資訊工程系【演算法概論】109-1 期末考

1. Consider inserting the keys 10, 22, 31, 4, 15, 28, 17, 88, 59 into a hash table of length $m = 11$ using open addressing with the auxiliary hash function $h'(k) = k$. Illustrate the result of inserting these keys (1) using linear probing, (2) using quadratic probing with $c_1 = 1$ and $c_2 = 3$, and (3) using double hashing with $h_1(k) = k$ and $h_2(k) = 1 + (k \bmod (m-1))$. (15%)
2. Suppose that we have numbers between 1 and 1000 in a binary search tree, and we want to search for the number 363. Which of the following sequences could not be the sequence of nodes examined? (15%)
   (a) 2, 252, 401, 398, 330, 344, 397, 363.
   (b) 924, 220, 911, 244, 898, 258, 362, 363.
   (c) 925, 202, 911, 240, 912, 245, 363.
   (d) 2, 399, 387, 219, 266, 382, 381, 278, 363.
   (e) 935, 278, 347, 621, 299, 392, 358, 363.
3. The sequence of keys 4, 7, 9, 11, 14, 17, 18, 19, 22 is obtained by an inorder traversal of binary search trees.
   (a) Please show out two different binary search trees which can produce the same sequence. (10%)
   (b) Given a binary search tree, please give an efficient procedure to generate the other binary search tree which has the same inorder ordering of keys. (10%)
   (c) If it is possible to have the third binary search tree has the same inorder ordering of keys. (10%)
4. Suppose that a node x is inserted into a red-black tree with RB-INSERT and then is immediately deleted with RB-DELETE. Is the resulting red-black tree the same as the initial red-black tree? Justify your answer. (15%)
5. Using dynamic programming to find an optimal parenthesization of a matrix-chain product whose sequence of dimensions is $\langle 5, 10, 3, 12, 5, 50, 6 \rangle$. Please construct the $\boldsymbol{m}$ and $\boldsymbol{s}$ tables in your process. (20%)
6. In the amortized analysis of a dynamic table, we should double the

table size upon inserting an item into a full table and halve the size when a deleting an item would cause the table to become less than 1/4 full. Consider a sequence of TABLE-INSERT and TABLE-DELETE operations as follows: 17 Insert, 11 Delete, 4 Insert, 7 Delete, 1 Insert, 1 Delete, 4 Insert, and 7 Delete. The potential function is defined as:

$$\Phi_i = \begin{cases} 2 \cdot num_i - size_i & \text{if } \alpha_i \geq 1/2\,, \\ size_i/2 - num_i & \text{if } \alpha_i < 1/2\,, \end{cases}$$

Please draw a function diagram to show the relationship among table size, number of items, and potentials in the sequence of operations. (20%)