Unit 7: Multiple linear regression

4. Transformations and case study

Sta 101 - Spring 2015

Duke University, Department of Statistical Science

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Slides posted at http://bitly.com/sta101sp15

1. Housekeeping

2. Transformations

3. Case study

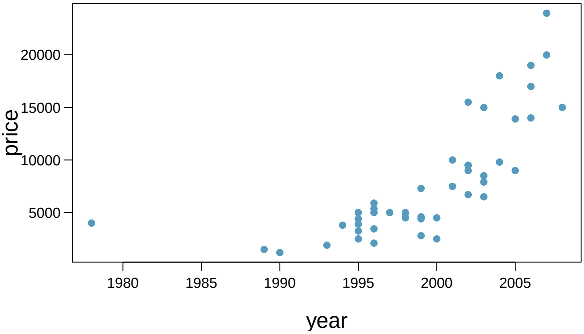
- Peer eval 3 by this Friday evening
- PA 7 opens today after class, due Friday evening
- Poster presentations will be in Link Classroom 3

1. Housekeeping

2. Transformations

3. Case study

The scatterplot below shows the relationship between year and price of a random sample of 43 pickup trucks. Describe the relationship between these two variables.



From: *http://faculty.chicagobooth.edu/robert.gramacy/teaching.html*

Let's remove trucks older than 20 years, and only focus on trucks made in 1992 or later.

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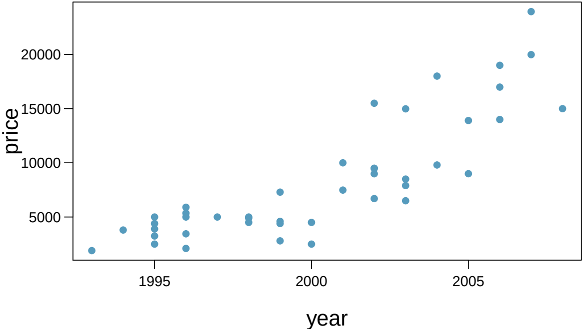
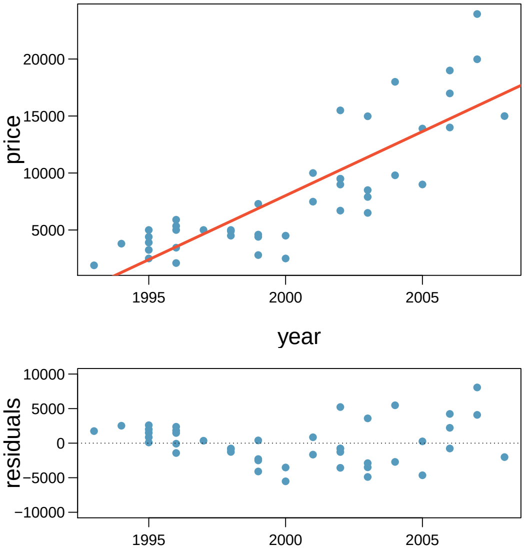
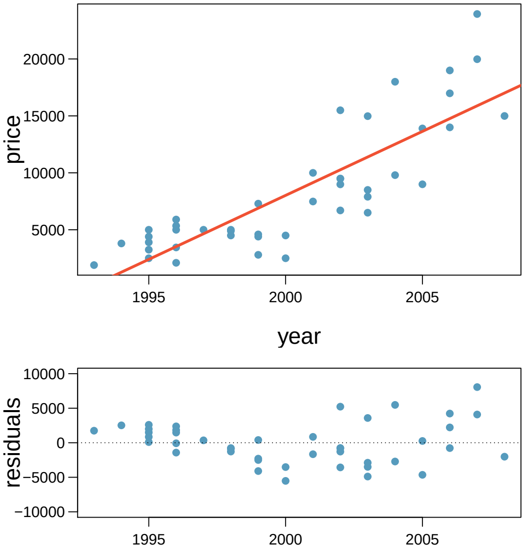
Now what can you say about the relationship?





