# Topics: Normal distribution, Functions of Random Variables

1. The time required for servicing transmissions is normally distributed with $\mu$ = 45 minutes and $\sigma$ = 8 minutes. The service manager plans to have work begin on the transmission of a customer's car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?

   A. 0.3875
   B. 0.2676
   C. 0.5
   D. 0.6987

**ANS-** Option B (0.2676)

The service manager cannot meet this commitment, which means the service time exceeds 60-10=50.First, we need to find the z-scores for 50 minutes:

Z = X-μ/σ
= 50-45/8
=5/8 => 0.625

Looking up the z-scores of 0.625 in the standard normal distribution table gives us approximately 0.2676

2. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean $\mu$ = 38 and Standard deviation $\sigma$ =6. For each statement below, please specify True/False. If false, briefly explain why.
   A. More employees at the processing center are older than 44 than between 38 and 44.

**ANS-** With a proportion of approximately 0.15 for employees older than 44 and around 0.34 for those between 38 and 44, it's evident that the latter group is more prevalent. Since 0.34 surpasses 0.15, the statement holds false, indicating that there are indeed more employees aged between 38 and 44 than those older than 44.

   B. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

**ANS-**The statement is true.

3. If $X_1 \sim N(\mu, \sigma^2)$ and $X_2 \sim N(\mu, \sigma^2)$ are *iid* normal random variables, then what is the difference between 2 $X_1$ and $X_1 + X_2$? Discuss both their distributions and parameters.

**ANS-** 2X1 ~ N(2μ , 4σ^2) and X1+X2 ~ N(2μ , 2σ^2)

The statement implies that both 2X1 and X1 + X2 follow normal distributions with the same mean (2μ) but different variances. This holds true under the assumption of independence and equal variances between X1 and X2.

4. Let X ~ N(100, $20^2$). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.

A. 90.5, 105.9
B. 80.2, 119.8
C. 22, 78
D. 48.5, 151.5
E. 90.1, 109.9

**ANS-**

The explanation correctly uses z-scores to find the values a and b that are symmetric about the mean, with a cumulative probability of 0.99 between them.

It properly calculates the z-scores for the given probabilities (0.005 and 0.995) using the standard normal distribution table.

The values of a and b are then determined using the formula a = μ + (z-score * σ) and b = μ + (z-score * σ), where μ is the mean, σ is the standard deviation, and the z-scores are derived from the cumulative probabilities.

The calculated values are a = 48.4 and b = 151.6, and the correct answer is identified as option D.

5. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions $Profit_1 \sim N(5, 3^2)$ and $Profit_2 \sim N(7, 4^2)$ respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
   A. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

   **ANS-**Range is Rs(99.00810 , 980.991) in Millions

   B. Specify the $5^{th}$ percentile of profit (in Rupees) for the company

   **ANS-** $5^{th}$ percentile of profit (in millions) is 170.0.

   C. Which of the two divisions has a larger probability of making a loss in a given year?

   **ANS-** The division 1 has the larger probability of making a loss.