

Intermediate Exam Logic and Proof

December 16 2024, 14:00

- Write your proofs using english sentences not just vague calculations.
- Write your **name** and **student number** on every piece of paper you use.
- Choose **only one** problem to solve per proof method!!!!

1 Direct proof

Problem 1.1

(3 points) Let a , b , c , and d be integers. Use direct proof to prove that if $a \mid b$ and $c \mid d$, then $ac \mid bd$.

Problem 1.2

(3 points) Use direct proof to prove that if two integers have opposite parity (that is, one is even, the other is odd), then their product is even.

2 Proof by contradiction

Problem 2.1

(3 points) Use proof by contradiction. Suppose $a, b, c \in \mathbb{Z}$. If $a^2 + b^2 = c^2$, then a or b is even.

Problem 2.2

(3 points) Use proof by contradiction to prove that the sum of a rational number and an irrational number is irrational.

3 Proof by induction

Problem 3.1

Prove with the help of the (weak) induction principle that $n! > n \cdot 2^{n-1}$ for every integer $n \geq 5$.

- a. (1 point) Write down the base case.
- b. (1 point) Formulate the induction hypothesis.
- c. (2 points) Now perform the inductive step. (**Hint:** $n! = n(n-1)!$)

Problem 3.2

Prove with the help of the (weak) induction principle that for all integers $n \geq 1$,

$$\sum_{i=1}^n 3^i = \frac{3^{n+1} - 3}{2}$$

- a. (1 point) Write down the base case.
- b. (1 point) Formulate the induction hypothesis.
- c. (2 points) Now perform the inductive step.