

## Chapter 2

### Boolean Algebra and Logic Gates

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## §2-4

1. Demonstrate the validity of the following identities by means of
  - i. truth tables
  - ii. postulates or proven theorems
  - iii. Venn diagrams

(a)  $XY + XZ + X'YZ' = XZ + YZ'$

(b)  $XYZ' + (X' + Z)(Y + Z') = X'Z' + Y$

EX-2

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(a)  $XY + XZ + X'YZ' = XZ + YZ'$

i. truth tables

XYZ							
0 0 0							
0 0 1							
0 1 0							
0 1 1							
1 0 0							
1 0 1							
1 1 0							
1 1 1							

EX-3

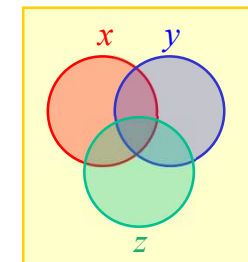
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(a)  $XY + XZ + X'YZ' = XZ + YZ'$

ii. postulates or proven theorems


$XY + XZ + X'YZ'$   
=

iii. Venn diagram



EX-4

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## §2-5


2. Draw the logic diagram for each of the following Boolean equations.

(a)  $F(X, Y, Z) = XY + XZ + X'YZ'$   
 $= XZ + YZ'$

(b)  $F(X, Y, Z) = XYZ' + (X' + Z)(Y + Z')$   
 $= X'Z' + Y$

EX-5

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## §2-5

\* Give an example (which may be designed as a combinational circuit) by yourself. Describe the problem, define the input and output variables, derive the Boolean equation(s), and simplify the Boolean equation(s) of the example.

EX-6

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## §2-5

3. Simplify the following Boolean expressions to a minimum number of literals according to the identities of Boolean algebra. Write the particular identities used in each step. For the simplified function you derive, how many literals does it have?

(a)  $F(X, Y, Z) = XY + XZ + X'YZ'$


(b)  $F(X, Y, Z) = XYZ' + (X' + Z)(Y + Z')$

(c)  $F(A, B, C, D) = A'BCD + A'BCD' + A'C'D + AB'D + ACD' + ABC + (A + B' + C + D)'$

Postulate/Theorem	(a)	(b)
Postulate 2	$x + 0 = x$	$x \cdot 1 = x$
Postulate 5	$x + x' = 1$	$x \cdot x' = 0$
Theorem 1	$x + x = x$	$x \cdot x = x$
Theorem 2	$x + 1 = 1$	$x \cdot 0 = 0$
Theorem 3, involution	$(x')' = x$	
Postulate 3, commutative	$x + y = y + x$	$xy = yx$
Theorem 4, associative	$x + (y + z) = (x + y) + z$	$x(yz) = (xy)z$
Postulate 4, distributive	$x(y + z) = xy + xz$	$x + (yz) = (x + y)(x + z)$
Theorem 5, DeMorgan	$(x + y)' = x'y'$	$(xy)' = x' + y'$
Theorem 6, absorption	$x + xy = x$	$x(x + y) = x$
Theorem 7, consensus	$xy + x'z + yz = xy + x'z$	$(x + y)(x' + z)(y + z) = (x + y)(x' + z)$

EX-7

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## §2-5

4. Complement the following functions by (i) applying DeMorgan's theorem and (ii) by using duals:

(a)  $XY + XZ + X'YZ'$

(b)  $XYZ' + (X' + Z)(Y + Z')$

EX-8

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## §2-6

5. Obtain the truth table of the following functions, and express each function in sum-of-minterms and product-of-maxterms form:

- (a)  $XY + XZ + X'YZ'$   
 (b)  $XYZ' + (X' + Z)(Y + Z')$

(a)

$XYZ$	$F$
0 0 0	
0 0 1	
0 1 0	
0 1 1	
1 0 0	
1 0 1	
1 1 0	
1 1 1	

EX-9

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## Technology Parameters

6. An integrated circuit logic family has NAND gates with a fan-out of 8 standard loads and buffers with a fan-out of 16 standard loads. Sketch a schematic showing how the output signal of a single NAND gate can be applied to  $N$  other gate inputs using as few buffers as possible. Assume that each input is one standard load.

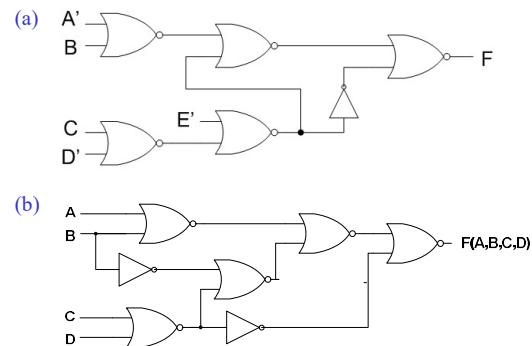
- (a)  $N = 38$   
 (b)  $N = 53$

EX-10

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## Technology Parameters

7. Assume that a NOR gate has propagation delay  $t_{pd\_NOR} = 0.078$  ns and an inverter has a propagation delay  $t_{pd\_inv} = 0.052$  ns. What is the propagation delay of the longest path through the following circuit.



EX-11

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## Brief Answers of the Exercises

3. (a)  $XZ + YZ'$ , 4 literals  
 (b)  $Y + X'Z'$ , 3 literals  
 (c)  $A'B + AC + B'C'D$ , 7 literals
4. (a)  $(X' + Y')(X' + Z')(X + Y + Z)$   
 (b)  $(X' + Y' + Z)(XZ' + Y'Z)$
5. (a)  $X'YZ' + XY'Z + XYZ' + XYZ$ ,  $(X + Y + Z)(X + Y + Z')(X + Y' + Z')(X' + Y + Z)$   
 (b)  $X'Y'Z' + X'YZ' + X'YZ + XYZ' + XYZ$ ,  $(X + Y + Z')(X' + Y + Z)(X' + Y' + Z)$
6. (a) 0.312ns  
 (b) 0.312ns
7. (a) 2 buffers  
 (b) 3 buffers

EX-12

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