DEPTH FIRST SEARCH

DFS

Depth-first search

- Depth-first search is a widely used graph traversal algorithm besides breadth-first search
- It was investigated as strategy for solving mazes by Trémaux in the 19th century
- It explores as far as possible along each branch before backtracking // BFS was a layer-by-layer algorithm
- Time complexity of traversing a graph with DFS: **O(V+E)**
- Memory complexity: a bit better than that of BFS !!!
- By itself the DFS isn't all that useful, but when augmented to perform other tasks such as count connected components, determine connectivity, or find bridges/articulation points then DFS really shines.

Depth-first search

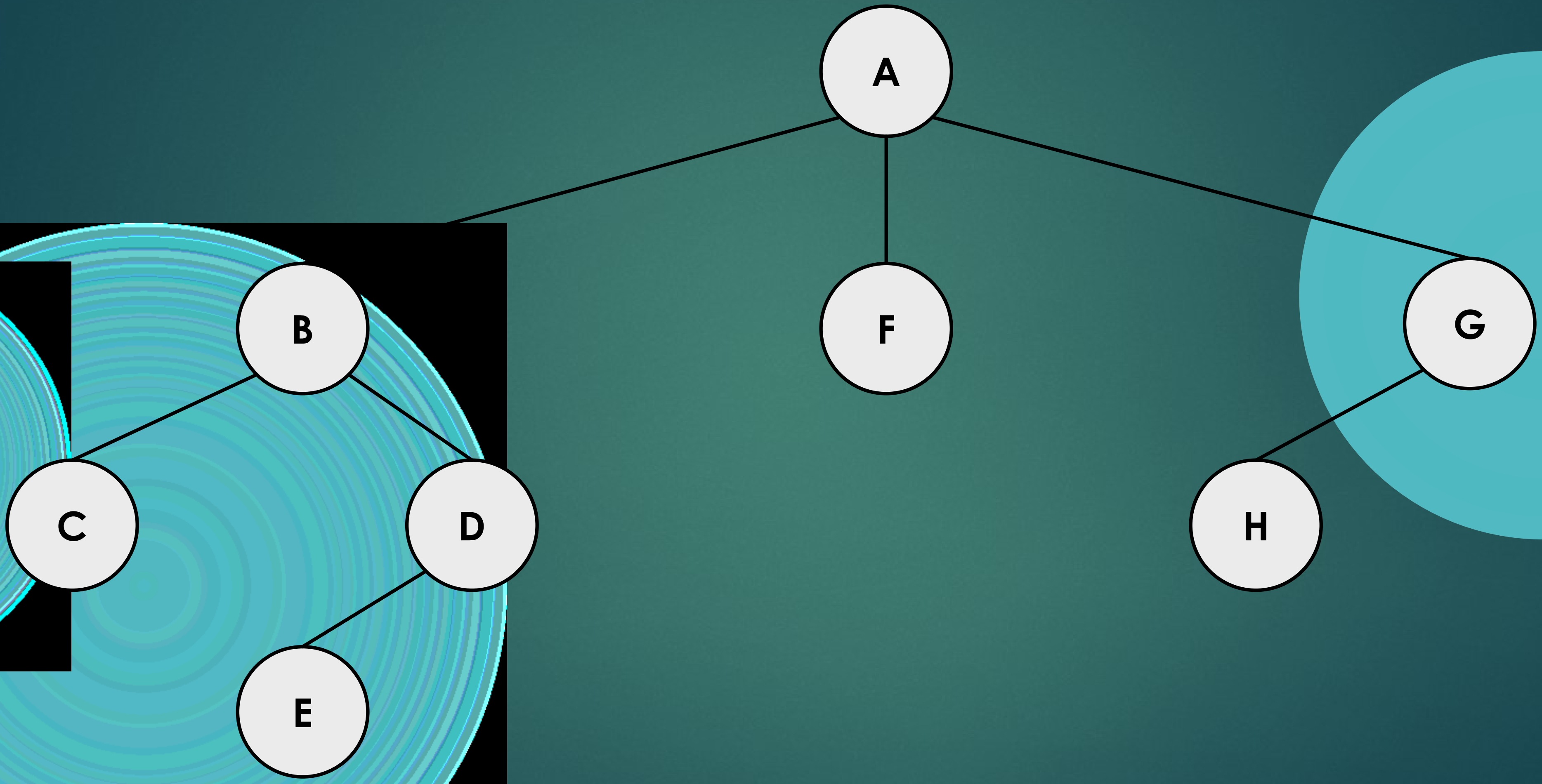
```
dfs(vertex)

        vertex set visited true
        print vertex

        for v in vertex neighbours
                if v is not visited
                        dfs(v)
```

RECURSION

```
dfs(vertex)

        Stack stack
        vertex set visited true
        stack.push(vertex)

        while stack not empty
                actual = stack.pop()

                for v in actual neighbours
                        if v is not visited
                                v set visited true
                                stack.push(v)
```

ITERATION

Recursively check every unmarked vertices!



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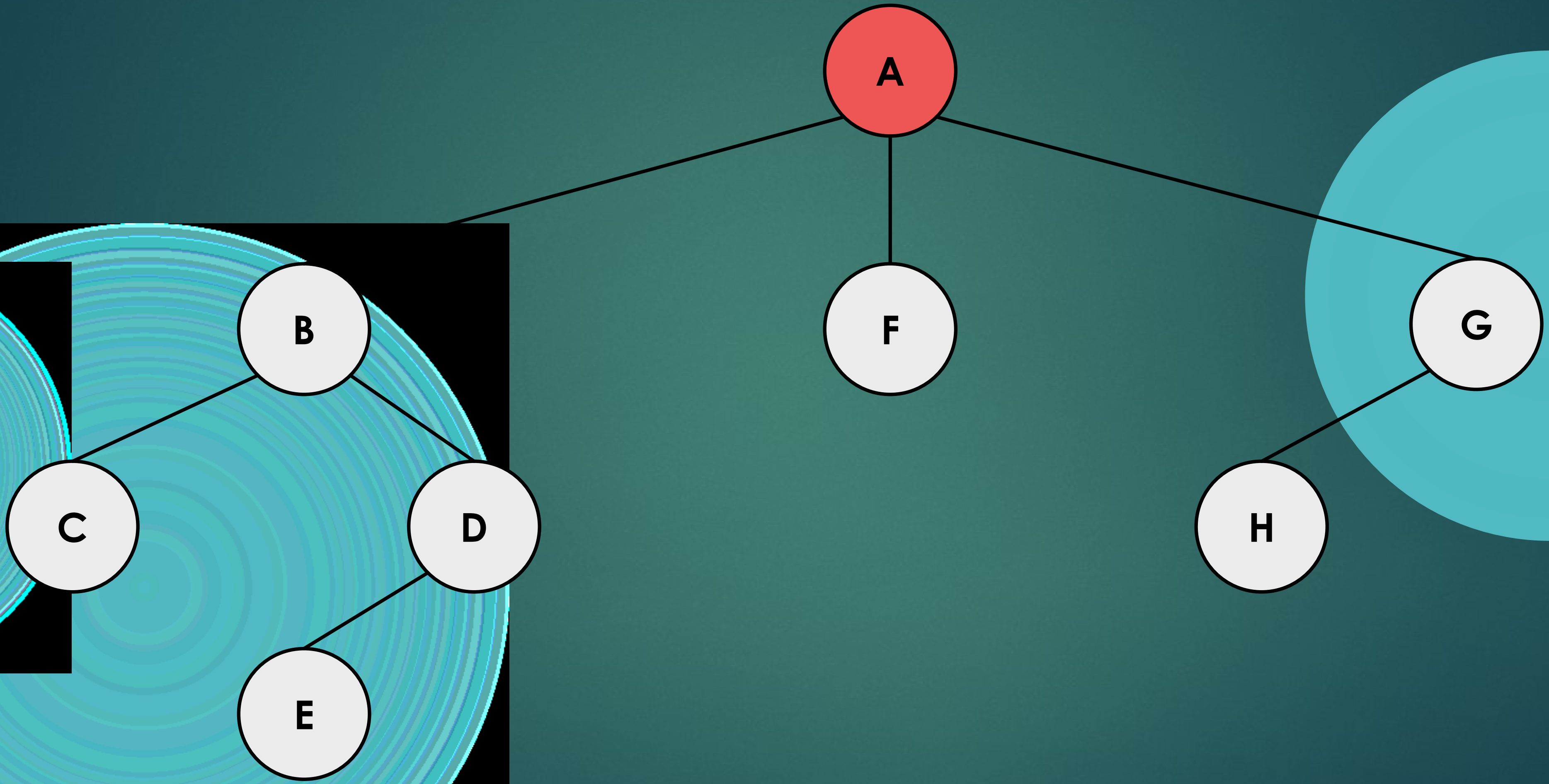

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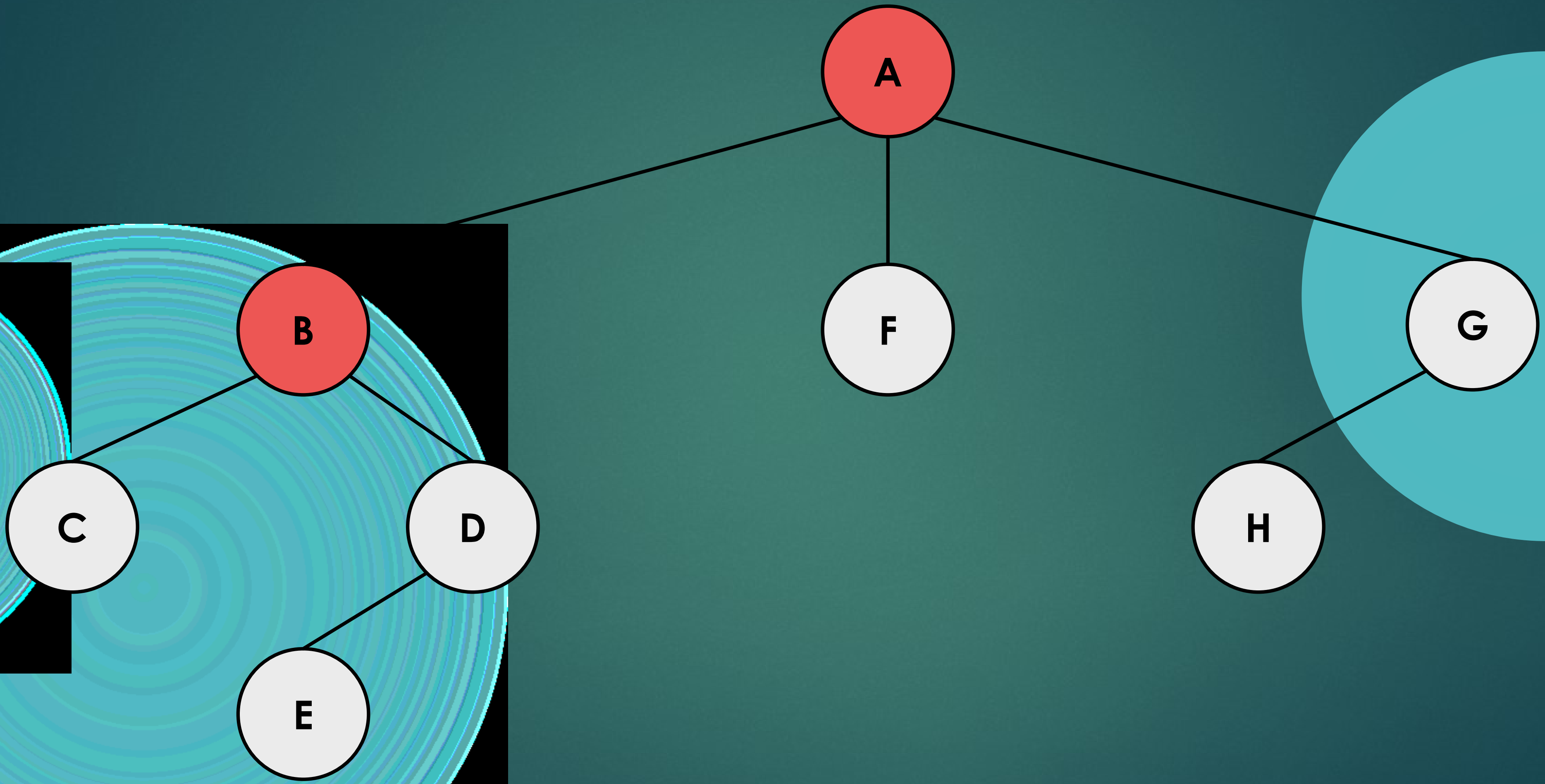

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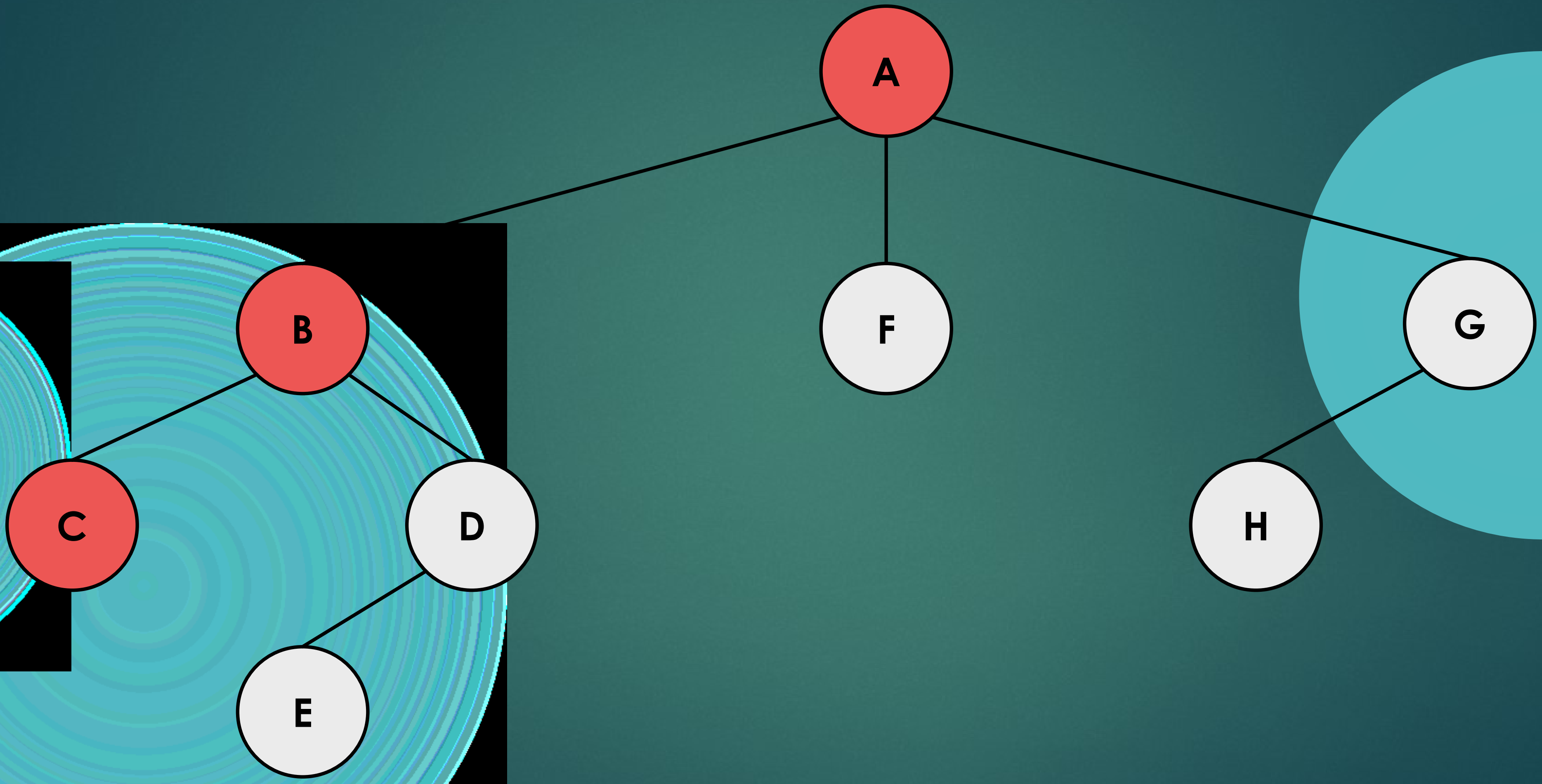

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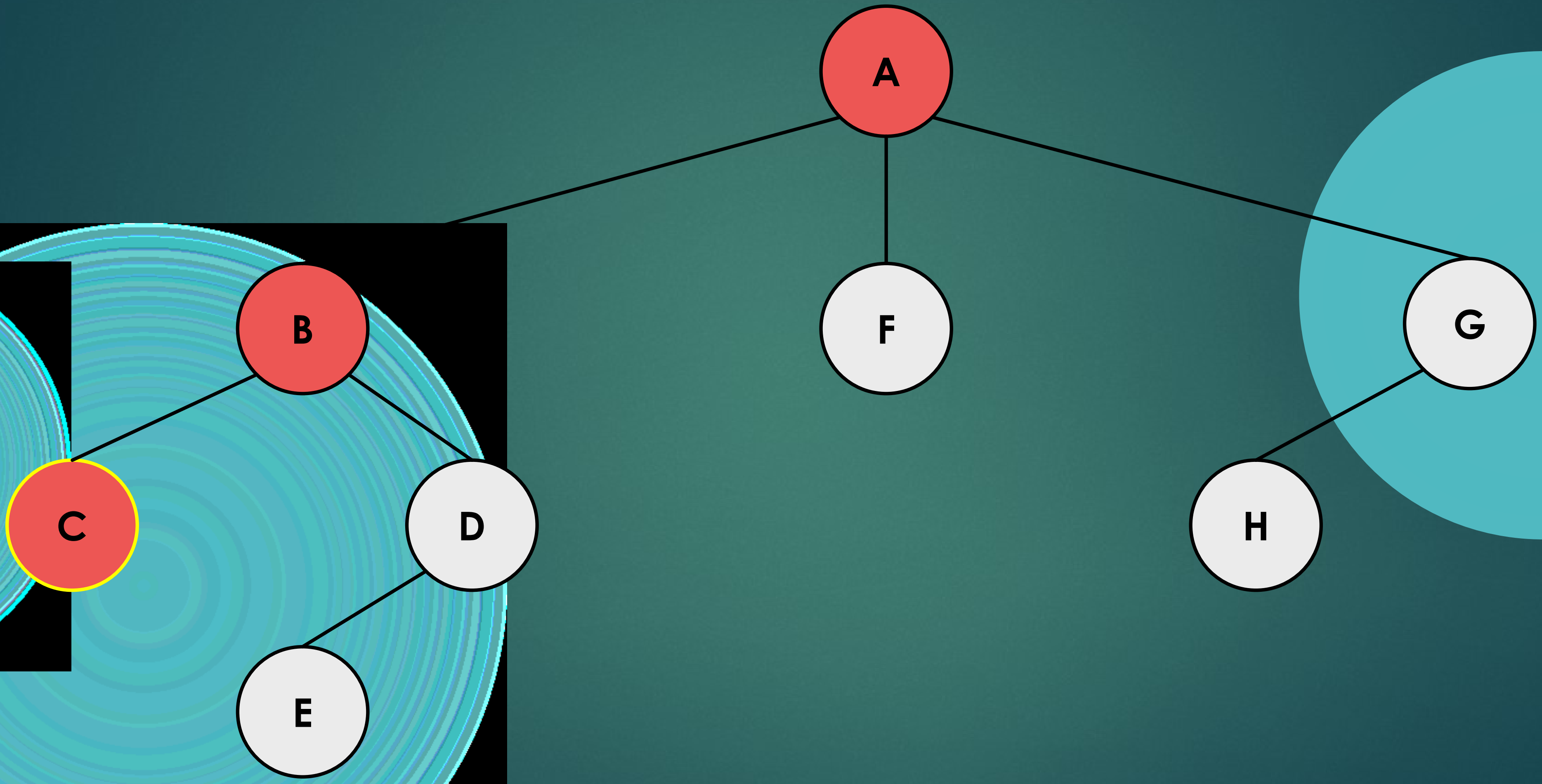

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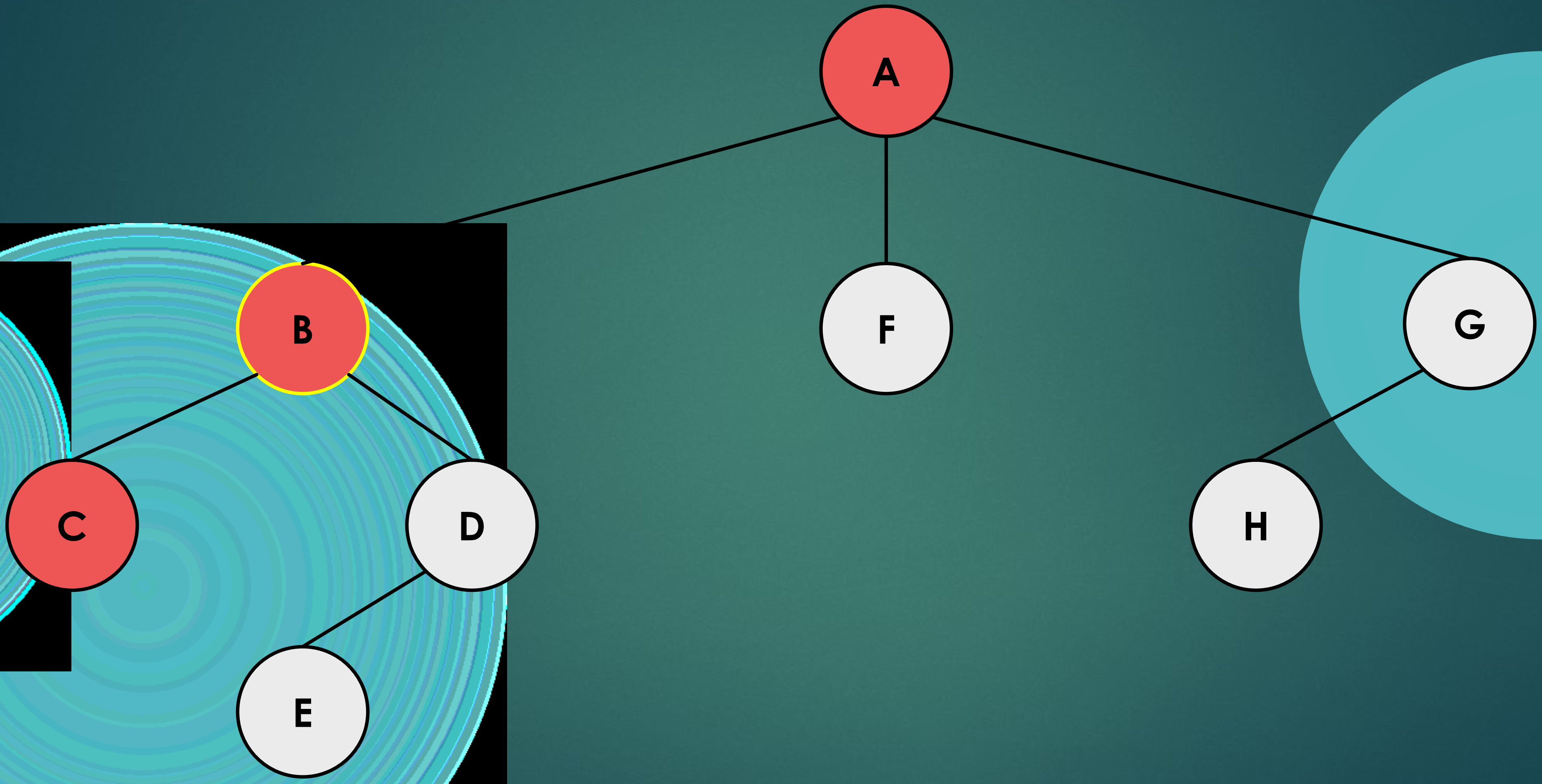

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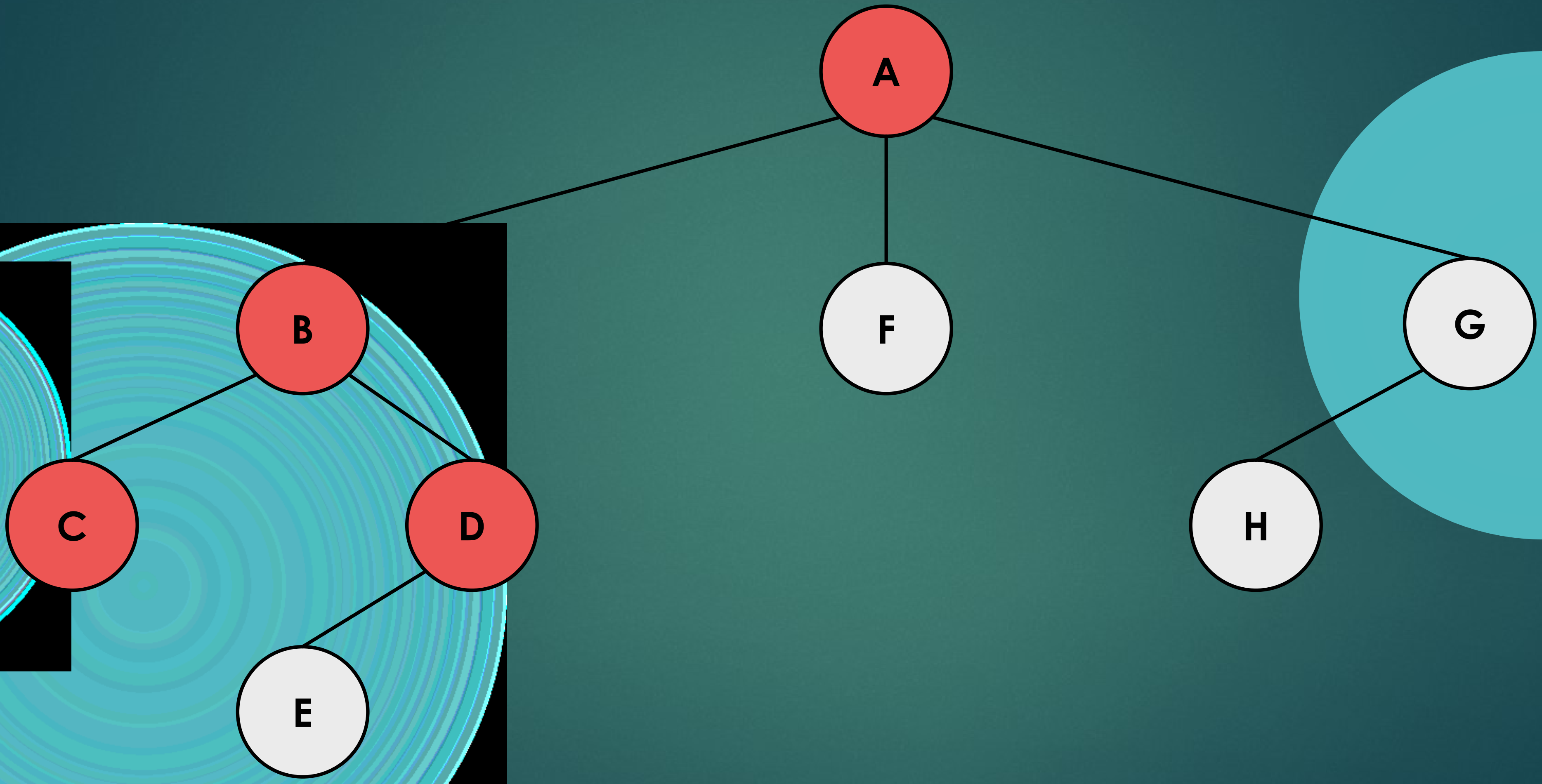

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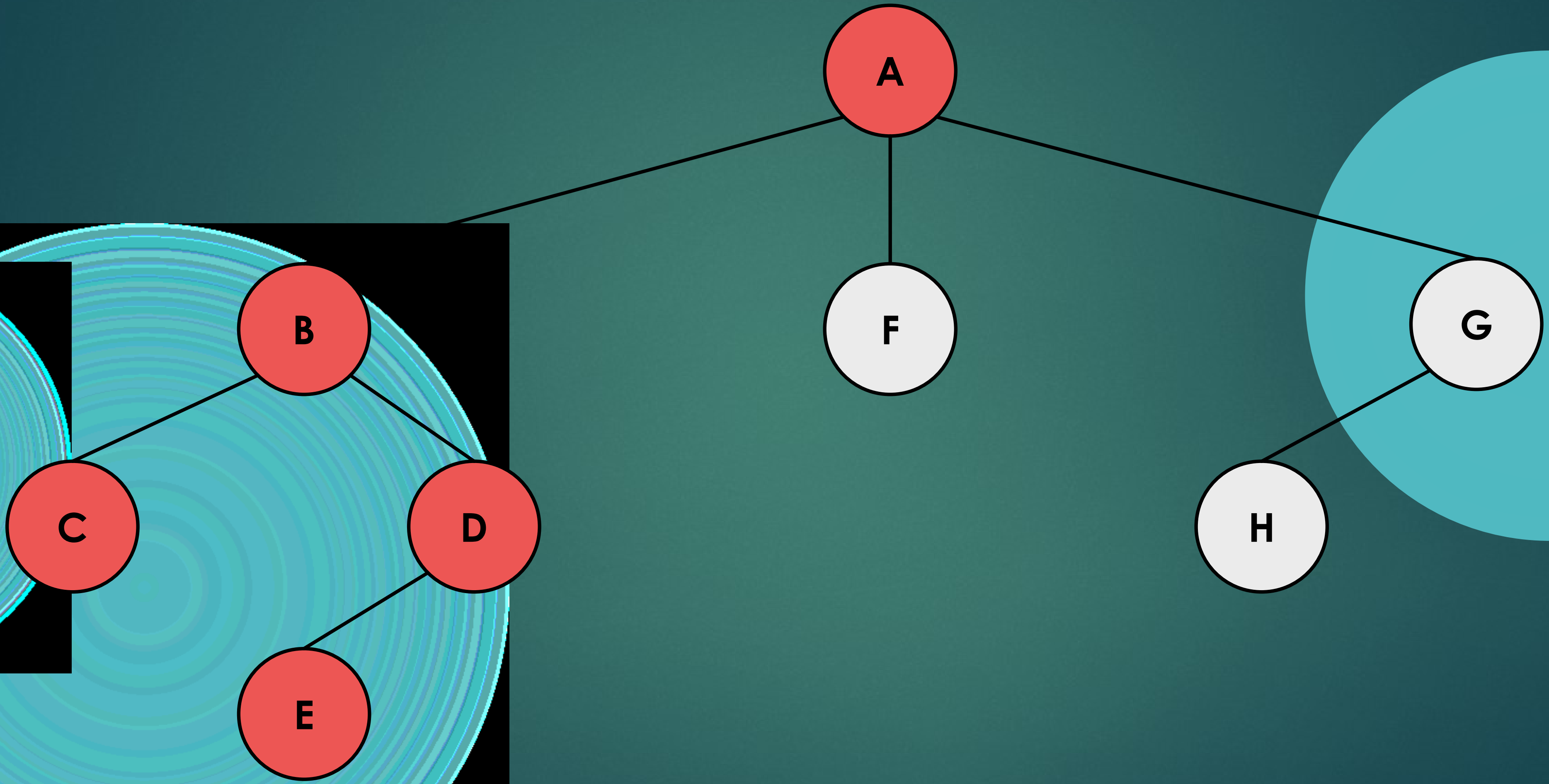

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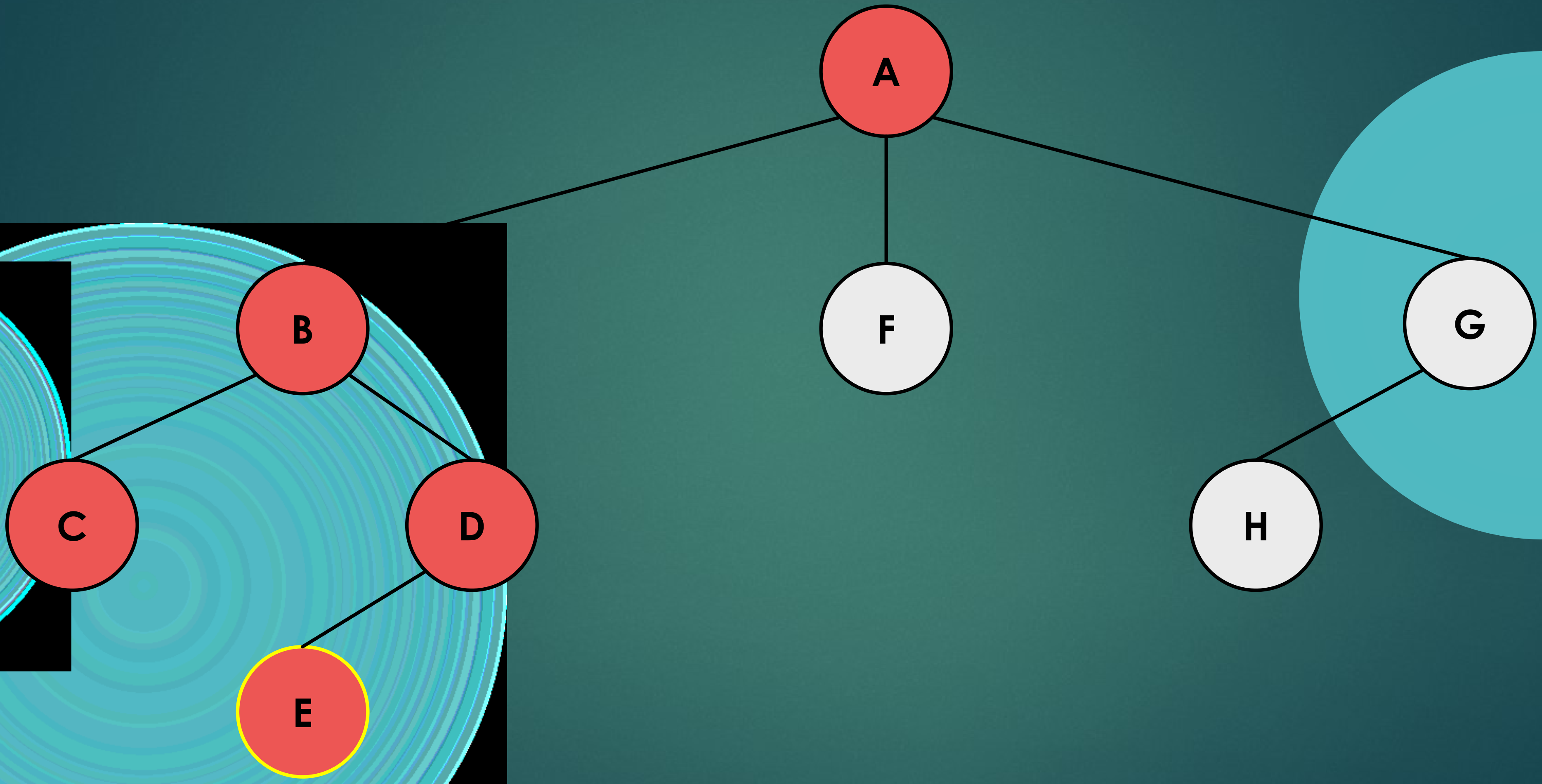

