Quiz 4 Numerical Method, 2020/6/11

ID:____________ Name:______________

1. (10) Based on the following table representing an unknown function $f(x)$, do the following:

a. (4) Compute $f'(2.4)$ using central-difference with h=0.1.

$$f'(x) \approx \frac{f(x+h)-f(x-h)}{2h}$$

$$f'(2.4) \approx \frac{f(2.5)-f(2.3)}{0.2}$$

$$= \frac{0.049126-0.074764}{0.2} = -0.12819$$

Table 5.5

i	x_i	f_i
0	2.0	0.123060
1	2.1	0.105706
2	2.2	0.089584
3	2.3	0.074764
4	2.4	0.061277
5	2.5	0.049126
6	2.6	0.038288
7	2.7	0.028722
8	2.8	0.020371
9	2.9	0.013164
10	3.0	0.007026

b. (6) Get an improved estimate of $f'(2.4)$ using Richardson extrapolation.

Repeat the calculation using $h=0.2$

2

$$f'(2.4) \approx \frac{f(2.6)-f(2.2)}{2\times 0.2} = \frac{0.038288-0.089584}{0.4} = -0.12824$$

4 Better - estimate

$$f'(2.4) = -0.12819 + \frac{1}{(0.2)/(0.1)^2-1}[-0.12819-(-0.12824)]$$

$$= -0.12817$$

2. (10)

a. (4) Use the data in the table to find the integral between x=1.0 and 1.8, using the trapezoidal rule with h=0.1

$$\int_{1.0}^{1.8} f(x) \approx \frac{0.1}{2}(1.543 + 2\times1.669 + 2\times1.811 + 2\times1.971$$
$$+ 2\times2.151 + 2\times2.352 + 2\times2.577$$
$$+ 2\times2.828 + 3.107)$$

$$= 1.7684$$

x	$f(x)$
1.0	1.543
1.1	1.669
1.2	1.811
1.3	1.971
1.4	2.151
1.5	2.352
1.6	2.577
1.7	2.828
1.8	3.107

b. (6) Extrapolate from the results of 2a. to get an improved value for the integral using Romberg integration.

using $h=0.2$

2

$$\int_{1.0}^{1.8} f(x) \approx \frac{0.2}{2}(1.543 + 2\times1.811 + 2\times2.151 + 2\times2.577 + 3.107)$$

$$= 1.7728$$

4 using this with the estimate when $h=0.1$

Better - estimate $= 1.7684 + \frac{1}{2^2-1}(1.7684 - 1.7728)$

$$= 1.7669333$$

46* For n an even integer, let T_h, T_{2h}, be trapezoidal rule integrals with step sizes h and 2h. It is easy to show that

$$T_h - T_{2h} = (h/2)(-f_0 + 2f_1 - 2f_2 + \ldots - f_n), \text{ from which}$$
$$T_h + (1/3)(T_h - T_{2h}) = (h/3)(f_0 + 4f_1 + 2f_3 + 4f_3 + \ldots + f_n),$$

which is Simpson's 1/3 rule. An equivalent relation is to subtract $T_{2h}/3$ from $4/3*T_h$.