Def **Proposition**

is a statement that is either true or false

e.g.

- "Amsterdam is a city"
- "Moons are made of cheese"
- "4 is an even number"

Def **Predicate**

is the "something" we can say about a proposition's subject

e.g.

subject	Predicate
Amsterdam	is a city
Moons	is made of cheese
4	is an even number

Def **Quantifier**

$\forall$ Every moon is made of cheese

$\exists$ Some moons are [ ]

$\nexists$ No moon is [ ]

$\exists!$ Exactly one moon is [ ]

Why? Algorithms i.e your code is in essence a set of logical gates

Our goal is to avoid ambiguity by being precise, and that your train of thought is reproduceable.

Def Let p and q be propositions

Then
$p \vee q \iff p$ "or" q
$p \wedge q \iff p$ "and" q
$\neg p \iff$ "not" p

} truth table

are also propositions where $\wedge, \vee$ and $\neg$ are conjunction disjunction and negation respectively.

e.g.

p = "I study data science"
q = "I dont study logic"

$p \vee \neg q \iff$ "I study logic and data science"

Why Symbols and not just and?

Example "I wanted to leave but did not leave." Here but means $\wedge$!

Def Compound Proposition

Let p, q and r be propositions

Then any propositions created by these proposition and logical operators is a compound proposition

e.g. $p \wedge q \wedge r$

$(p \vee q) \wedge r$

$(\neg(p \wedge (q \vee r))$

etc.

Note Always add parentheses where ambiguity may arise!

Make the order of operations clear

Def Let p and q be propositions. Then the logical operators $\Rightarrow$, $\Leftrightarrow$ and $\oplus$ are defined as

p	q	$p \Rightarrow q$	$p \Leftrightarrow q$	$p \oplus q$
T	T	T	T	F
T	F	F	F	T
F	T	T	F	T
T	T	T	T	F

also $p \Rightarrow q \equiv \neg p \vee q$

$\Rightarrow$ means roughly "then" i.e. if p then q $\Leftrightarrow$ $p \Rightarrow q$

$\Leftrightarrow$ means if and only if
p if and only if q $\Leftrightarrow$ $p \Leftrightarrow q$

$\oplus$ means this either or but not both
p or q but not both

Implication ← $\Rightarrow$ conditional Operator

$\Leftrightarrow$ Biconditional

$\oplus$ Exclusive OR

e.g. p = "Its is raining"
q = "The ground is wet"

$(p \Rightarrow q) \Leftrightarrow$ "if its raining then the ground is wet"

p = "The shape is a square"
q = "$\square$ has 4 equal right angles and sides"

$(p \Leftrightarrow q) \Leftrightarrow$ "A shape is a square only if it has 4 equal right angles and sides"

p = "Bob goes to the meeting"
q = "Alice goes to the meeting"

$p \oplus q \Leftrightarrow$ "Either Bob goes to the meeting or Alice but not both"

Def **Tautology**

A compound proposition is said to be a tautology if and only if for all values of the propositions the compound proposition is true.

e.g. $\neg p \vee p$

Tautology

p	q	$p \vee q$	$\neg q$	$(p \vee q) \wedge \neg q$	$((p \wedge q) \wedge \neg q) \Rightarrow p$
T	T	T	F	F	T
T	F	T	T	T	T
F	T	T	F	F	T
F	F	F	T	F	T

Def **Contradiction**

Opposite of a tautology hence for all values of prop. is the compound false.

e.g.

p	$p \wedge \neg p$
T	F
F	F

Def Logical Equivalence $\equiv$

Let p and q be prop. then p and q are logically equivalent only if $p \iff q$ is a tautology $\square$

Boolean algebra is the algebra of of logic

Def Laws of Boolean algebra

Double Negation	$\neg(\neg p) \equiv p$	
Excluded Middle	$p \vee \neg p \equiv T$	for any
Contradiction	$p \wedge \neg p \equiv F$	p, q and r
Identity	$T \wedge p \equiv p$ $T \vee p \equiv p$	
Idempotent	$p \wedge p \equiv p$ $p \vee p \equiv p$	
Commutative "Abelian"	$p \wedge q \equiv q \wedge p$ $p \vee q = q \vee p$	
Associative	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$ $(p \vee q) \vee r = p \vee (q \vee r)$	
Distr.	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$ $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$	
De Morgan	$\neg(p \wedge q) \equiv (\neg p) \vee (\neg q)$ $\neg(p \vee q) \equiv (\neg p) \wedge (\neg q)$	

Def Contrapositive

$p \Rightarrow q \equiv \neg q \Rightarrow \neg p$

p	q	$p \Rightarrow q$	$\neg q \Rightarrow \neg p$
T	T	T	T
T	F	F	F
F	T	T	T
F	F	T	T

why? Sometimes its easier to prove $\neg q \Rightarrow \neg p$ than $p \Rightarrow q$

* Theorem 2.1 First subst. law

Suppose that Q is any proposition and P is a prop. variable

consider any tautology R.

If all P in R are subst. for Q denote this R'. Then

R' is also a tautology.

* Theorem 2.2 Second subst law

Suppose P, Q and R are any prop. s.t. $P \equiv Q$

R is a compound prop. including P

let R' be R with Q subst for P in R

Then $R \equiv R'$

e.g. $P \wedge (P \Rightarrow q) \equiv P \wedge (\neg P \vee q)$ def $P \Rightarrow q$ and theorem 2.2

$\equiv (P \wedge \neg P) \vee (P \wedge q)$ Distr.

$\equiv F \vee (P \wedge q)$ Contradiction Theorem 2.2

$\equiv P \wedge q$ Identity law

eg. $((P \vee q) \wedge \neg P) \Rightarrow q$

$\equiv (\neg((P \vee q) \wedge \neg P)) \vee q$ def $\Rightarrow$

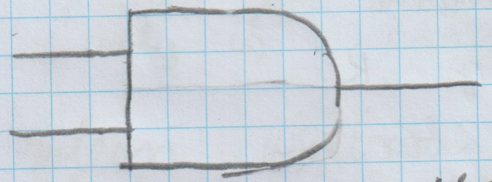
$\equiv (\neg(P \vee q) \vee P) \vee q$ Demorgan and double negation

$\equiv \neg(P \vee q) \vee (P \vee q)$ Associative

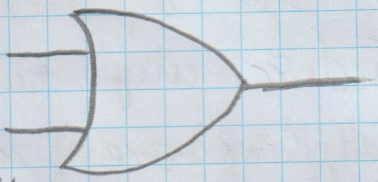
$\equiv T$ law of Excluded middle

Application logic Gates

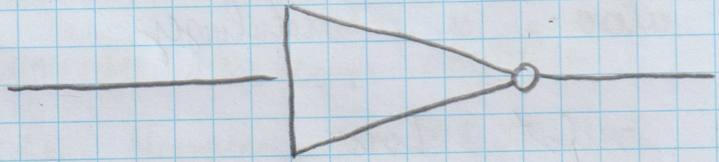
$\wedge$ "And"



$\vee$ "Or"



$\neg$ "not"



eg.