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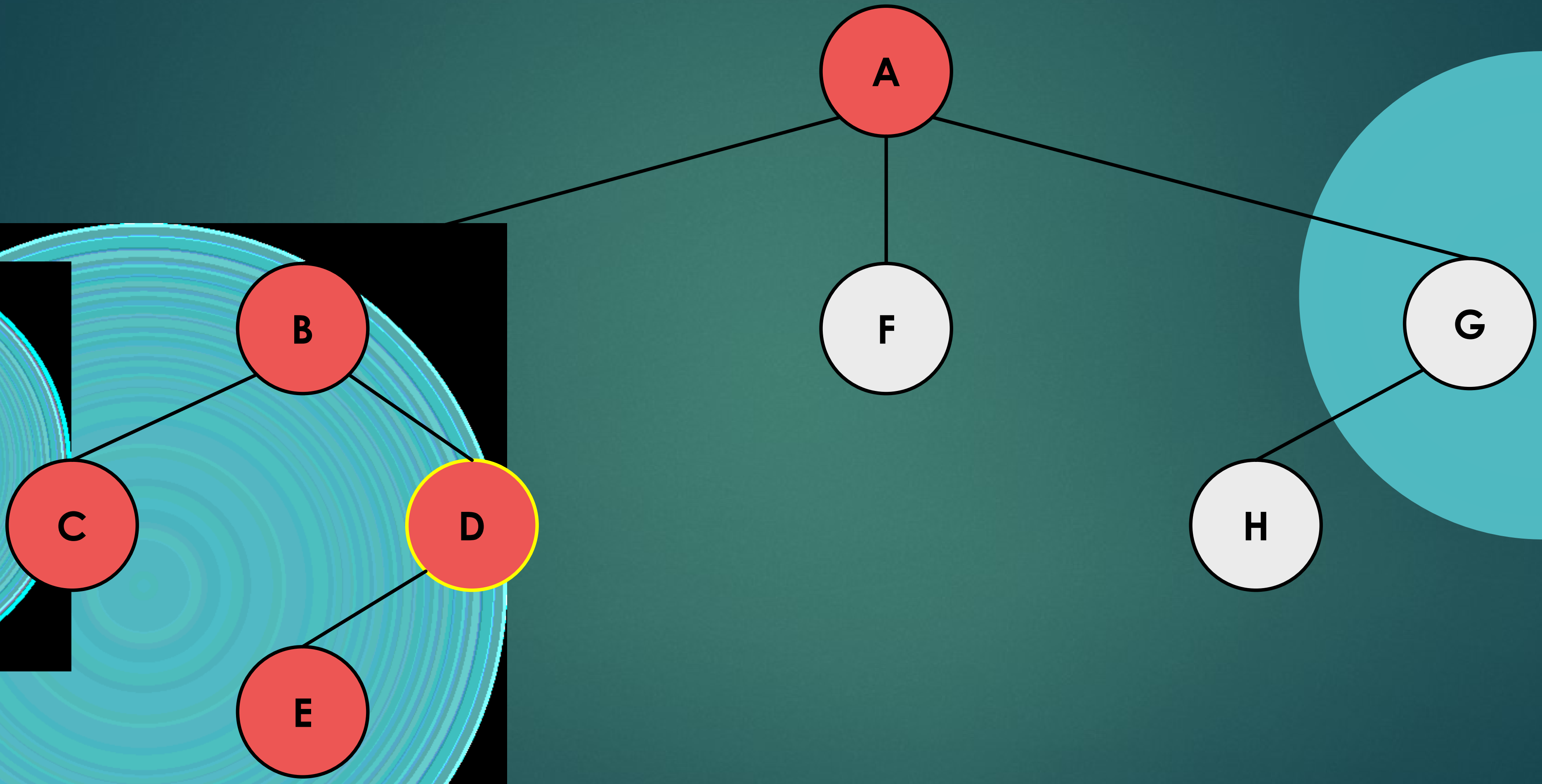

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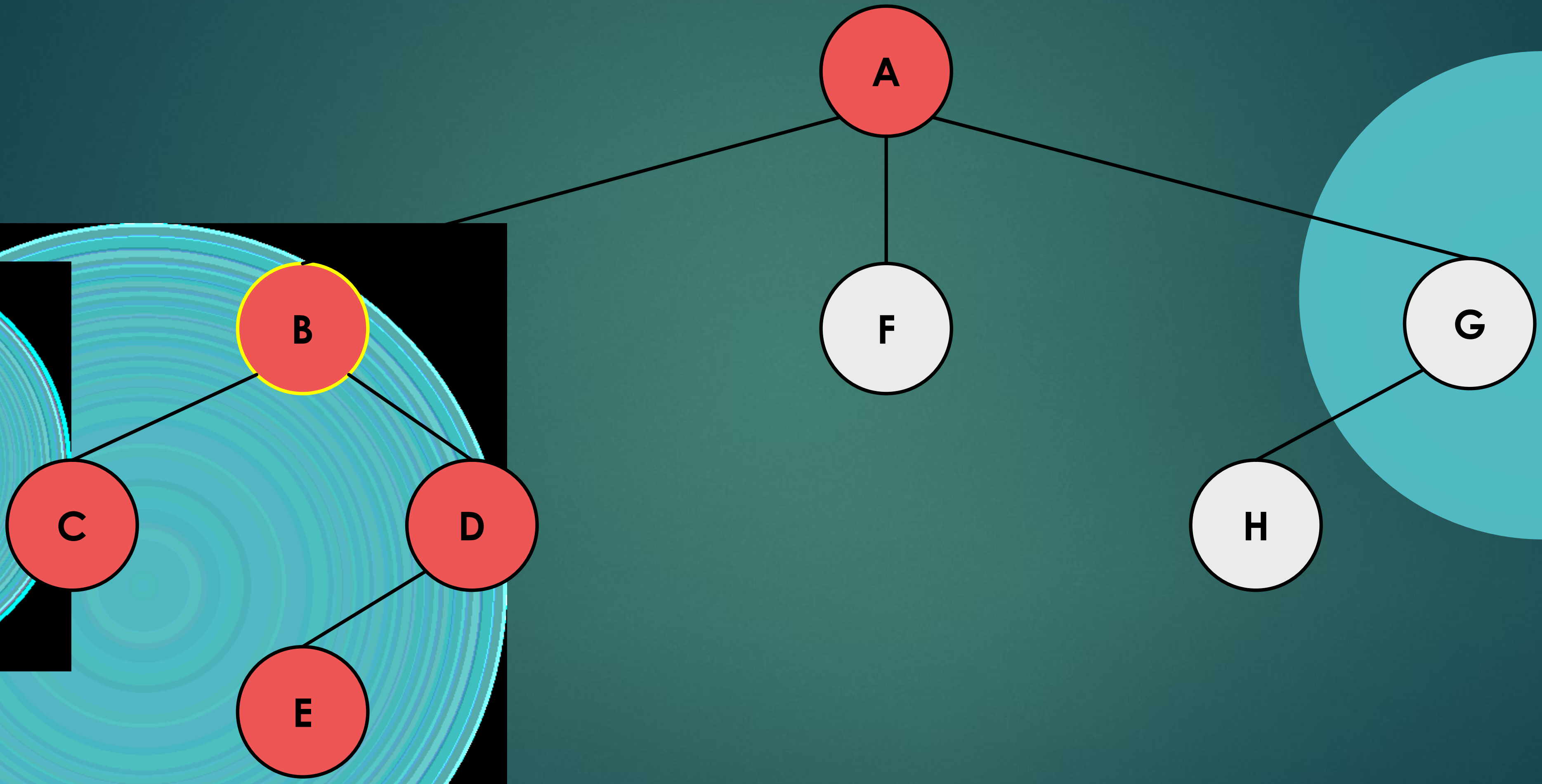

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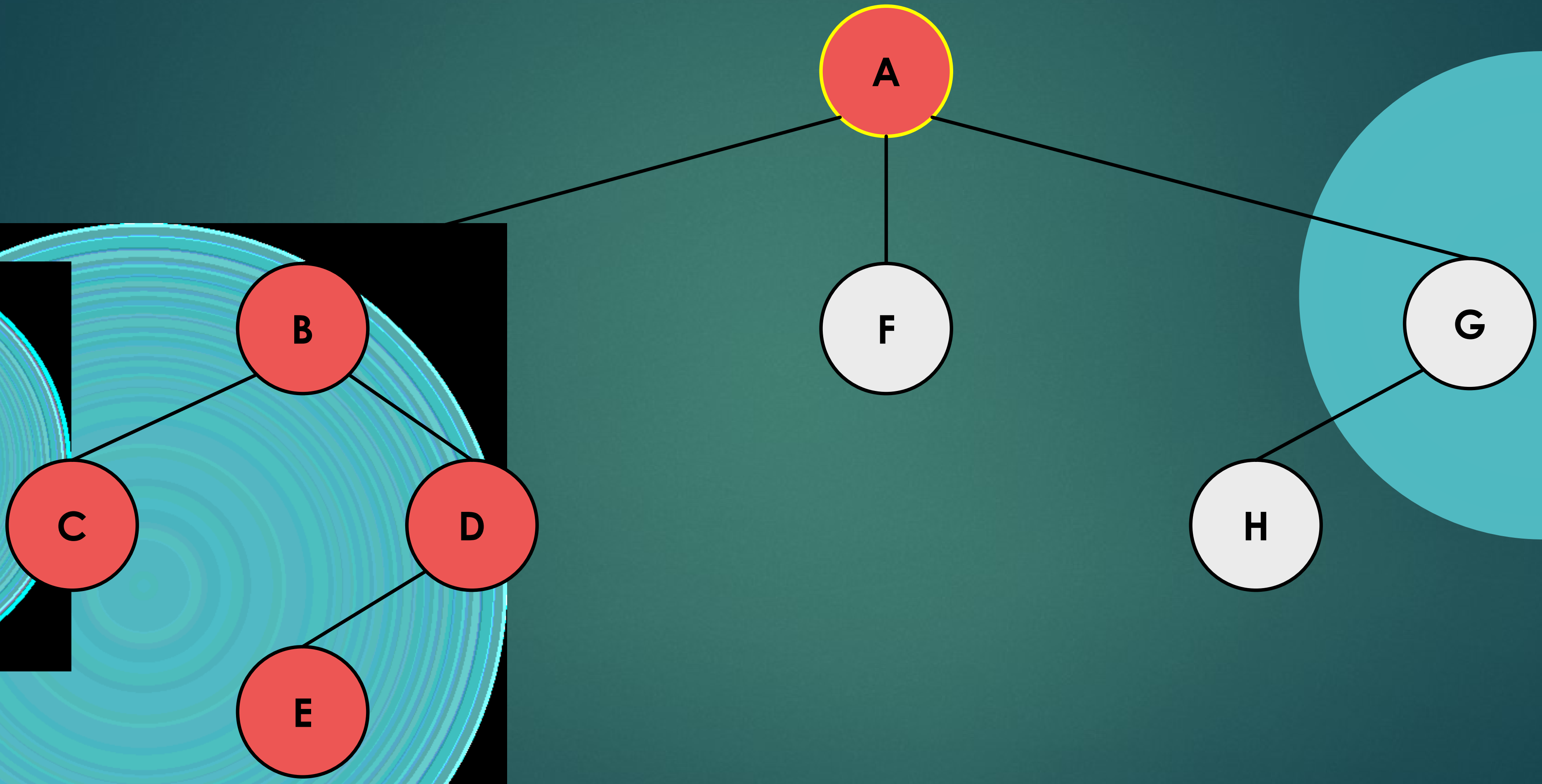

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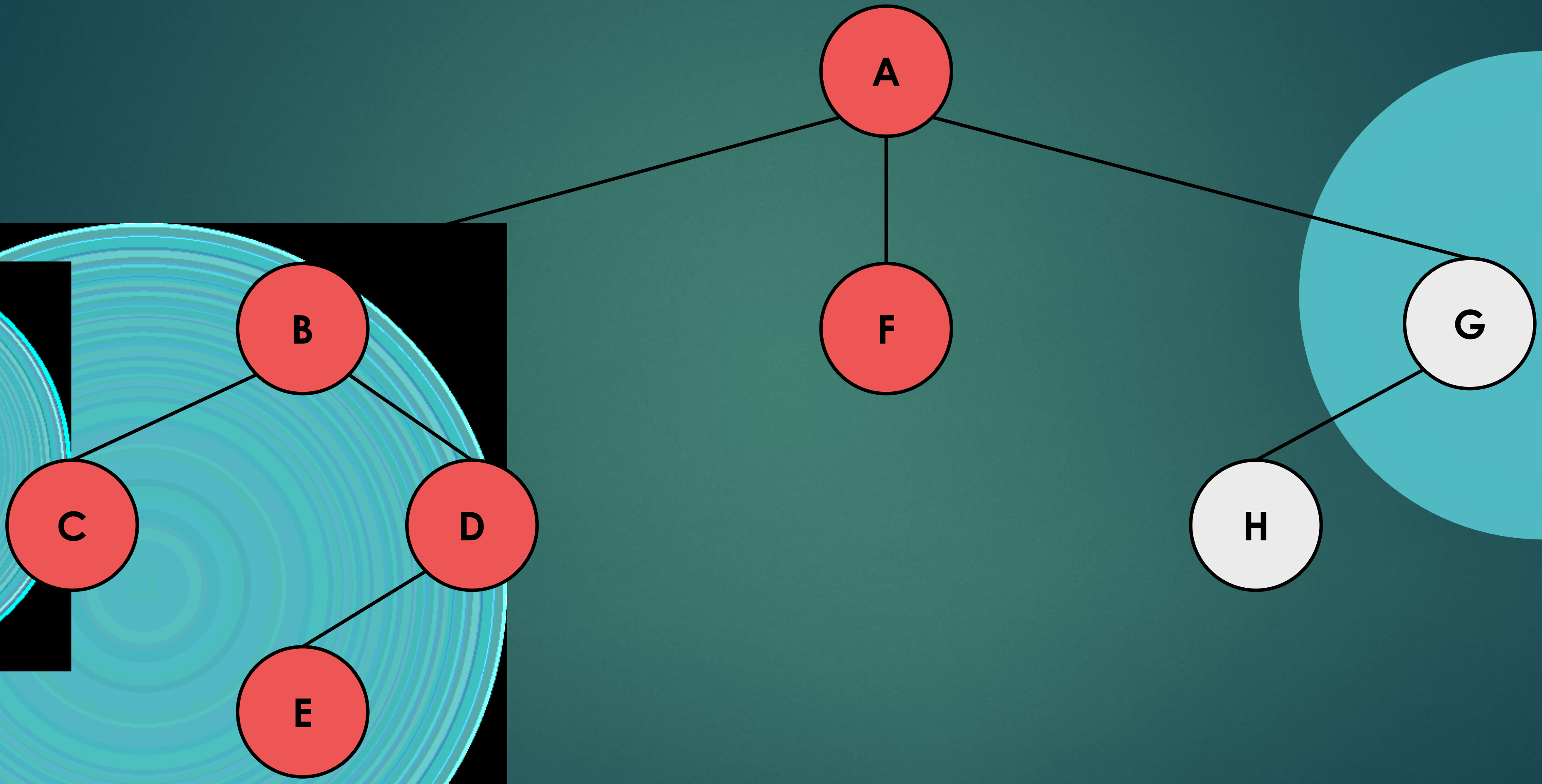

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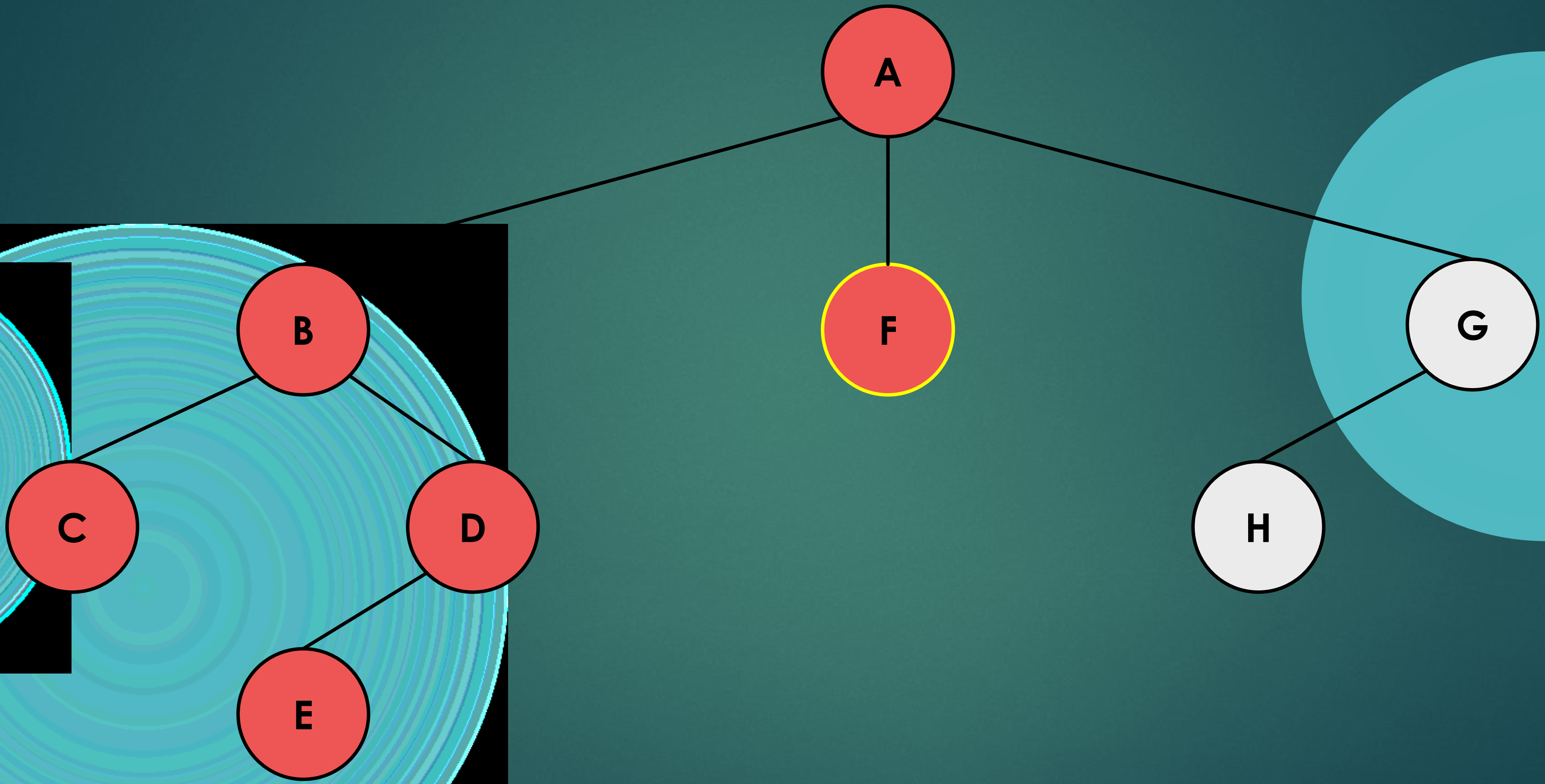

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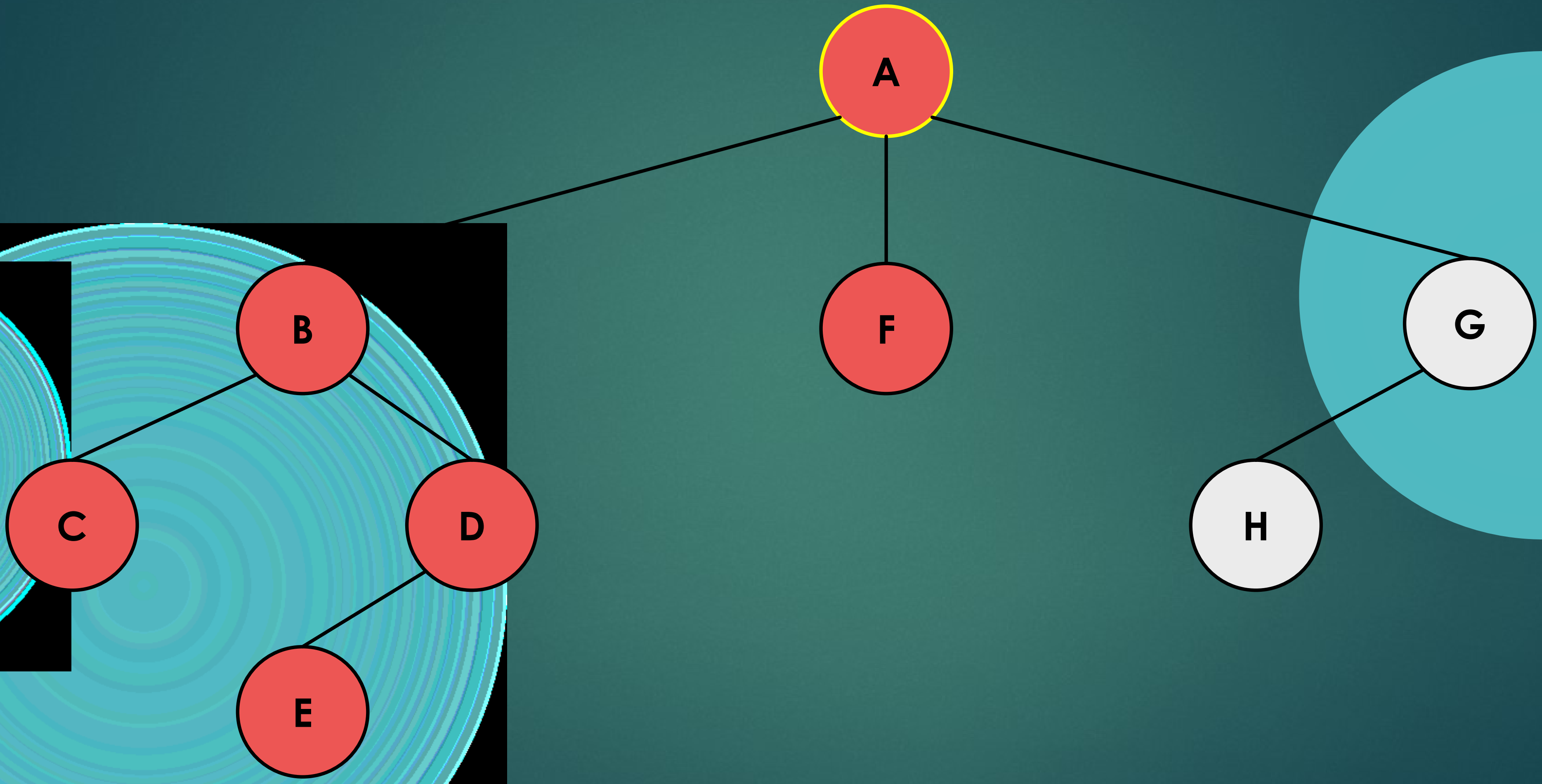

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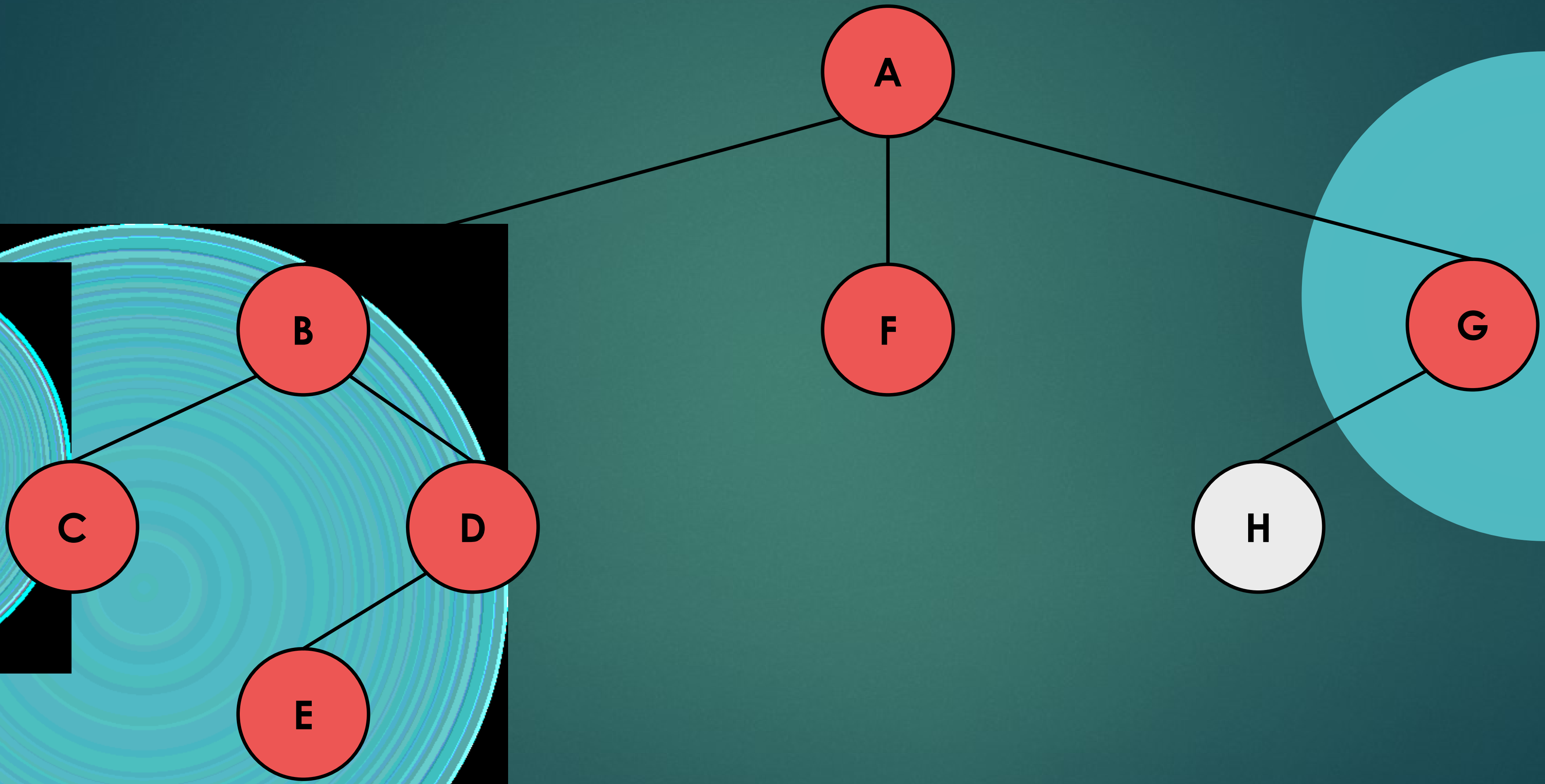

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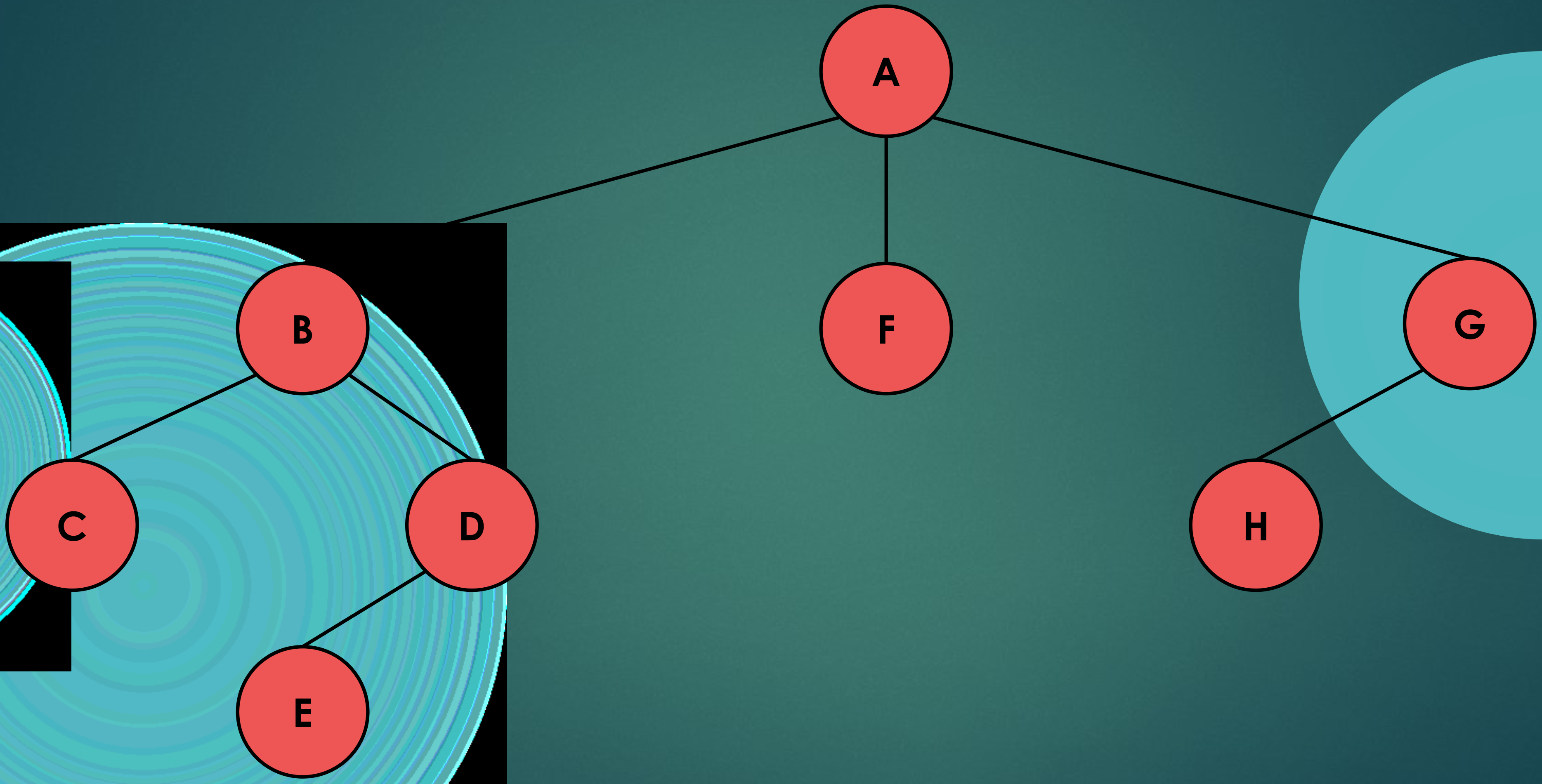

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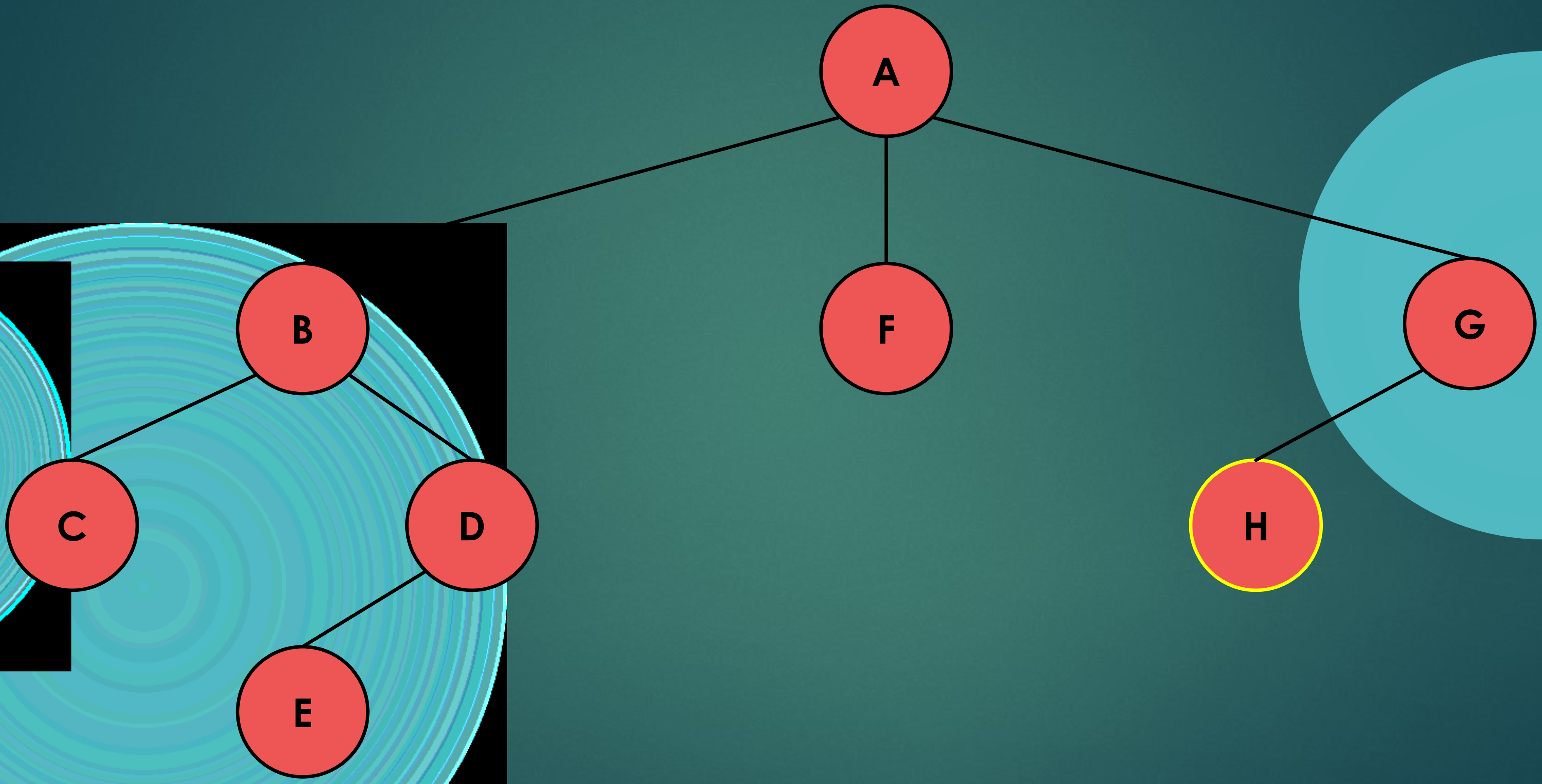

