

# Hypothesis Testing

INFERENTIAL STATISTICS

# Inferential statistics

- Inferential statistics allow us to directly test our hypothesis by evaluating our research question based on a sample, with the goal of generalizing the results to the larger population from which the sample was drawn.
- Statistical hypothesis testing is defined as assessing evidence provided by the raw data in favor of or against each hypothesis about the population.
- Probability is the underlying foundation of all statistical methods, so we must talk about it first.

# Probability

- Ideally, the sample should be random so that it might better represent the entire population.
- Not all random samples are ideal, no random sample will be the exactly the same as any other.
- We cannot know how “ideal” a random sample from a population is. This uncertainty means that we must quantify how much we expect a random sample to vary.
- This allows us to draw conclusions about the population in the face of uncertainty when we use random samples.

# Probability

- We are interested in estimating the percentage of US adults who favour the death penalty.
- We choose a random sample of 1,200 US adults and ask their opinion.
- 744 out of 1200, or 62% are in favour.
- Our goal is to infer/draw conclusions about the opinions of US adults regarding the death penalty.
- Based on 1200 US adults can we absolutely conclude that 62% of the population favours the death penalty?

# Probability

- Another random sample could give a very different result, so we are uncertain.
- Since our sample is random we know the uncertainty is due to chance. Not due to sampling method.
- Therefore we can use probability to describe the likelihood that our sample is within the desired level of accuracy.
- How likely is it that our conclusion is within 3% of the ACTUAL percentage of ALL US adults who are in favour of the death penalty?



# Probability

- The answer to that question has an important impact on the confidence we can attach to the inference step.
- Probability is the likelihood of something occurring; of an event occurring; the chance of something happening.
- It is a mathematical description of randomness and uncertainty.
- It is a way to measure or quantify uncertainty.
- It is the official name for chance.

# Probability

- Gambling industry:
  - Cutting a deck of cards:
    - what are the chances that it going to be red?
    - what are the chances of it being a spade?
    - what are the chances of it being a King?
    - what are the chances of it being a King or a Club?
    - what are the chances of it being a King and being a Club?

# Probability

- The probability of an event ranges from 0 to 1.
- A probability of 0 means that the event has zero chances of happening. It will never occur.
- A probability of 1 means that the event will occur for certain.
- A probability of .5 indicates that the event has a 50% chance of happening. The event is as likely to occur as not to occur.



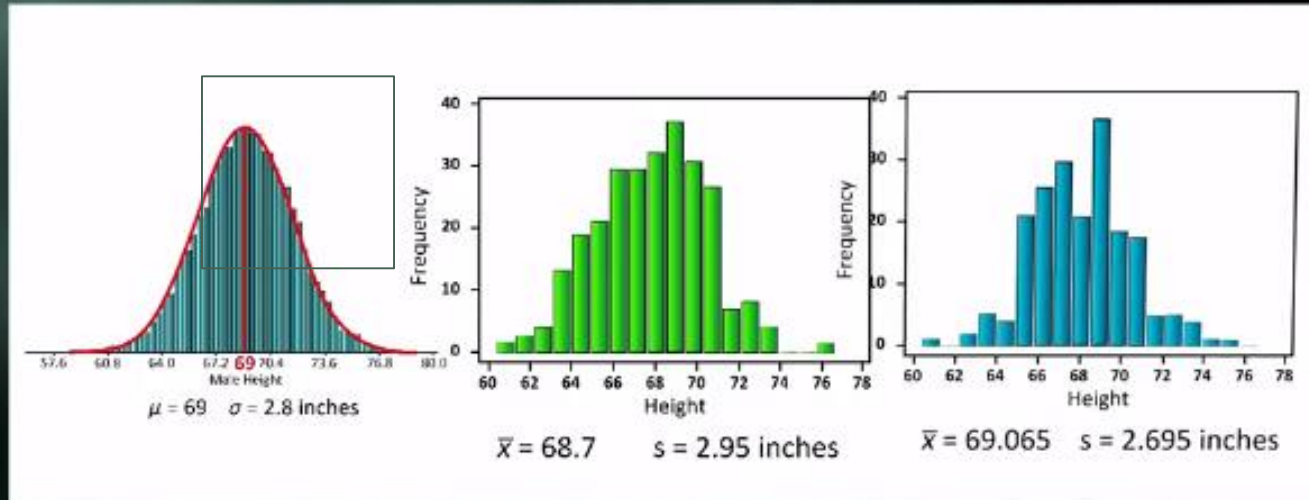
# Probability

- Any probability that is greater than .5 indicates that the event is more likely to occur than it is not to occur.
- Any probability that is less than .5 indicates that the event is more likely not to occur than it is to occur.
- Expressing probability in percentages is very common.
- The chance that an event will occur is between 0% and 100%.

# Central Limit Theorem

- The parameter is 69 inches.
- The statistic for sample 1 is 68.7 inches.
- The statistic for sample 2 is 69.25 inches

A parameter is a number that describes the population.  
A statistic is a number that is computed from a sample.

