90 + 10) = 100.$$

Hence, $x = 100$.

c. $(n+1)! = 90 \times (n-1)!$, *find n.*

SOLUTION We have

$$(n+1)! = 90 \times (n-1)!$$

$$\Rightarrow (n+1) \times n \times (n-1)! = 90 \times (n-1)!$$

$$\Rightarrow (n+1)n = 90$$

$$\Rightarrow (n+1)n = 10 \times 9$$

[writing 90 as product of two consecutive integers]

$$\Rightarrow n = 9.$$

Hence, $n = 9$.

e. If $(n+1)! = 12 \times (n-1)!$, find the value of n.

$$(n+1)n \times (n-1)! = 12 \times (n-1)! \Rightarrow (n+1) \times n = 12 = 4 \times 3 \Rightarrow n = 3.$$