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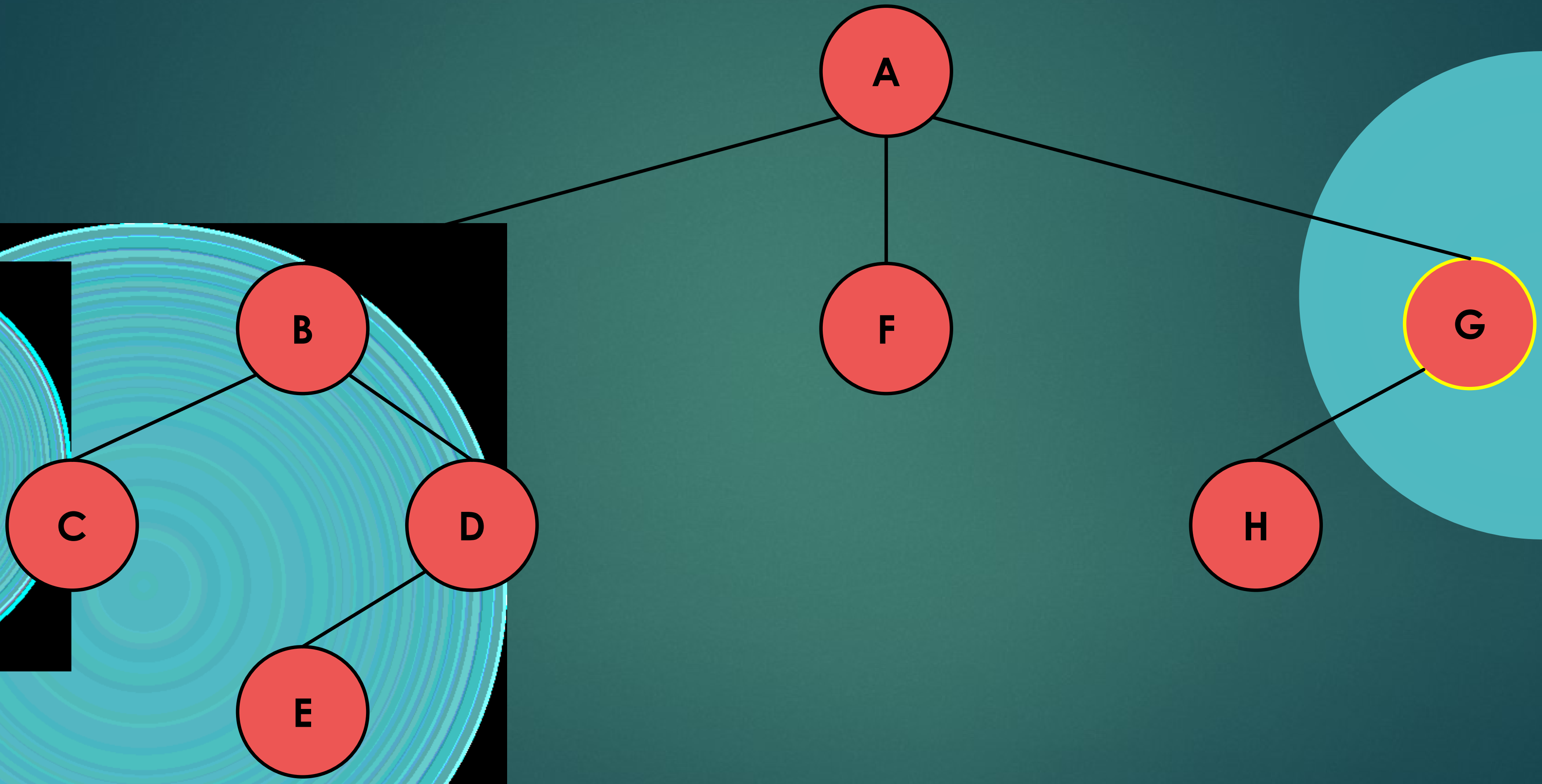

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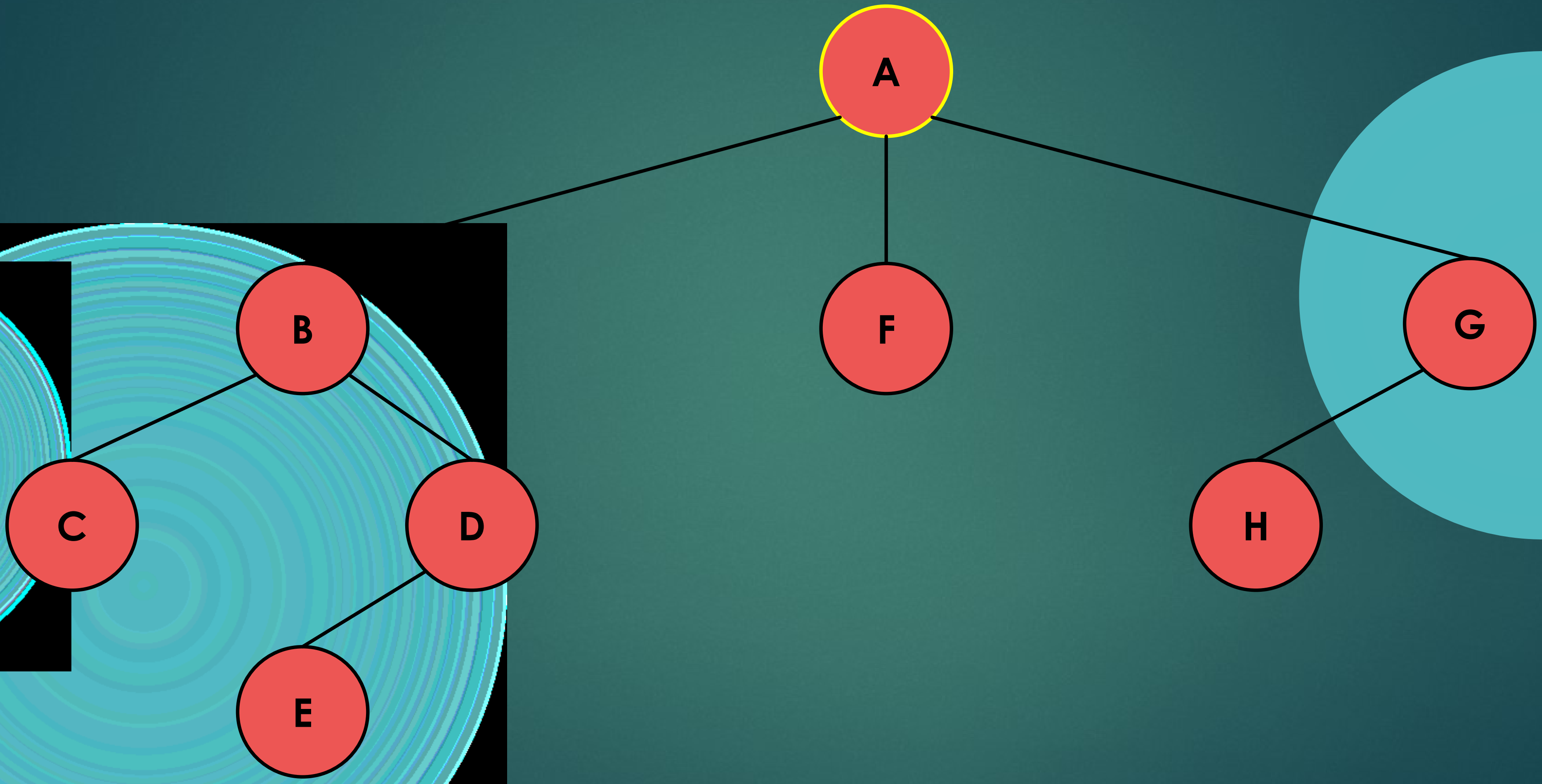

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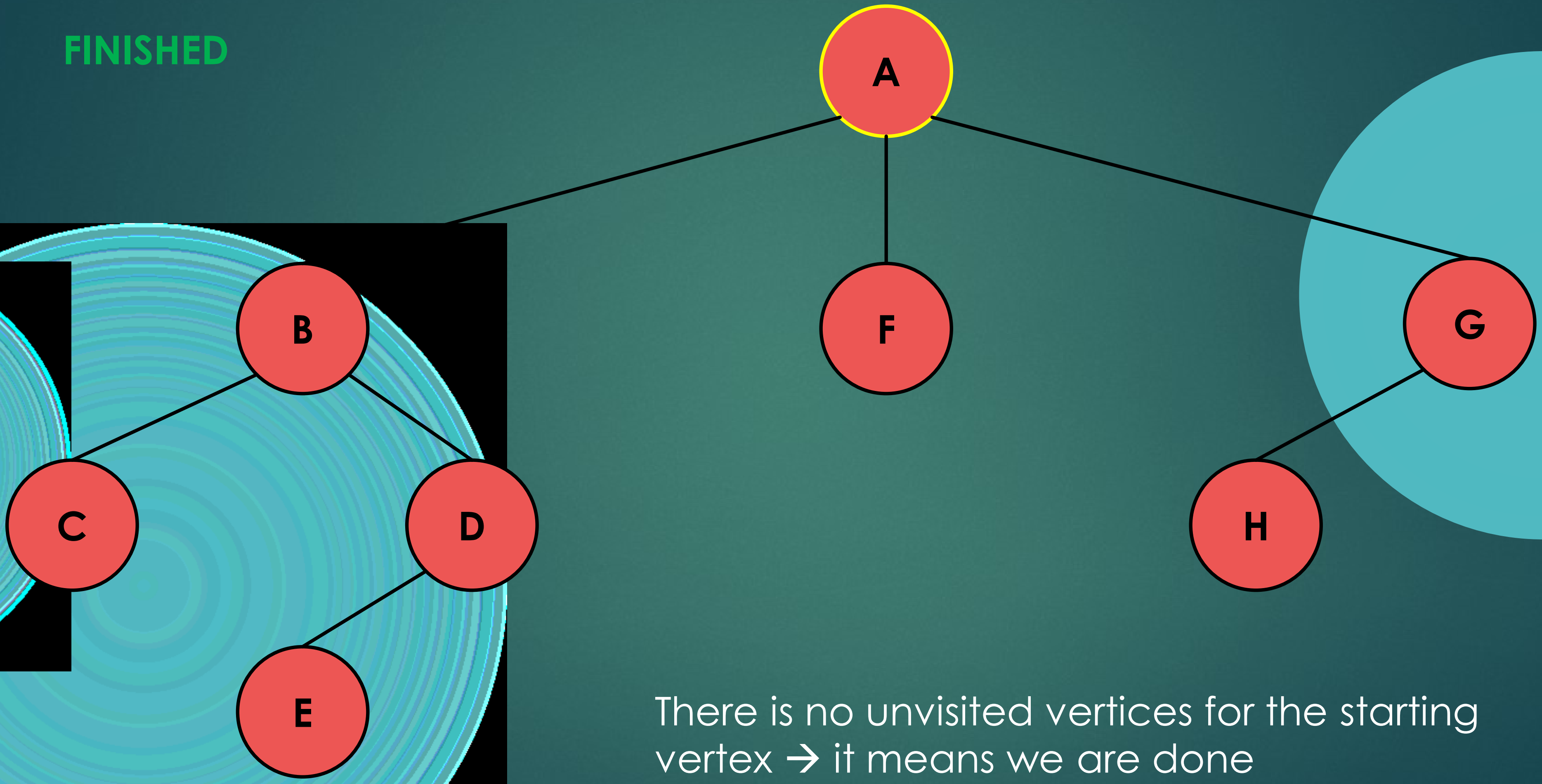
FINISHED

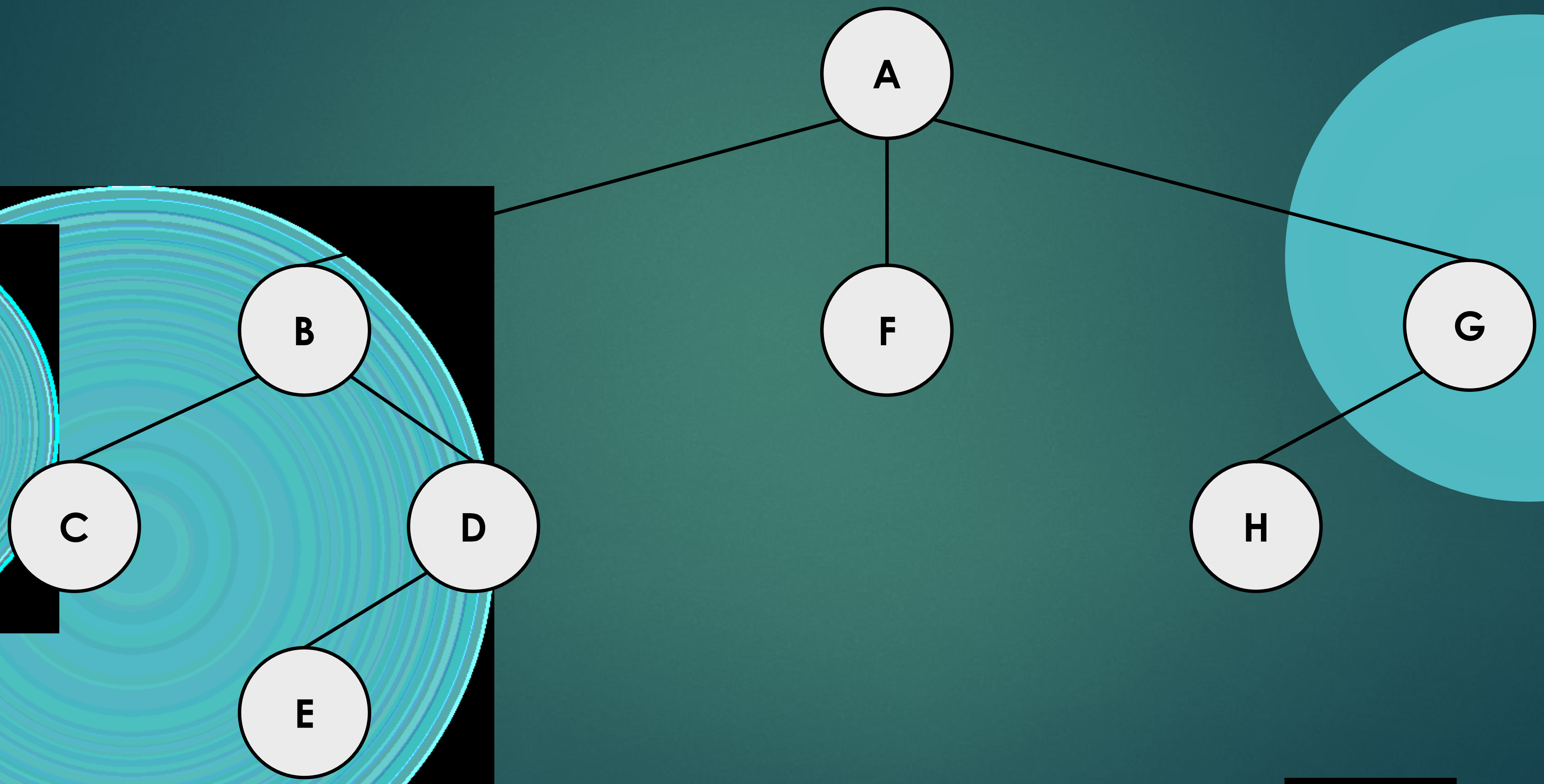


There is no unvisited vertices for the starting
vertex → it means we are done

Applications

- Topological ordering
- Kosaraju algorithm for finding strongly connected components in a graph which can be proved to be very important in recommendation systems (youtube)
- Detecting cycles (checking whether a graph is a DAG or not)
  - Processes waiting for each other ➔ this is a cycle
- Generating mazes OR finding way out of a maze

Revisiting breadth-first search



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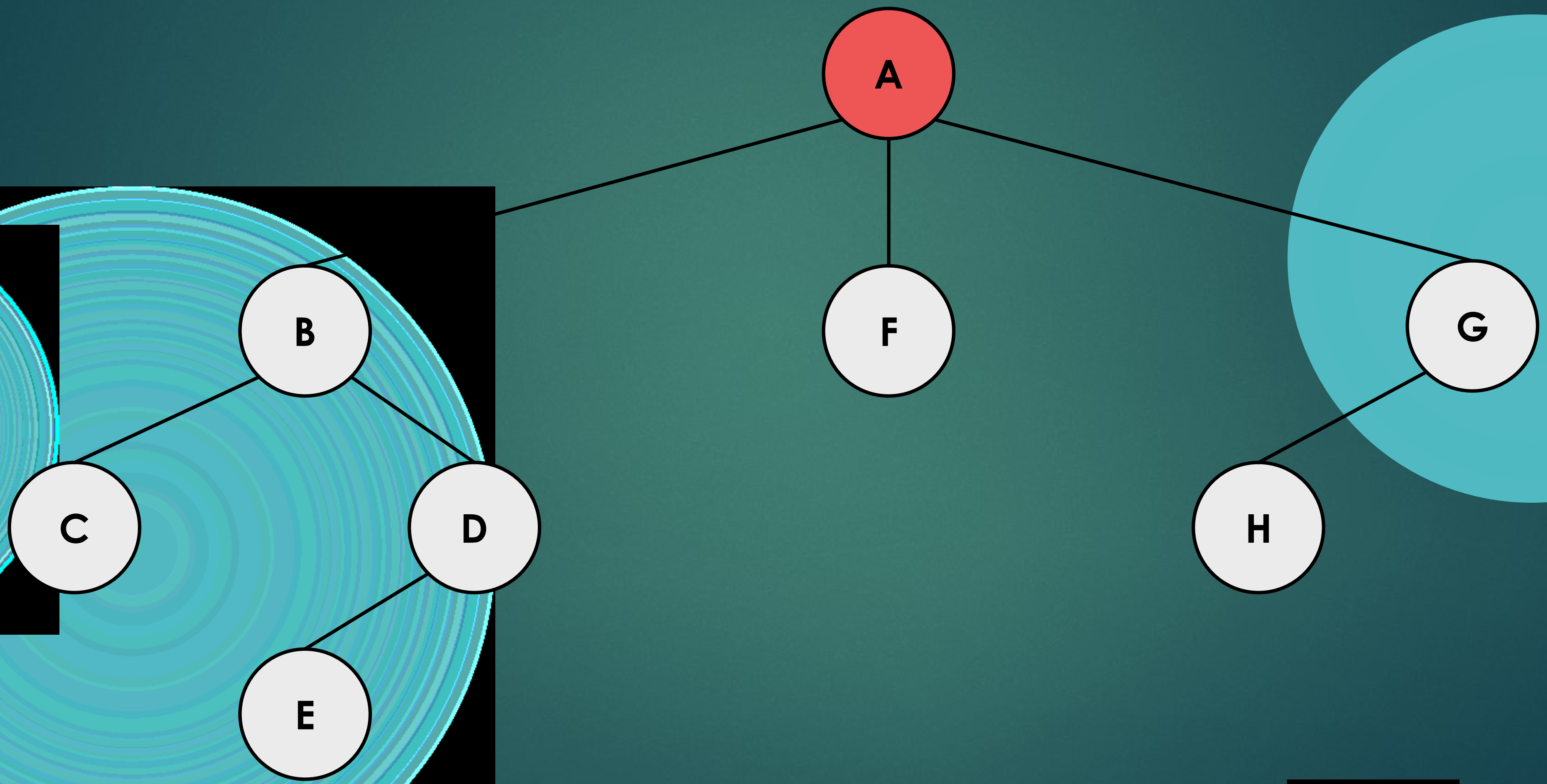

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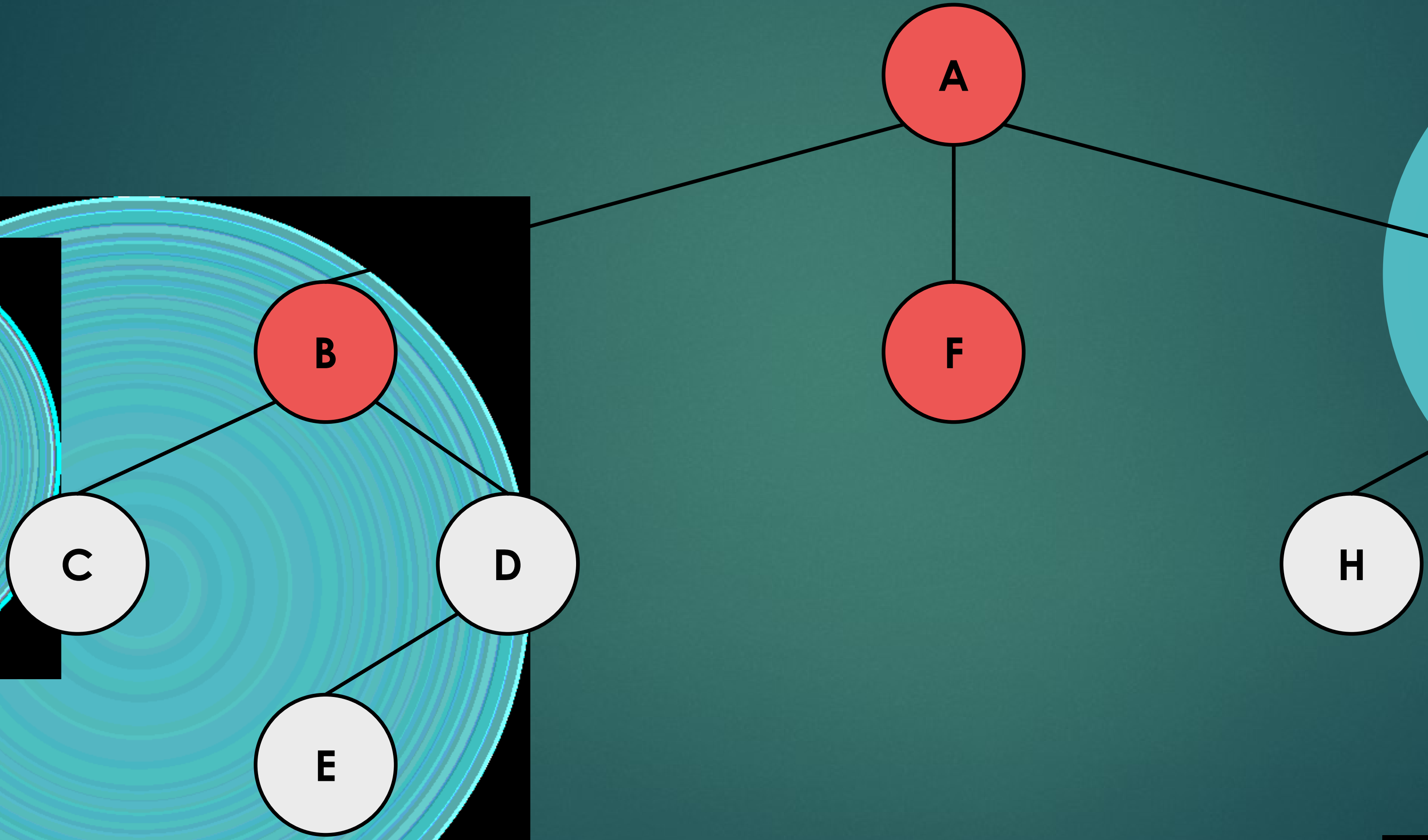

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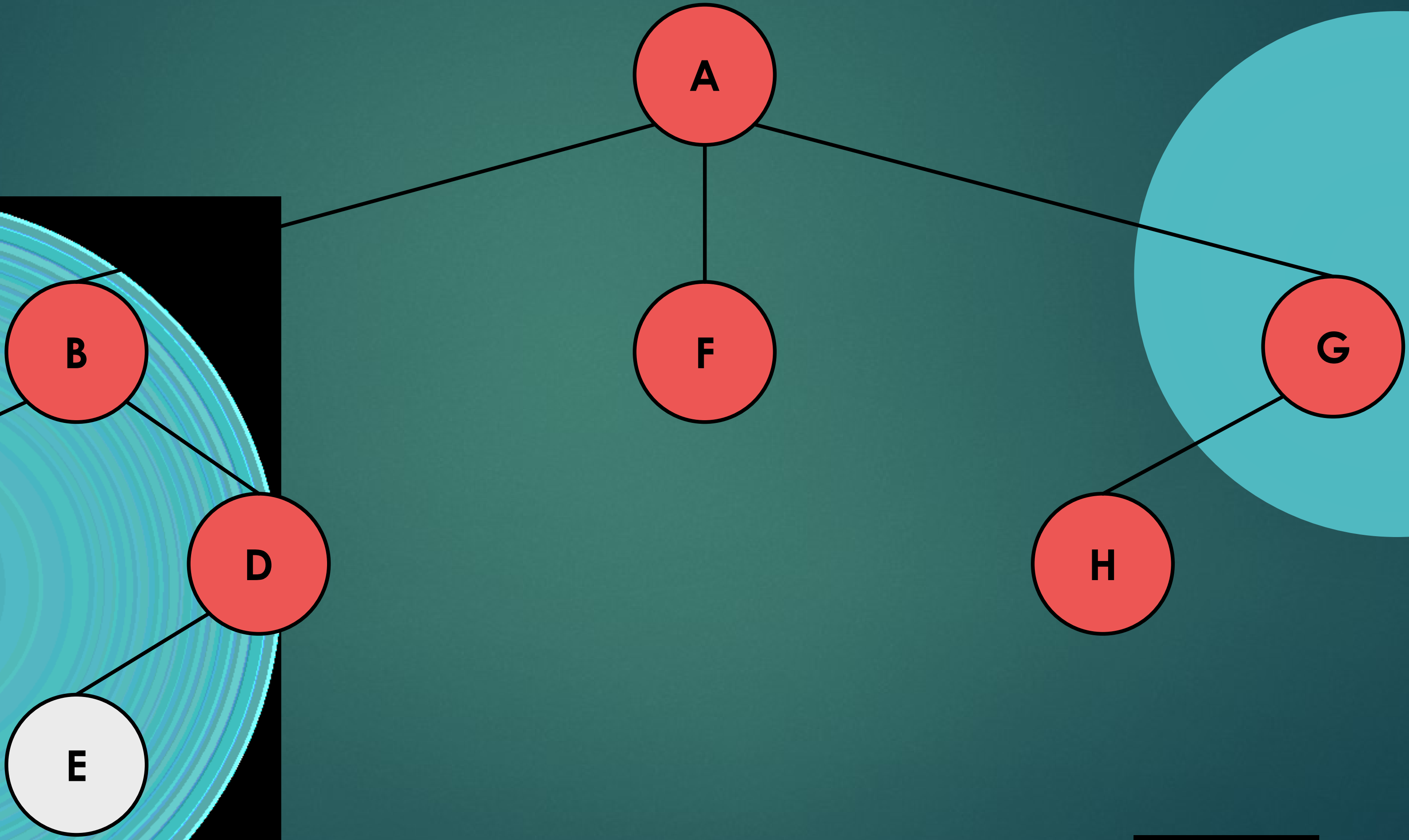

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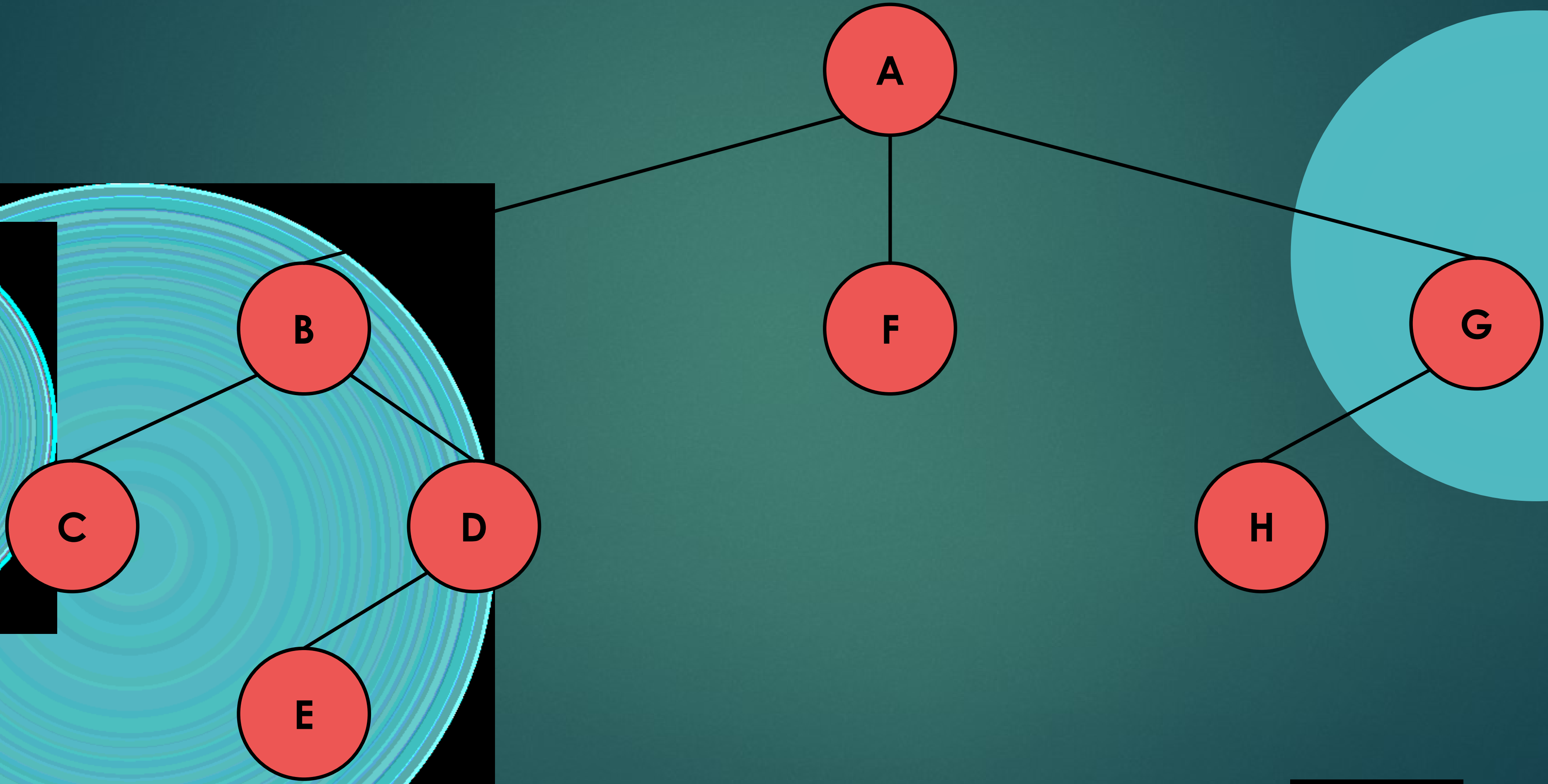
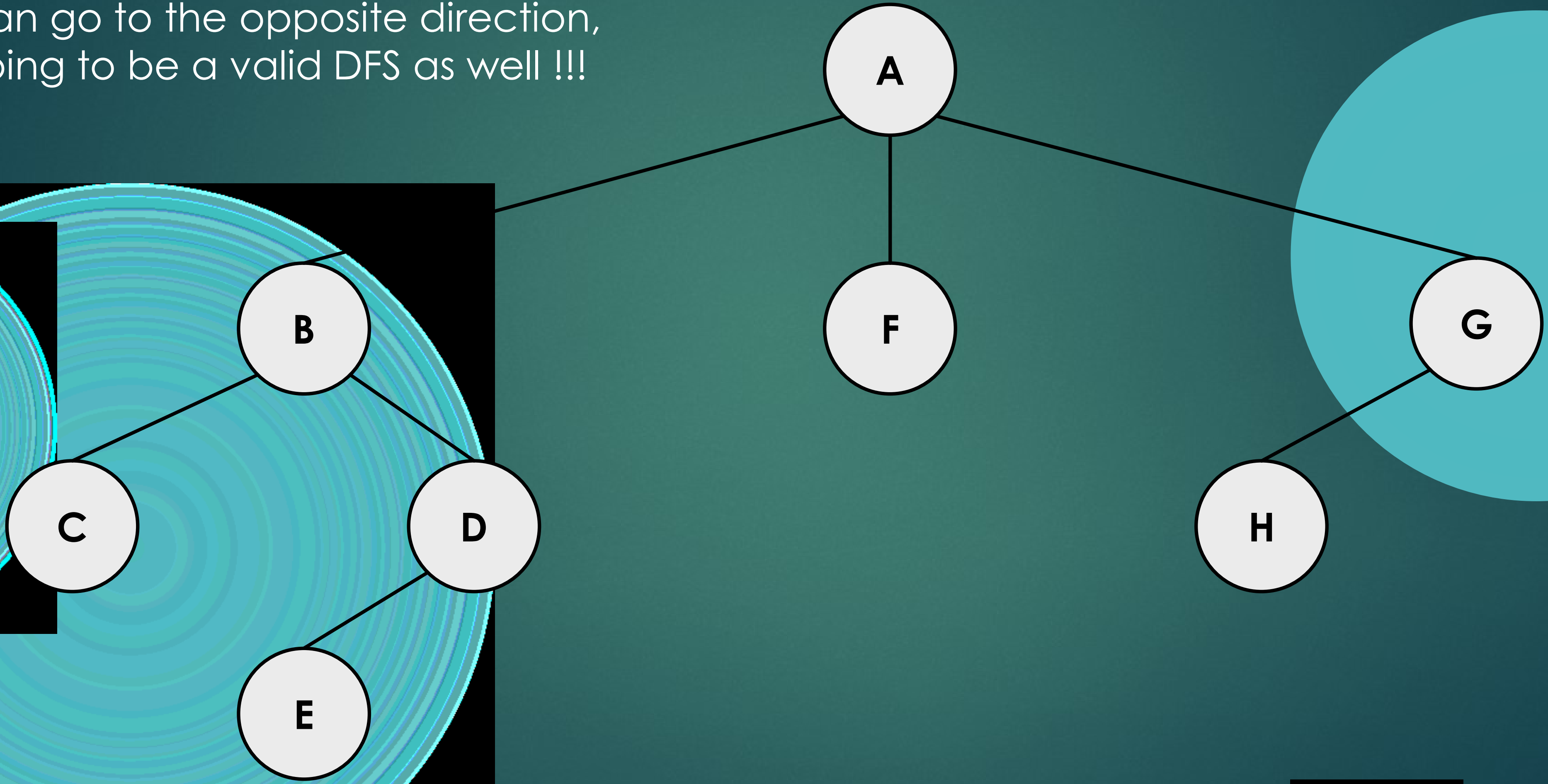


Symmetry in DFS

We can go to the opposite direction,
it is going to be a valid DFS as well !!!