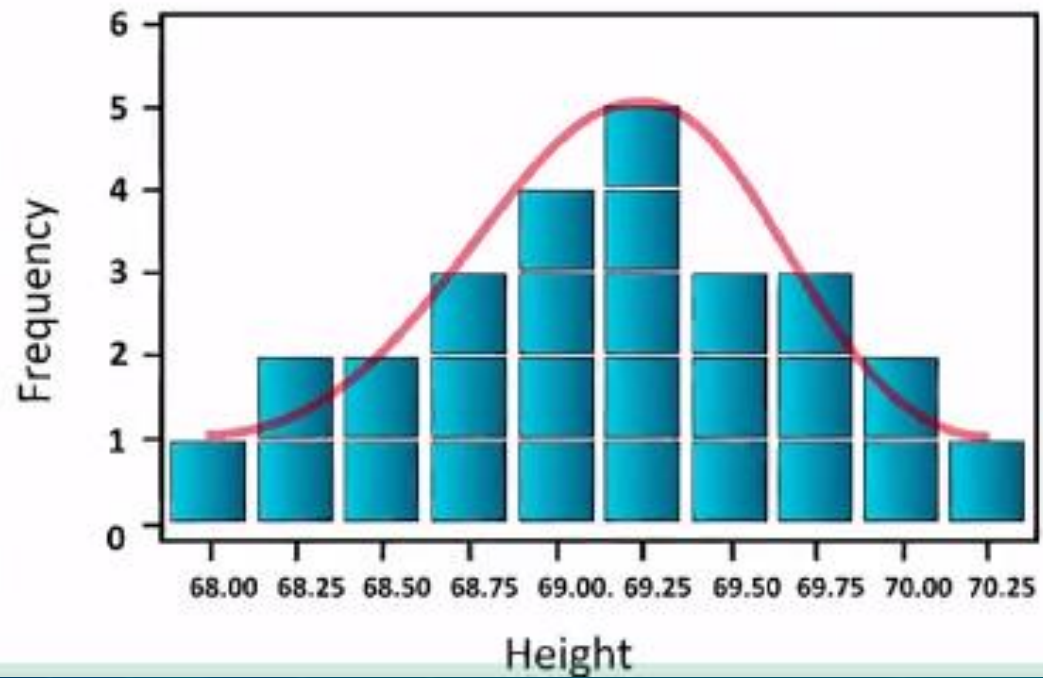


**Parameters are typically unknown.**

# Central Limit Theorem

- Sampling variability
  - As we have already stated each random sample is different. The more random samples we use then the greater the picture of the population.
  - We calculate statistics from each sample, such as mean, std dev etc
  - Suppose we know the actual mean of the total male population height to be 69 inches. We select 30 random samples from the population, each sample size being 500.
  - Each sample would have slightly different mean, 68.5, 69.25 etc.
  - If we plotted these together we would have a normal distribution.

# Central Limit Theorem



# Central Limit Theorem

- As long as adequately large samples and an adequately large number of samples are drawn from a population, the distribution of the statistics of the samples, whether of mean, proportion, standard deviation, or any other statistic, will be normally distributed.



# Hypothesis Testing

- This method is used when we want to make decisions concerning populations on the basis of only sample information.
- Analysis of variance test, ANOVA, Chi-Square test of independence.
- Same steps:
  - Specifying null hypothesis  $H_0$
  - specifying the alternate hypothesis  $H_a$
  - Choose sample, assess evidence, draw conclusion.

# Hypothesis Testing

- NESARC data set
- Representative sample of 43,093 adults in U.S
- Evaluate if there is an association between a diagnosis of major depression and how much a person smokes.
- Step 1: Hypotheses
  - $H_0$  There is no difference in smoking quantity between people with and without depression.
  - $H_a$  There is a difference in smoking quantity between people with and without depression.

# Hypothesis Testing

- The null hypothesis says that nothing special is going on between the two variables, they are unrelated.
- The alternative says there is a relationship, it could be positive or negative.
- Step 2 Choose a sample:
  - We are going to use the nesarc data set. We are going to evaluate the hypothesis among individuals who are smokers and who are younger (aged between 18 and 26).
  - The sample size is 1859

# Hypothesis testing

- Assess evidence – draw conclusion
- The p-value for a test is an estimate of how often we would get the obtained result by chance if in fact the null hypothesis is true.
- The mean number of cigarettes smoked is 11.40 std dev 9.67 for those with depression.
- the mean number of cigarettes smoked is 10.31 std dev 8.88 for those without depression.
- The probability of getting a difference of 1.09 cigarettes smoked per day in a random sample of 1859 is roughly 2%
- This means if we took 100 different random samples from the population and we rejected the null hypothesis each time, we would be wrong 2 times. (there would be 2 times the results would show a relationship but in fact there is none).
- Is 2% small enough to reject the null hypothesis?