Recap known number sets

$\mathbb{Z} = \{\dots, -1, 0, 1, \dots\}$

$\mathbb{N} = \{1, 2, 3, \dots\}$

$\mathbb{Q} = \{\frac{a}{b} \mid a, b \in \mathbb{Z}$ s.t. $b \neq 0$ and $\gcd(a,b) = 1\}$

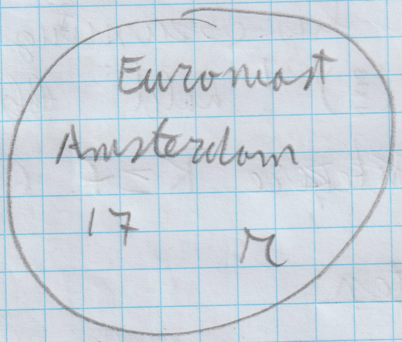
etc.

def A <u>set</u> is a collection of <u>unique</u> elements.

denoted capital A, B, C, .. etc

ex. $A = \{$Euromast, Amsterdam, 17, $\pi\}$ Notice is finite!

Visualization using a Venn diagram



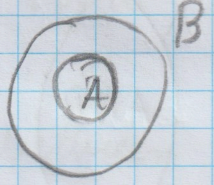
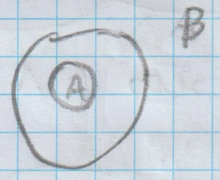
e.g. Abstract example $\{a, b, c, d\}$

e.g. elements of a set can themselves be sets!

$\{\{a,b\}, \{c\}, \{a,b,c\}\}$

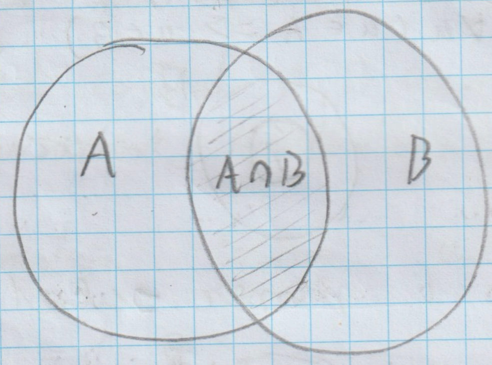
Set Notations

Let A and B be any set. Then

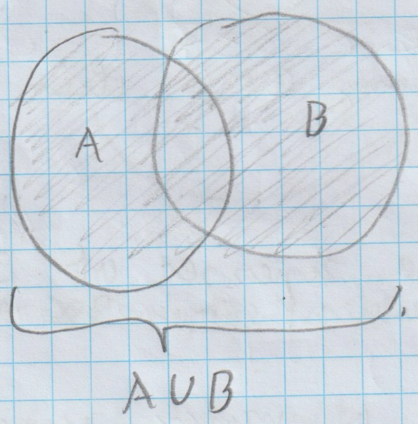
Notation	Definition
$a \in A$	a is a member of A
$a \notin A$	a is not a member of A.
$\emptyset$	denotes the empty sets i.e. it contains no elements.
$A \subseteq B$	A is a subset of B i.e. Given any $a \in A$ then $a \in B$. or $\forall a \, (a \in A \Rightarrow a \in B)$ i.e  or $A = B$
$A \subsetneq B$	A is a proper subset of B i.e.  and $A \neq B$ Given any $a \in A$ then $a \in B$ s.t. $A \neq B$.
$A \supseteq B$	A is a superset of B which is equivalent to. $B \subseteq A$.
$A \supsetneq B$	A is a proper superset of B which is again equivalent to $B \subsetneq A$
$A = B$	A and B have the same members i.e. $A \subseteq B$ and $B \subseteq A$.
$A \cup B$	Union of A and B i.e. $A \cup B = \{x \mid (x \in A \vee x \in B)\}$
$A \cap B$	Intersection of A and B i.e. $A \cap B = \{x \mid (x \in A \wedge x \in B)\}$
$A \setminus B$	Set difference between A and B i.e. $A \setminus B = \{x \mid x \in A \wedge x \notin B\}$

Notation	Definition
$A \Delta B$	symmetric set difference of A and B i.e. $A \Delta B = \{x \mid (x \in A \wedge x \notin B) \vee (x \notin A \wedge x \in B)\}$
$P(A)$	Powerset of A i.e. $P(A) = \{X \mid X \subseteq A\}$
A^c or $\bar{A}$	complement of A i.e. all elements <u>not</u> in A.