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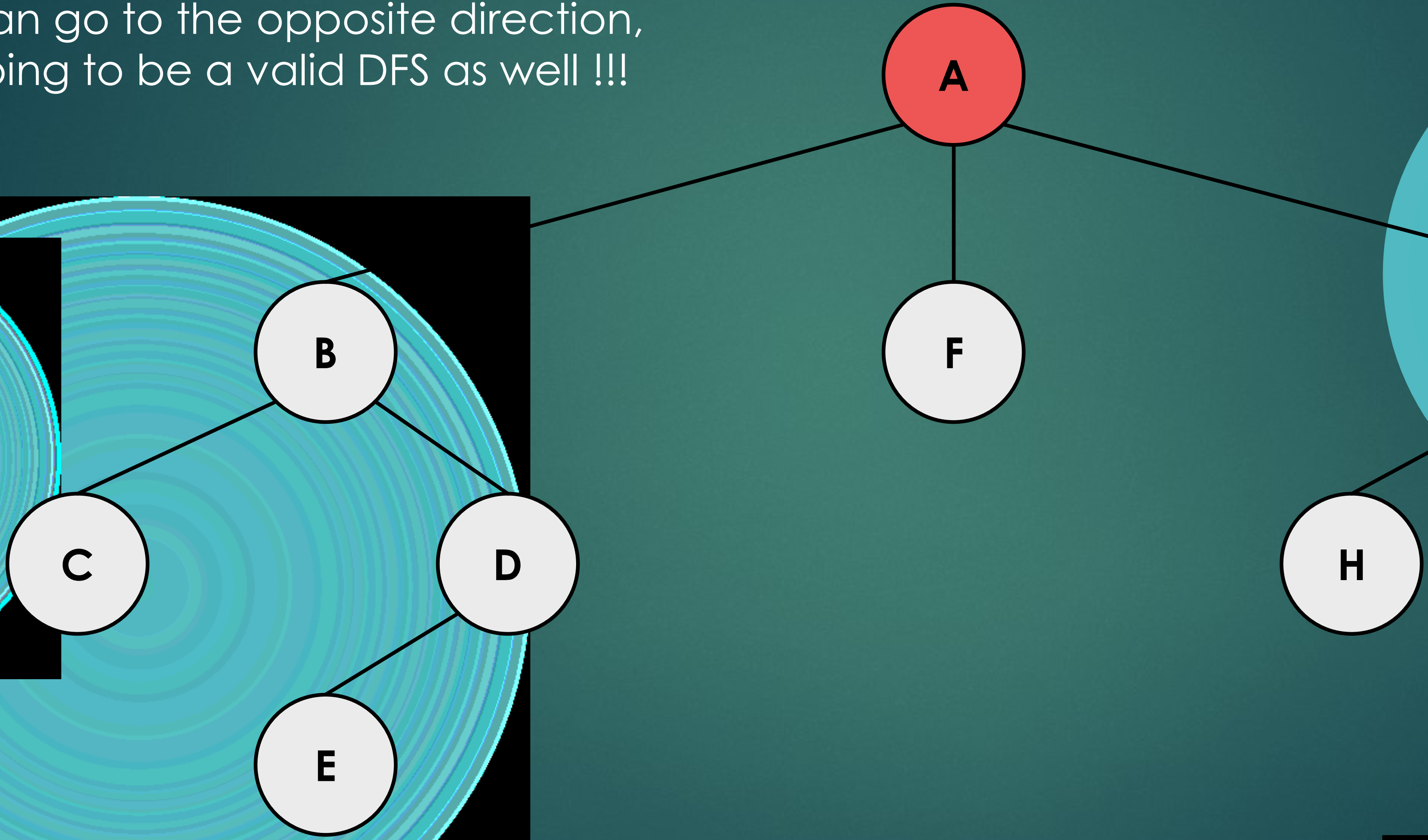

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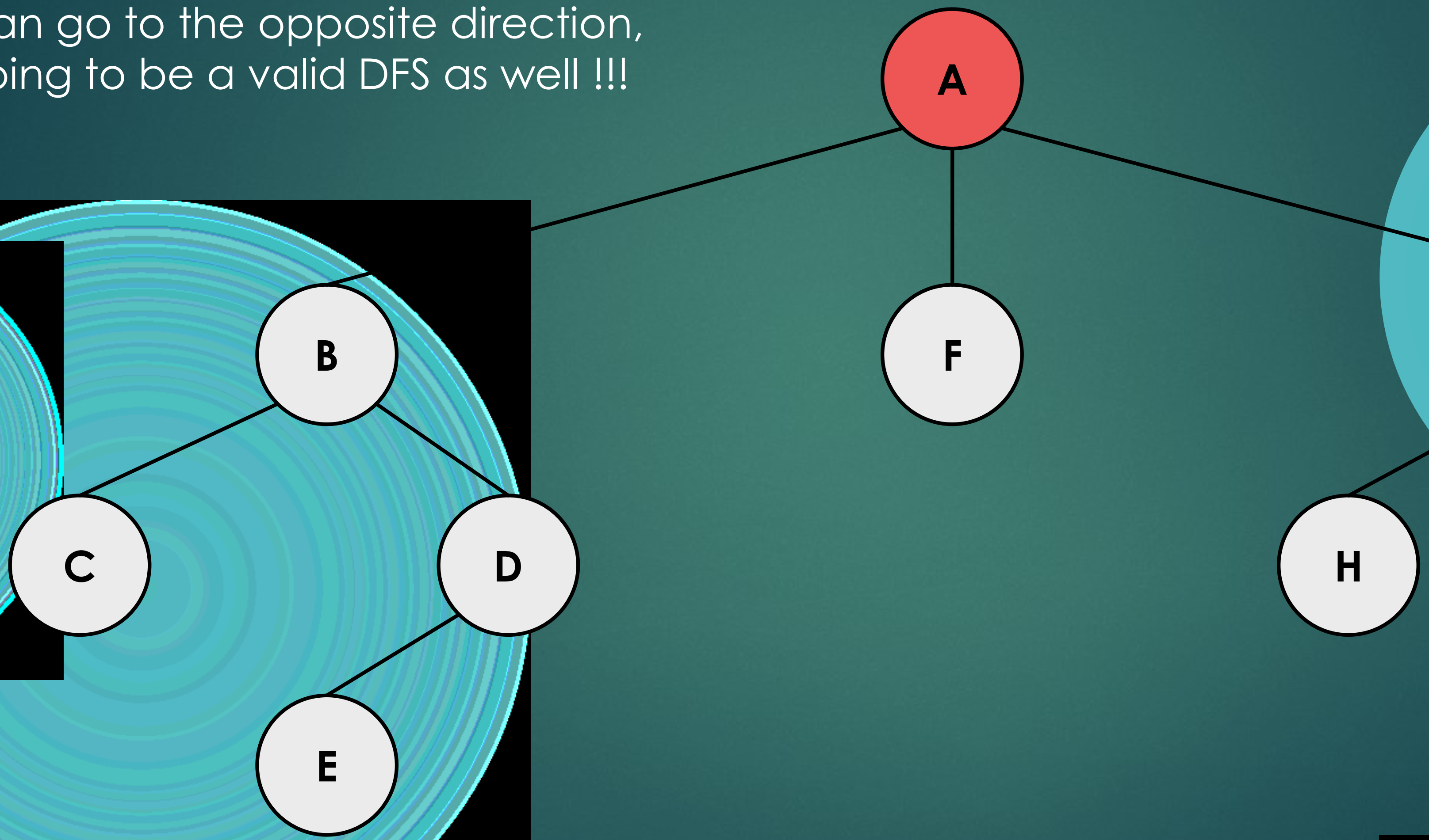

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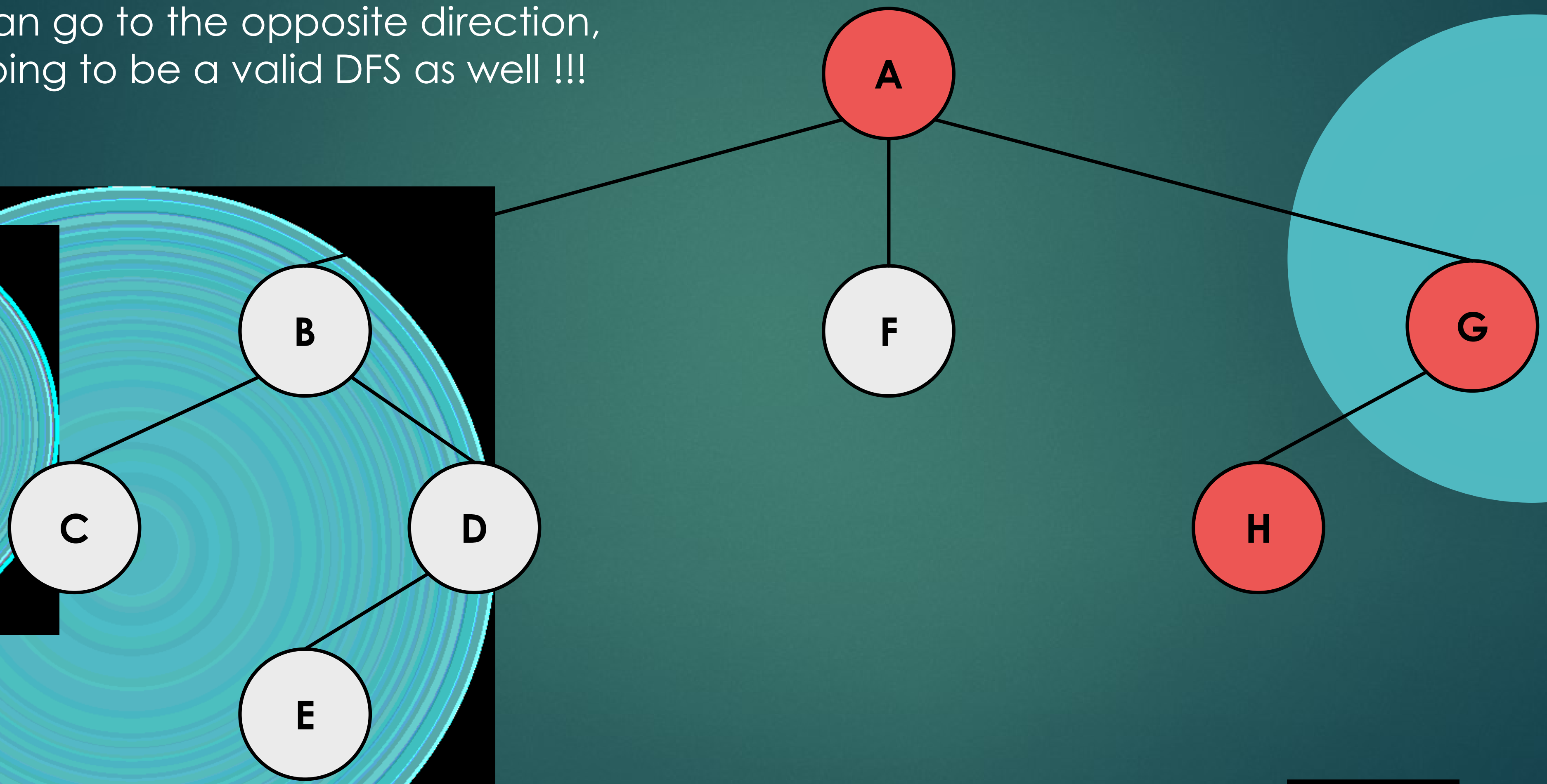

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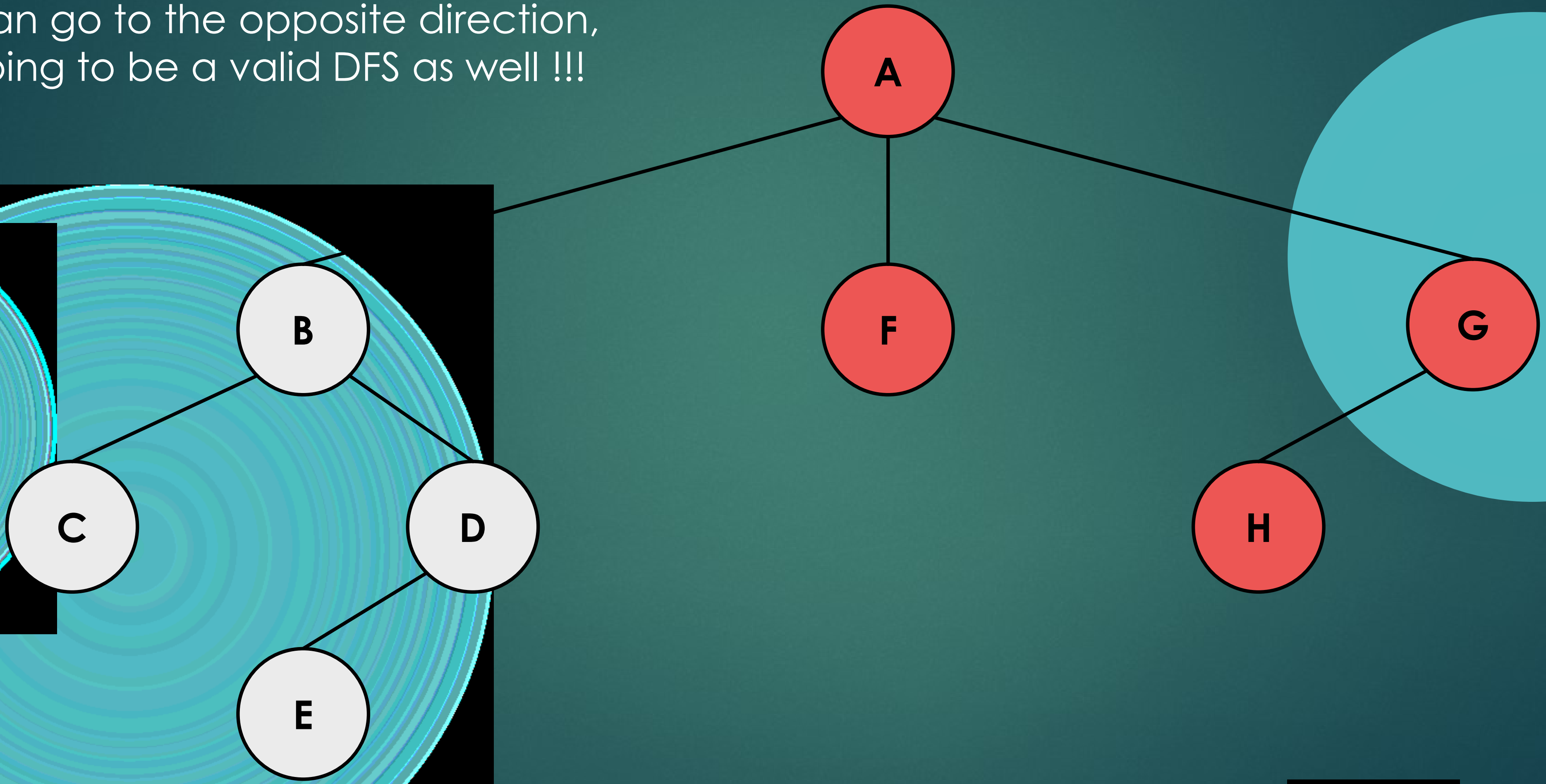

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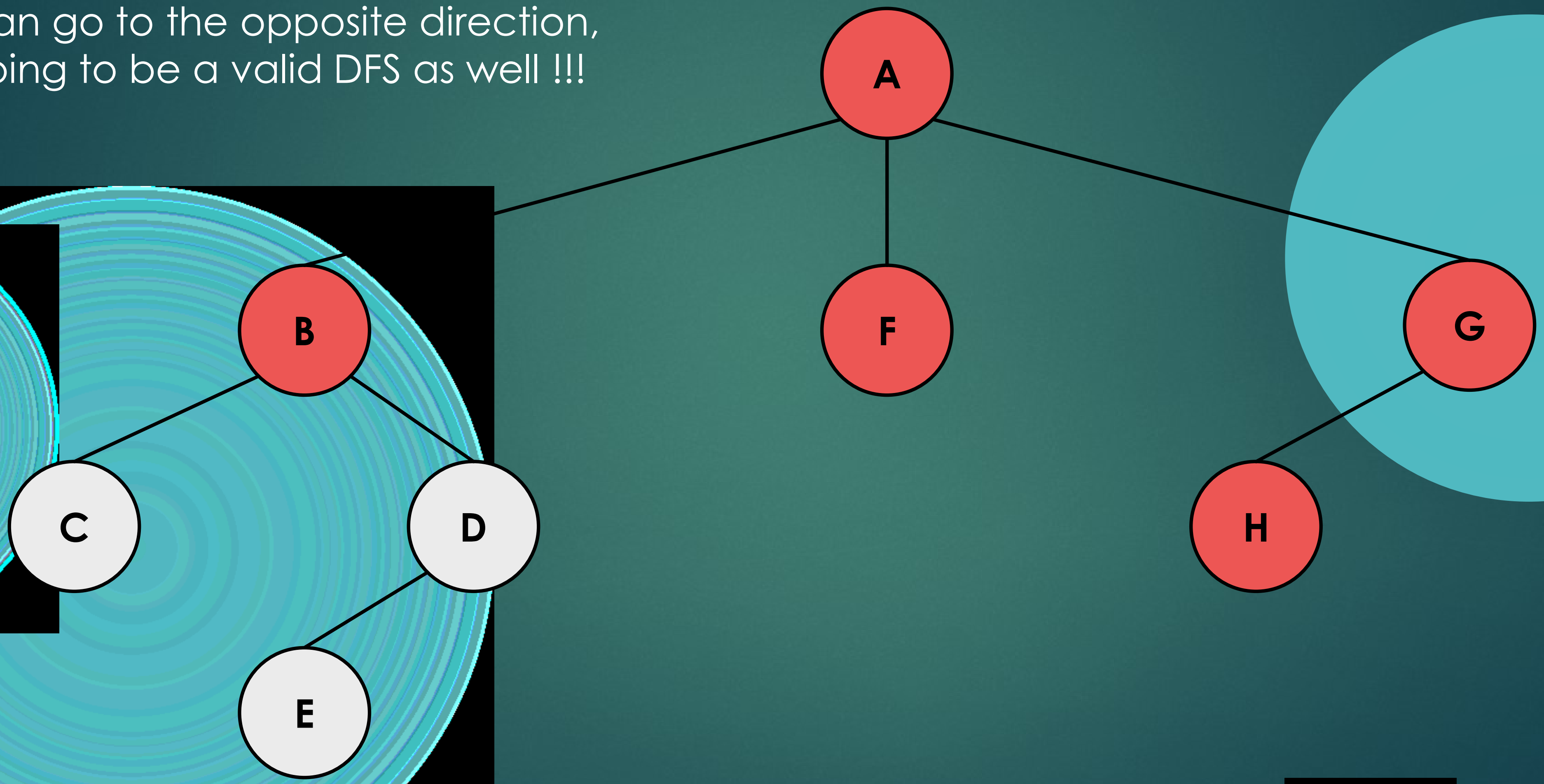

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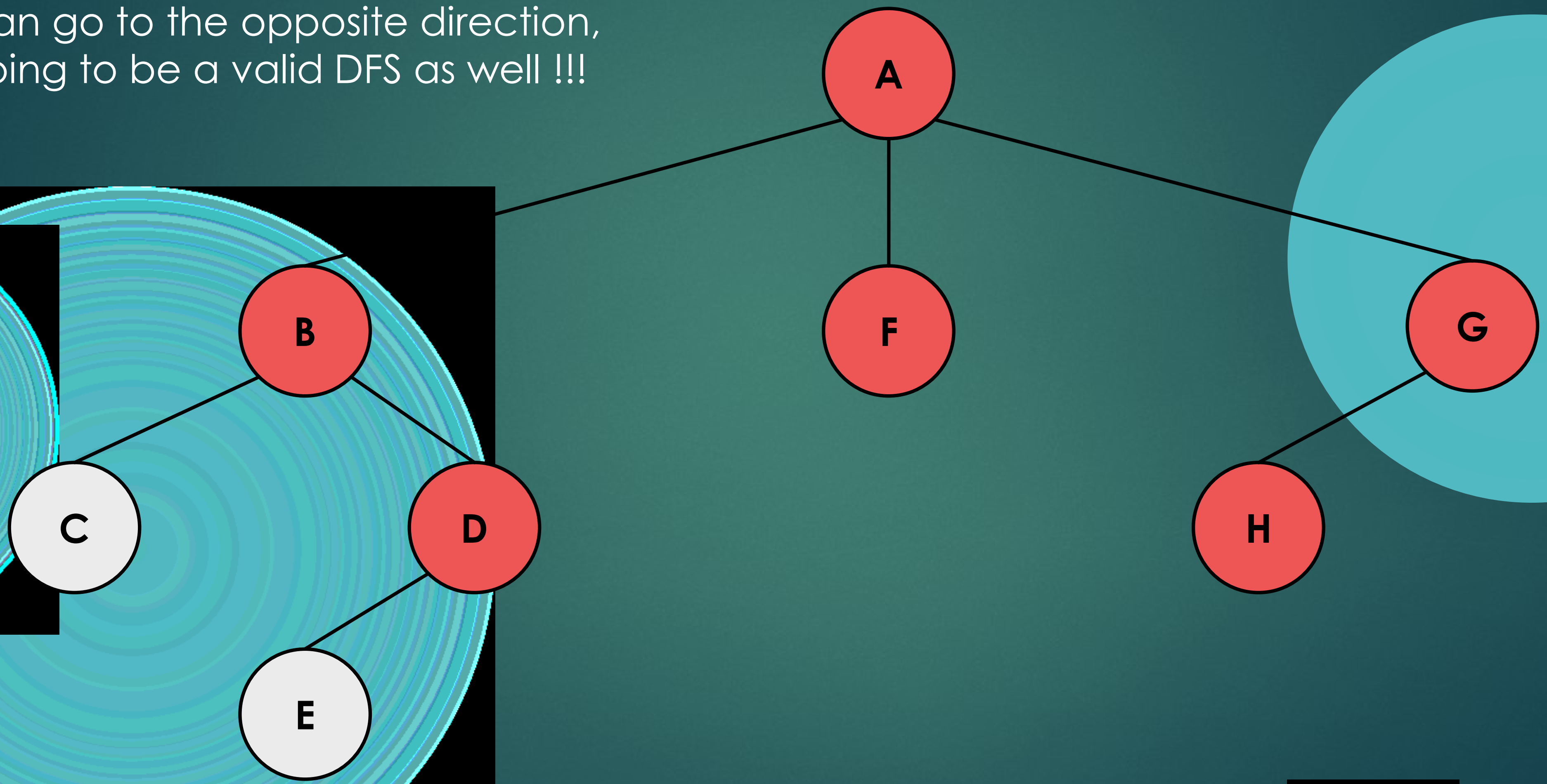

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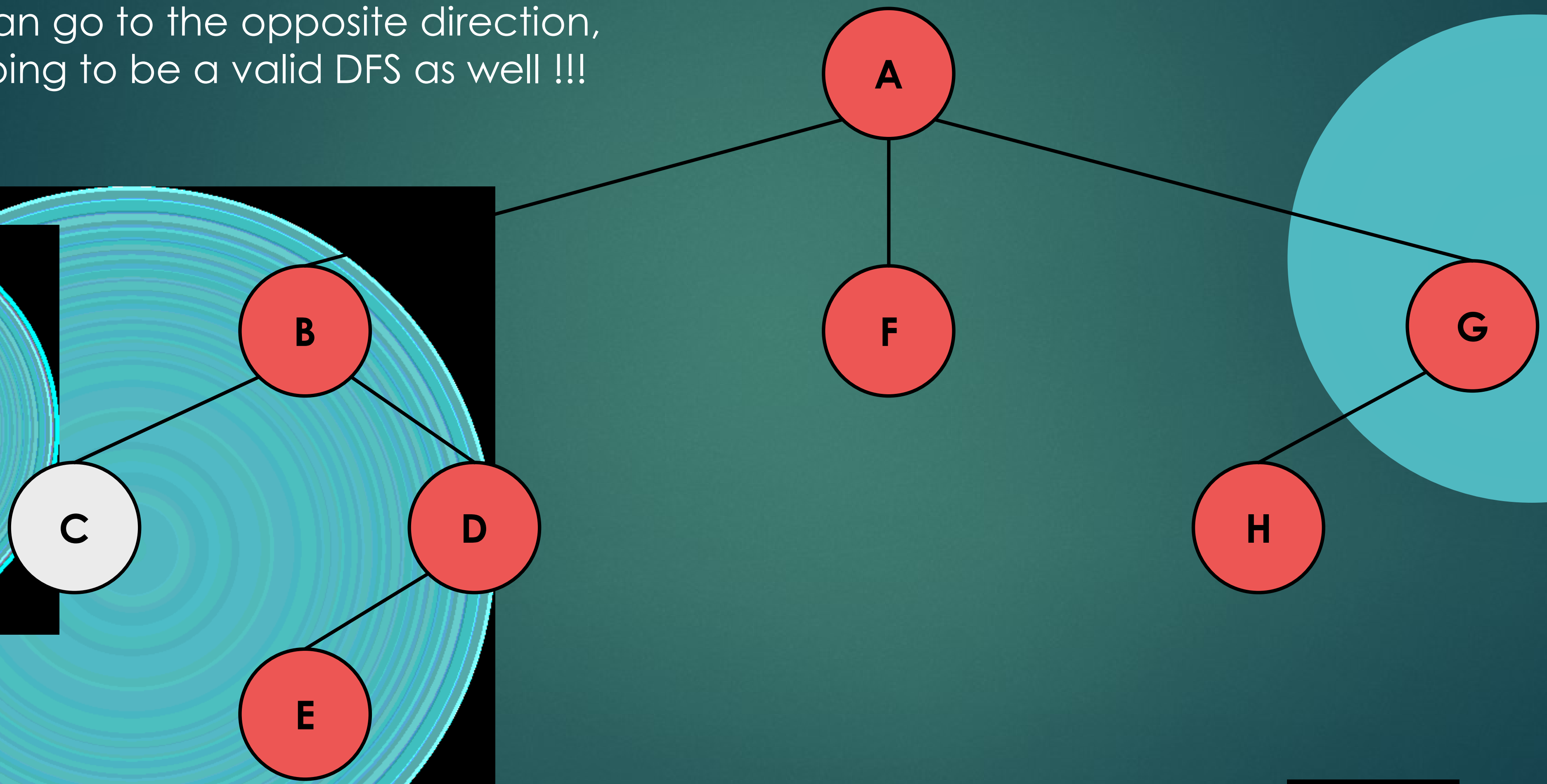

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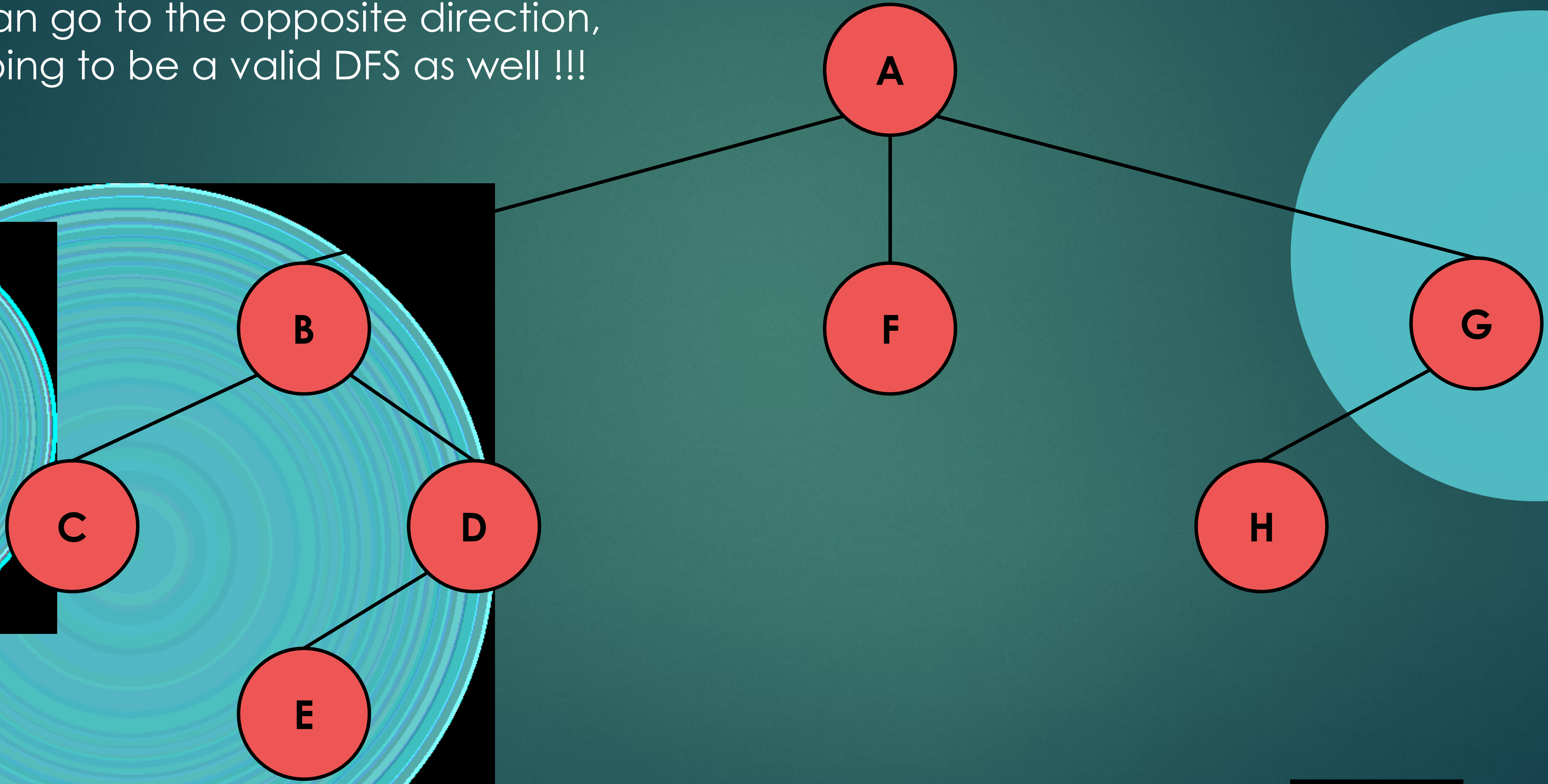
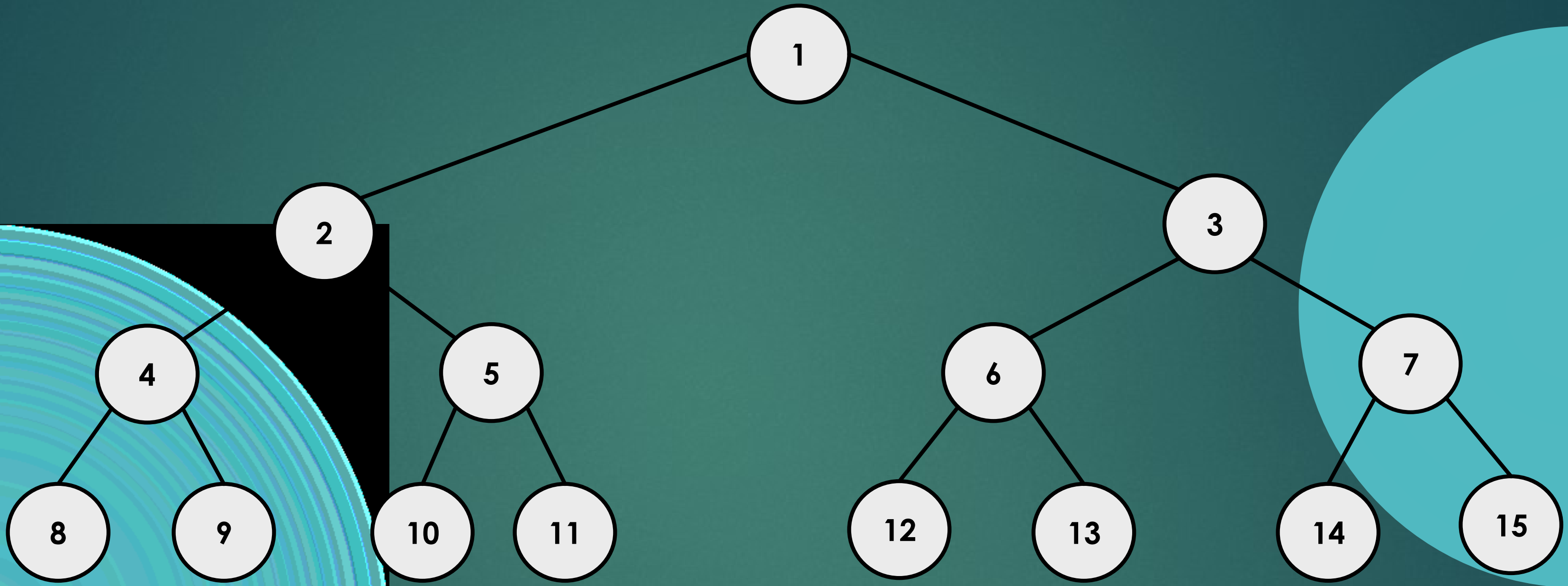


DFS IMPLEMENTATION

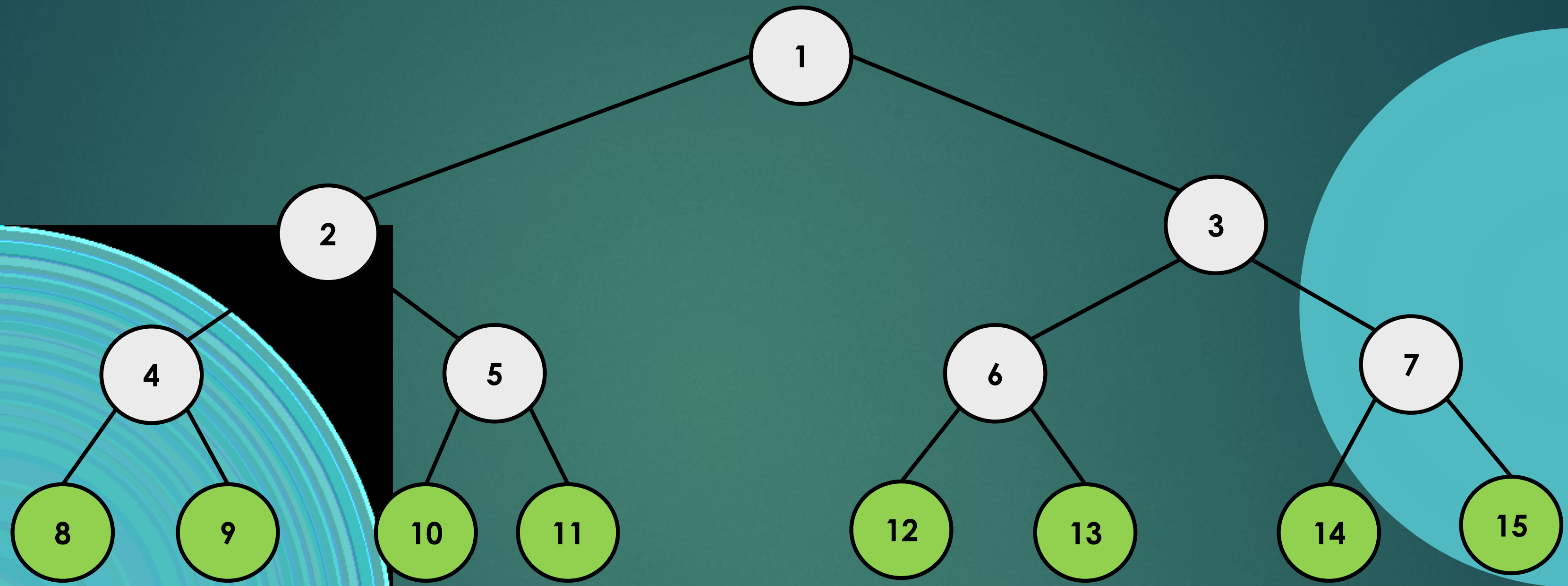
Memory Complexity

BFS VS DFS

Memory complexity: BFS vs DFS



Memory complexity: BFS vs DFS

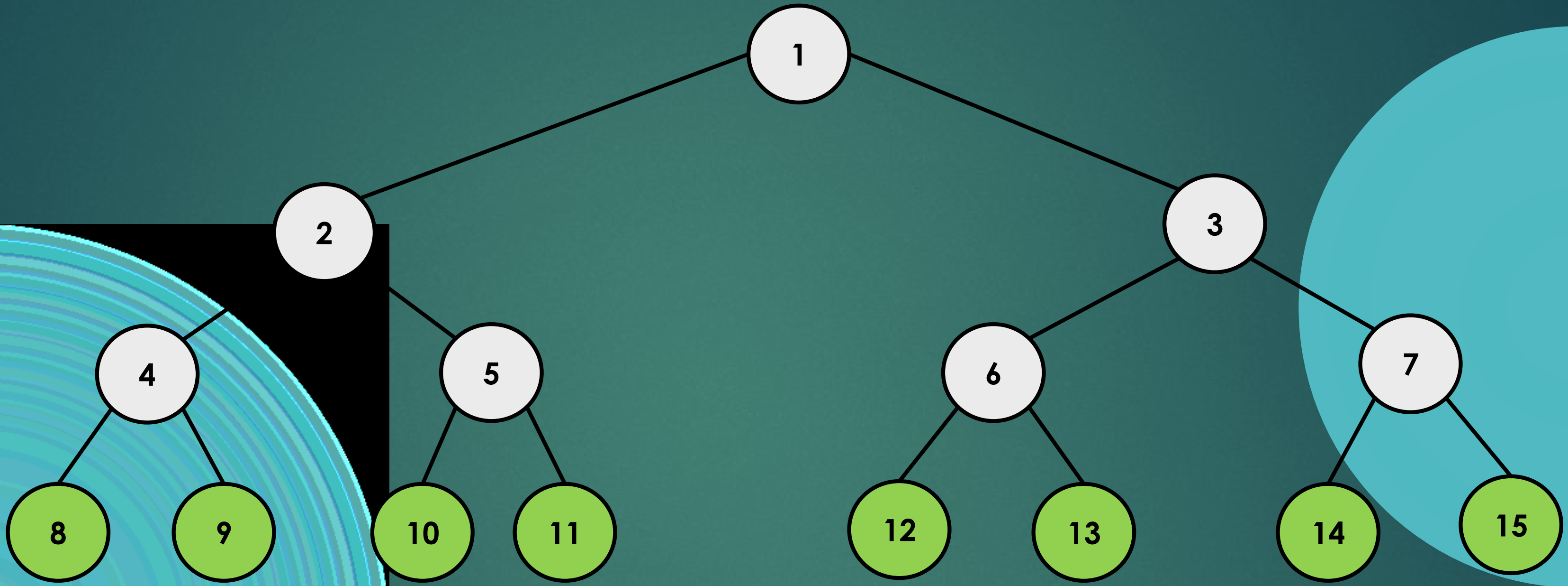


What does it mean?

