Problem Chosen	2022 MCM/ICM Summary Sheet	Team Control Number
C		2222938

Earn the Greatest Profit Through a Rational Trading Strategy

Summary

Influenced by the continuous inflation explosion, more and more people choose to buy economic resources, like short-term assets, fixed assets, financial investment, etc. Amongst those economic resources, the riskiest, as well as the most beneficial one, must be the financial investment. The volatile and unpredictable market can make someone rich overnight, but also can push somebody indebted heavily. How to choose an investment with high profit and low risk is the research focus of many quantitative investment practitioners and scholars.

In the absence of any other social information, it is very challenging to trade only using the current and past price data. We split the investment problem into two parts. One is to determine the right time/date of trading, and the other is to determine the proportion of investment (cash, gold, bitcoin) for trading operation.

In the strategy construction part, a trading rule and a portfolio strategy is established. based on the past and current data, we can compute **Technical Analysis Indicators (TAIs)** —- Moving average Convergence/Divergence (MACD) and Relative Strength Index (RSI) are used in this essay. They are popular indicators often combined together in the market to determine a proper trading time, thereby working as the key indicator in our trading rule. Then, following the principle **maximizing return and minimizing risk**, we establish a **two-objective nonlinear programming model** inspired by the Modern Portfolio Theory of Harry Markowitz. And some tricks are used, making it to a **solvable one-objective optimization problem**. In our essay, we also discuss two scenarios, when the mental market is closed and when it is opened. Using this trading strategy, the initial $1000 turned out to be **$11619.9937** on 09/11/2021. The total return is more than 10 time of the initial funds, which experimentally proves the superiority of our strategy.

In our model validation part, three topics are discussed. The first is about the prediction model not applied in our strategy. The second is the utilization of MACD and RSI. And the last is the validation of our portfolio strategy.

In the last sensitivity test, the result shows that our strategy is sensitive to transaction cost. It will reach $14820.1596 when there is not transaction cost, and drop to $4168.5452 when transaction costs of bitcoin and gold are 0.030 and 0.020.

In summary, although our trading strategy has a little lag, we have fully dig into the every day's information and established a rational and reasonable strategy that has been experimentally validated.

Keywords: Technical Analysis; Portfolio; Quantitative Analysis; Optimization;

Contents

1 Introduction

1.1 Problem Background

Financial Market offers traders opportunities by buying investments and becoming a holder to earn from the long-term profits or trading on them. gold and bitcoin are the stars of the current financial trading market. The former is a wide-known old-school capital preservation method. It provides a refuge for investors in a downtown trend market, thereby being a favor for those who prefer low-risk investments. The latter one, a digital currency, is quite more young and volatile than Gold. Hence, it is favored by those who prefer high-risk and high-profit investments.

The stock market is dynamic and volatile[4]. It has been proven that there are no perfect investment strategies that are able to maximize the return as well as minimize the risk[8]. However, diversification may allow for the same portfolio expected return with reduced risk[7]. Therefore, it seems quite important for investors to build a proper portfolio asset to greater return. With the development of technical analysis, the construction of automated financial trading systems (FTSs) is a subject of high interest for both the academic environment and the financial one due to the potential promises by self-learning methodologies[2].

1.2 Restatement of the Problem

- Provide a daily trading strategy based only on price data up to that day. Compute the worth of the initial $1000 on 9/10/2021 using the strategy.
- Prove that the strategy model is the best.
- Test the sensitivity of the strategy to transaction cost. How do transaction costs affect the strategy and results.
- Write an short memorandum to report the strategy, model and result to the trader.

1.3 Our Approach

According to the requirement of the topic for maximizing the final return, our work mainly includes the following:

- A trading rule, which determines the trading day for us to adjust the portfolio setting.
- A portfolio strategy, which attempts to maximize portfolio expected return and minimize a potential risk ratio
- Evaluate the result, effectiveness and sensitivity of the whole strategies.

The model overview is showed in Fig.1.

