0.1   Linear Algebra Background

Consider the following matrix **X** and vectors **y** and **z**,

$$X = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}, \quad y = \begin{pmatrix} 3 \\ 1 \end{pmatrix}, \quad z = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

- What is $\mathbf{y} \cdot \mathbf{z}$?

- What is matrix product $\mathbf{X}y$ and $\mathbf{z}^T X$?

- Find the inverse of the matrix X

- What is the determinant of X?

- What is the rank of X?

- Perform the eigen decomposition for X

0.2   Calculus

Consider the following function $f(x) = x^3 - 3x + 7$

- Find the derivative $\frac{df}{dx}$

- Find the critical points of the function

- Find the maximum and minimum of the interval [0,2]

- Find the maximum and minimum on the interval [-2,0]

Now consider the function $f(x, y) = x^2 + y^2 + xy$

- Find the gradient and Hessian of $f(x, y)$

- Find the critical points of the function. What can you say about them

Let $f(x, y, z) = (x + 1)^2 + (y + 2)^2 + (z - 2)^2$ and $g(x, y, z) = x^2 + y^2 + z^2 - 36$
  Using the method of Lagrange multipliers, find the critical point of $f(x, , y, z)$
subject to the constraint $g(x, y, z) = 0$
  Find the critical points of $f(x) = x^4$. Show your work.

0.3   Probability

Calculate the first four moments of a uniform distribution on [0,2].