Model:

$$\widehat{price} = b_0 + b_1 \; year$$



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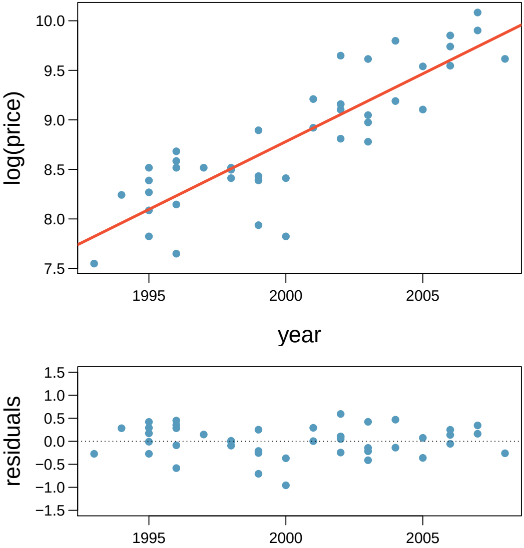
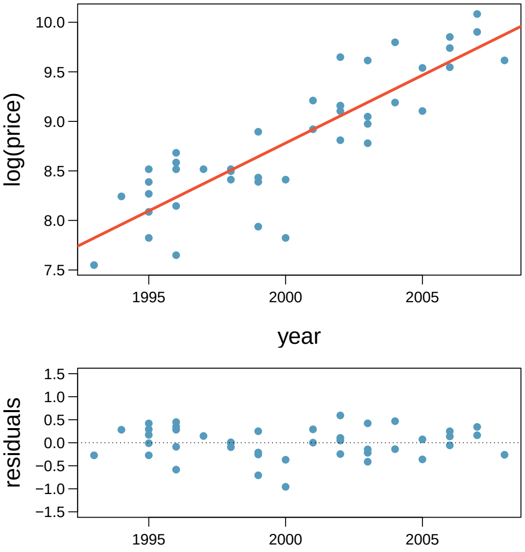
The linear model doesn't appear to be a good fit since the residuals have non-constant variance.



Model:

$$\widehat{log(price)} = b_0 + b_1 \; year$$



Model:

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We applied a log transformation to the response variable. The relationship now seems linear, and the residuals no longer have non-constant variance.

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-265.07	25.04	-10.59	0.00
pu$year	0.14	0.01	10.94	0.00

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- which is not very useful...

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- We can these identities to ``undo" the log transformation

The slope coefficient for the log transformed model is 0.14, meaning the log price difference between cars that are one year apart is predicted to be 0.14 log dollars.

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\end{aligned}
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For each additional year the car is newer (for each year decrease in car's age) we would expect the price of the car to increase on average *by a factor of 1.15*.

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- Another useful transformation is the square root: $\sqrt{y}$, especially useful when the response variable is counts.
- These transformations may also be useful when the relationship is non-linear, but in those cases a polynomial regression may also be needed.

1. Housekeeping
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1. `income`: Yearly income (wages and salaries)
2. `employment`: Employment status, not in labor force, unemployed, or employed
3. `hrs_work`: Weekly hours worked
4. `race`: Race, White, Black, Asian, or other
5. `age`: Age
6. `gender`: gender, male or female
7. `citizens`: Whether respondent is a US citizen or not
8. `time_to_work`: Travel time to work
9. `lang`: Language spoken at home, English or other
10. `married`: Whether respondent is married or not
11. `edu`: Education level, hs or lower, college, or grad
12. `disability`: Whether respondent is disabled or not
13. `birth_qrtr`: Quarter in which respondent is born, jan thru mar, apr thru jun, jul thru sep, or oct thru dec

```
download("http://stat.duke.edu/~mc301/data/acs.RData",
    destfile = "acs.RData")
load("acs.RData")
acs_sub = subset(acs, acs$employment == "employed"
    & acs$income > 0)
```

```
table(acs_sub$employment)
```

```
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```

```
not in labor force          unemployed            employed
                 0                   0                 787
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not in labor force         unemployed             employed
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```
acs_sub = droplevels(acs_sub) # overwrite acs_sub
table(acs_sub$employment)
```

```
employed
     787
```

Suppose we only want to consider the following explanatory variables: `hrs_work`, `race`, `age`, `gender`, `citizen`.