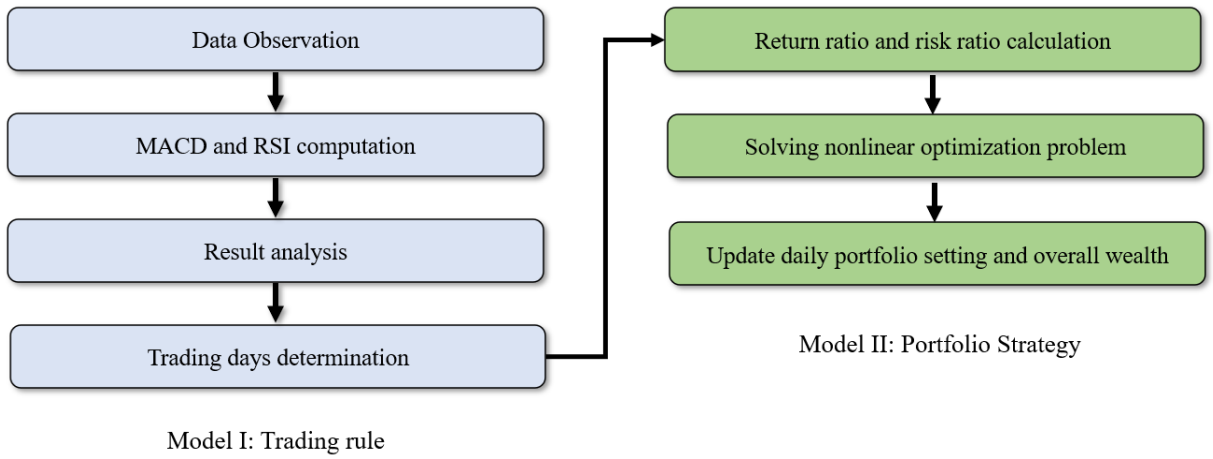

Figure 1: Strategy Overview

2 Assumptions and Notations

2.1 Assumptions

To simplify our model and eliminate the complexity, we make the following main assumptions in this literature.

Assumption 1. *The asset returns are jointly normally distributed, and there is no correlation among them*

Assumption 2. *The trader would not choose a portfolio setting with risk ratio beyond the threshold he hold.*

Assumption 3. *Investors are risk averse, meaning that given two portfolios that offer the same expected return, investors will prefer the less risky one.*

2.2 Notation

In this work, we use the nomenclature in Table.1 in the model construction. Other none-frequent used symbols will be introduced once they are used.

3 Trading rule

When developing our trading rule, we have referred to a lot of analysis indicators in financial transactions, including MA, MACD, RSI and BIAS, etc, and analyzed their uses, advantages, and disadvantages. For example, MACD absorbed the advantages of MA, using the trend to judge the buy and sell signals, and got rid of MA's frequent false signals at the same time; Both RSI and BIAS can judge Overbought and Oversold behaviors, but the buying and selling signals of BIAS is not clear, and there is no unified or certified judgment standard, which is risky. Therefore, after a comprehensive analysis, we decided to combine MACD and RSI as our trading rule.

Symbols	Definition	Range
d	Index of day from 9/11/2016 to 9/11/2021.	Integer, [0, 1826]
M_d	the total assets on day d. Defined as the sum of the value of cash, gold and bitcoin	Float, $[0, \infty)$
A_i	Asset i. 0 for cash, 1 for on bitcoin and 2 for gold	
w_i	Weight of A_i in the portfolio.	Float, [0, 1]
$P_{i,d}$	Price of A_i at day d	Float, $[0, \infty)$
r_i	Return ratio of A_i	Float, [0, 1]
Q_i	Risk ratio of A_i	Float, [0, 1]
R_i	Return of A_i	Float, $[0, \infty)$
R_p	Expected total of return of the portfolio setting	Float, $[0, \infty)$
Q_p	Risky ratio of the portfolio setting	Float, [0, 1]
V_d	Cumulative return at day d	Float, $[0, \infty)$
α_i	Transaction cost of A_i.	Float, [0, 1]
L	The greatest acceptable risky ratio.	Float, [0, 1]
T	The number of days using in trading rule	Integer, [0, 1826]

Table 1: Notation

3.1 Technical Analysis Indicators

These are trading indicators used in technical analysis for forecasting the direction of prices through the study of past market data, especially price and volume[6]. We use them to study gold and bitcoin's movement in both the short term and long term, recognizing if it's time to make a new portfolio.

3.1.1 MACD (Moving Average Convergence/Divergence)

Indicate content: MACD is essentially the convergence and divergence of two EMA(exponential moving average), showing the fast change and slow change of the prices respectively. It represents less noise compared to simple EMA. [11]

Trading signal: The MACD contains two analysis indicators, DIF and DEM (MACD). We obtain transaction signals in the following two ways[5]:

1. Intersection of DIF and DEM: When DIF passes through DEM from bottom to top, it is a buy signal; On the contrary, if from the top through, for the sell signal.

2. Intersection of DIF and 0 axis: if DIF exceeds the 0 axis upward, it is a buy signal; If the DIF value falls below the 0 axis, it is a sell signal.