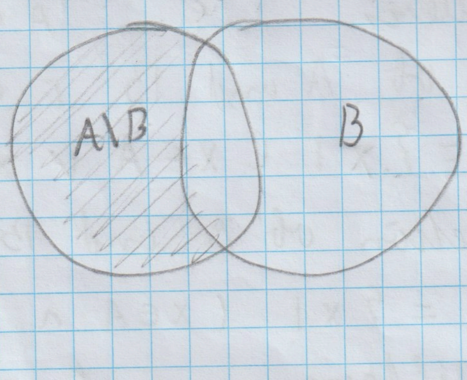
e.g. Intersection of sets A and B.



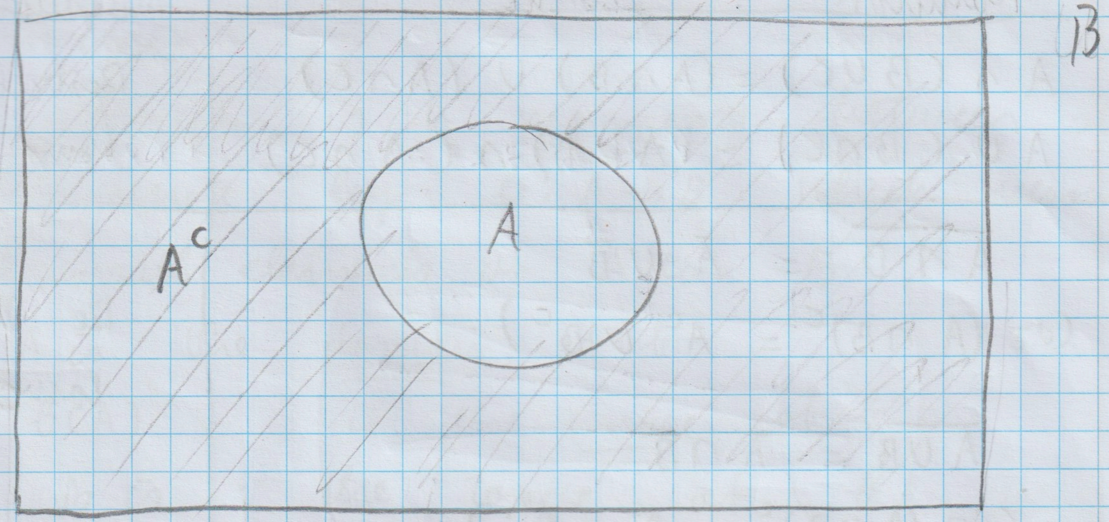
e.g. union of sets A and B



e.g. set difference of A and B.



e.g.



Suppose $A \subseteq B$. Then $A^c = B \setminus A$.

def cartesian product between any sets A and B denoted $A \times B$, is the set

$$A \times B = \{(a,b) \mid a \in A \text{ and } b \in B\}$$

def Laws of Boolean Algebra of sets

Notation	Name
$\bar{\bar{A}} = A$ or $(A^c)^c = A$	double complement law
$A \cup A^c = U$ $A \cap A^c = \emptyset$ $\emptyset \cup A = A$ $\emptyset \cap A = \emptyset$	Miscellaneous laws
$A \cap A = A$ $A \cup A = A$	Idempotent laws
$A \cap (B \cap C)$ $= (A \cap B) \cap C$ $A \cup (B \cup C)$ $= (A \cup B) \cup C$	Associative laws

Solution	Notation
$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ $A \cup (B \cap C) = (A \cup B) \cap (A \cap C)$	Distributive Laws
$\overline{A \cap B} = \bar{A} \cup \bar{B}$ (or $(A \cap B)^c = A^c \cup B^c$) $\overline{A \cup B} = \bar{A} \cap \bar{B}$ (or $(A \cup B)^c = A^c \cap B^c$)	De morgan laws

where $A, B, C \subseteq U$ i.e. $A, B,$ and C are subsets of U.

