Guiding Center Equation

reference:: https://agupubs.onlinelibrary.wiley.com/doi/full/10.1029/2010JA015682

CGS

$$\frac{\partial f}{\partial t} + \vec{v} \cdot \frac{\partial f}{\partial \vec{x}} = 0$$

$$ec{v}=rac{ec{D}}{B_{\shortparallel}^{*}} imesec{b}$$

$$ec{D}=ec{E}-rac{\mu}{q}
ablaec{B}-rac{m}{q}(rac{\partialec{v_E}}{\partial t}+
ablarac{v_E^2}{2})$$

$$ec{v_E} = rac{c}{B}ec{E} imesec{b}$$

$$ec{B^*} = ec{B} + rac{m}{a} (
abla imes ec{v_E})$$

$$B_{\parallel}^{*} = \vec{B^{*}} \cdot \vec{b}$$

$$B = |\vec{B}|$$

$$ec{B}(ec{m},ec{r})=rac{\mu_0}{4\pi}rac{3(ec{m}\cdotec{\hat{r}})ec{\hat{r}}-ec{m}}{r^3}$$

$$ec{\hat{r}}=rac{ec{r}}{r}$$

Below We use cylindrical coordinates

$$\vec{x} = (r, \theta, z)$$

$$abla f = rac{\partial f}{\partial r} ec{e_r} + rac{1}{r} rac{\partial f}{\partial heta} ec{e_ heta} + rac{\partial f}{\partial z} ec{e_z}$$

$$abla \cdot ec{A} = rac{1}{r}(rac{\partial (rA_r)}{\partial r} + rac{\partial A_ heta}{\partial heta} + rrac{\partial A_z}{\partial z})$$

$$abla imes ec{A} = (rac{1}{r}rac{\partial A_z}{\partial heta} - rac{\partial A_ heta}{\partial z})ec{e_r} + (rac{\partial A_r}{\partial z} + rac{\partial A_z}{\partial r})ec{e_ heta} + rac{1}{r}(rac{\partial (rA_ heta)}{\partial r} - rac{\partial A_r}{\partial heta})ec{e_z}$$

Toroidal mode wave

ULF waves

$$ec{B}_{wave} = ec{e_{ heta}} B_A sin(\omega(t-rac{r heta}{v_a}))$$

$$=ec{e_{ heta}}B_{A}sin(m\omega_{d}(t-rac{r heta}{v_{a}}))$$

$$=ec{e_{ heta}}B_{A}sin(m2\pi(rac{t}{T}-rac{r heta}{\lambda}))$$

$$ec{E}_{wave} = ec{e_r} E_A sin(m2\pi(rac{t}{T} - rac{r heta}{\lambda}) + rac{\pi}{2})$$

$$E_{wave} = E_{A} sin(m2\pi(rac{t}{T} - rac{r heta}{\lambda}) + rac{\pi}{2})$$

$$ec{E}_{wave} = ec{e_r} E_{wave}$$

$$ec{B_0}=rac{\mu_0}{4\pi}rac{3(ec{m}\cdot\hat{ec{x}})ec{\hat{x}}-ec{m}}{|ec{x}|^3}$$

$$ec{m}=mec{e_z}$$

$$ec{B_0}=rac{\mu_0}{4\pi}rac{3mz\hat{\hat{x}}-mec{e_z}}{|ec{x}|^3}$$

$$\vec{E_0} = \vec{0}$$

$$ec{E} = ec{E}_0 + ec{E}_{wave} = ec{E}_{wave}$$

$$ec{B} = ec{B_0} + ec{B}_{wave}$$

Apply to the fomula

$$ec{v_E} = rac{ec{E}}{B} imes ec{b}$$

$$=rac{ec{E}_{wave}}{|B|^2} imesec{B}$$

$$ec{B^*} = ec{B} + rac{m}{a} (
abla imes ec{v_E})$$

$$=ec{B}+rac{m}{a}(
abla imes(rac{ec{E}_{wave}}{|B|^2} imesec{B}))$$

$$=ec{B}+rac{m}{q}((
abla\cdotec{B})rac{ec{E}_{wave}}{|B|^2}-(
abla\cdotrac{ec{E}_{wave}}{|B|^2})ec{B})$$

$$ec{D}=ec{E}-rac{\mu}{q}
ablaec{B}-rac{m}{q}(rac{\partialec{v_E}}{\partial t}+
ablarac{v_E^2}{2})$$

$$=ec{E}_{wave}-rac{\mu}{q}
ablaec{B}-rac{m}{q}(rac{\partial}{\partial t}(rac{ec{E}_{wave}}{|B|^2} imesec{B})+rac{1}{2}
abla|rac{ec{E}_{wave}}{|B|^2} imesec{B}|^2)$$

Assumption

•
$$\vec{B}_{wave} << \vec{B_0}$$

• $\frac{\partial B_z}{\partial z} = 0$

•
$$\frac{\partial B_z}{\partial z} = 0$$

•
$$\vec{B} = \vec{B_0}$$

•
$$\frac{\partial \vec{B}}{\partial t} = \vec{0}$$

•
$$\vec{B} = (B_r, B_\theta, B_z) = (0, 0, B_z)$$

•
$$\frac{\partial \vec{B}}{\partial \vec{x}} = (\xi_r, 0, 0)$$

• $\vec{b} = \vec{e_z}$

•
$$\vec{b} = \vec{e}$$

$$ec{D} = ec{E}_{wave} - rac{\mu}{q}
abla ec{B} - rac{m}{q} (rac{\partial ec{E}_{wave}}{\partial t} imes rac{ec{B}}{|B|^2} + rac{1}{2}
abla | rac{ec{E}_{wave}}{|B|^2} imes ec{B}|^2)$$

$$ec{D} = ec{e_r} E_{wave} - rac{\mu}{q} \xi_r ec{e_r} - rac{m}{q} (rac{\partial (ec{e_r} E_{wave})}{\partial t} imes rac{ec{e_z}}{B_z} + rac{1}{2}
abla | rac{ec{e_r} E_{wave}}{|B|^2} imes ec{B}|^2)$$

$$ec{D}=ec{e_r}E_{wave}-rac{\mu}{q}\xi_rec{e_r}-rac{m}{q}(-rac{\partial E_{wave}}{\partial t}rac{1}{B_z}e_ heta+rac{1}{2}
abla|rac{ec{e_r}E_{wave}}{|B|^2}ec{B}+rac{ec{e_r}E_{wave}}{|B|^2} imesec{B}|^2)$$

$$ec{v}=rac{ec{D}}{B_{\scriptscriptstyle \parallel}^*} imesec{b}$$

$$ec{v} = rac{ec{E} - rac{\mu}{q}
abla ec{B} - rac{m}{q} (rac{\partial ec{E}_{wave}}{\partial t} imes rac{ec{B}}{|B|^2} + rac{1}{2}
abla |rac{ec{E}_{wave}}{|B|^2} imes ec{B}|^2) + +}{(ec{B} + rac{m}{q} ((
abla \cdot ec{B}) rac{ec{E}_{wave}}{|B|^2} - (
abla \cdot rac{ec{E}_{wave}}{|B|^2}) ec{B}) \cdot ec{e_z}} imes ec{e_z}$$