At the leaves → if we have N items stored in the balanced tree

→ then there will be N/2 leave nodes

Memory complexity: BFS vs DFS



What does it mean?
At the leaves → if we have N items stored in the balanced tree
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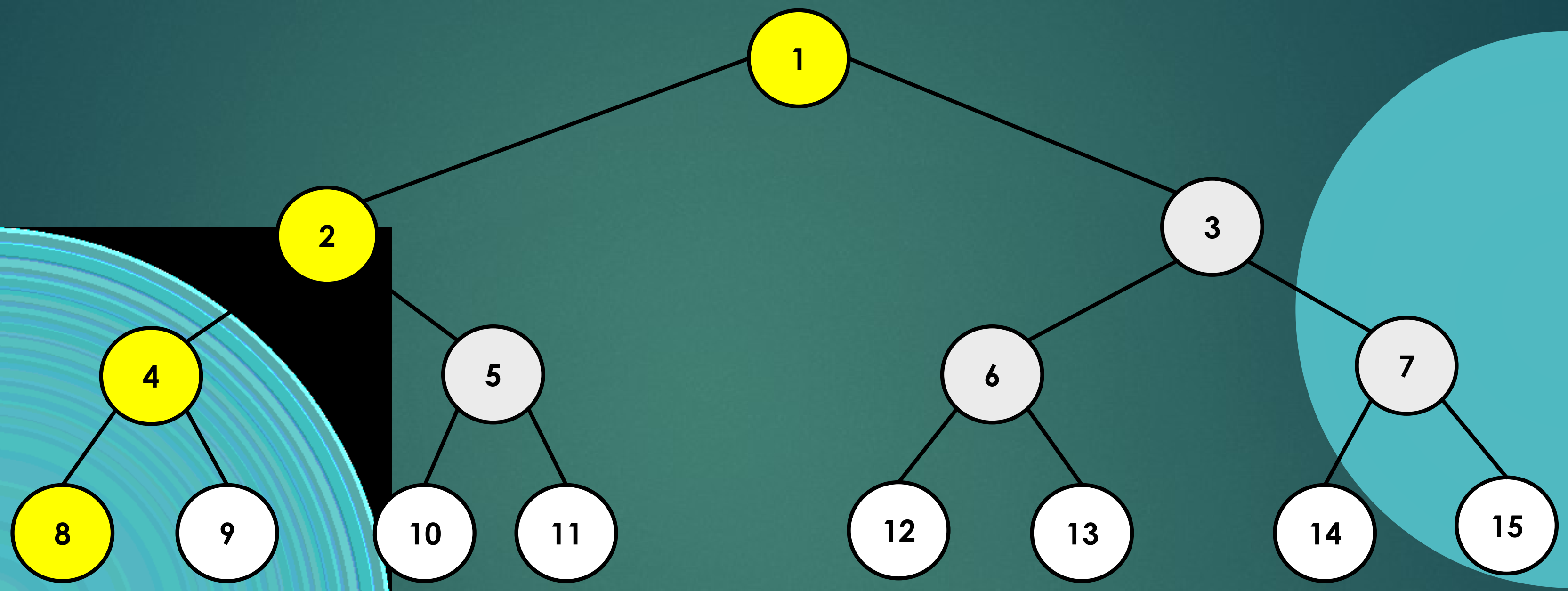
So we have to store **O(N)** times if we want to traverse a tree that contains N items!!!

Memory complexity: BFS vs DFS



Here we have to backtrack (pop item from stack)

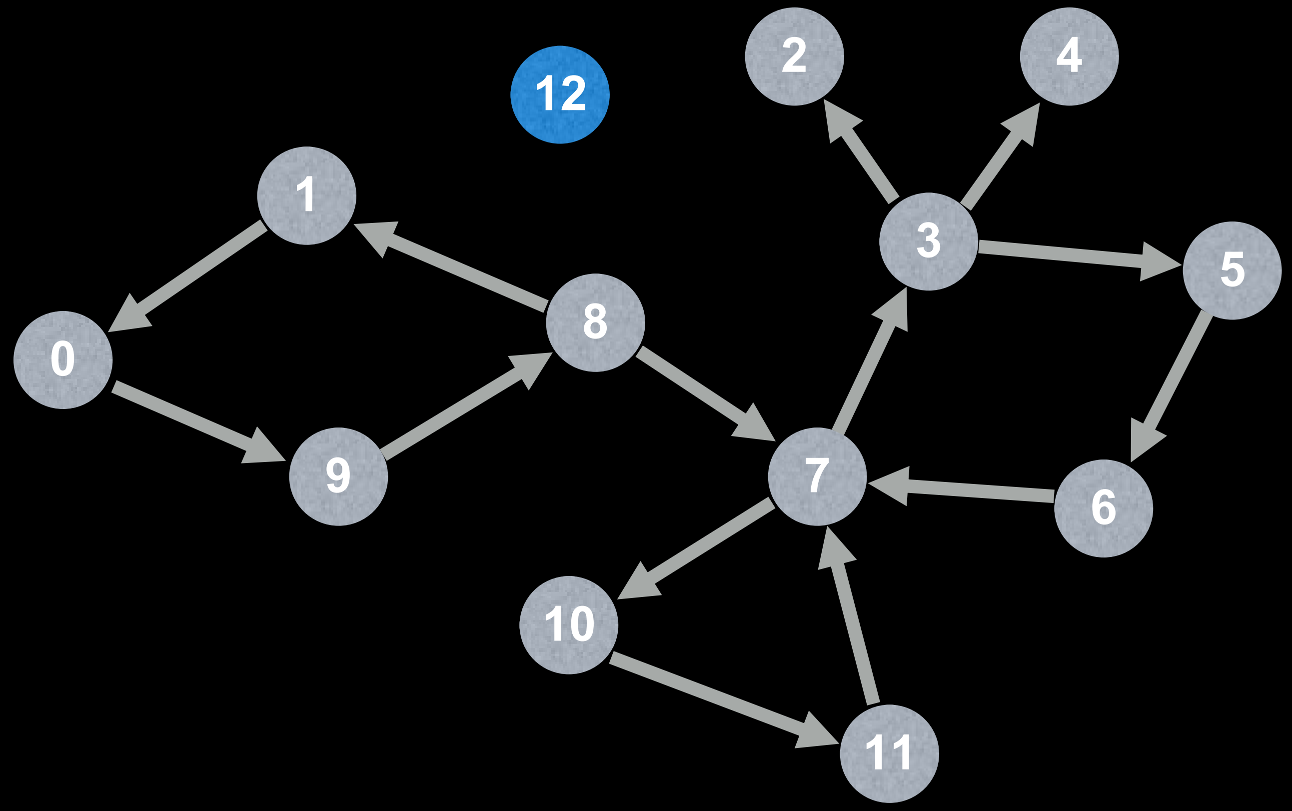
- We just have to store as many items in the stack as the height of the tree
- Which is log N !!!
- The memory complexity will be **O(logN)**

Memory complexity: BFS vs DFS

- Breadth-First Search: **O(N)**
- Depth-First Search: **O(logN)**

- Depth-First Search is **preferred** most of the time
- There may be some situations where BFS is better
  - Artificial intelligence
  - Robot movements

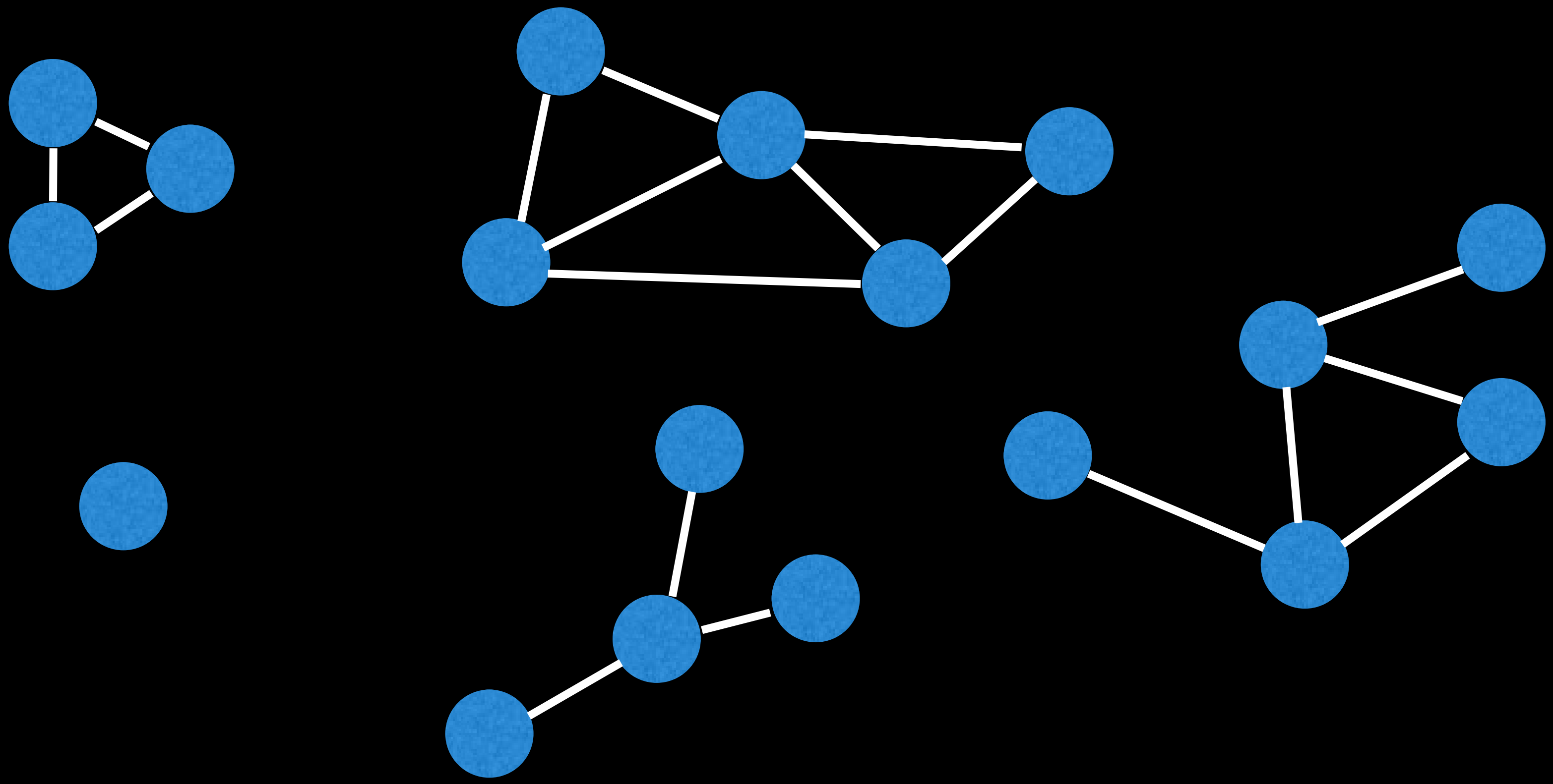
Basic DFS



```
# Global or class scope variables
n = number of nodes in the graph
g = adjacency list representing graph
visited = [false, …, false] # size n

function dfs(at):
  if visited[at]: return
  visited[at] = true

  neighbours = graph[at]
  for next in neighbours:
    dfs(next)

# Start DFS at node zero
start_node = 0
dfs(start_node)
```

Connected Components

Sometimes a graph is split into multiple components. It’s useful to be able to identify and count these components.



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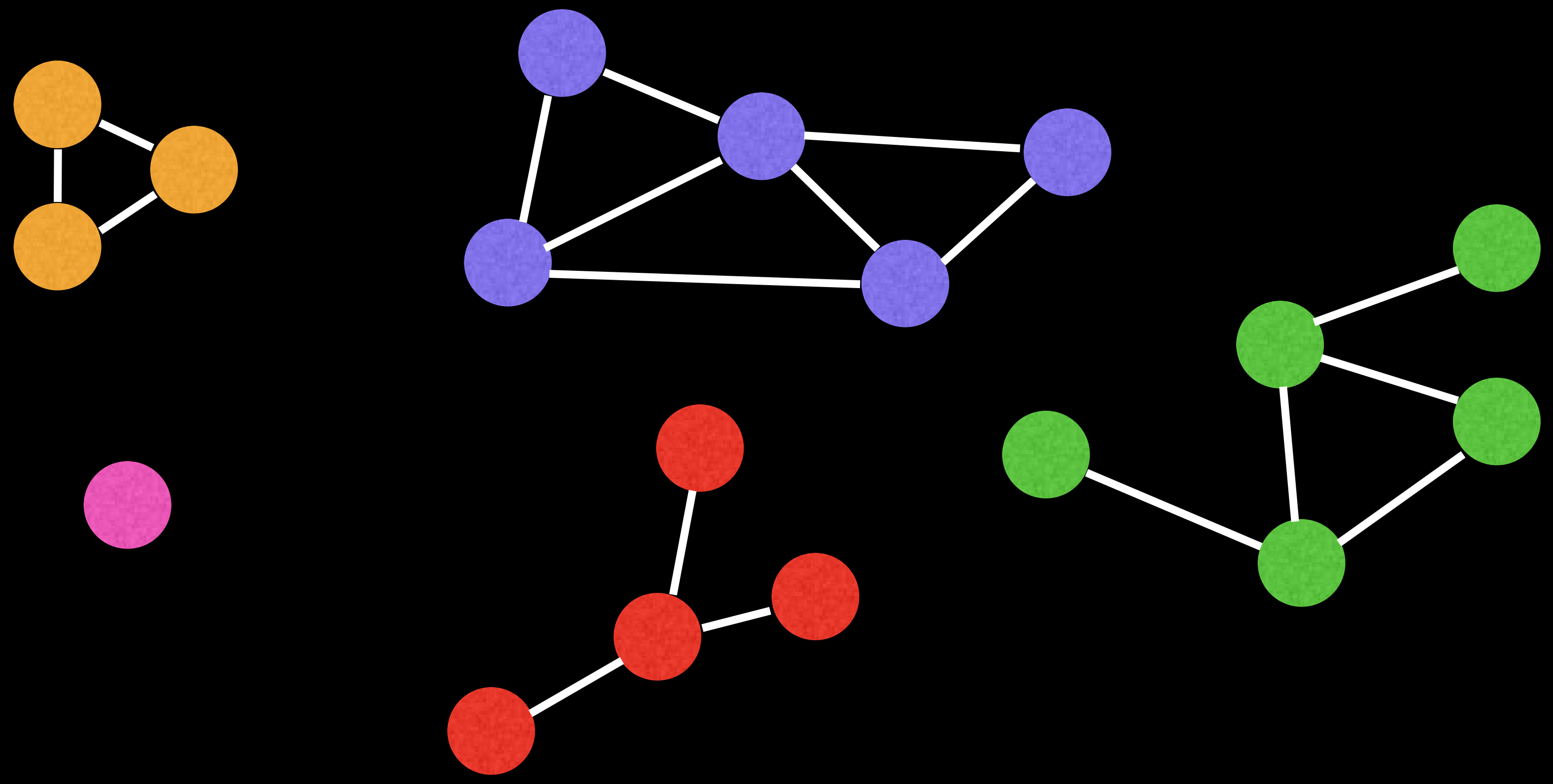
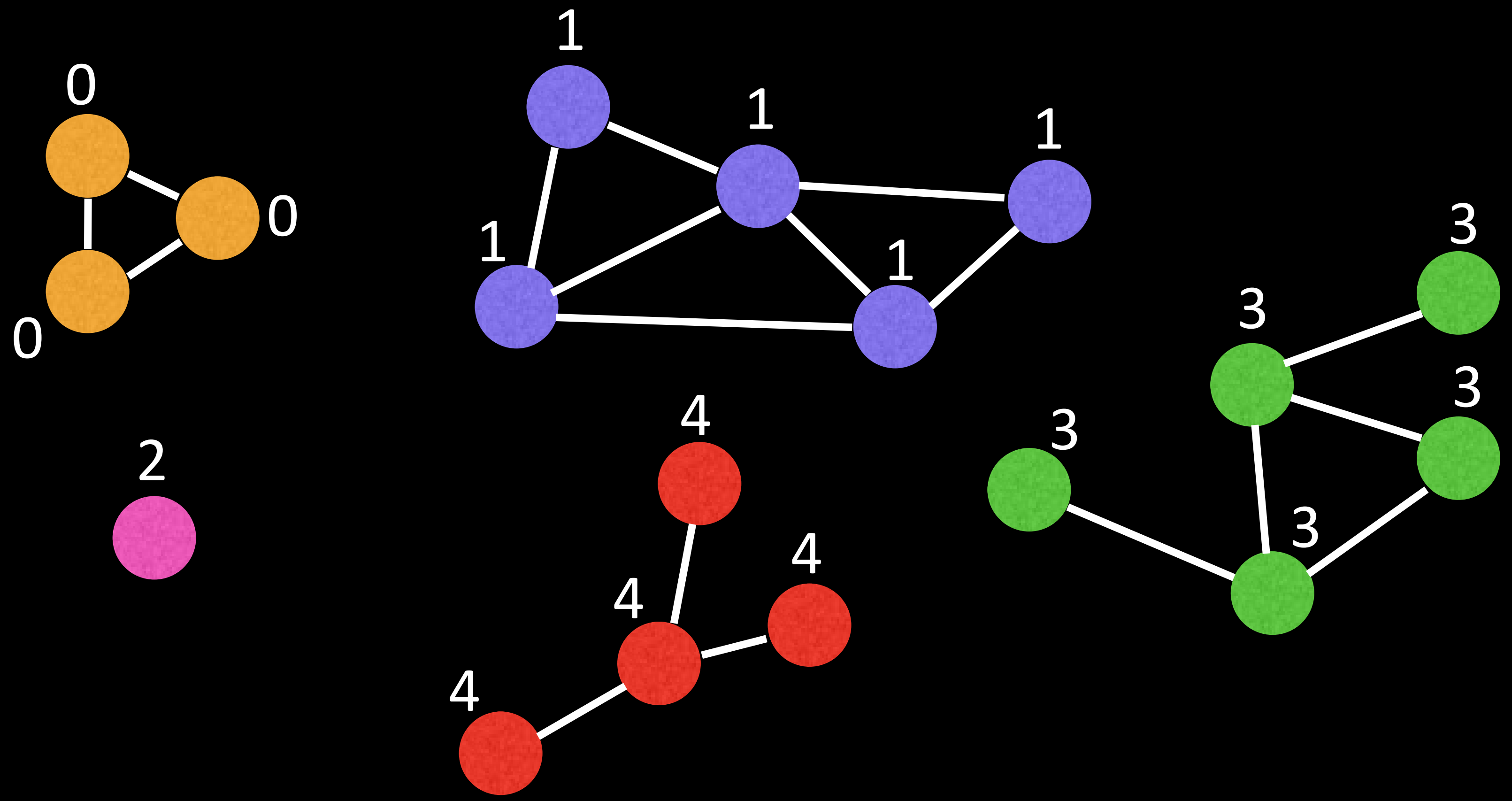
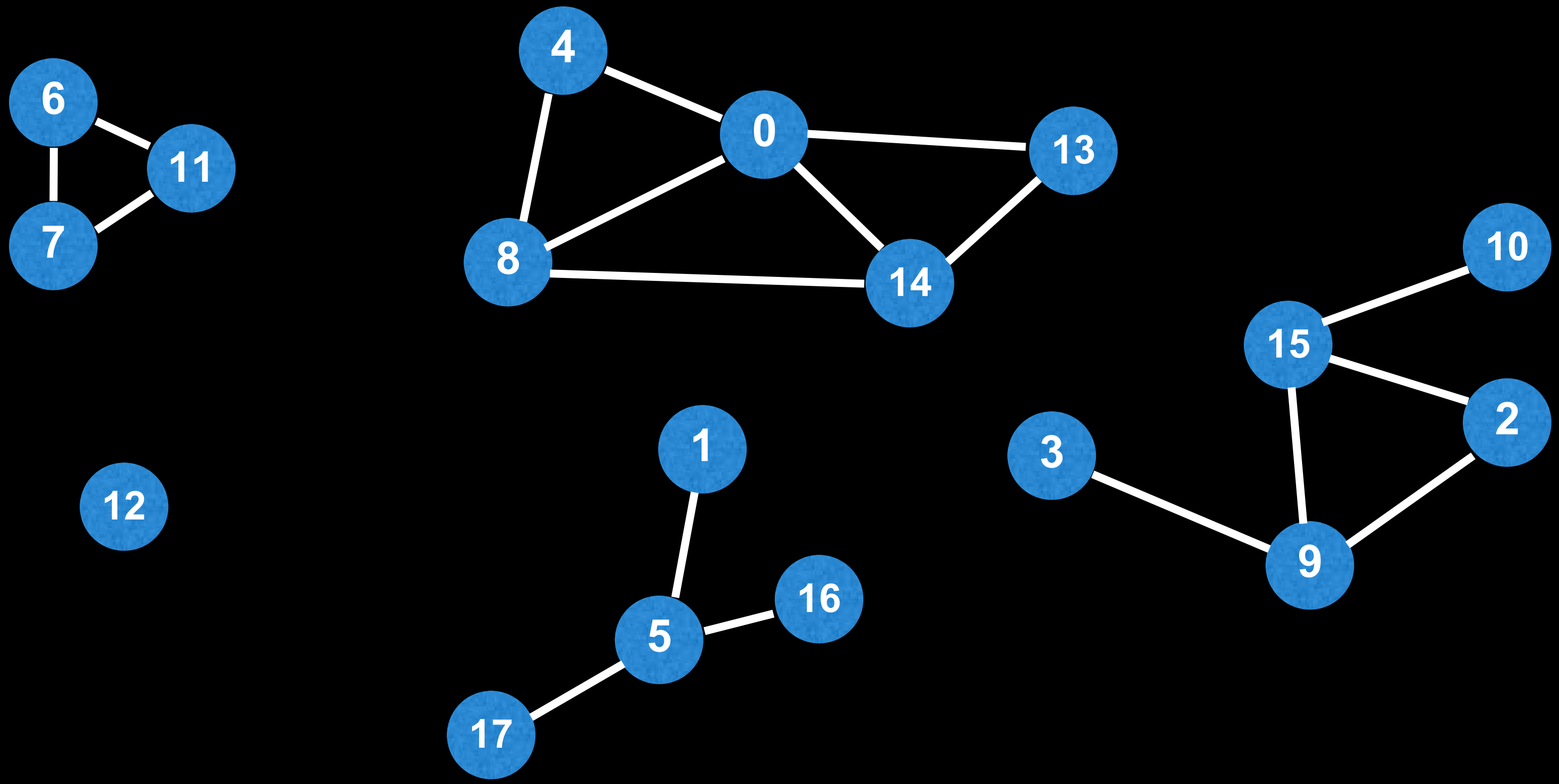


Connected Components

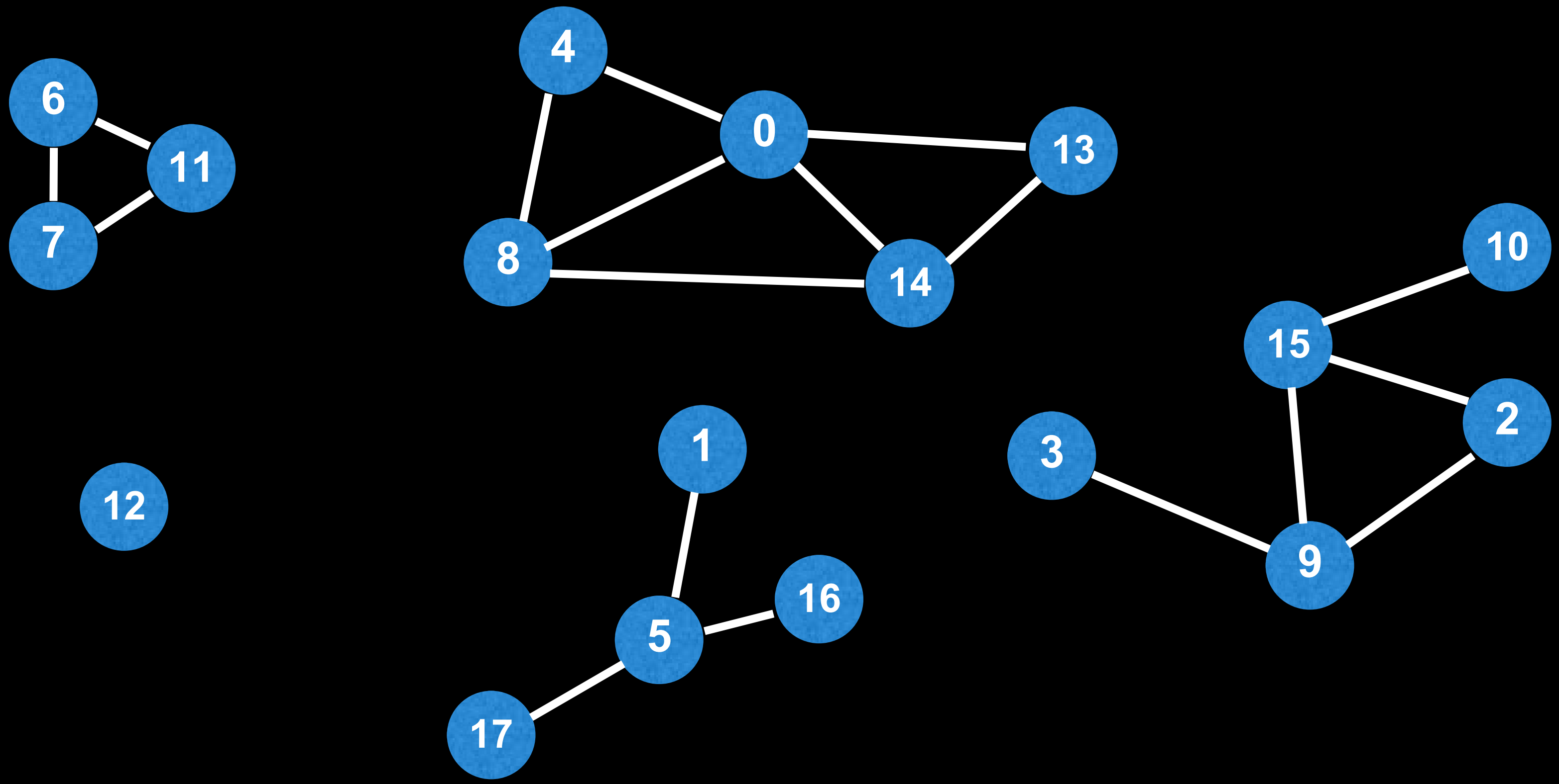
Assign an integer value to each group to be able to tell them apart.



We can use a DFS to identify components. First, make sure all the nodes are labeled from [0, n) where n is the number of nodes.



Algorithm: Start a DFS at every node (except if it's already been visited) and mark all reachable nodes as being part of the same component.



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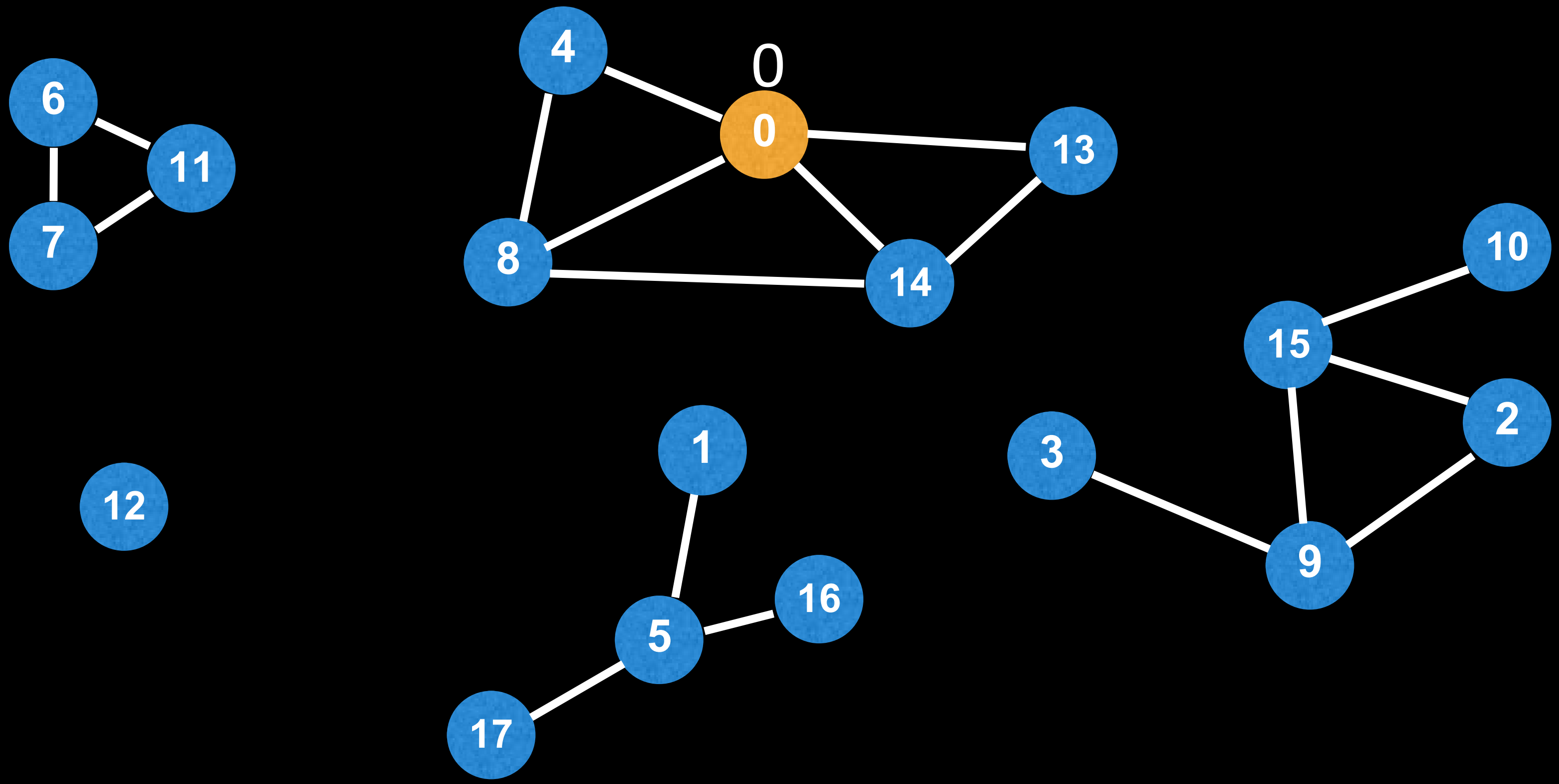

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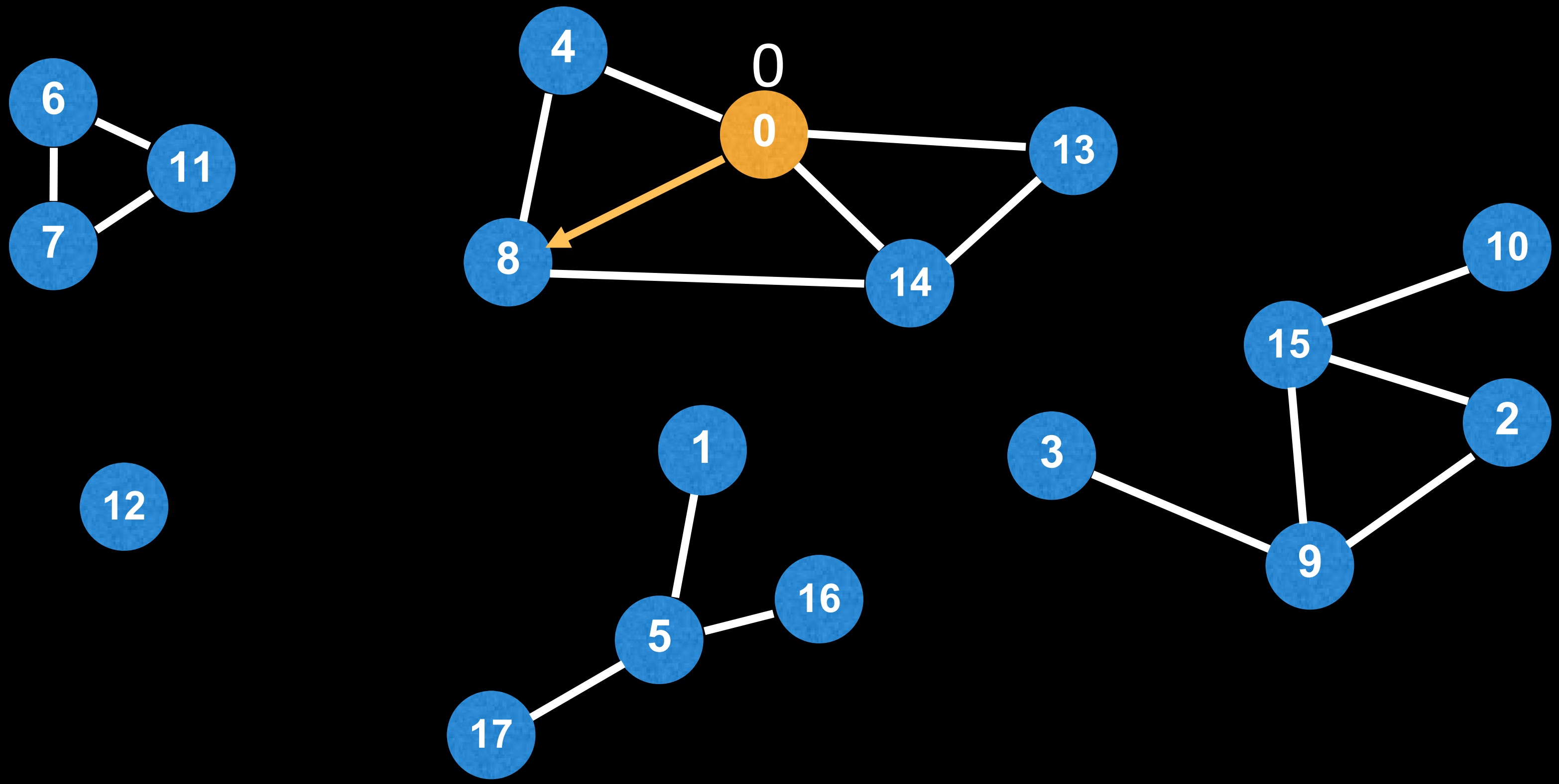