Parameter decision: "Fast" refers to a shorter EMA, while "slow" refers to a longer EMA, and the most commonly used being the **12** and **26** days[1].

3.1.2 RSI (Relative Strength Index)

Indicate content: RSI is a momentum-based oscillator used to measure how fast and how much the price moves. Buying or selling according to the RSI is based on the principle which says a long rise must fall, a long fall must rise. The RSI can tell us to buy when gold or bitcoin is Oversold and to sell when it is Overbought[3].

Trading signal & Parameter decision: According to Wilder's measurements[9], the index is most representative when $T = 14$. He points out that when the RSI rises to **70**, The financial product is Overbought and traders should consider selling it. Conversely, when the RSI falls to **30**, it means that the financial product is Oversold and traders should buy it.

Supplemental instruction: There two variables in RIS's formula: U and D. Both of them express the change of the price. Assume P_d and P_{t-d} is today's price and yesterday's price respectively.
If the price is increase,

$$U = P_d - P_{d-1}$$
$$D = 0$$

If the price is decrease,

$$U = 0$$
$$D = P_{d-1} - P_d$$

If there is no change,

$$U = 0$$
$$D = 0$$

EMA(Exponential Moving Averages)

According to Table.2, to calculate MACD and RSI, we should know how to calculate EMA first. EMA represents the momentum and trend of price.

The weights of values in EMA decreases exponentially with time. The more recent the data, the greater the weight.

The calculation of EMA with value P at time t is defined as:

$$EMA_d = \alpha \times P_d + (1 - \alpha) \times EMA_{d-1}$$

Indicator	Formula
MACD	$DIF = EMA_{d(P,12)} - EMA_{d(P,26)}$
	$DEM = EMA_{(EMA_{d(P,12)} - EMA_{d(P,26)},9)}$
RSI	$\frac{EMA_{d(U,14)}}{EMA_{d(U,14)} + EMA_{d(D,14)}} \times 100\%$ (U/D: price change)

Table 2: Technical Analysis Indicators' Formulas

It's a recursion, but it's impractical to calculate it from a very first day. According to Johann Bernoulli in 1697, Assume T is the number od days we set, when $\alpha = \frac{2}{T+1}$, only calculate the first $T+1$ terms has covered 86% of the complete weight.

Thus,

$$EMA_{d(P,T)} = P_d(\frac{2}{T+1}) + EMA_{d-1(P,T)}(1 - \frac{2}{T+1})$$

The degree of weighting is determined by the constant α.

Cause SMA(Simple Moving Averages) can also reflect the trend of price, and its disadvantage of hysteresis will help us in some extent, we use SMA to approximate the EMA of zeroth day (s.t at time $d-(T+1)$). For convenience, we represent this item as $EMA_{0(P,T)}$, note that it's not at real time 0, but time $d-(T+1)$.

$$EMA_{0(P,T)} = SMA_{0(P,T)} = \frac{1}{T-1}\sum_{-T+1}^{-1} P_i$$

Then we can calculate EMA smoothly.

3.2 Comprehensive Utilization

It is unreasonable to use MACD or RSI alone. If we only use MACD, it can provide false reversal signals. Sometimes reversal signals shown by the MACD does not mean a significant reversal would happen, so we need RSI to confirm it. If we only use RSI, it can only provide signals when Overbought and Oversold, which isn't highly credible in trend zones, so we need MACD's help. Then we can achieve a complementary effect.

How to combine MACD and RSI? One principle is that, when both MACD and RSI provide the same buy or sell signal, we will start our portfolio strategy to transact, otherwise, when MACD and RSI diverge, we won't do any transaction. Besides, considering the same signal of MACD and RSI may appear in very short duration, which can also be valid. We allow trading when there is the same signal from another indicator within **3** days.

Because there are two financial products, gold and bitcoin, needing us to determine the trading day, and gold is not traded everyday. As long as one of them needs a transaction, we will use the portfolio strategy to trade.

4 Portfolio Strategy

Inspired by the Modern Portfolio Theory (MPT) of Harry Markowitz, we customized a specific portfolio strategy in this case. The mathematical model is as follows.

Given that on the day $d-1$ ($d > 0$) , the weight of cash, bitcoin and gold are respectively $w_{0,d-1}, w_{1,d-1}, w_{2,d-1}$. And the worth of the portfolio is M_{d-1}. On the day 0, which is 9/10/2016, $w_{0,0} = 1, w_{1,0} = 0, w_{2,d-1} = 0$ and $M_0 = \$1000$.

