## Exercises for Chapter 4

Use the method of direct proof to prove the following statements.

**1.** If $x$ is an even integer, then $x^2$ is even.

**2.** If $x$ is an odd integer, then $x^3$ is odd.

**3.** If $a$ is an odd integer, then $a^2+3a+5$ is odd.

**4.** Suppose $x, y \in \mathbb{Z}$. If $x$ and $y$ are odd, then $xy$ is odd.

**5.** Suppose $x, y \in \mathbb{Z}$. If $x$ is even, then $xy$ is even.

**6.** Suppose $a, b, c \in \mathbb{Z}$. If $a \mid b$ and $a \mid c$, then $a \mid (b+c)$.

**7.** Suppose $a, b \in \mathbb{Z}$. If $a \mid b$, then $a^2 \mid b^2$.

**8.** Suppose $a$ is an integer. If $5 \mid 2a$, then $5 \mid a$.

**9.** Suppose $a$ is an integer. If $7 \mid 4a$, then $7 \mid a$.

**10.** Suppose $a$ and $b$ are integers. If $a \mid b$, then $a \mid (3b^3 - b^2 + 5b)$.

**11.** Suppose $a, b, c, d \in \mathbb{Z}$. If $a \mid b$ and $c \mid d$, then $ac \mid bd$.

**12.** If $x \in \mathbb{R}$ and $0 < x < 4$, then $\frac{4}{x(4-x)} \geq 1$.

**13.** Suppose $x, y \in \mathbb{R}$. If $x^2 + 5y = y^2 + 5x$, then $x = y$ or $x + y = 5$.

**14.** If $n \in \mathbb{Z}$, then $5n^2 + 3n + 7$ is odd. (Try cases.)

**15.** If $n \in \mathbb{Z}$, then $n^2 + 3n + 4$ is even. (Try cases.)

**16.** If two integers have the same parity, then their sum is even. (Try cases.)

**17.** If two integers have opposite parity, then their product is even.

**18.** Suppose $x$ and $y$ are positive real numbers. If $x < y$, then $x^2 < y^2$.

**19.** Suppose $a, b$ and $c$ are integers. If $a^2 \mid b$ and $b^3 \mid c$, then $a^6 \mid c$.