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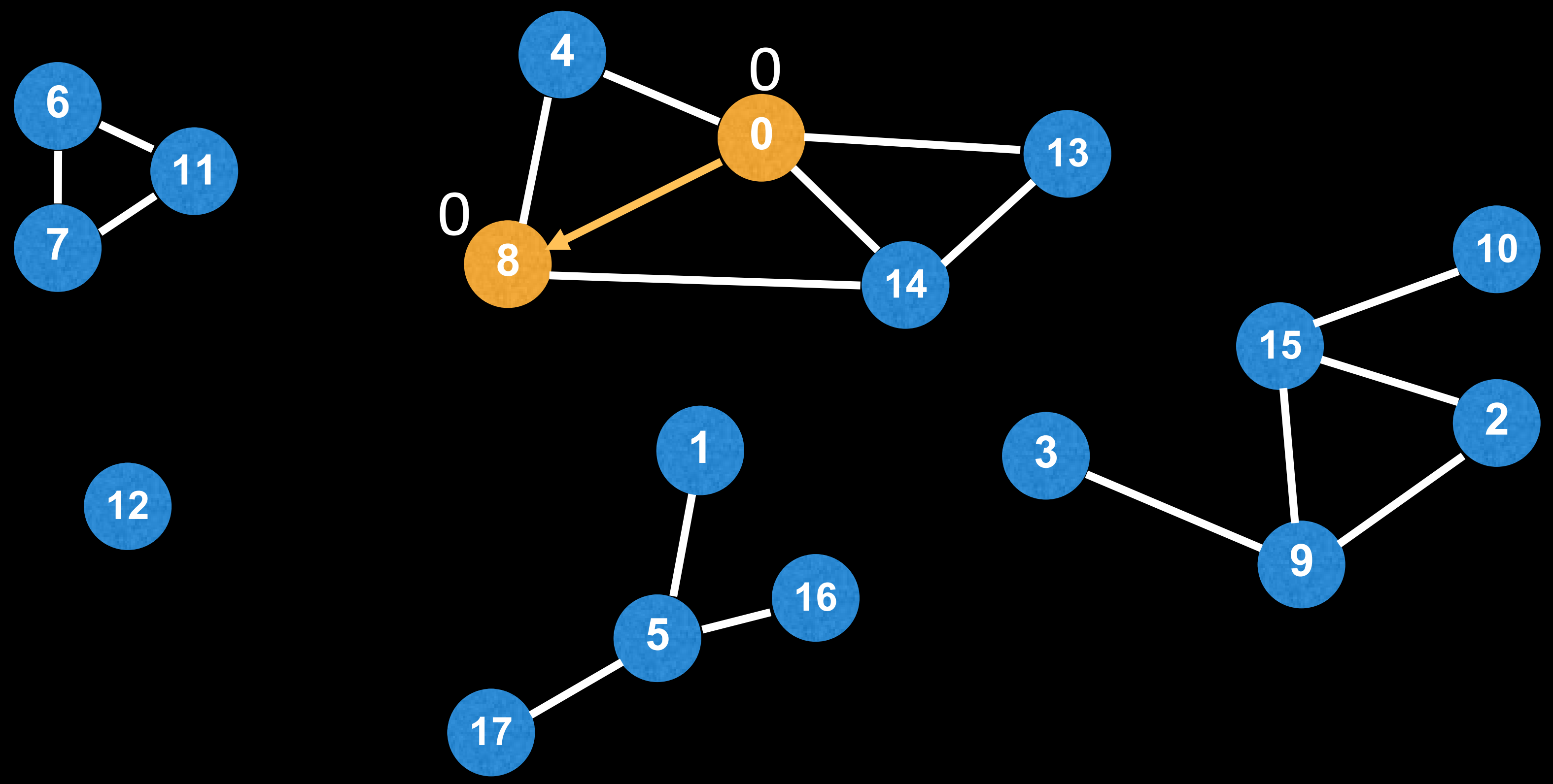

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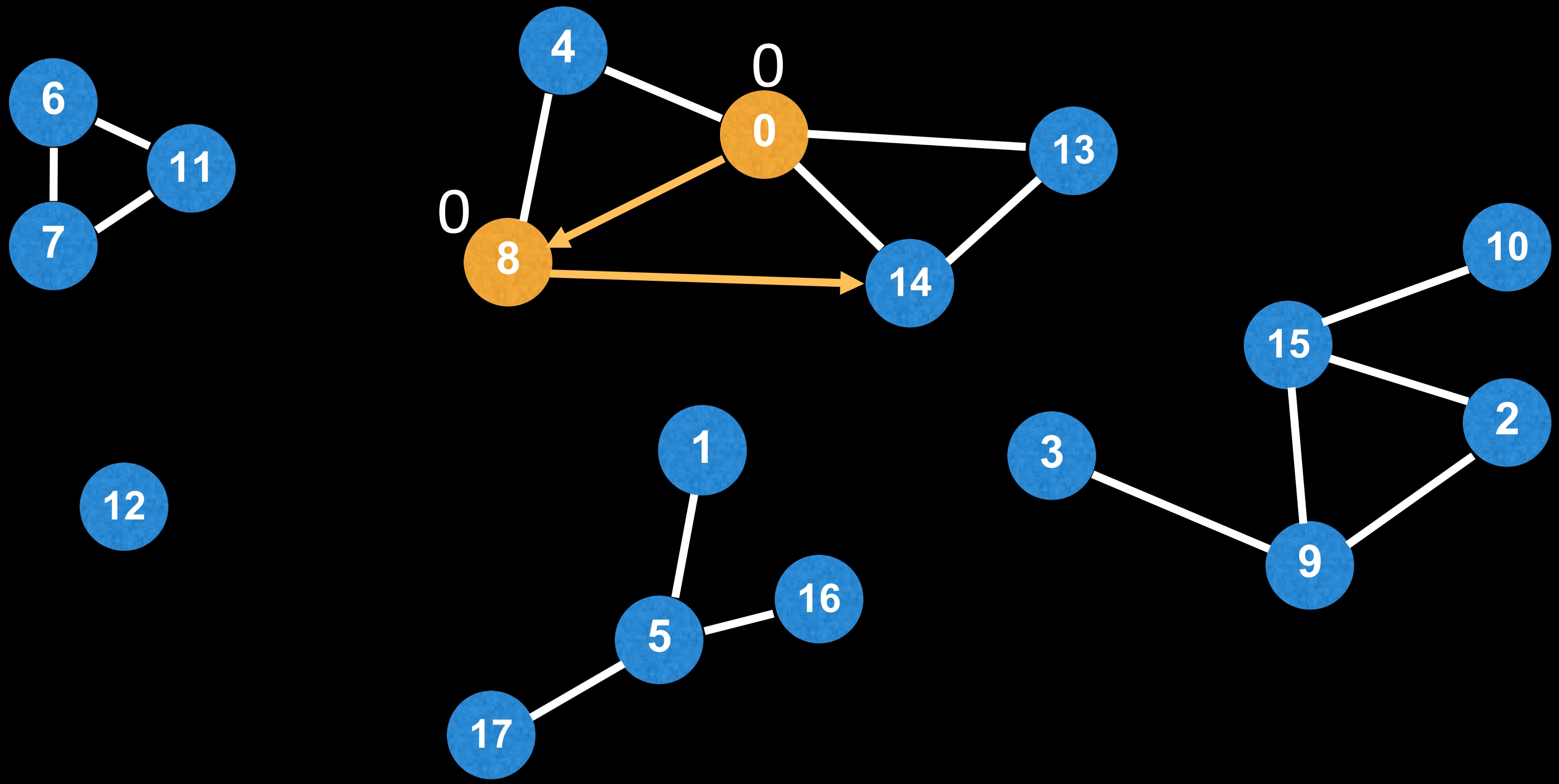

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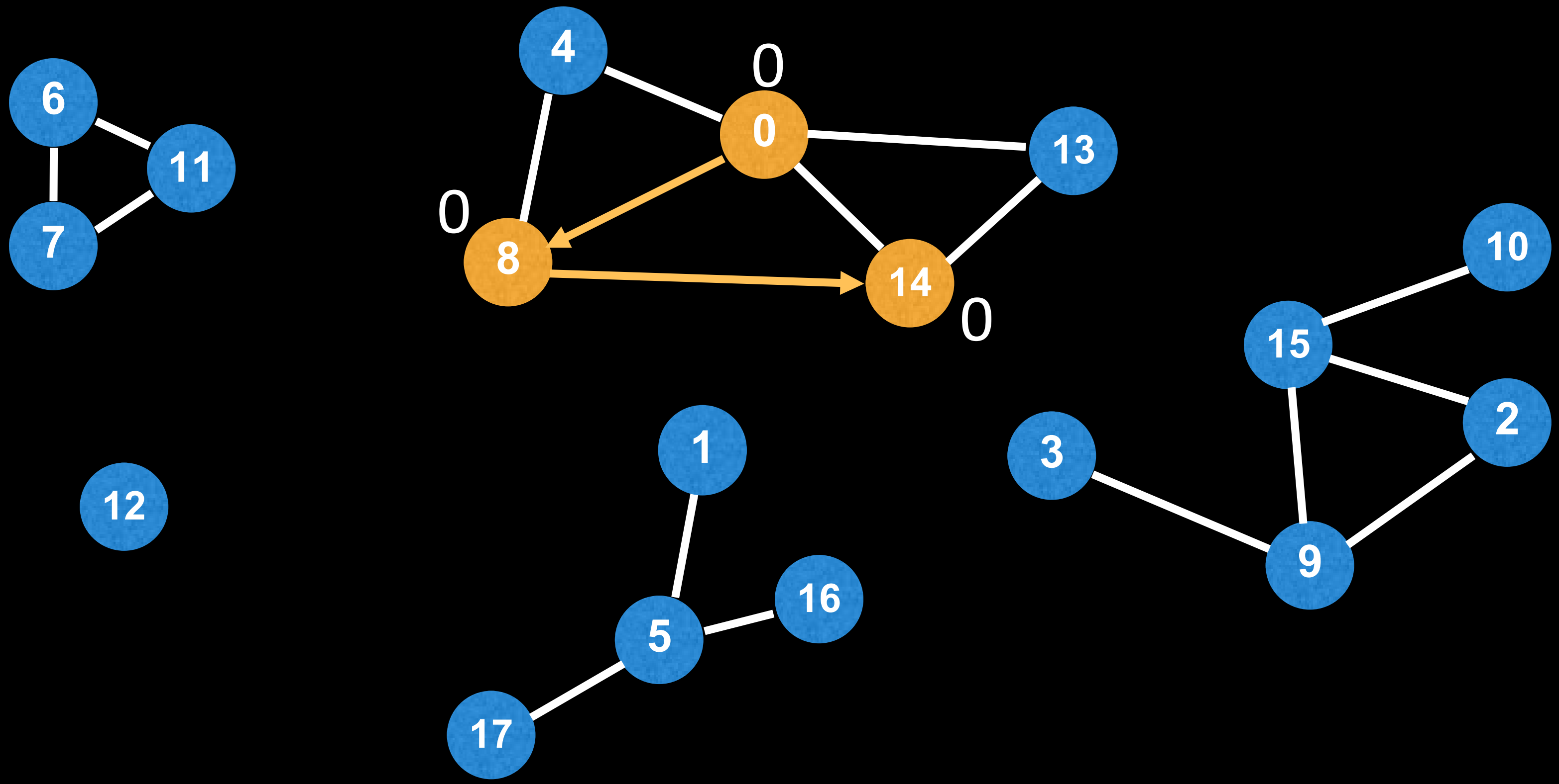

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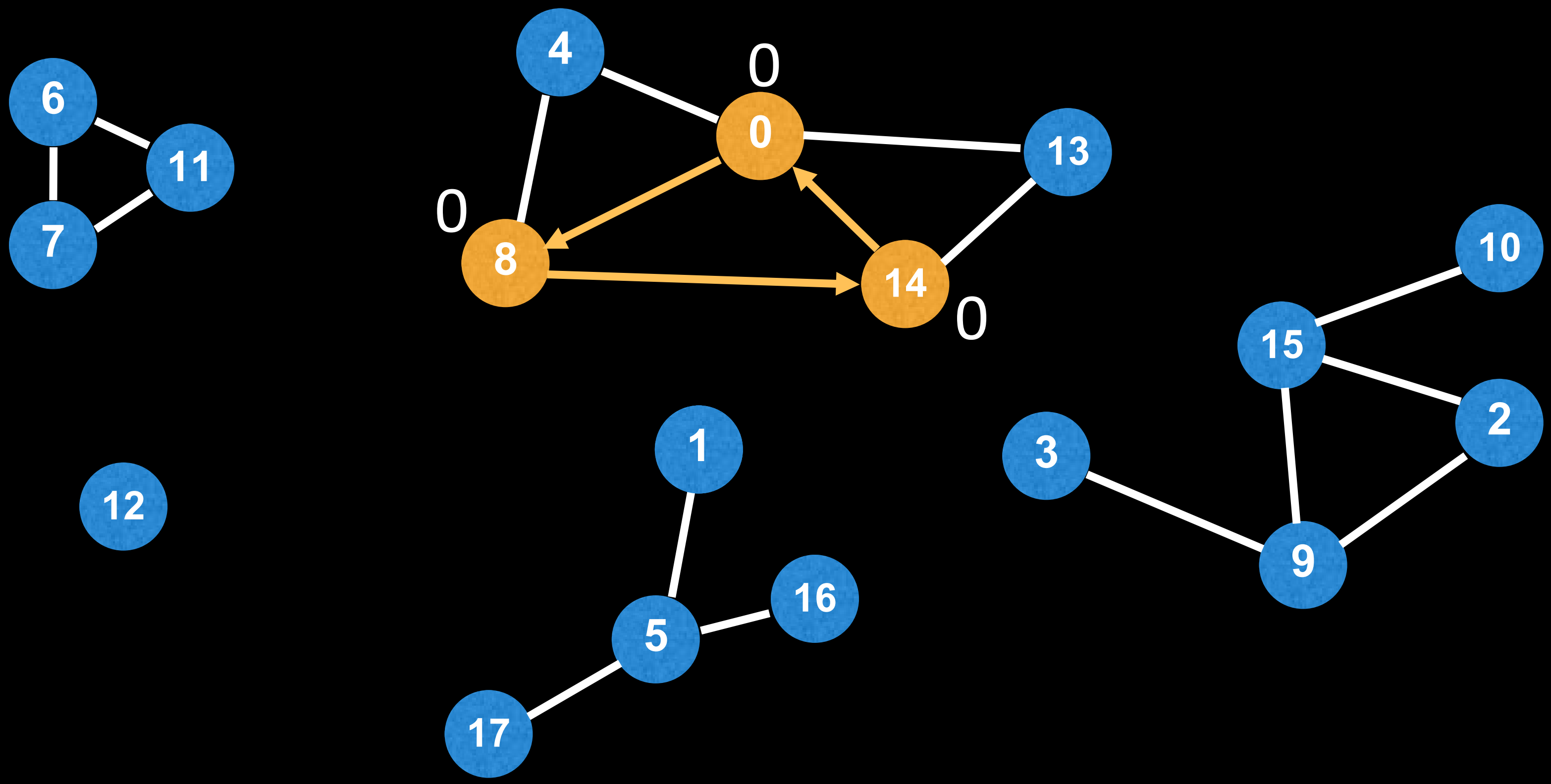

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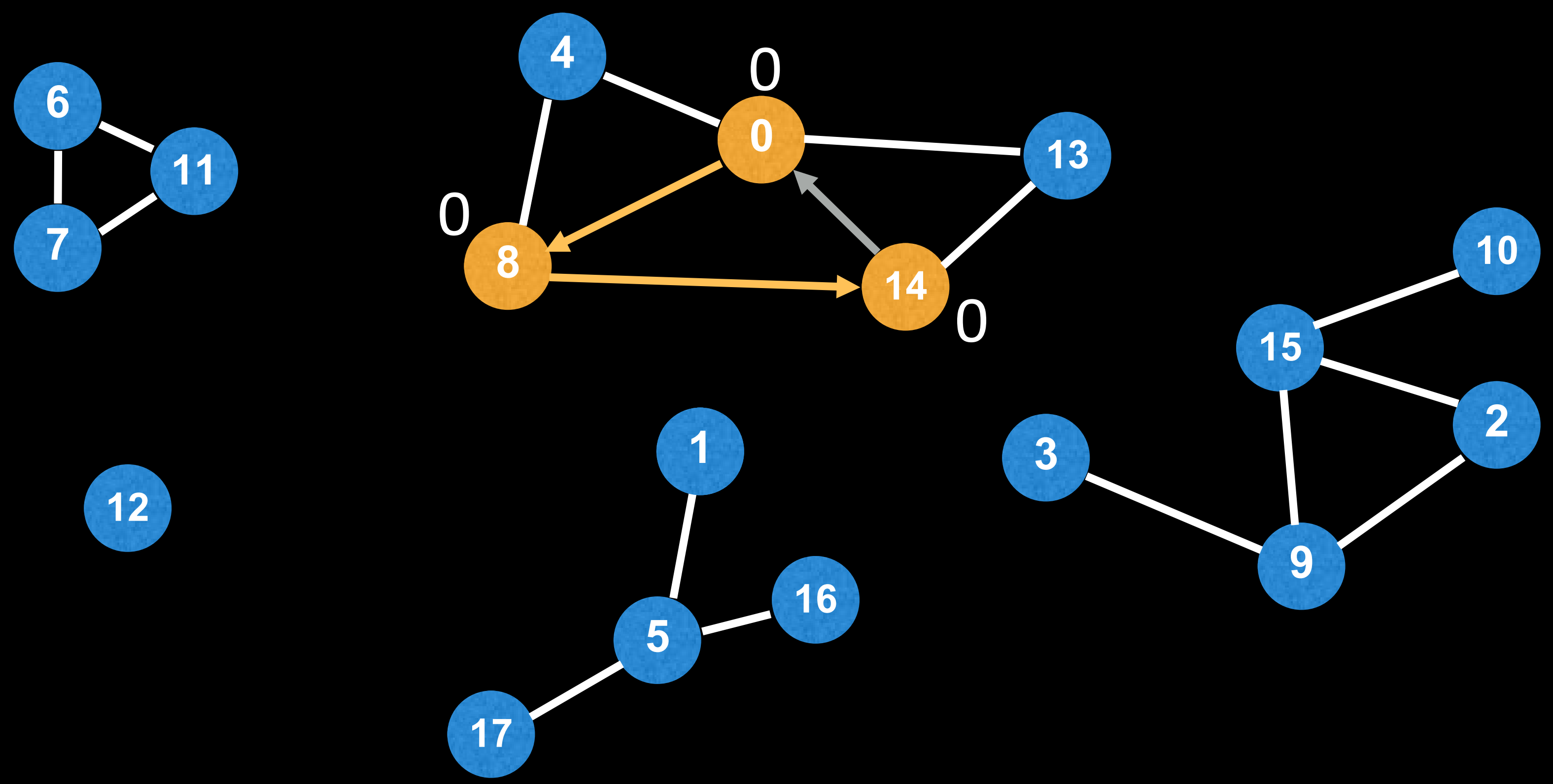

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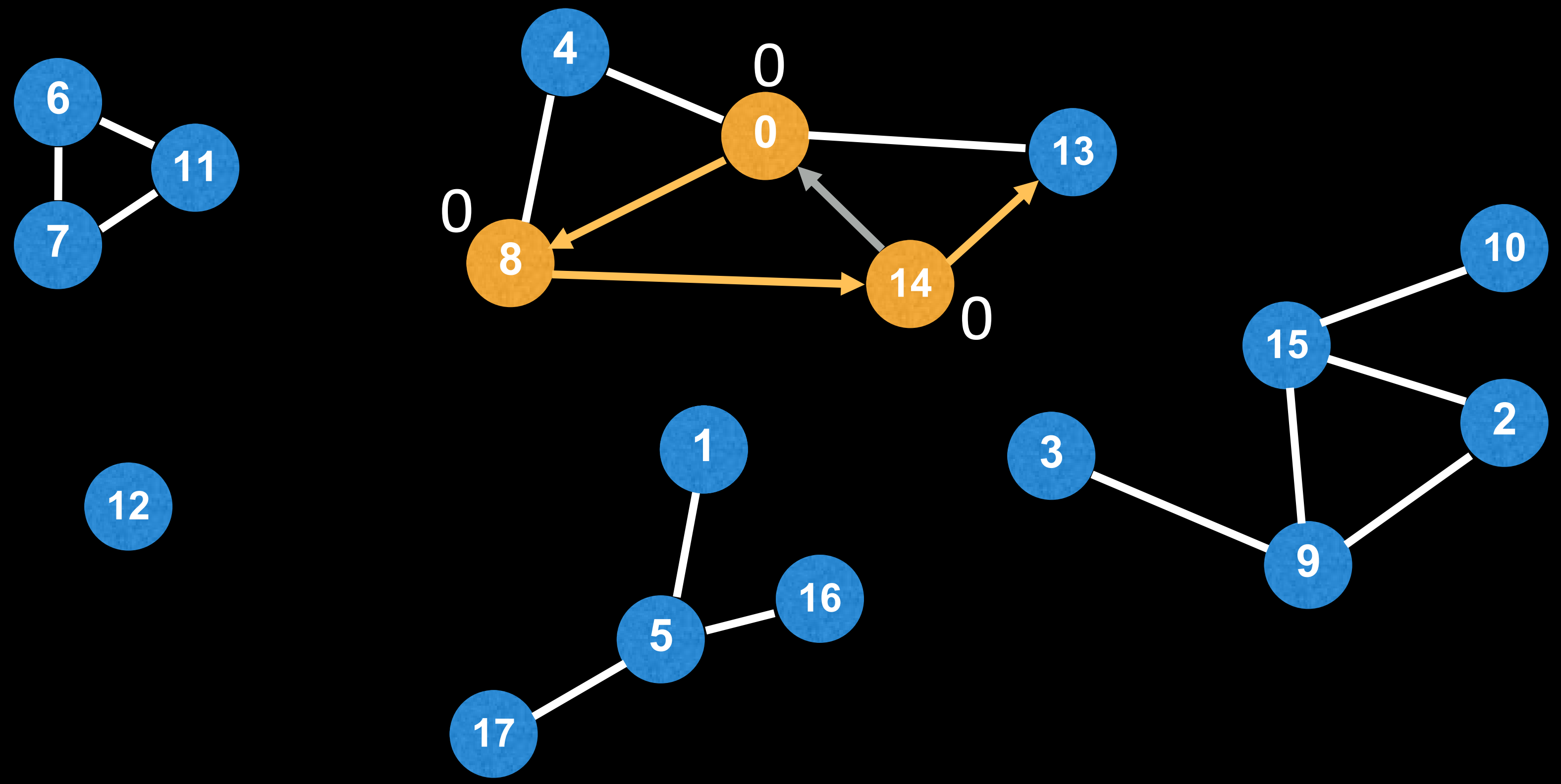

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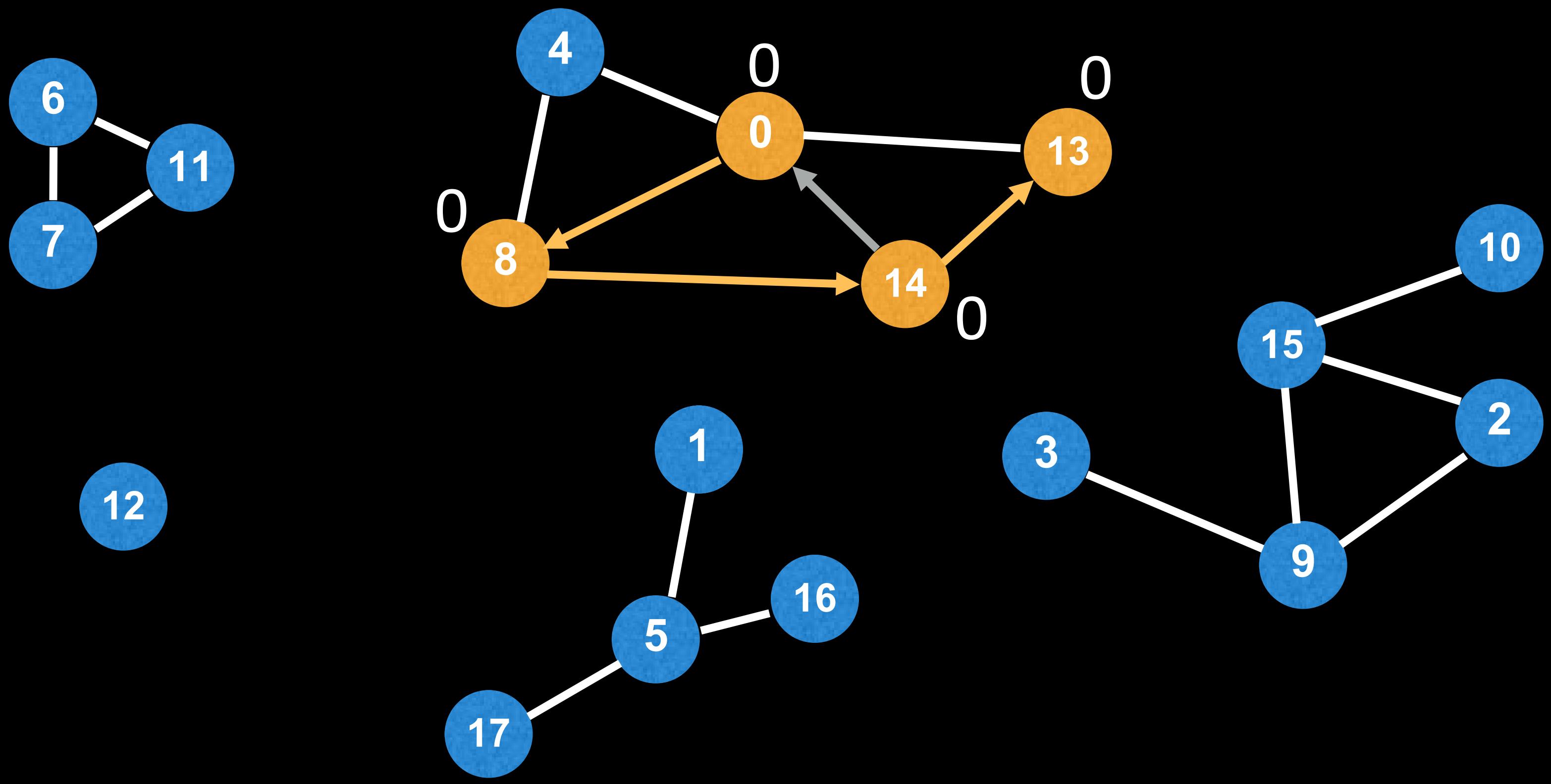

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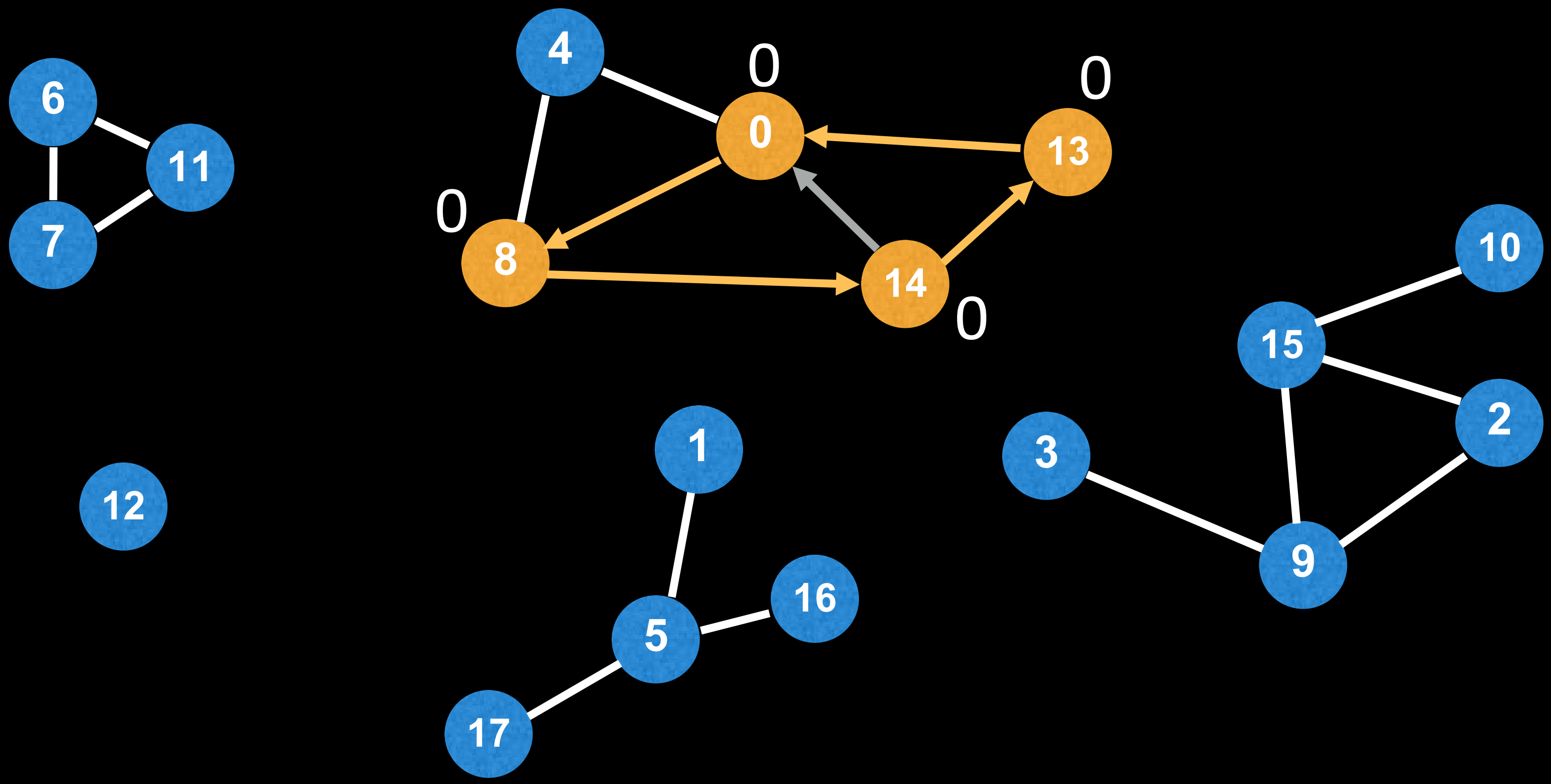

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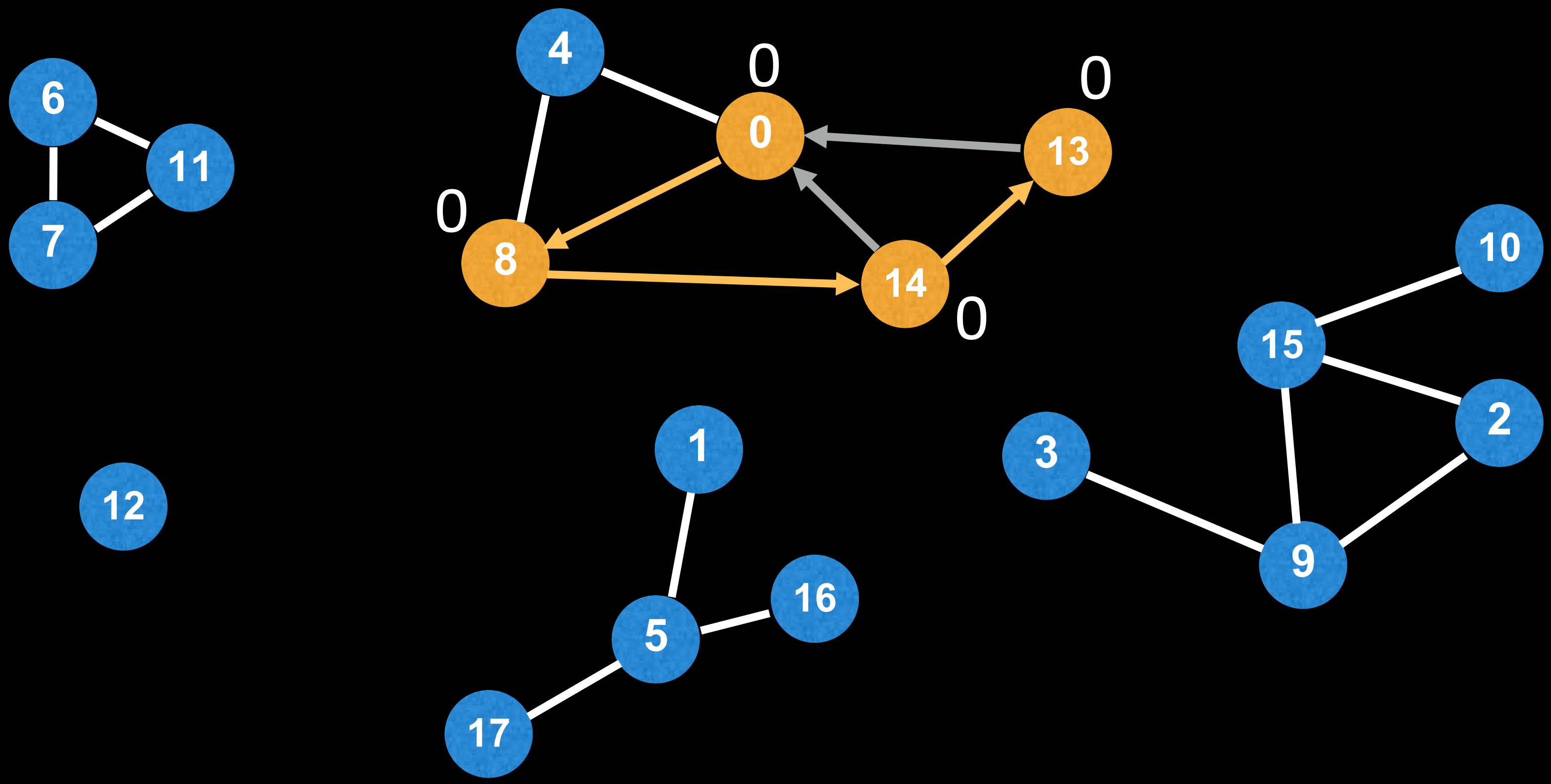

