ID:_____   Name:_____

1. (10) Based on the following table representing an unknown function $f(x)$, do the following:

   a. (4) Compute $f'(2.4)$ using central-difference with $h=0.1$.

Table 5.5

i	x_i	f_i
0	2.0	0.123060
1	2.1	0.105706
2	2.2	0.089584
3	2.3	0.074764
4	2.4	0.061277
5	2.5	0.049126
6	2.6	0.038288
7	2.7	0.028722
8	2.8	0.020371
9	2.9	0.013164
10	3.0	0.007026

   b. (6) Get an improved estimate of $f'(2.4)$ using Richardson extrapolation.

2. (10)

   a. (4) Use the data in the table to find the integral between $x=1.0$ and 1.8, using the trapezoidal rule with $h=0.1$

x	$f(x)$
1.0	1.543
1.1	1.669
1.2	1.811
1.3	1.971
1.4	2.151
1.5	2.352
1.6	2.577
1.7	2.828
1.8	3.107

   b. (6) Extrapolate from the results of 2a. to get an improved value for the integral using Romberg integration.

3. (10) Show that extrapolating once with the trapezoidal rule is equivalent to using Simpson's 1/3 rule with a comparable value for h. You need to indicate the change of error order.

Useful information:

The trapezoidal rule

$$\int_a^b f(x)dx \cong \frac{h}{2}(f_0 + 2f_1 + 2f_2 + \cdots + 2f_{n-1} + f_n)$$

Simpson's 1/3 rule

$$\int_a^b f(x)dx = \frac{h}{3}[f_1 + 4f_2 + 2f_3 + 4f_4 + 2f_5 + \cdots + 4f_n + f_{n+1}]$$

$$= \frac{h}{3}[f_1 + 4\sum f_{even} + 2\sum f_{odd} + f_{n+1}]$$