```
m_full = lm(income ~ hrs_work + race + age + gender
    + citizen, data = acs_sub)
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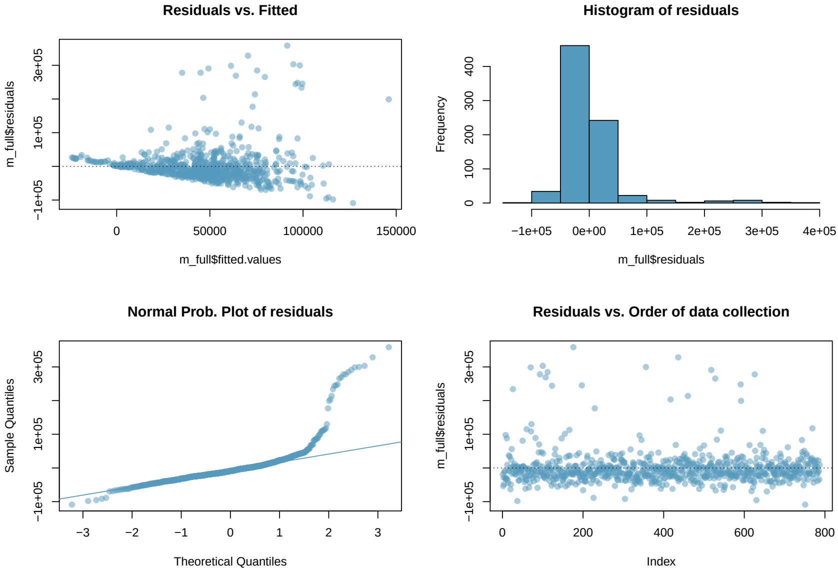
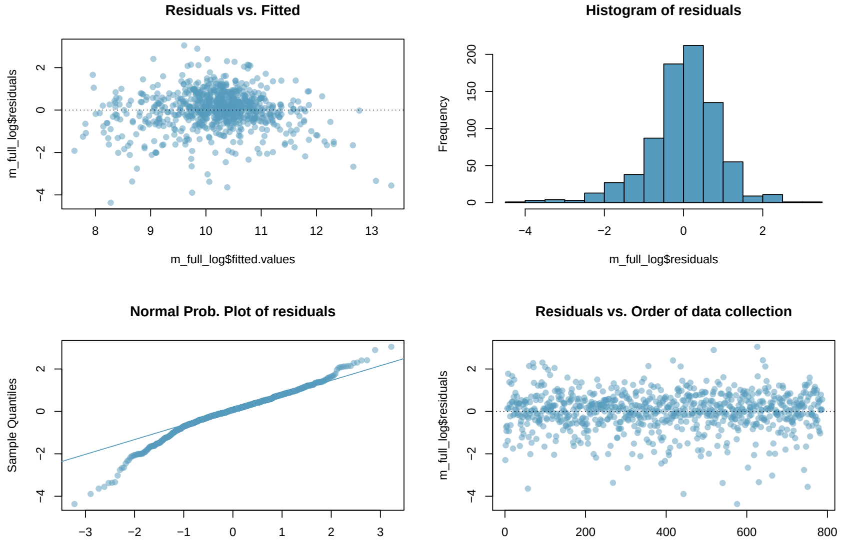
```

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-17215.60	11399.81	-1.51	0.13
hrs_work	1251.31	153.14	8.17	0.00
raceblack	-13202.39	6373.05	-2.07	0.04
raceasian	32699.34	8903.66	3.67	0.00
raceother	-12032.88	7556.78	-1.59	0.11
age	760.99	129.71	5.87	0.00
genderfemale	-17246.91	3887.17	-4.44	0.00
citizenyes	-9537.20	8360.85	-1.14	0.25

What do you think?



```
m_full_log = lm(log(income) ~ hrs_work + race + age
    + gender + citizen, data = acs_sub)
```



Application exercise: 7.4 Interpreting models with a transformed response

See course website for more details