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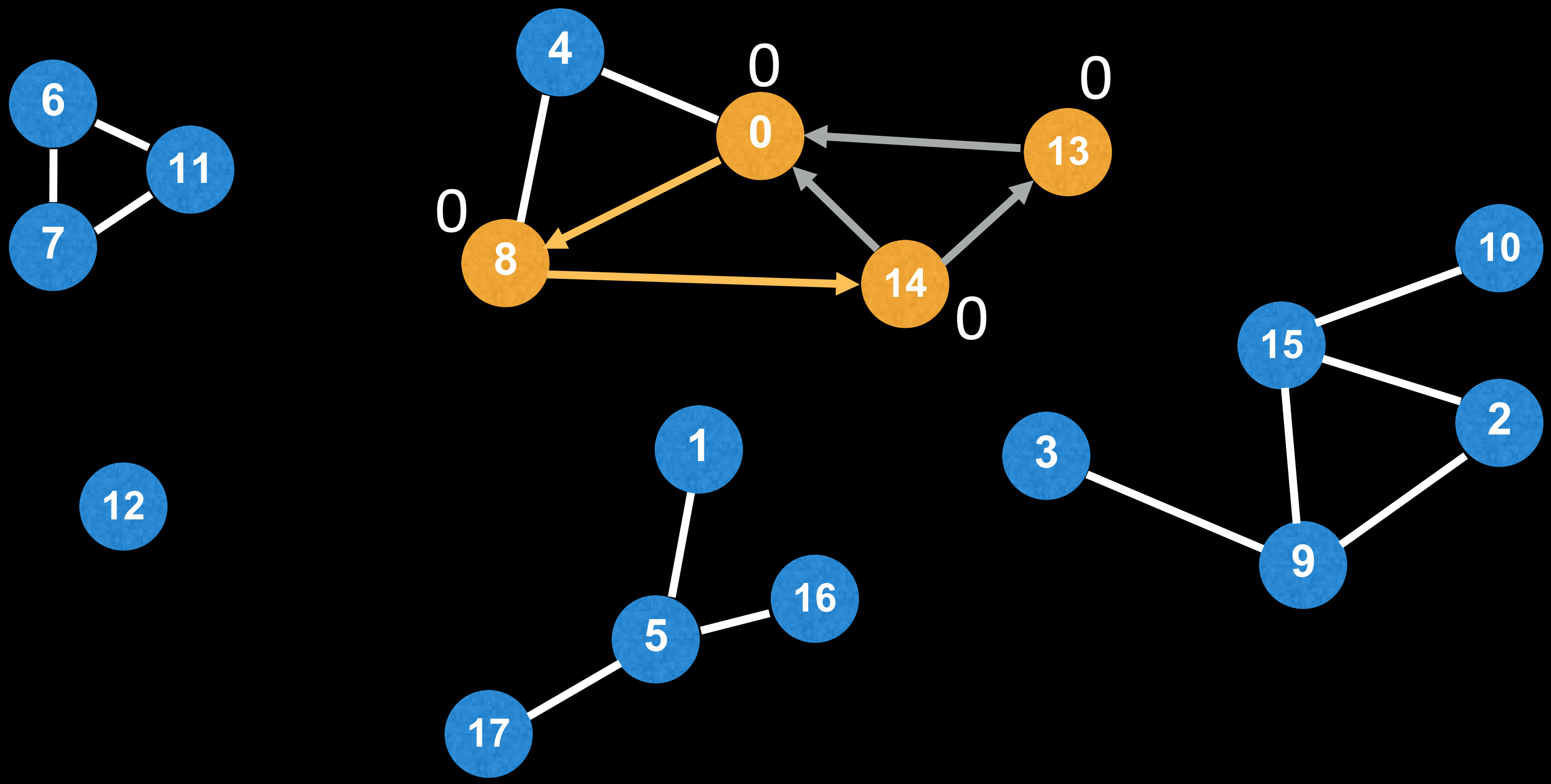

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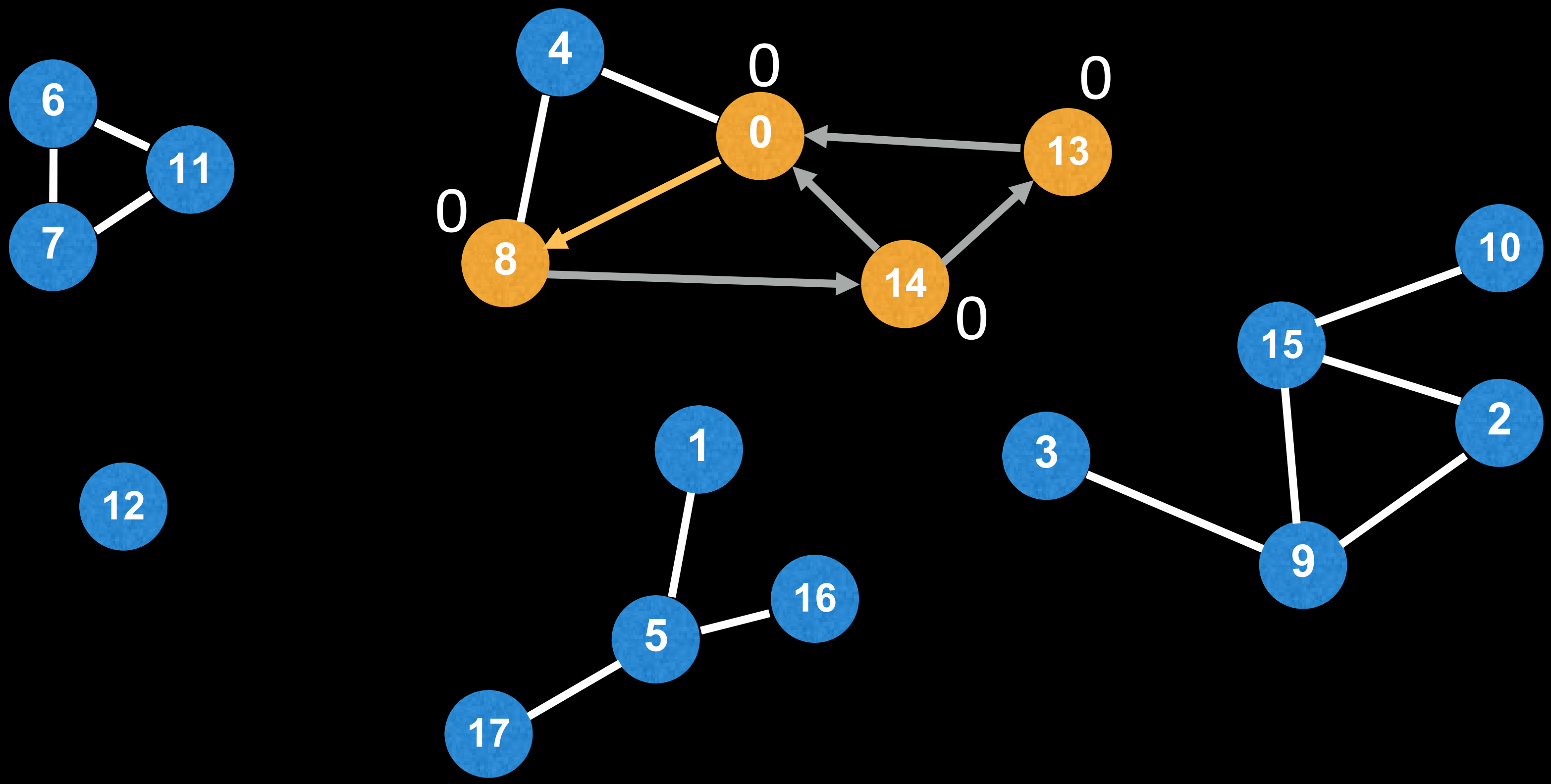

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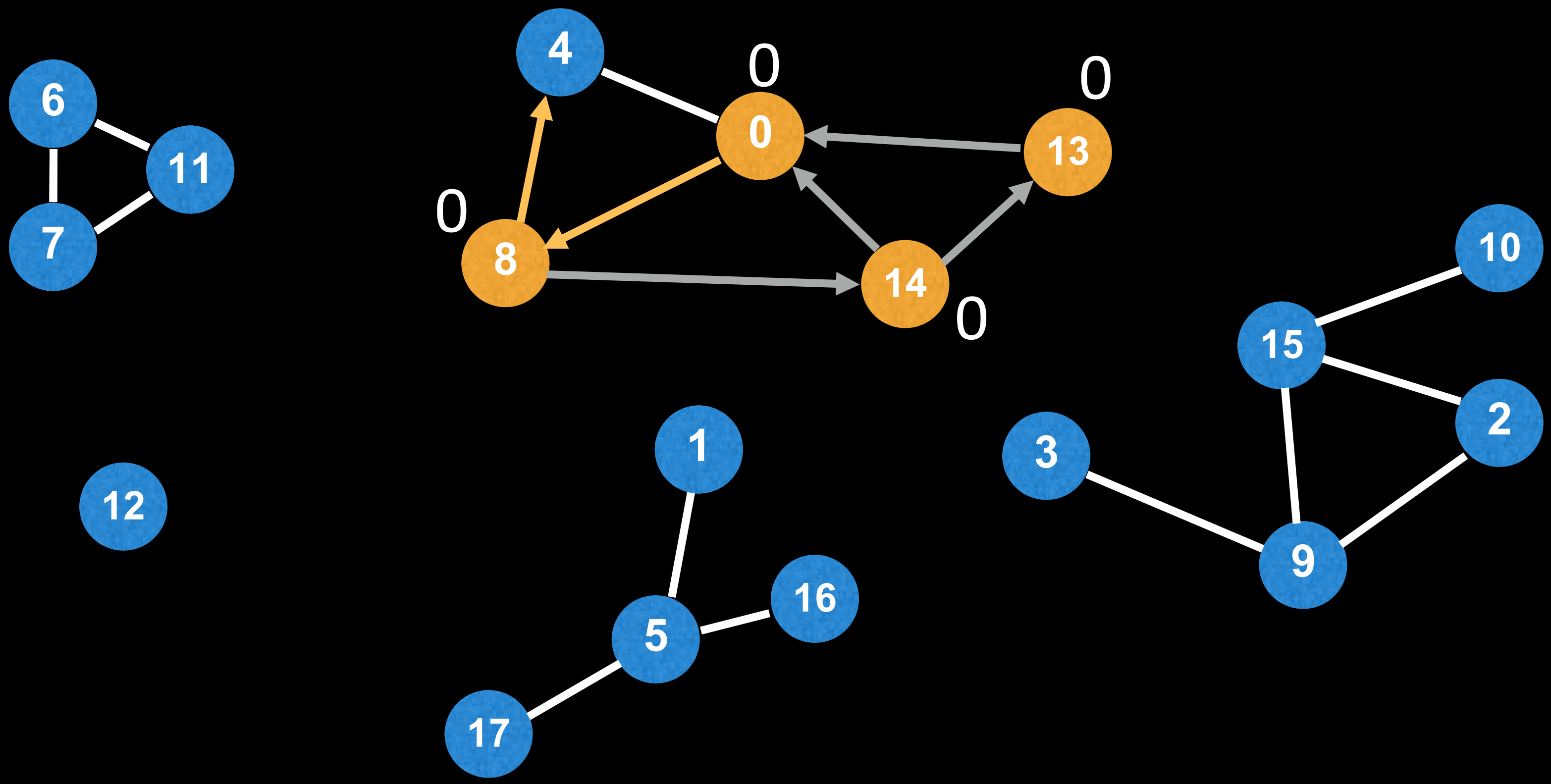

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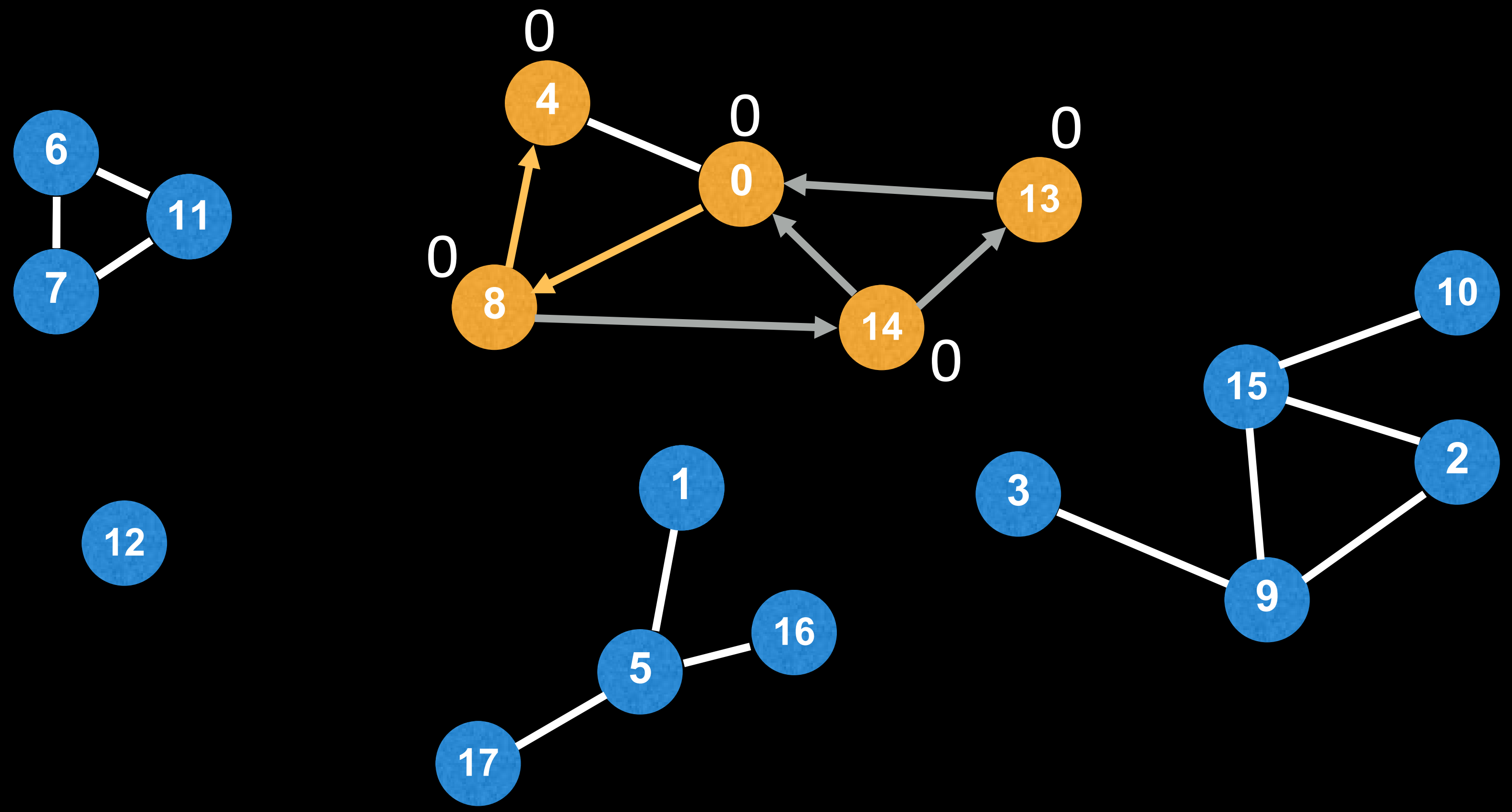

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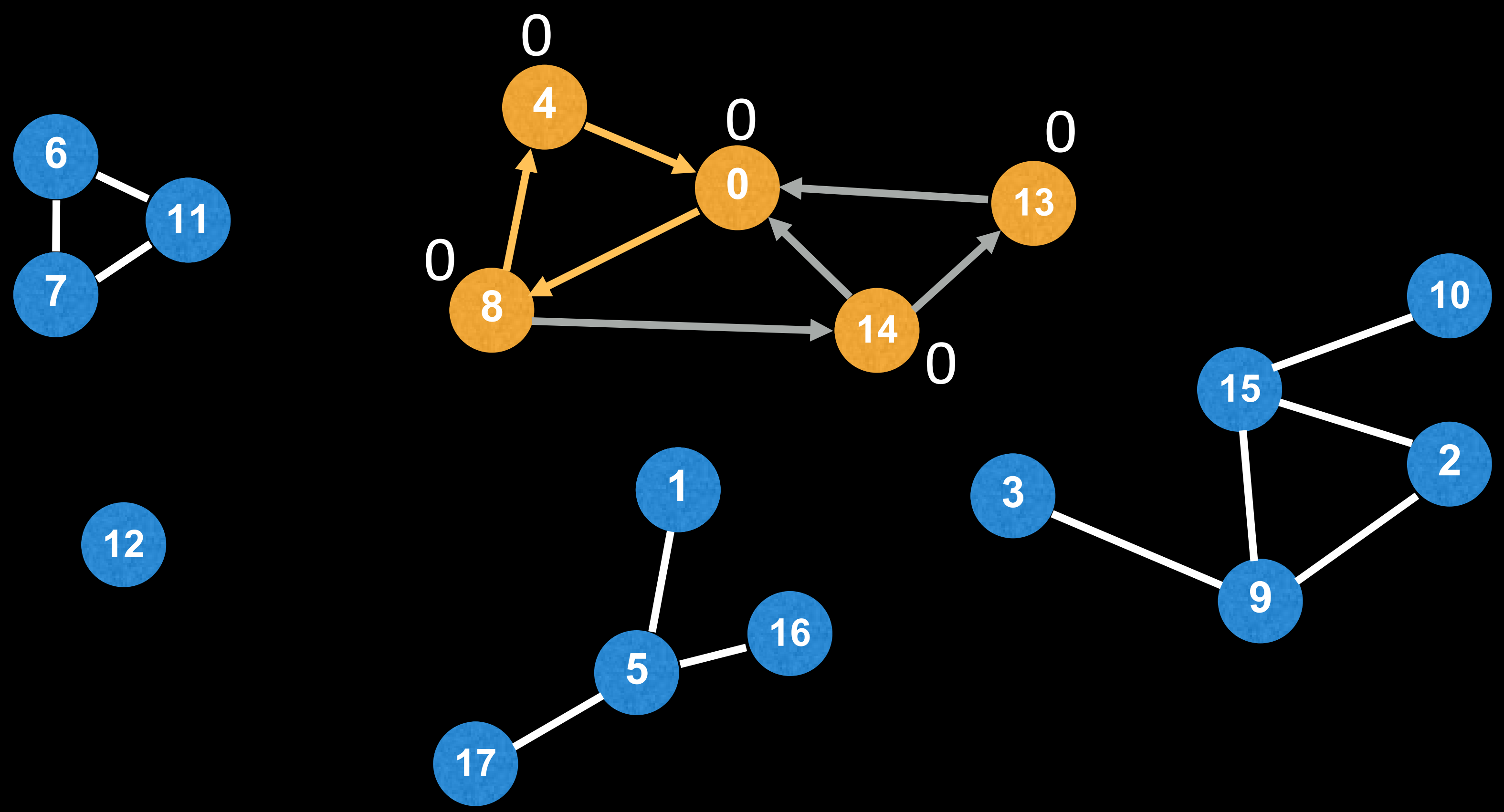

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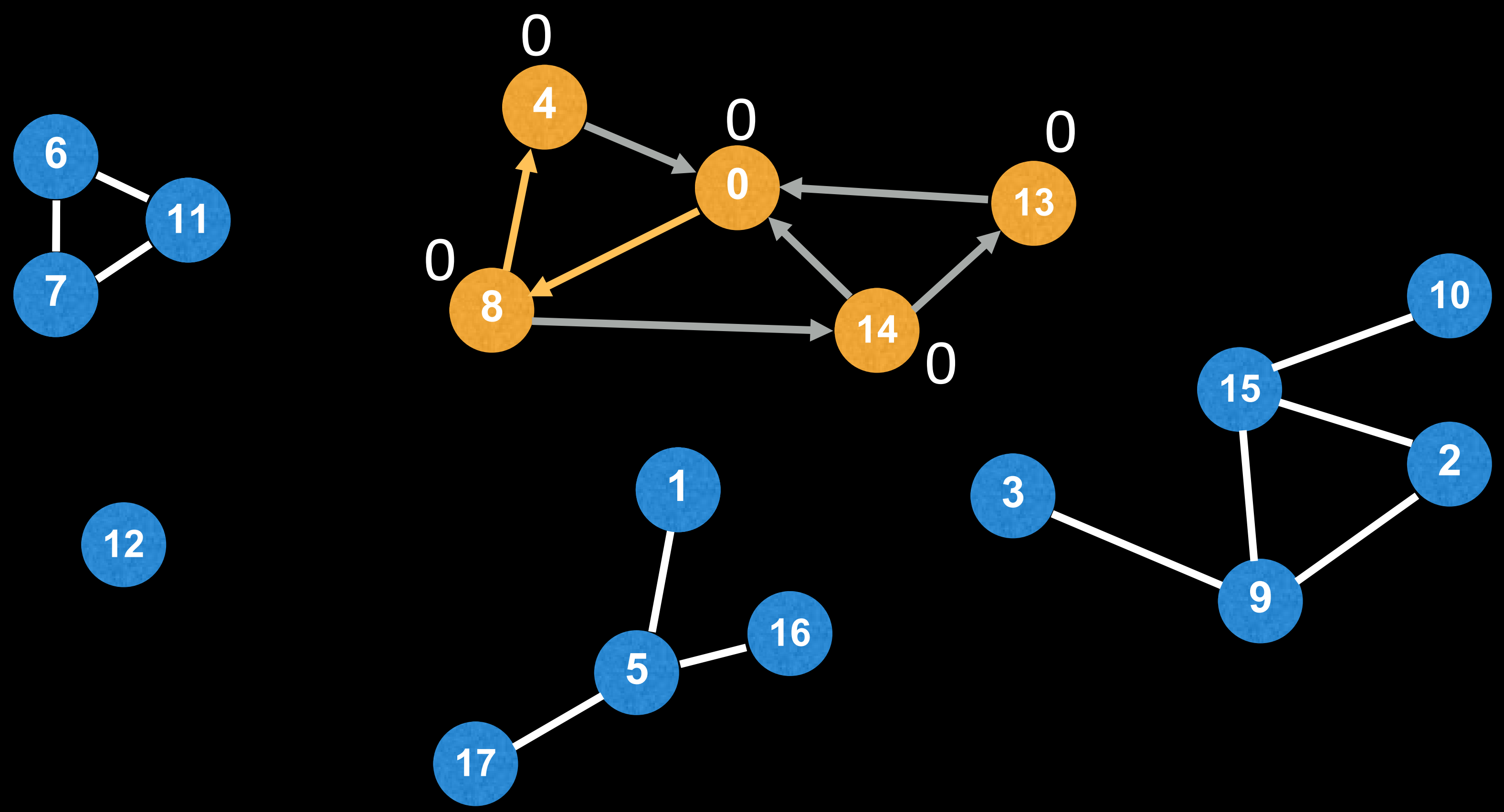

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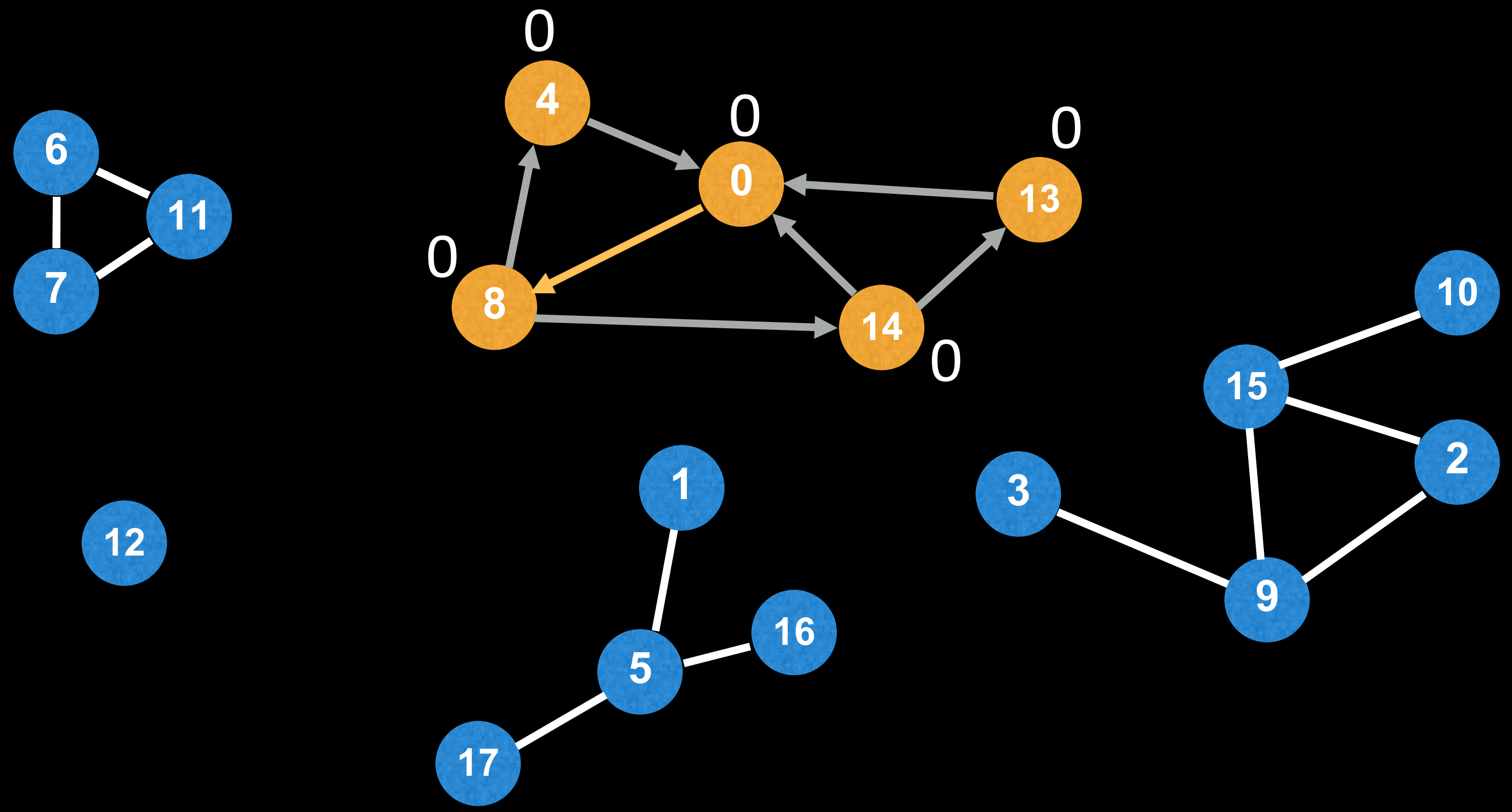

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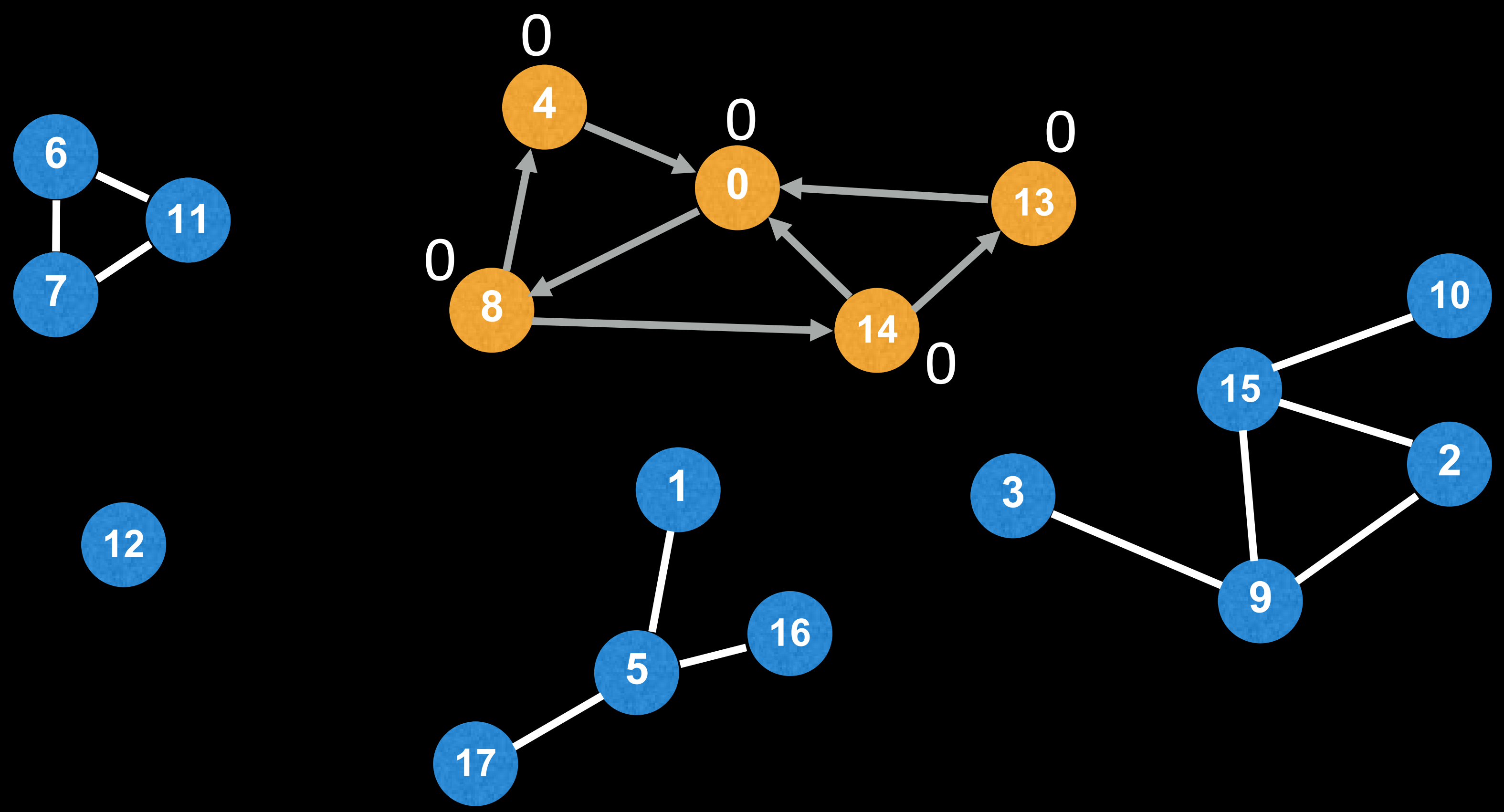

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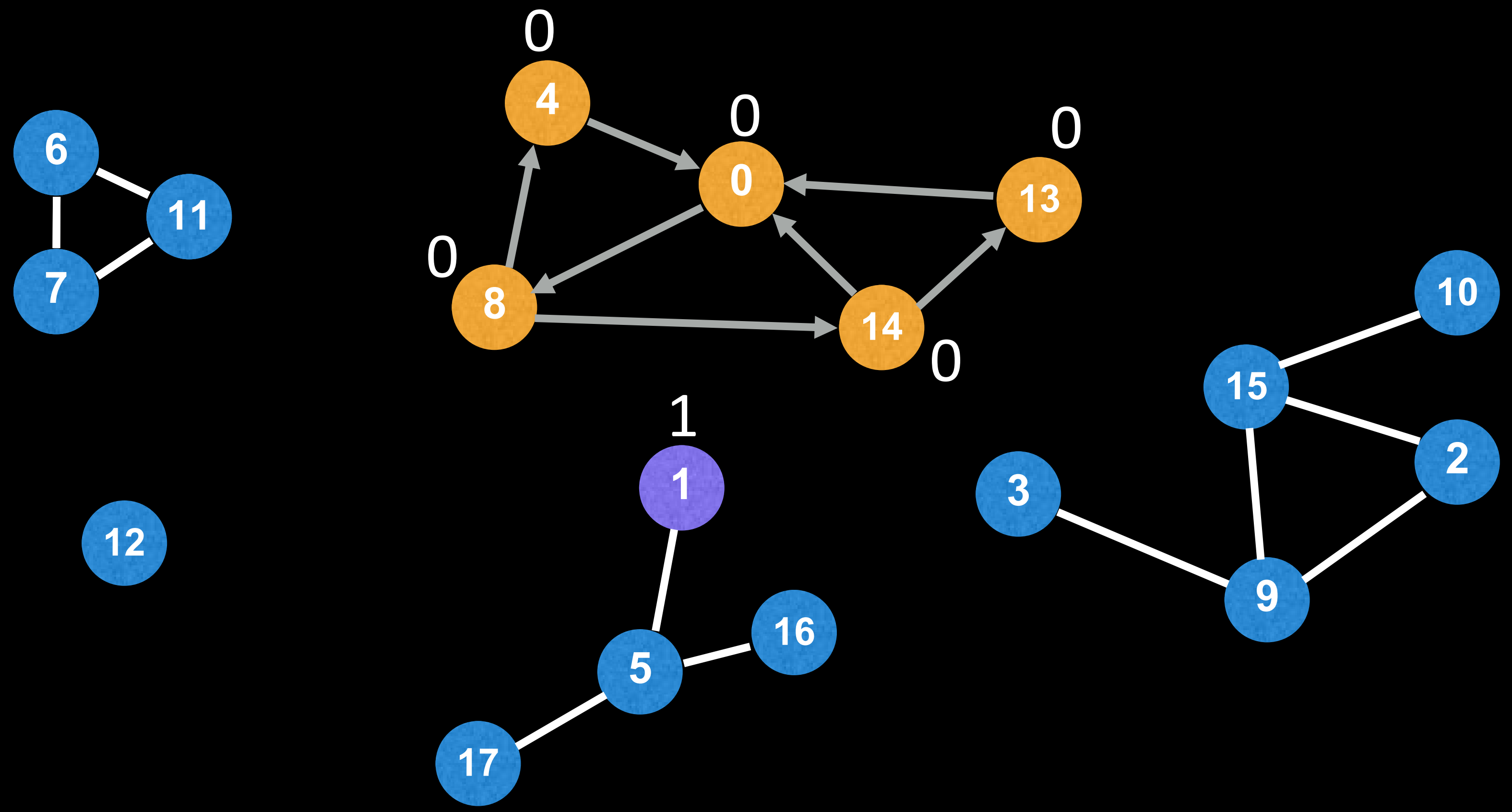

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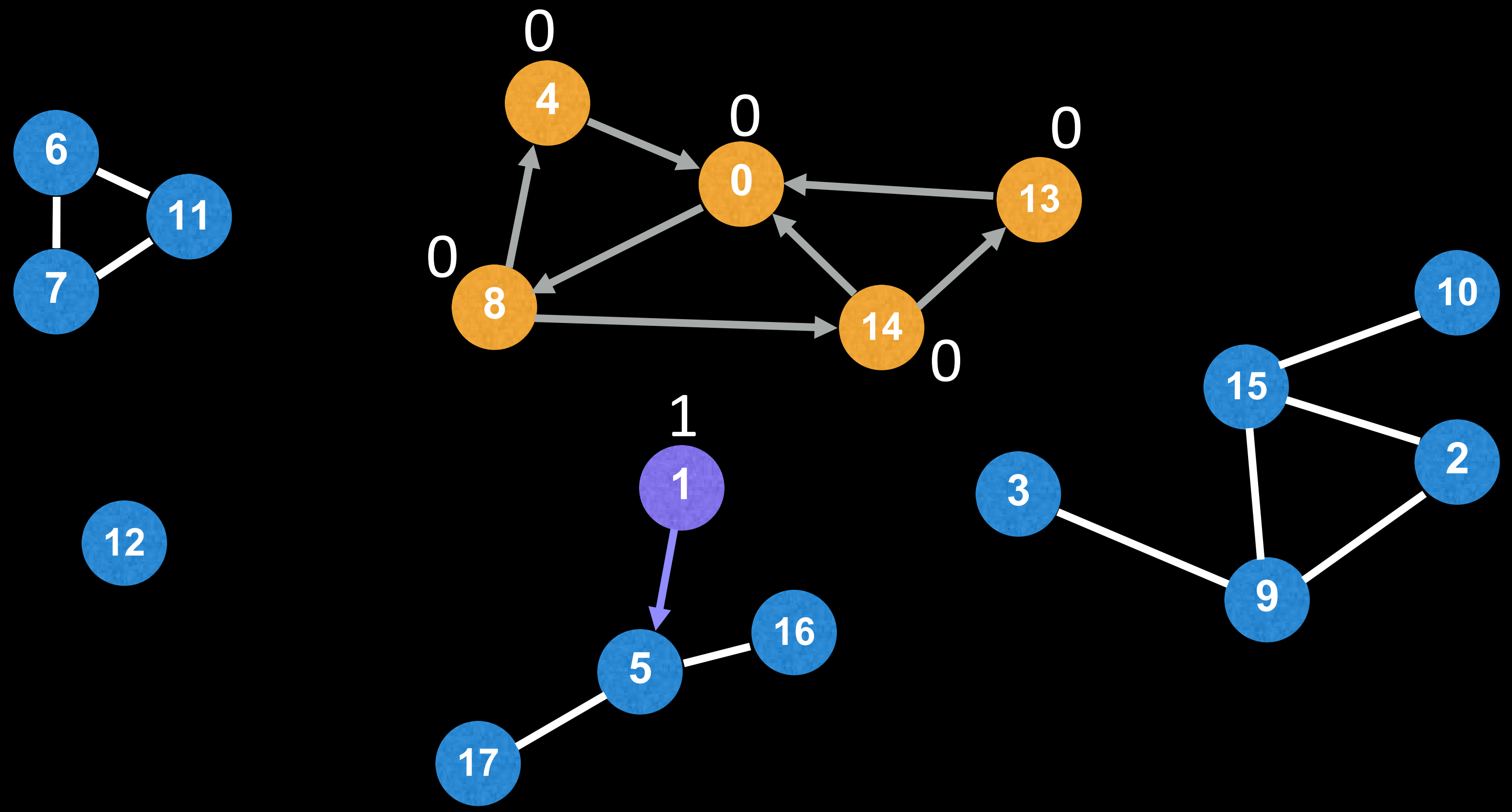

