$$egin{aligned} rac{\partial f}{\partial t} + ec{v} \cdot rac{\partial f}{\partial ec{x}} &= 0 \ & ec{v} = c rac{ec{D}}{B_{\parallel}^*} imes ec{b} \ & ec{D} &= ec{E} - rac{\mu}{q}
abla B - rac{m}{q} (rac{\partial ec{v_E}}{\partial t} +
abla rac{v_E^2}{2}) \end{aligned}$$

$$ec{v_E} = rac{cec{E}}{B} imes ec{b}$$

$$ec{B^*} = ec{B} + rac{mc}{q} (
abla imes ec{v_E})$$

$$B_{||}^* = \vec{B^*} \cdot \vec{b}$$

$$B = |\vec{B}|$$

$$ec{E}=ec{E_{1r}}$$

$$ec{B} = ec{B_{0z}} + ec{B_{1 heta}}$$

$$rac{1}{B} = rac{1}{\sqrt{B_{0z}^2 + 2B_{1 heta}B_0 + B_{1 heta}^2}} pprox rac{1}{\sqrt{B_{0z}^2 + 2B_{1 heta}B_0}} = (B_{0z}^2 + 2B_{1 heta}B_{0z})^{-rac{1}{2}} = rac{1}{B_{0z}}(1 + 2rac{B_{1 heta}}{B_{0z}})^{-rac{1}{2}} pprox rac{1}{B_{0z}}(1 - rac{1}{2}rac{B_{1 heta}}{B_{0z}})$$

$$\begin{split} \vec{b} &= \frac{\vec{B}}{B} = \vec{B} \frac{1}{B} = (\vec{B_{0z}} + \vec{B_{1\theta}}) \frac{1}{B_{0z}} (1 - \frac{1}{2} \frac{B_{1\theta}}{B_{0z}}) \\ &= \frac{1}{B_{0z}} (\vec{B_{0z}} + \vec{B_{1\theta}} - \frac{1}{2} \frac{B_{1\theta}}{B_{0z}} \vec{B_{0z}} - \frac{1}{2} \frac{B_{1\theta}}{B_{0z}} \vec{B_{1\theta}}) \\ &= \frac{1}{B_{0z}} (\vec{B_{0z}} + \vec{B_{1\theta}} - \frac{1}{2} B_{1\theta} \hat{e}_z - \frac{1}{2} \frac{B_{1\theta}}{B_{0z}} \vec{B_{1\theta}}) \approx \frac{1}{B_{0z}} (\vec{B_{0z}} + \vec{B_{1\theta}} - \frac{1}{2} B_{1\theta} \hat{e}_z) \\ &= \vec{b_{0z}} + \vec{b_{1\theta}} + \vec{b_{1z}} \\ &(\vec{b_{1z}} = -\frac{1}{B_{0z}} \frac{1}{2} B_{1\theta} \hat{e}_z) \end{split}$$

$$ec{v_E} = rac{cec{E}}{B} imes ec{b}$$

 $|b_{0z}| = 1$

$$=rac{c}{B_{0z}}(1-rac{1}{2}rac{B_{1 heta}}{B_{0z}})ec{E_{1r}} imesec{b}$$

$$ec{E_{1r}} imesec{b}=ec{E_{1r}} imes(ec{b_{0z}}+ec{b_{1 heta}}+ec{b_{1z}})$$

$$pprox -E_1 b_{0z} \hat{e_ heta}$$

$$ec{v_E} = rac{c}{B_{0z}} (1 - rac{1}{2} rac{B_{1 heta}}{B_{0z}}) (-E_1 b_{0z} \hat{e_ heta})$$

$$pprox rac{c}{B_{0z}}(-E_1b_{0z}\hat{e_ heta})$$

$$=E_1b_{1 heta}\hat{e_z}-E_1(b_{0z}+b_{1 heta})\hat{e_ heta}$$

so, $\vec{v_E}$ is not affected by $B_{1\theta}$ to the first order.

$$ec{B^*} = ec{B} + rac{mc}{a} (
abla imes ec{v_E})$$

 $ec{B^*}$ is not affected by $B_{1 heta}$ to the first order.

