1(a)
Finite representation in computers
Floating-point of fixed word length
Numbers are round when stored as floating-point numbers

1(b)
Errors propagated in the succeeding steps of a process due to the occurrence of an earlier error

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1(c)
Not sensitive to input inaccuracy
The change (error) in the output is not greater than the change (error) in the input

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1(d)
Sensitive to input inaccuracy
A small change (error) in the input causes a large change (error) in the output

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1(e)
Newton > Muller > Secant > False Position > Bisection
Newton > Muller > Secant > False Position = Bisection

1(f)
linearly
wrong answer: (k-1)/k

1(g)
(1 point) f'(x) will always be 0 at a root.
(1 point) Computers will find f(x) equal to 0 throughout the neighborhood of the root.
Program cannot distinguish which value is really the root

2(a)

$$P(x) = (x-r)\,Q(x) + R$$

$$P'(x) = Q(x) + (x-r)\,Q'(x)$$

$$P'(r) = Q(r)$$

$= $ remainder of $Q(x) \div (x-r)$ (by remainder theorem)

2(b)

$$g(x_n) = x_n - \frac{f(x_n)}{f'(x_n)} = x_{n+1}$$

$$g'(x_n) = \frac{f(x_n)f''(x_n)}{[f'(x_n)]^2}$$

$$g(x_n) = g(r) + g'(r)(x_n - r) + \frac{g''(\varepsilon_n)}{2}(x_n - r)^2, \quad \varepsilon_n \in [x_n, r]$$

(by Taylor Expansion of $g(x_n)$ at r)

$$g'(r) = \frac{f(r)f''(r)}{[f'(r)]^2} = 0 \quad (\text{since } f(r) = 0)$$

$$g(x_n) = g(r) + 0 + \frac{g''(\varepsilon_n)}{2}(x_n - r)^2$$

$$g(x_n) - g(r) = \frac{g''(\varepsilon_n)}{2}(x_n - r)^2$$

$$x_{n+1} - r = \frac{g''(\varepsilon_n)}{2}(x_n - r)^2$$

$$e_{n+1} = \frac{g''(\varepsilon_n)}{2} e_n^2$$

$$\lim_{n\to\infty} \frac{|e_{n+1}|}{|e_n^2|} = \lim_{n\to\infty} \left|\frac{g''(\varepsilon_n)}{2}\right| = \left|\frac{g''(r)}{2}\right| \neq 0 \quad (\text{for simple root})$$

$\Rightarrow$ Newton method is quadratically convergent

3(a)

1.5	1.1	4.6	6.6	-12	-16
		1.65	9.375	23.9625	17.94375
1.5	1.1	6.25	15.975	11.9625	$\underline{1.94375}$
		1.65	11.85	41.7375	
	1.1	7.9	27.825	$\underline{53.7}$	

$P(1.5) = 1.94375.$

$P'(1.5) = 53.7.$

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3(b)

$$x_1 \cong x_0 - \frac{P(1.5)}{P'(1.5)} = 1.4638$$

3(c)

$$y = \frac{P(1.6) - P(1.5)}{1.6 - 1.5} \cdot (x - 1.5) + P(1.5)$$

$$x = 1.5 - P(1.5) \cdot \frac{1.6 - 1.5}{P(1.6) - P(1.5)} \cong 1.4665$$

next interval : $[1.4665, 1.5]$

3(d)

$$P(x) = 1.1x^4 + 4.6x^3 + 6.6x^2 - 12x - 16$$

① $x = g_1(x) = \sqrt[4]{\dfrac{-4.6x^3 - 6.6x^2 + 12x + 16}{1.1}}$

check $|g'(p)| < 1$ converges , $|g'(p)| > 1$ diverges

$$g_1'(x) = \frac{1}{4}\left(\frac{-4.6x^3 - 6.6x^2 + 12x + 16}{1.1}\right)^{-\frac{3}{4}} \cdot \left(\frac{-13.8x^2 - 13.2x + 12}{1.1}\right)$$

$g_1'(1.5) = -3.60995$ diverges

② $x = g_2(x) = \sqrt[3]{\dfrac{-1.1x^4 - 6.6x^2 + 12x + 16}{4.6}}$

$$g_2'(x) = \frac{1}{3}\left(\frac{-1.1x^4 - 6.6x^2 + 12x + 16}{4.6}\right)^{-\frac{2}{3}} \cdot \left(\frac{-4.4x^3 - 13.2x + 12}{4.6}\right)$$

$g_2'(1.5) = -0.79750$ converges

③ $x = g_3(x) = \sqrt{\dfrac{-1.1x^4 - 4.6x^3 + 12x + 16}{6.6}}$

$$g_3'(x) = \frac{1}{2}\left(\frac{-1.1x^4 - 4.6x^3 + 12x + 16}{6.6}\right)^{-\frac{1}{2}} \cdot \left(\frac{-4.4x^3 - 13.8x^2 + 12}{6.6}\right)$$

$g_3'(1.5) = -1.83652$ diverges

④ $x = g_4(x) = \dfrac{-1.1x^4 - 4.6x^3 - 6.6x^2 + 16}{-12}$

$g_4'(1.5) = 5.475$ diverges

⑤ $x = g_5(x) = \dfrac{16}{1.1x^3 + 4.6x^2 + 6.6x - 12}$

$$g_5'(x) = -16(1.1x^3 + 4.6x^2 + 6.6x - 12)^{-2} \cdot (3.3x^2 + 9.2x + 6.6)$$

$g_5'(1.5) = -3.11108$ diverges

② is convergent to the root 1.46272 with starting value $x_0 = 1.5$ #

(3 point) 寫出任三種g(x)
(1 point) 寫出 |g'(p)| < 1 這個公式
(1 point) 透過 |g'(x0)| < 1 找到目標form