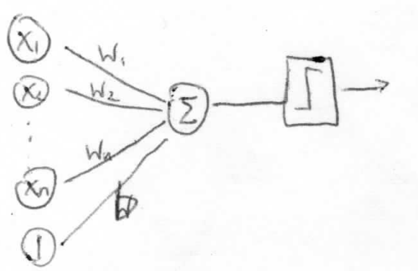
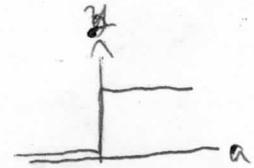
CODEN 296A HW3 Yuan-lin Hsu W1279028

1.



$$a = \sum_i w_i x_i + b$$

$$y = \begin{cases} 1 & a \geq 0 \\ 0 & 0 \end{cases}$$



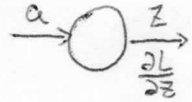
$$\frac{\partial L}{\partial w_i} = \frac{\partial L}{\partial y} \cdot \frac{\partial y}{\partial w_i}$$

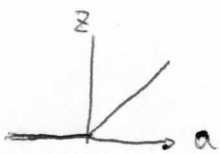
$$\frac{\partial y}{\partial w_i} = \underbrace{\frac{\partial y}{\partial a}}_{0} \cdot \frac{\partial a}{\partial w_i}$$

It turns out the gradient is always 0.

2.

$$z = \begin{cases} a & a \geq 0 \\ 0 & a < 0 \end{cases}$$

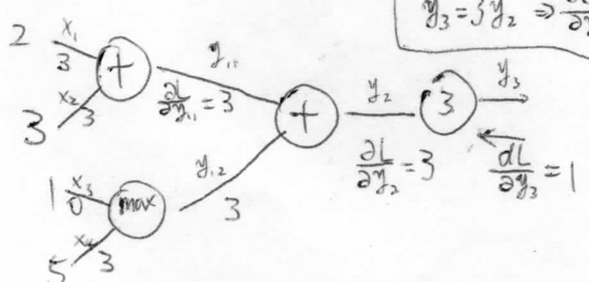




$$\frac{\partial L}{\partial a} = \frac{\partial L}{\partial z} \cdot \frac{\partial z}{\partial a} = \begin{cases} \frac{\partial L}{\partial z} & a \geq 0 \\ 0 & a < 0 \end{cases}$$

$$\frac{dz}{\partial a} = \begin{cases} 1 & a \geq 0 \\ 0 & a < 0 \end{cases}$$

3.



$$y_3 = 3y_2 \Rightarrow \frac{\partial y_3}{\partial y_2} = 3$$

$$\Rightarrow \frac{\partial L}{\partial y_2} = \frac{\partial L}{\partial y_3} \cdot \frac{\partial y_3}{\partial y_2} = 3$$

$$y_2 = y_{11} + y_{12}$$

$$\frac{\partial y_2}{\partial y_{11}} = 1 \quad \Rightarrow \frac{\partial L}{\partial y_{11}} = \frac{\partial L}{\partial y_2} \cdot \frac{\partial y_2}{\partial y_{11}} = 3 \cdot 1 = 3$$

$$\frac{\partial y_2}{\partial y_{12}} = 1 \quad \Rightarrow \frac{\partial L}{\partial y_{12}} = \frac{\partial L}{\partial y_2} \cdot \frac{\partial y_2}{\partial y_{12}} = 3$$

$$y_{11} = x_1 + x_2$$

$$\vdots$$

$$\frac{\partial L}{\partial x_1} = 3$$

$$\frac{\partial L}{\partial x_2} = 3$$

$$y_{12} = \max(x_3, x_4) = \begin{cases} x_3 & x_3 \geq x_4 \\ x_4 & x_3 < x_4 \quad \checkmark \end{cases}$$

$$5 = x_4 > x_3 = 1$$

$$y_{12} = x_4$$

$$\frac{\partial y_{12}}{\partial x_3} = 0 \qquad \frac{\partial y_{12}}{\partial x_4} = 1 \quad \Rightarrow \quad \frac{\partial L}{\partial x_3} = 0 \qquad \frac{\partial L}{\partial x_4} = 3$$

Problem 4

1. Number of Parameter (Using bias)
Dense layer(input = 10, output =50) : (10+1) * 50 = 550
Relu layer: None
Dense layer(input = 50, output = 3): (50+1)*3 = 153
Softmax layer: None
Total : 703 parameters

2.
The code is attached, basically it has two files:
layer.py defines computing layers that can compute 1-D output from 1-D input.
Main.py initilize the input and target and model, perform backprop training.

layer.py
```
import numpy as np
# A layer handles one dimensional input x, apply its computation and return 1-D result y
class Layer:
	# apply layer computation
	def forward(self, x):
		pass
	#  returns dL/dx
	#  saves gradient
	def backward(self, dy):
		pass
	# adjust weight by gradient descend
	def adjust_w(self, lr):
		pass

class dense_layer(Layer):
	def __init__(self, inputNum, unitNum):
		self.w = np.random.uniform(0., 0.1, (inputNum, unitNum))
		self.b = np.random.uniform(0., 0.1, (1, unitNum))
		self.input = None
		self.gradient = None
		self.b_grad =None
	def forward(self, x):
		self.input = x
		return np.dot (x ,self.w)+self.b
	def backward(self, dy):
		self.gradient = (dy*self.input).T
		self.b_grad = dy.T
		return np.dot(self.w , dy)
	def adjust_w(self, lr):
		self.w -= lr*self.gradient
		self.b -= lr*self.b_grad

class relu_layer(Layer):
	def __init__(self):
		self.input = None
	def forward(self, x):
		self.input = x
```

```
        y = x.copy()
        y[y<0]=0
        return y
    def backward(self, dy):
        y = self.input.copy()
        y[y>=0] = 1
        y[y<0] = 0
        return dy*y.T

class softmax_layer(Layer):
    def __init__(self):
        self.input = None
        self.output= None
    def forward(self, x):
        self.input = x
        y = np.exp(x)
        self.output = y/np.sum(y)
        return self.output
    def backward(self, dy):
        res = np.dot((np.eye(dy.shape[0]) - self.output * self.output.T), dy)
        return np.dot((np.eye(dy.shape[0]) - self.output*self.output.T),dy)
```

