$$\frac{\partial f}{\partial t}+\vec{v}\cdot\frac{\partial f}{\partial\vec{x}}=0$$

$$\vec{v}=c\frac{\vec{D}}{B_{\parallel}^{*}}\times\vec{b}$$

$$\vec{D}=\vec{E}-\frac{\mu}{q}\nabla B-\frac{m}{q}(\frac{\partial\vec{v_E}}{\partial t}+\nabla\frac{v_E^2}{2})$$

$$\vec{v_E}=\frac{c\vec{E}}{B}\times\vec{b}$$

$$\vec{B^*}=\vec{B}+\frac{mc}{q}(\nabla\times\vec{v_E})$$

$$B_{\parallel}^{*}=\vec{B^*}\cdot\vec{b}$$

$$B=|\vec{B}|$$

$$\vec{E}=\vec{E_{1r}}$$

$$\vec{B}=\vec{B_{0z}}+\vec{B_{1\theta}}$$

$$\frac{1}{B}=\frac{1}{\sqrt{B_{0z}^2+2B_{1\theta}B_0+B_{1\theta}^2}}\approx\frac{1}{\sqrt{B_{0z}^2+2B_{1\theta}B_0}}=(B_{0z}^2+2B_{1\theta}B_{0z})^{-\frac{1}{2}}=\frac{1}{B_{0z}}(1+2\frac{B_{1\theta}}{B_{0z}})^{-\frac{1}{2}}\approx$$
$$\frac{1}{B_{0z}}(1-\frac{1}{2}\frac{B_{1\theta}}{B_{0z}})$$

$$\vec{b}=\frac{\vec{B}}{B}=\vec{B}\frac{1}{B}=(\vec{B_{0z}}+\vec{B_{1\theta}})\frac{1}{B_{0z}}(1-\frac{1}{2}\frac{B_{1\theta}}{B_{0z}})$$
$$=\frac{1}{B_{0z}}(\vec{B_{0z}}+\vec{B_{1\theta}}-\frac{1}{2}\frac{B_{1\theta}}{B_{0z}}\vec{B_{0z}}-\frac{1}{2}\frac{B_{1\theta}}{B_{0z}}\vec{B_{1\theta}})$$

$$=\frac{1}{B_{0z}}(\vec{B_{0z}}+\vec{B_{1\theta}}-\frac{1}{2}B_{1\theta}\hat{e}_z-\frac{1}{2}\frac{B_{1\theta}}{B_{0z}}\vec{B_{1\theta}})\approx\frac{1}{B_{0z}}(\vec{B_{0z}}+\vec{B_{1\theta}}-\frac{1}{2}B_{1\theta}\hat{e}_z)$$

$$=\vec{b_{0z}}+\vec{b_{1\theta}}+\vec{b_{1z}}$$
$$(\vec{b_{1z}}=-\frac{1}{B_{0z}}\frac{1}{2}B_{1\theta}\hat{e}_z)$$

$$|b_{0z}|=1$$

$$\vec{v_E}=\frac{c\vec{E}}{B}\times\vec{b}$$

$= \frac{c}{B_{0z}}(1 - \frac{1}{2}\frac{B_{1\theta}}{B_{0z}})\vec{E_{1r}} \times \vec{b}$

$\vec{E_{1r}} \times \vec{b} = \vec{E_{1r}} \times (\vec{b_{0z}} + \vec{b_{1\theta}} + \vec{b_{1z}})$

$\approx -E_1 b_{0z}\hat{e_\theta}$

$\vec{v_E} = \frac{c}{B_{0z}}(1 - \frac{1}{2}\frac{B_{1\theta}}{B_{0z}})(-E_1 b_{0z}\hat{e_\theta})$

$\approx \frac{c}{B_{0z}}(-E_1 b_{0z}\hat{e_\theta})$

$= E_1 b_{1\theta}\hat{e_z} - E_1(b_{0z} + b_{1\theta})\hat{e_\theta}$

so, $\vec{v_E}$ is not affected by $B_{1\theta}$ to the first order.

$\vec{B^*} = \vec{B} + \frac{mc}{q}(\nabla \times \vec{v_E})$

$\vec{B^*}$ is not affected by $B_{1\theta}$ to the first order.