Logic analogues

Set	Logic
U	T
$\emptyset$	F
A	P
$\bar{A}$	$\neg P$
$A \cup B$	$P \vee q$
$A \cap B$	$P \wedge q$

def Function

A function is a mapping between sets, we denote by $f: A \to B$ which means f maps all elements of A to B

A is the domain of f

$f(A) \subseteq B$ is the image of f.

B is the range of f.

e.g. $f: \mathbb{R} \to \mathbb{R}$, $f(x) = x^2$

$f(\mathbb{R}) = \mathbb{R}_{\geq 0}$ is the image since $f(x) \geq 0$ for all $x \in \mathbb{R}$.

$\mathbb{R}$ is the domain of f

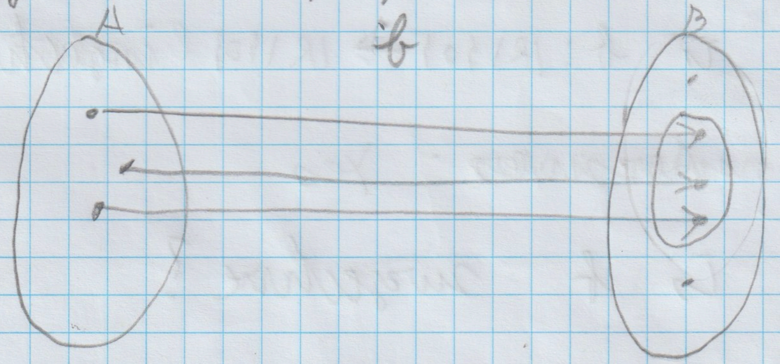
$\mathbb{R}$ is the range of f

Question: What is the domain of $f(x) = \frac{1}{x}$?

$\mathbb{R} \setminus \{0\}$ is its domain

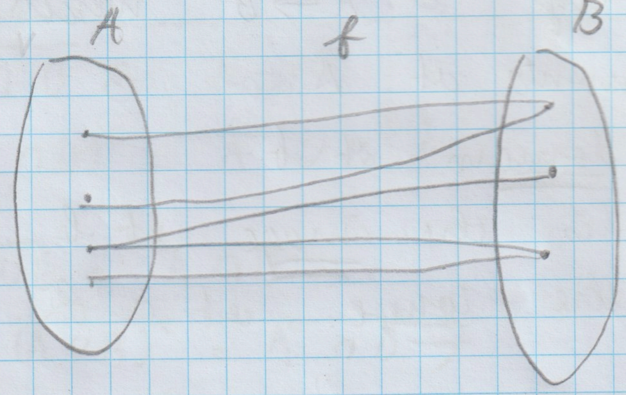
def A function $f: A \to B$ is injective iff

Formally $\forall x \in A \ \forall y \in Y \ (f(x) = f(y) \Rightarrow x = y)$



def A function $f: A \to B$ is surjective iff

formally $\forall y \in B \ \exists x \in A$ s.t. $f(x) = y$



def A function is bijective if its injective and surjective.

Question Is $f: \mathbb{R} \to \mathbb{R}$, $f(x) = x^2$ injective?

Answer: No

Is f surjective?

Answer: Yes

Question Is $f: \mathbb{R} \setminus \{0\} \to \mathbb{R} \setminus \{0\}$ injective?

Answer: Yes

Is f surjective?

Answer: Yes

Is f bijective

Answer: Yes