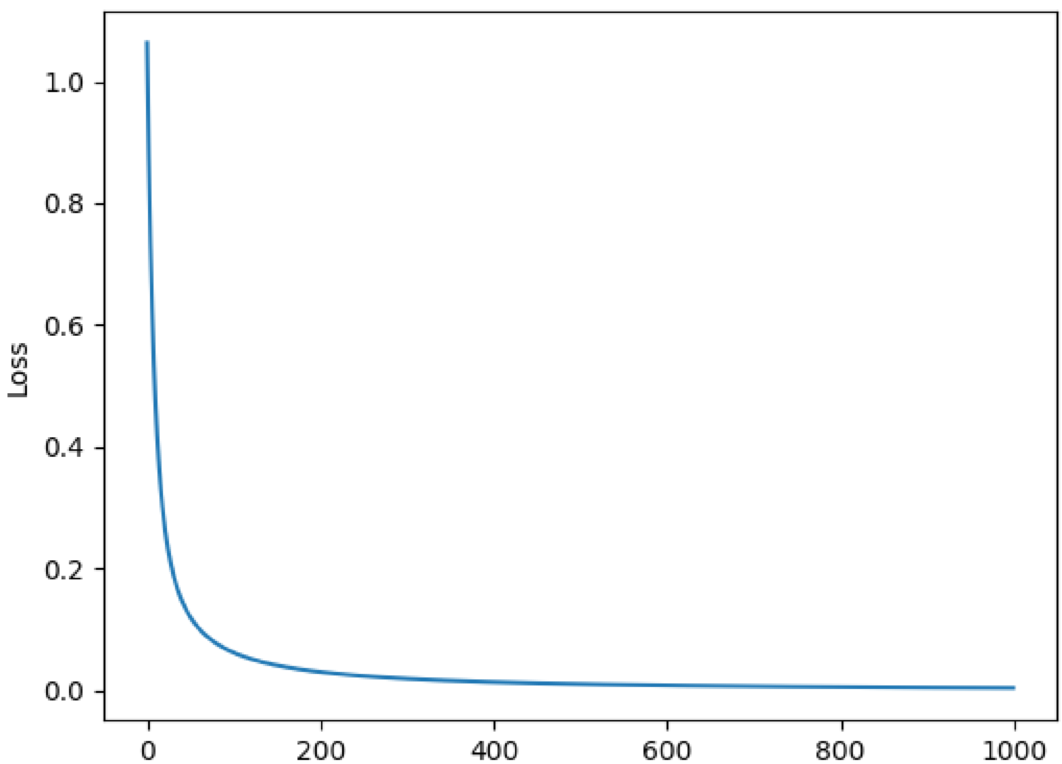
main.py

```
from layer import *
import numpy as np
import matplotlib.pyplot as plt
# Model is a list of layers
# compute the result of input_x put into model
def forward_pass(input_x, model):
    for layer in model:
        input_x = layer.forward(input_x)
    return input_x
# compute the result of dy back propagated through the model
# the result is somewhat meaningless, what's important is it calls the backward method in each layer
# that method will compute and save the gradient change of weight (if applicable)
def backward_pass(dy, model):
    for layer in model[::-1]:
        dy = layer.backward(dy)
    return dy
# Calles the adjust weight method in each layer, gradient descend.
def adjust_w(model,lr):
    for layer in model:
        layer.adjust_w(lr)
learning_rate = 0.02
num_iter =1000
test_run_times = 1
# Test run for test_run_times times
for test_time in range(test_run_times):
    # create input
    input_x = np.asarray([[0.5, 0.6, 0.1, 0.25, 0.33, 0.9 , 0.88, 0.76, 0.69, 0.95]], dtype= np.float32)
    # create target
    Target = np.asarray([[1,0,0]], dtype= np.float32)
    # create model
    model = [dense_layer(10, 50), relu_layer(), dense_layer(50, 3), softmax_layer()]
    loss_rec = []
```

```
        # training for num_iter iterations
        for i in range(num_iter):
                # forward pass
                y = forward_pass(input_x,model)
                # Loss function = -sum(t log y)
                loss_rec.append( -float(np.dot(np.log(y), Target.T)))
                dy = (y-Target).T
                backward_pass(dy,model)
                adjust_w(model, learning_rate)
        # print the output
        # print (y)
# # print out final parameter
# print ('W1--')
# print (model[0].w)
# print ('b1--')
# print (model[0].b)
# print ('W2--')
# print (model[2].w)
# print ('b2--')
# print (model[2].b)
# # plot loss function
# plt.plot(loss_rec)
# plt.ylabel('Loss')
# plt.show()
```

RESULTS: (Trained after 1000 iterations)

W1
--
[[
0.0
580
895
6
0.0
911
314
5
0.0
989
536
0.0
160
188
5
0.0
797
988
3
0.0



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 0.12812481 0.1218433 0.09617899 0.07134238 0.01483019 0.03396974

```
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```

```
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```

```
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b2--
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```