

# Intermediate Exam Logic and Proof

## December 16 2024, 14:30

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- Write your proofs using english sentences not just vague calculations.
- Write your **name** and **student number** on every piece of paper you use.
- Choose **only one** problem to solve per proof method!!!!

### 1 Direct proof

#### Problem 1.1

(3 points) Use direct proof to prove that if  $a \mid b$  and  $c \mid d$ , then  $ac \mid bd$ .

#### Problem 1.2

(3 points) Use direct proof to prove that if two integers have opposite parity (that is, one is even, the other is odd), then their product is even.

### 2 Proof by contradiction

#### Problem 2.1

(3 points) Use proof by contradiction to prove that  $4 \nmid (a^2 + 2b^2)$ .

#### Problem 2.2

(3 points) Use proof by contradiction to prove that the sum of a rational number and an irrational number is irrational.

### 3 Proof by induction

#### Problem 3.1

Prove with the help of the (weak) induction principle that  $n! > n \cdot 2^{n-1}$  for every integer  $n \geq 5$ .

- a. (1 point) Write down the base case.
- b. (1 point) Formulate the induction hypothesis.
- c. (2 points) Now perform the inductive step. (**Hint:**  $n! = n(n-1)!$ )

#### Problem 3.2

Prove with the help of the (weak) induction principle that for all integers  $n \geq 1$ ,

$$\sum_{i=1}^n 3^i = \frac{3^{n+1} - 3}{2}$$

- a. (1 point) Write down the base case.
- b. (1 point) Formulate the induction hypothesis.
- c. (2 points) Now perform the inductive step.