$$egin{aligned} B_{\parallel}^* &= ec{B^*} \cdot ec{b} \ \ &= (ec{B_{0z}} + ec{B_{1 heta}} + rac{mc}{q} (
abla imes ec{v_E})) \cdot (ec{b_{0z}} + ec{b_{1 heta}} + ec{b_{1z}}) \ \ &= (B_{0z} + rac{mc}{q} (
abla imes ec{v_E})) (b_{0z} + b_{1z}) \ \ &+ (B_{1 heta} + rac{mc}{q} (
abla imes ec{v_E})) (b_{1 heta}) \end{aligned}$$

 v_E is a first-order term,

$$=(B_{0z}+rac{mc}{q}(
abla imesec{v_E}))b_{0z}+b_{1z}B_{0z}$$

$$=B_{0z}((1+rac{1}{B_{0z}}rac{mc}{q}(
abla imesec{v_E}))b_{0z}+b_{1z})$$

$$B_{\parallel}^{*} = B_{0z}(b_{0z} + b_{1
abla imes v_E}^{*} + b_{1z})$$

$$ec{D}=ec{E}-rac{\mu}{q}
abla B-rac{m}{q}(rac{\partial ec{v_E}}{\partial t}+
ablarac{v_E^2}{2})$$

in terms of $\vec{v_E}$, $B_{1\theta}$ is not affected in terms up to the first order.

$$egin{aligned}
abla B \ &=
abla (B_{0z}^2 + B_{1 heta}^2)^{rac{1}{2}} \ &=
abla (B_{0z}(1 + rac{B_{1 heta}}{B_{0z}}^2)^{rac{1}{2}}) \ &pprox
abla (B_{0z}) \ &= rac{\partial B_{0z}}{\partial r} \hat{e_r} \end{aligned}$$

The \vec{D} is not affected by $B_{1\theta}$ to the first order.

$$\begin{split} &\text{in,} \\ &\vec{v} = c\frac{\vec{D}}{B_{\parallel}^*} \times \vec{b} \\ &= (B_{0z}(b_{0z} + b_{1\nabla \times v_E}^* + b_{1z}))^{-1} \\ &= \frac{1}{B_{0z}b_{0z}}(1 + \frac{b_{1\nabla \times v_E}^*}{b_{0z}} + \frac{b_{1z}}{b_{0z}})^{-1} \\ &\approx \frac{1}{B_{0z}b_{0z}}(1 - \frac{b_{1\nabla \times v_E}^*}{b_{0z}} - \frac{b_{1z}}{b_{0z}}) \\ &\stackrel{1}{=} \frac{1}{B_0b_{0z}}(1 - \frac{b_{1\nabla \times v_E}^*}{b_{0z}} - \frac{b_{1z}}{b_{0z}}) \\ &\approx \frac{1}{B_0b_{0z}}(1 - \frac{b_{1\nabla \times v_E}^*}{b_{0z}} - \frac{b_{1z}}{b_{0z}})(\vec{b_{0z}} + \vec{b_{1\theta}} + \vec{b_{1z}}) \\ &\approx \frac{1}{B_{0z}b_{0z}}(\vec{b_{0z}} - \hat{e_z}b_{1\nabla \times v_E}^* - \hat{e_z}b_{1z} + \vec{b_{1\theta}} + \vec{b_{1z}}) \\ &= \frac{1}{B_{0z}}(\vec{b_{0z}} - \hat{e_z}b_{1\nabla \times v_E}^* + \vec{b_{1\theta}}) \\ &= \frac{1}{B_{0z}}(\vec{b_{0z}} - \hat{e_z}b_{1\nabla \times v_E}^* + \vec{b_{1\theta}}) \end{split}$$

Only $\vec{b_{1\theta}}$ is related to $\vec{B_{1\theta}}$. $\hat{e_z}\,b_{1\nabla imes v_E}^*$ is not related to $\vec{B_{1\theta}}$.

Only zero-order ∇B in \vec{D} .

$$egin{aligned} cec{D} imesrac{ec{b_{1 heta}}}{B_{0z}}\ &=crac{\mu}{q}rac{\partial B_{0z}}{\partial r}\hat{e_r} imesrac{ec{b_{1 heta}}}{B_{0z}} \end{aligned}$$

$$=crac{\mu}{q}rac{\partial B_{0z}}{\partial r}rac{ec{b_{1 heta}}}{B_{0z}}\hat{e_z}$$