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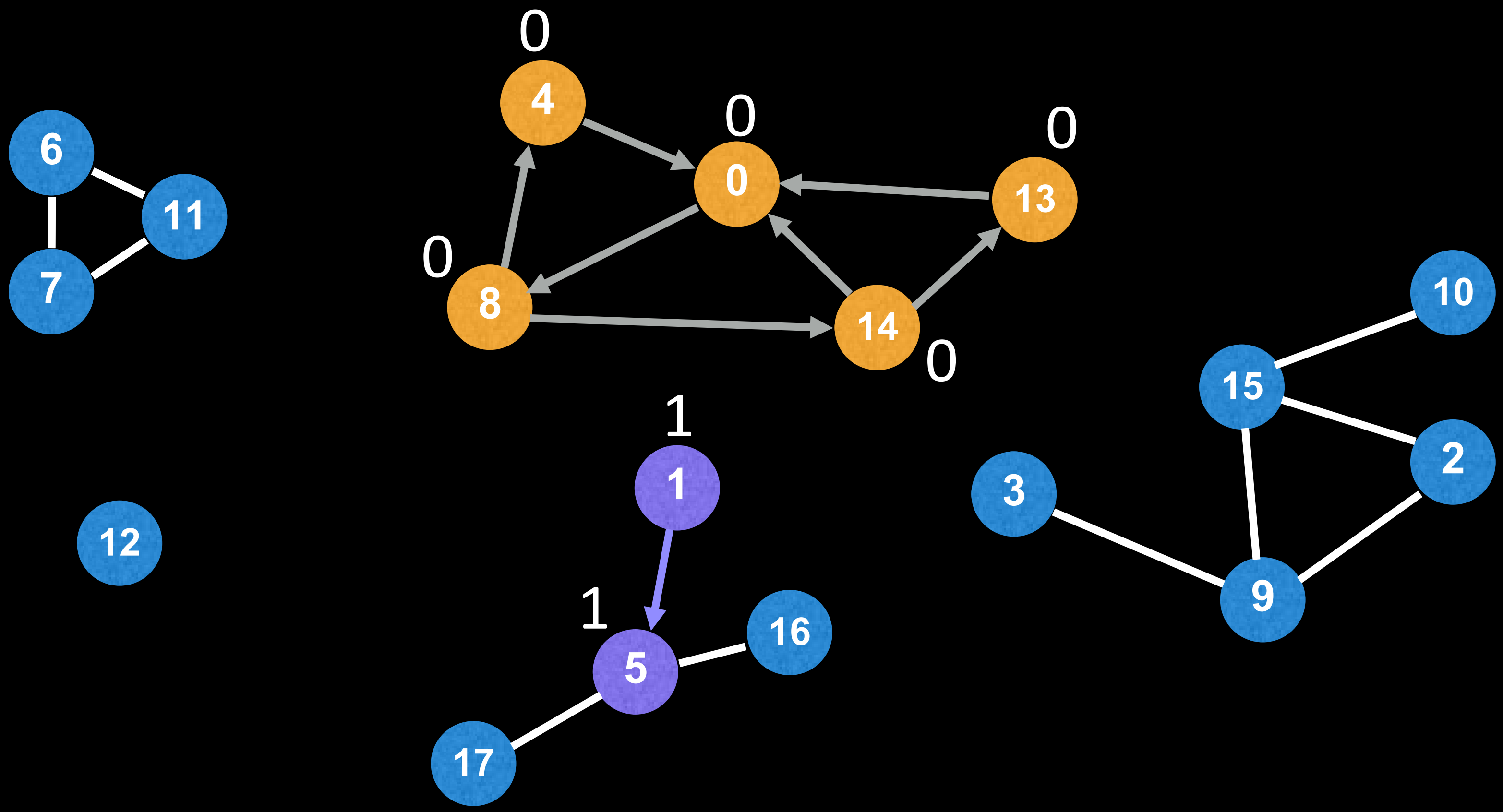

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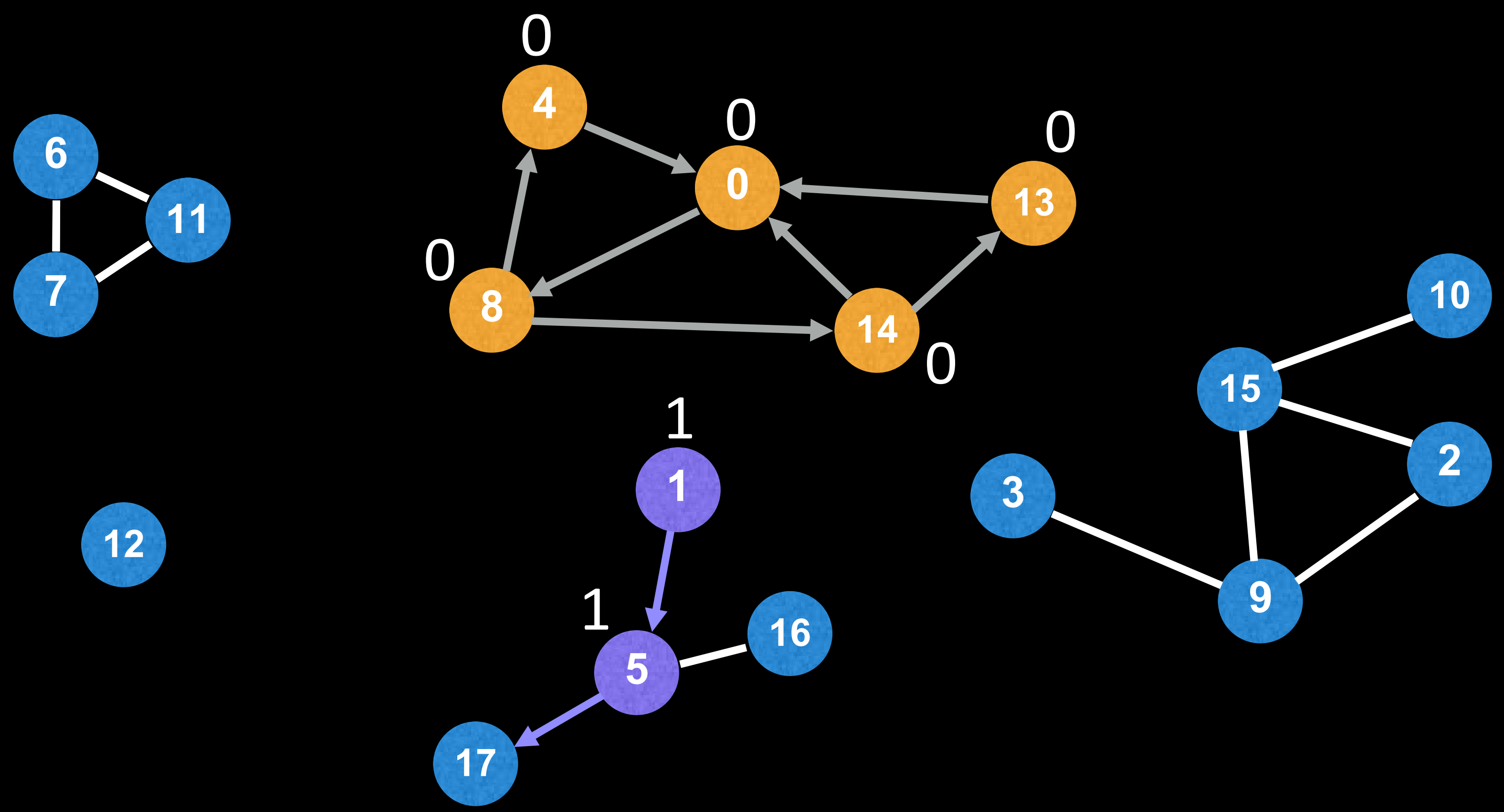

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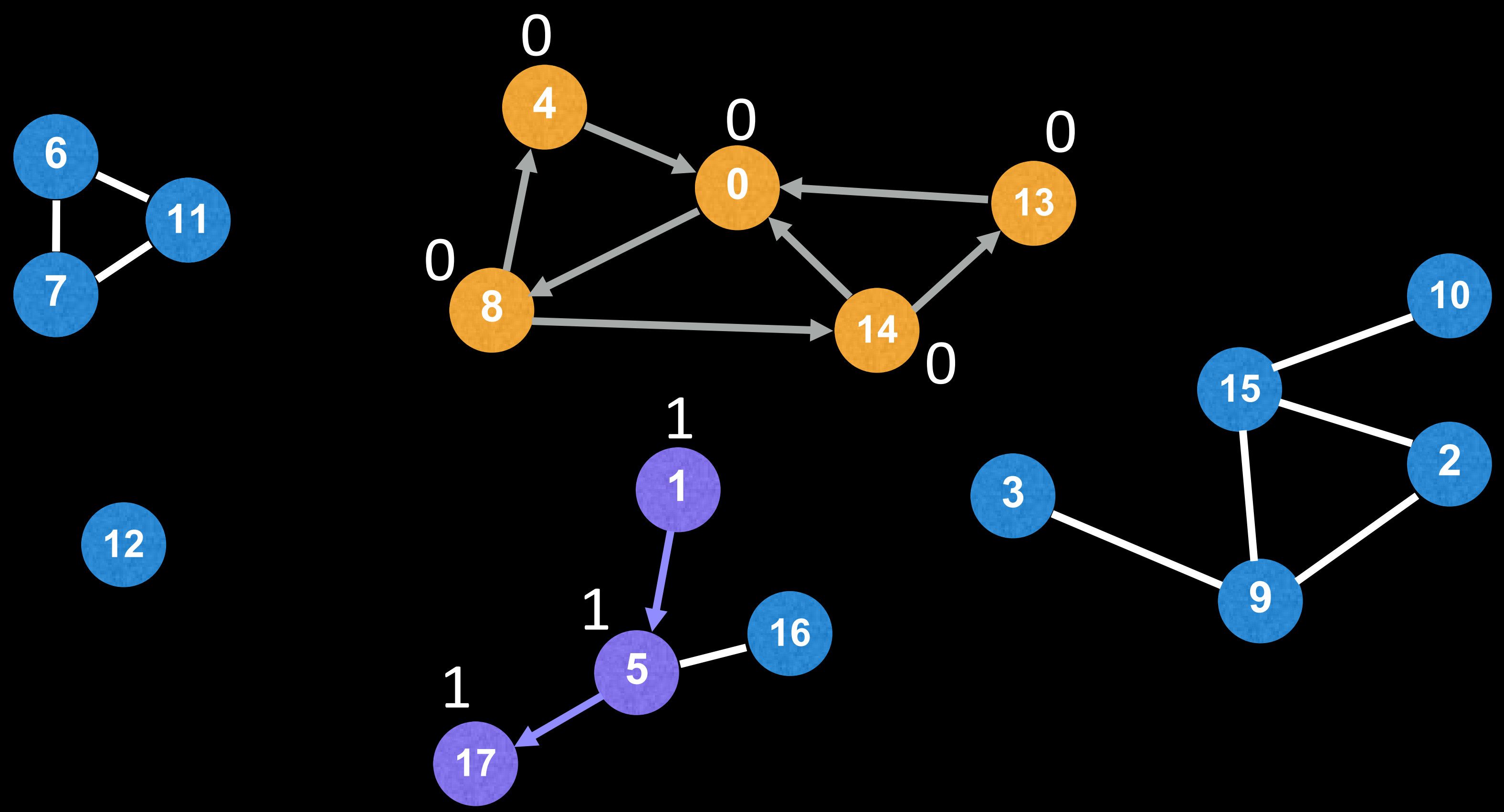

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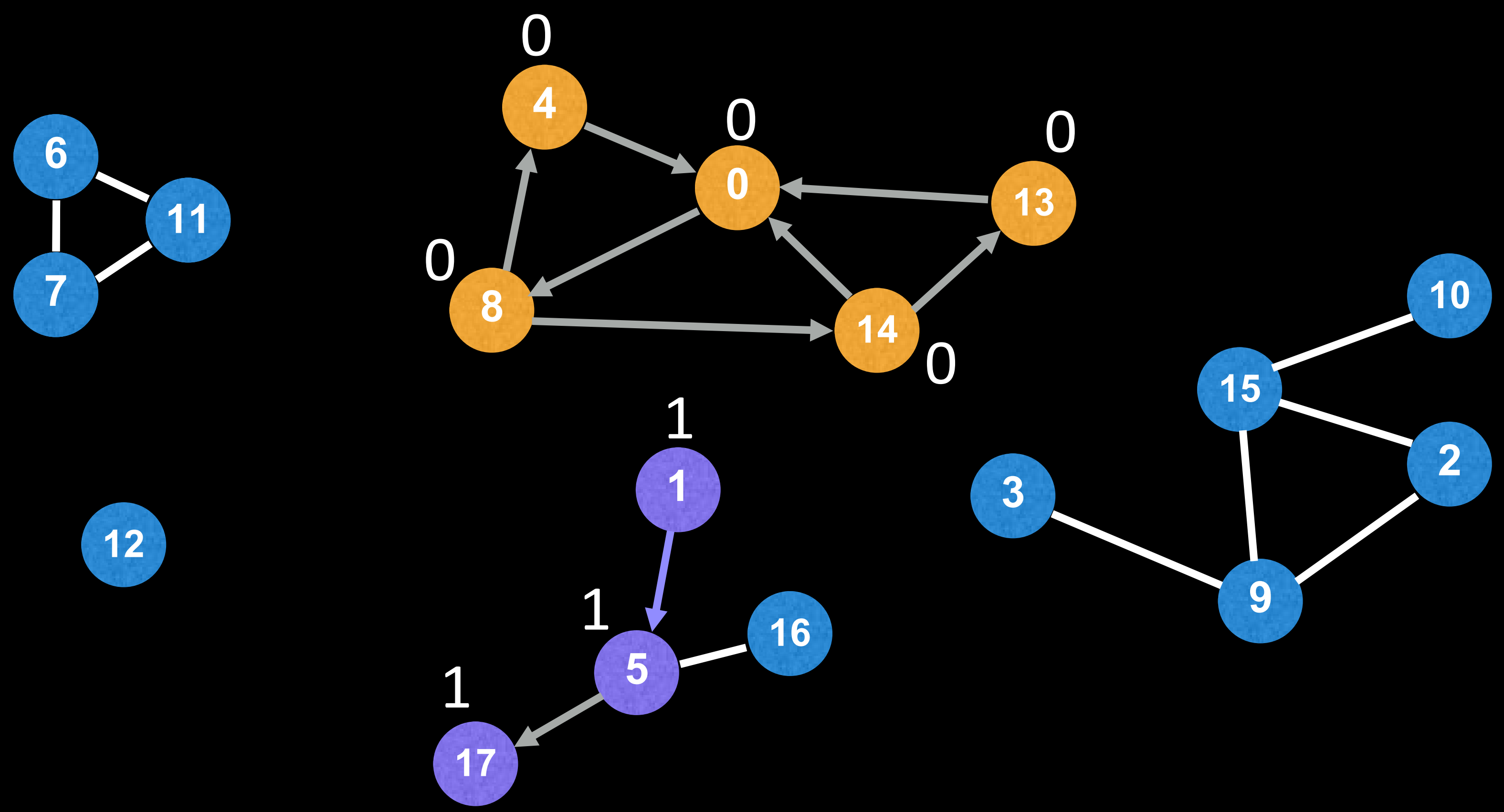

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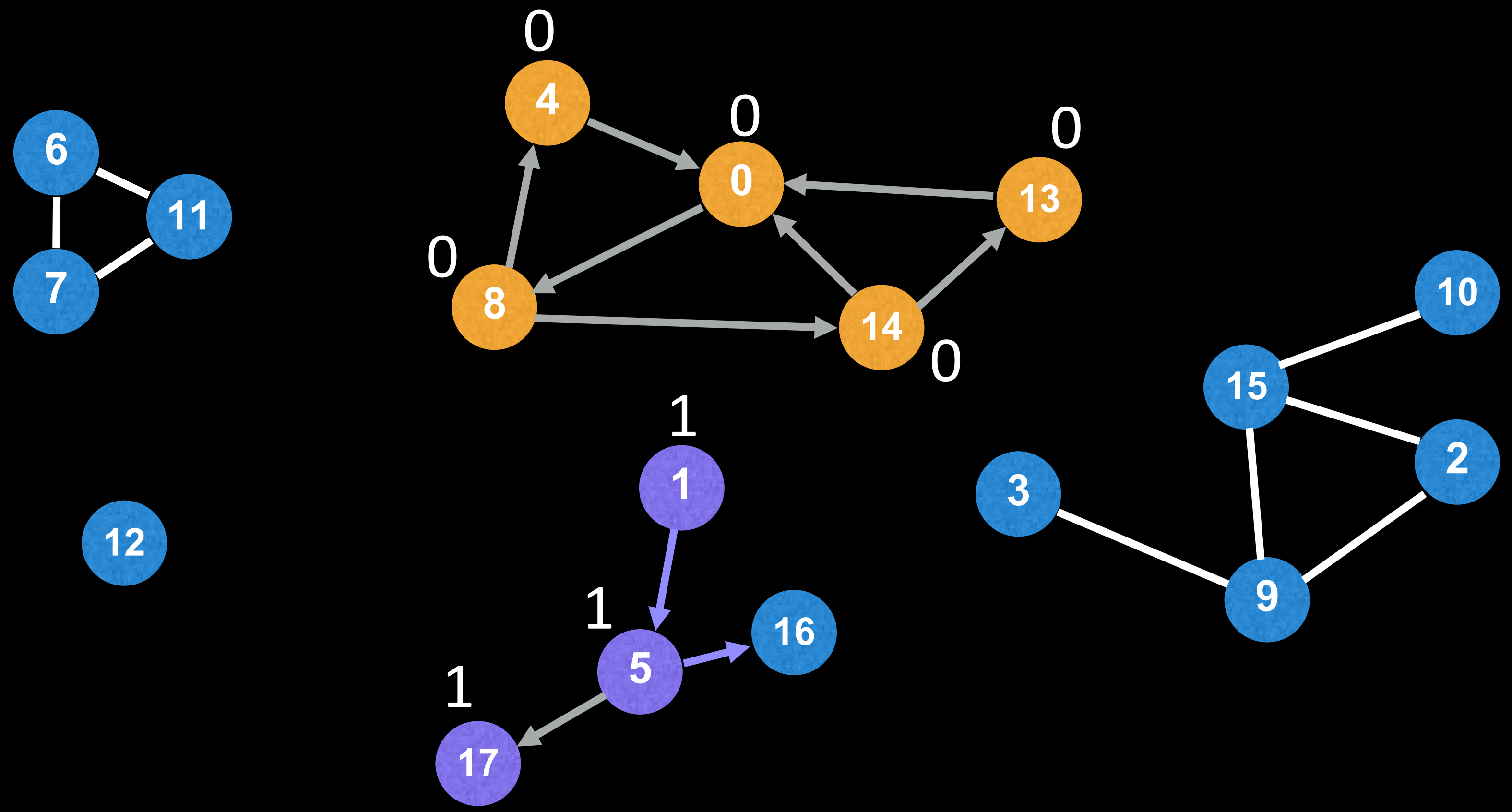

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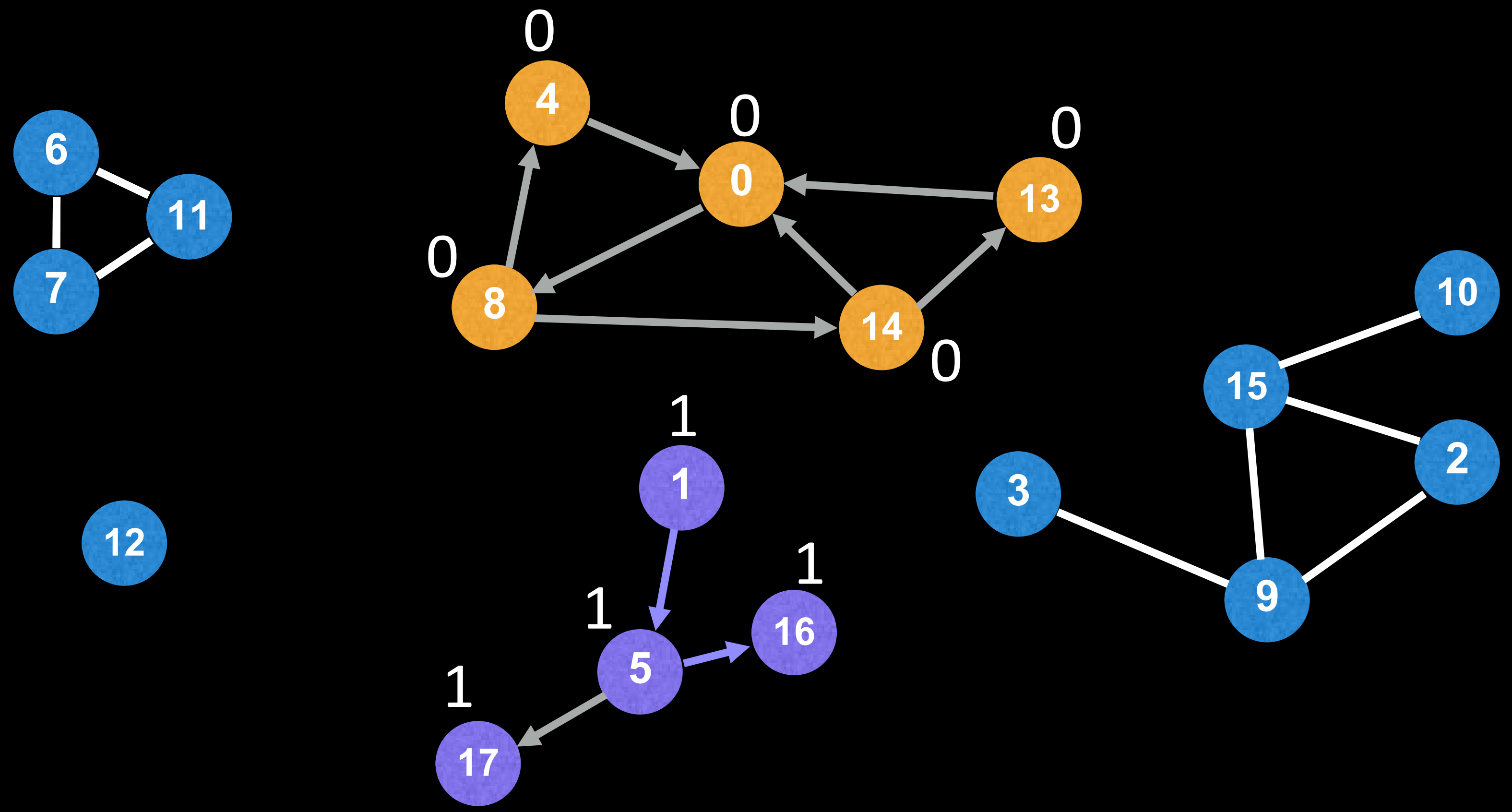

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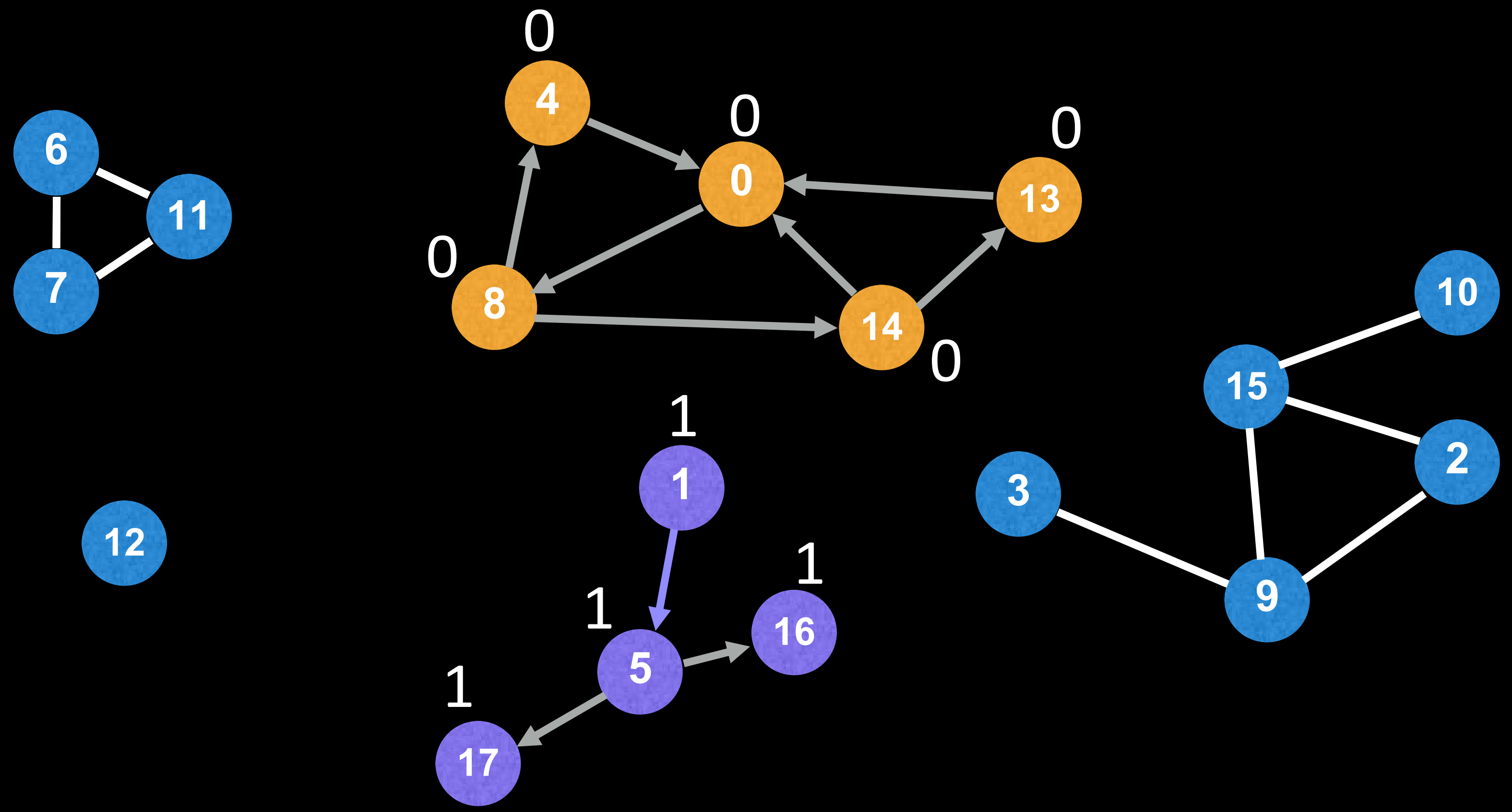

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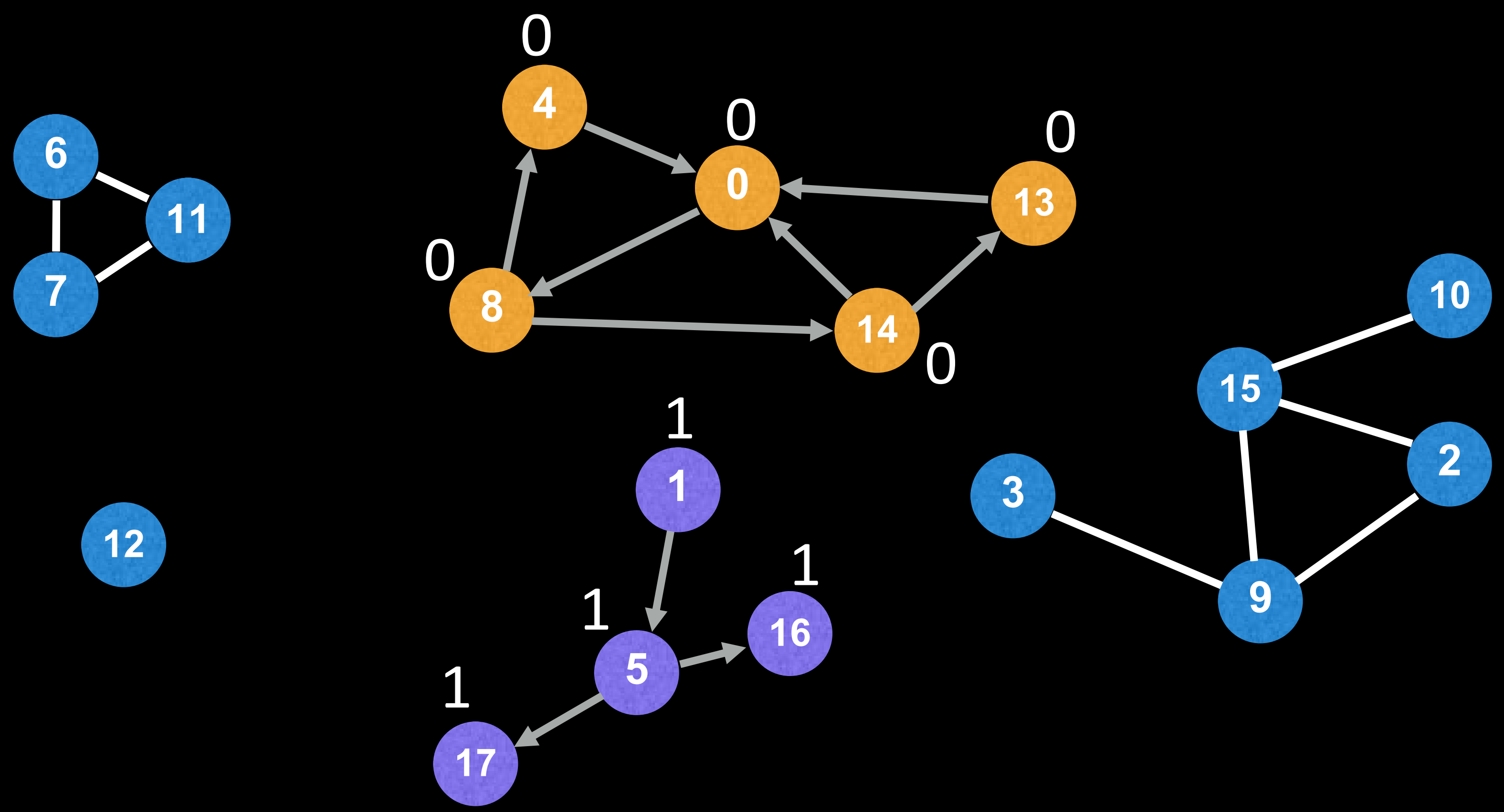
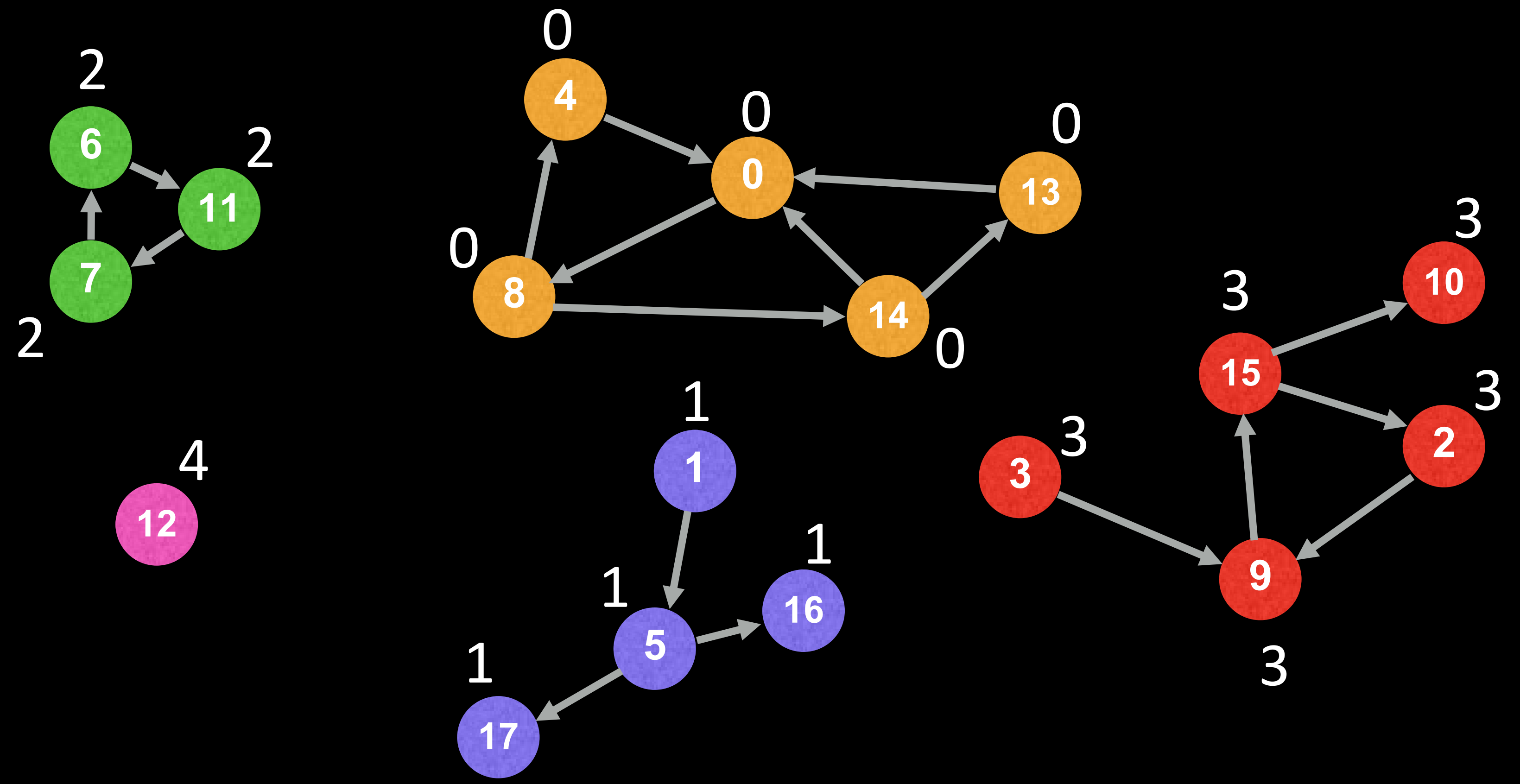


… and so on for the other components



```
# Global or class scope variables
n = number of nodes in the graph
g = adjacency list representing graph
count = 0
components = empty integer array # size n
visited = [false, …, false] # size n

function findComponents():
  for (i = 0; i < n; i++):
    if !visited[i]:
      count++
      dfs(i)
  return (count, components)

function dfs(at):
  visited[at] = true
     components[at] = count
  for (next : g[at]):
    if !visited[next]:
      dfs(next)
```

What else can DFS do?

We can augment the DFS algorithm to:

- Compute a graph's minimum spanning tree.
- Detect and find cycles in a graph.
- Check if a graph is bipartite.
- Find strongly connected components.
- Topologically sort the nodes of a graph.
- Find bridges and articulation points.
- Find augmenting paths in a flow network.
- Generate mazes.