Quiz 1

Numerical method, 2020/3/30

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1. (10) The function $f(x) = x^2 - 0.8x + 0.15$ has two zeros at 0.3 and 0.5. Starting with [0, 0.49],

   a. (2) Apply bisection method to get next interval.

   $X_3 = \frac{0+0.49}{2} = 0.245$ $f(X_2) = f(0.49) = -0.0019$ $f(X_2)f(X_3) < 0$

   $f(X_1) = f(0) = 0.15$ $f(X_3) = f(0.245) = 0.014025$ A: [0.245 0.49]

   b. (4) Apply the secant method to get next interval.

   $X_2 = X_1 - f(X_1)\frac{(X_0 - X_1)}{f(X_0) - f(X_1)} = 0.49 - (-0.0019)\frac{(-0.49)}{0.15-(-0.0019)} = 0.48387$

   A: $\overset{X_2 \to X_1}{0.49}$, $\overset{X_3 \to X_2}{0.48387}$

   c. (4) Which root will be reached with bisection and secant method? Which method will be faster?

   (a) bisection = 0.3

   secant = 0.5

   (b) secant will be faster

2. (10) Given a degree 4 polynomial $P(x) = 1.1x^4 + 4.6x^3 + 6.6x^2 - 12x - 16$,

   a. (4) Use synthetic division to evaluate P(1.5) and P'(1.5) for P(x).

1.5	1.1	4.6	6.6	-12	-16
		1.65	9.375	23.9625	17.94375
	1.1	6.25	15.975	11.9625	1.94375

   $P(1.5) = 1.94375$

1.5	1.1	6.25	15.975	11.9625
		1.65	11.85	41.7375
	1.1	7.9	27.825	53.7

   $P'(1.5) = 53.7$

   b. (2) One of the real roots of P(x) is near 1.5. Perform Newton's method to get the next approximate x_1 using initial point x_0=1.5.

   $$X_1 = 1.5 - \frac{1.94375}{53.7} = 1.4638$$

   c. (4) For Problem 2(a), to get P'(1.5) we apply synthetic division to the reduced polynomial resulted from dividing P(x) by (x-1.5). Explain why?

   $P(X) = (X-a)Q(X) + R$

   $P'(X) = Q(X) + (X-a)Q'(X)$

   $X = a$ 代入

   $P'(a) = Q(a) + 0$

   $= Q(a)$

3. (10) Given $f(x) = x^2 + 2x - 1$,

a. (3) Convert f(x)=0 to possible forms for fixed-point iteration.

① $X = g_1(X) = \sqrt{-2X+1}$

② $2X = -X^2+1$, $X = g_1(X) = \frac{-X^2+1}{2}$

③ $X(X+2)-1=0$, $X = g_1(X) = \frac{1}{X+2}$

b. (2) Choose one fixed-point iteration x=g(x) and compute the first approximate x_1 using initial point x_0=1.0.

X_0 代入 ②

$X = \frac{-1+1}{2} = 0$

X_0 代入 ③

$X = \frac{1}{1+2} = \frac{1}{3}$

c. (5) What is Aitken acceleration of this fixed-point iteration? How faster it is compared to x_{n+1}=g(x_n)?

① $R = X_n - \frac{(\Delta X_n)^2}{\Delta^2 X_n}$

$\Delta X_n = X_{n+1} - X_n$

$\Delta^2 X_n = \Delta X_{n+1} - \Delta X_n$

② 參考講義 P.87