Guiding Center Equation

reference:: https://agupubs.onlinelibrary.wiley.com/doi/full/10.1029/2010JA015682

CGS

$$\frac{\partial f}{\partial t} + \vec{v} \cdot \frac{\partial f}{\partial \vec{x}} = 0$$

$$ec{v} = rac{ec{D}}{B_{\shortparallel}^*} imes ec{b}$$

$$ec{D}=ec{E}-rac{\mu}{q}
ablaec{B}-rac{m}{q}(rac{\partialec{v_E}}{\partial t}+
ablarac{v_E^2}{2})$$

$$ec{v_E} = rac{c}{B} ec{E} imes ec{b}$$

$$ec{B^*} = ec{B} + rac{m}{q} (
abla imes ec{v_E})$$

$$B_{||}^* = \vec{B^*} \cdot \vec{b}$$

$$B = |\vec{B}|$$

$$ec{B}(ec{m},ec{r})=rac{\mu_0}{4\pi}rac{3(ec{m}\cdotec{\hat{r}})ec{\hat{r}}-ec{m}}{r^3}$$

$$ec{\hat{r}}=rac{ec{r}}{r}$$

Below We use cylindrical coordinates

$$\vec{x} = (r, \theta, z)$$

$$abla f = rac{\partial f}{\partial r} ec{e_r} + rac{1}{r} rac{\partial f}{\partial heta} ec{e_ heta} + rac{\partial f}{\partial z} ec{e_z}$$

$$abla \cdot ec{A} = rac{1}{r}(rac{\partial (rA_r)}{\partial r} + rac{\partial A_ heta}{\partial heta} + rrac{\partial A_z}{\partial z})$$

$$abla imes ec{A} = (rac{1}{r}rac{\partial A_z}{\partial heta} - rac{\partial A_ heta}{\partial z})ec{e_r} + (rac{\partial A_r}{\partial z} + rac{\partial A_z}{\partial r})ec{e_ heta} + rac{1}{r}(rac{\partial (rA_ heta)}{\partial r} - rac{\partial A_r}{\partial heta})ec{e_z}$$

Toroidal mode wave

ULF waves

$$ec{B}_{wave} = ec{e_{ heta}} B_A sin(\omega(t-rac{r heta}{v}))$$

$$=ec{e_{ heta}}B_{A}sin(m\omega_{d}(t-rac{r heta}{v}))$$

$$=ec{e_{ heta}}B_{A}sin(m2\pi(rac{t}{T}-rac{r heta}{\lambda}))$$

$$ec{E}_{wave} = ec{e_r} E_A sin(m2\pi(rac{t}{T} - rac{r heta}{\lambda}) + rac{\pi}{2})$$

$$ec{B_0} = rac{\mu_0}{4\pi} rac{3(ec{m} \cdot \hat{ec{x}}) ec{\hat{x}} - ec{m}}{|ec{x}|^3}$$

$$ec{m}=mec{e_z}$$

$$ec{B_0}=rac{\mu_0}{4\pi}rac{3mzec{\hat{x}}-mec{e_z}}{|ec{x}|^3}$$

$$\vec{E_0} = \vec{0}$$

$$\vec{E} = \vec{E_0} + \vec{E}_{wave} = \vec{E}_{wave}$$

$$ec{B} = ec{B_0} + ec{B}_{wave}$$

Apply to the fomula

$$ec{v_E} = rac{ec{E}}{B} imes ec{b}$$

$$=rac{ec{E}_{wave}}{|B|^2} imesec{B}$$

$$ec{B^*} = ec{B} + rac{m}{g} (
abla imes ec{v_E})$$

$$=ec{B}+rac{m}{a}(
abla imes(rac{ec{E}_{wave}}{|B|^2} imesec{B}))$$

$$=ec{B}+rac{m}{a}((
abla\cdotec{B})rac{ec{E}_{wave}}{|B|^2}-(
abla\cdotrac{ec{E}_{wave}}{|B|^2})ec{B})$$

$$ec{D}=ec{E}-rac{\mu}{q}
ablaec{B}-rac{m}{q}(rac{\partialec{v_E}}{\partial t}+
ablarac{v_E^2}{2})$$

$$=ec{E}-rac{\mu}{q}
ablaec{B}-rac{m}{q}(rac{\partial}{\partial t}(rac{ec{E}_{wave}}{|B|^2} imesec{B})+rac{1}{2}
abla|rac{ec{E}_{wave}}{|B|^2} imesec{B}|^2)$$

Assumption $\vec{B}_{wave} << \vec{B}_0$

$$\vec{B} = \vec{B_0}$$

 $ec{b}=ec{e_z}$ (only in the case of no spatial differentiation)

$$ec{D} = ec{E} - rac{\mu}{q}
abla ec{B} - rac{m}{q} ig(rac{\partial ec{E}_{wave}}{\partial t} imes rac{ec{B}}{|B|^2} + rac{1}{2}
abla ig| rac{ec{E}_{wave}}{|B|^2} imes ec{B} ig|^2 ig)$$

$$ec{v} = rac{ec{D}}{B_{\scriptscriptstyle \parallel}^*} imes ec{b}$$

$$\vec{v} = \frac{\vec{E} - \frac{\mu}{q} \nabla \vec{B} - \frac{m}{q} (\frac{\partial \vec{E}_{wave}}{\partial t} \times \frac{\vec{B}}{|B|^2} + \frac{1}{2} \nabla |\frac{\vec{E}_{wave}}{|B|^2} \times \vec{B}|^2) + +}{(\vec{B} + \frac{m}{q} ((\nabla \cdot \vec{B}) \frac{\vec{E}_{wave}}{|B|^2} - (\nabla \cdot \frac{\vec{E}_{wave}}{|B|^2}) \vec{B}) \cdot \vec{e_z}} \times \vec{e_z}$$