$B^*_{\|} = \vec{B^*} \cdot \vec{b}$

$= (\vec{B_{0z}} + \vec{B_{1\theta}} + \frac{mc}{q}(\nabla \times \vec{v_E})) \cdot (\vec{b_{0z}} + \vec{b_{1\theta}} + \vec{b_{1z}})$

$= (B_{0z} + \frac{mc}{q}(\nabla \times \vec{v_E}))(b_{0z} + b_{1z})$

$+(B_{1\theta} + \frac{mc}{q}(\nabla \times \vec{v_E}))(b_{1\theta})$

v_E is a first-order term,

$= (B_{0z} + \frac{mc}{q}(\nabla \times \vec{v_E}))b_{0z} + b_{1z}B_{0z}$

$= B_{0z}((1 + \frac{1}{B_{0z}}\frac{mc}{q}(\nabla \times \vec{v_E}))b_{0z} + b_{1z})$

$B^*_{\|} = B_{0z}(b_{0z} + b^*_{1\nabla \times v_E} + b_{1z})$

$\vec{D} = \vec{E} - \frac{\mu}{q}\nabla B - \frac{m}{q}(\frac{\partial \vec{v_E}}{\partial t} + \nabla \frac{v_E^2}{2})$

in terms of $\vec{v_E}$,$B_{1\theta}$ is not affected in terms up to the first order.

∇B
$= \nabla(B_{0z}^2 + B_{1\theta}^2)^{\frac{1}{2}}$

$= \nabla(B_{0z}(1 + \frac{B_{1\theta}}{B_{0z}}^2)^{\frac{1}{2}})$

$\approx \nabla(B_{0z})$

$= \frac{\partial B_{0z}}{\partial r}\hat{e_r}$

The $\vec{D}$ is not affected by $B_{1\theta}$ to the first order.

in,
$\vec{v} = c\frac{\vec{D}}{B_{\parallel}^*} \times \vec{b}$

$\frac{1}{B_{\parallel}^*}$

$= (B_{0z}(b_{0z} + b_{1\nabla\times v_E}^* + b_{1z}))^{-1}$

$= \frac{1}{B_{0z}b_{0z}}(1 + \frac{b_{1\nabla\times v_E}^*}{b_{0z}} + \frac{b_{1z}}{b_{0z}})^{-1}$

$\approx \frac{1}{B_{0z}b_{0z}}(1 - \frac{b_{1\nabla\times v_E}^*}{b_{0z}} - \frac{b_{1z}}{b_{0z}})$

$\frac{1}{B_{\parallel}^*}\vec{b}$

$= \frac{1}{B_0b_{0z}}(1 - \frac{b_{1\nabla\times v_E}^*}{b_{0z}} - \frac{b_{1z}}{b_{0z}})(\vec{b_{0z}} + \vec{b_{1\theta}} + \vec{b_{1z}})$

$\approx \frac{1}{B_{0z}b_{0z}}(\vec{b_0}z - \hat{e_z}b_{1\nabla\times v_E}^* - \hat{e_z}b_{1z} + \vec{b_{1\theta}} + \vec{b_{1z}})$

$= \frac{1}{B_{0z}b_{0z}}(\vec{b_0}z - \hat{e_z}b_{1\nabla\times v_E}^* + \vec{b_{1\theta}})$

$= \frac{1}{B_{0z}}(\vec{b_0}z - \hat{e_z}b_{1\nabla\times v_E}^* + \vec{b_{1\theta}})$

Only $\vec{b_{1\theta}}$ is related to $\vec{B_{1\theta}}$.
$\hat{e_z}b_{1\nabla\times v_E}^*$ is not related to $\vec{B_{1\theta}}$.

Only zero-order ∇B in $\vec{D}$.

$c\vec{D} \times \frac{\vec{b_{1\theta}}}{B_{0z}}$

$= c\frac{\mu}{q}\frac{\partial B_{0z}}{\partial r}\hat{e_r} \times \frac{\vec{b_{1\theta}}}{B_{0z}}$

$$= c\frac{\mu}{q}\frac{\partial B_{0z}}{\partial r}\frac{\vec{b_{1\theta}}}{B_{0z}}\hat{e_z}$$