Then on day d, assume that the weight of cash, gold, and bitcoin are $w_{0,d}, w_{1,d}, w_{2,d}$ and the worth of portfolio is M_d. We will establish a model to compute the optimal value of these four parameters.

Since the international precious metal market is closed during major international holiday, we can not trade on gold on these days. The portfolio strategy will be discussed under two scenario.

4.1 Situation I: metal market is open

Since the worth of portfolio on day $d-1$ is equal to the worth of portfolio on day$d$ plus transaction cost, they should satisfy the following wealth balance equation, Equ.1.

$$\begin{aligned} &w_{0,d-1}M_{d-1} + w_{1,d-1}M_{d-1}\frac{P_{1,d}}{P_{1,d-1}} + w_{2,d-1}M_{d-1}\frac{P_{2,d}}{P_{2,d-1}} \\ &= M_d + 0.02|w_{1,d}M_d - w_{1,d-1}M_{d-1}\frac{P_{1,d}}{P_{1,d-1}}| + 0.01|w_{2,d}M_d - w_{2,d-1}M_{d-1}\frac{P_{2,d}}{P_{2,d-1}}| \end{aligned} \tag{1}$$

To maximize return, we need the return ratio of gold and bitcoin, r_1, r_2 to compute the expected portfolio returnR_1, R_2. Here r_1 and r_2 is defined as the return ratio in the latest 28 days.

$$\begin{aligned} r_1 &= \frac{P_{1,d} - P_{1,d-27}}{P_{1,d-27}} \\ r_2 &= \frac{P_{2,d} - P_{2,d-27}}{P_{2,d-27}} \\ R_1 &= r_1 * w_{1,d}M_d \\ R_2 &= r_2 * w_{2,d}M_d \end{aligned} \tag{2}$$

Then, we have the expected portfolio return

$$R_p = R_1 + R_2 \tag{3}$$

And the value function needed to be maximized is as follow.

$$max : V_p = R_p - 0.02|w_{1,d}M_d - w_{1,d-1}M_{d-1}\frac{P_{1,d}}{P_{1,d-1}}| - 0.01|w_{2,d}M_d - w_{2,d-1}M_{d-1}\frac{P_{2,d}}{P_{2,d-1}}| \tag{4}$$

Except maximizing the value function, we also need to minimize the risk function. According to our assumption 1, we computed the monthly risk ratio of gold and bitcoin. The philosophy behind is the 95% confidence interval $[P_{1,d} + r_1 - 1.96 * \sigma_{1,(d-27,d)}, P_{1,d} + r_1 + 1.96 * \sigma_{1,(d-27,d)}]$. We use 2 to approximate 1.96 in the real computation.

$$\begin{aligned} Q_1 &= \frac{2 * \sigma_{1,(d-27,d)}}{P_{1,d}} \\ Q_2 &= \frac{2 * \sigma_{2,(d-27,d)}}{P_{2,d}} \end{aligned} \tag{5}$$

where $\sigma_{1,(d-27,d)}$ and $\sigma_{1,(d-27,d)}$ are the standard deviation of price based on historical data.

And the risk function needed to be minimized is as follows.

$$min : Q_p = Q_1 w_{1,d} + Q_2 w_{2,d} \tag{6}$$

Combined Equ.1, Equ.4, Equ.6, we have our portfolio strategy Equ.7 when the mental market is open.

$$
\begin{aligned}
&max: V_p = R_p - 0.02|w_{1,d}M_d - w_{1,d-1}M_{d-1}\frac{P_{1,d}}{P_{1,d-1}}| - 0.01|w_{2,d}M_d - w_{2,d-1}M_{d-1}\frac{P_{2,d}}{P_{2,d-1}}| \\
&min: Q_p = q_1 w_{1,d} + q_2 w_{2,d} \\
&s.t \begin{cases} w_{0,d} + w_{1,d} + w_{2,d} = 1, \\ w_{0,d-1}M_{d-1} + w_{1,d-1}M_{d-1}\frac{P_{1,d}}{P_{1,d-1}} + w_{2,d-1}M_{d-1}\frac{P_{2,d}}{P_{2,d-1}} \\ = M_d + 0.02|w_{1,d}M_d - w_{1,d-1}M_{d-1}\frac{P_{1,d}}{P_{1,d-1}}| + 0.01|w_{2,d}M_d - w_{2,d-1}M_{d-1}\frac{P_{2,d}}{P_{2,d-1}}|, \\ 0 \le w_{0,d}, \\ 0 \le w_{1,d}, \\ 0 \le w_{2,d}, \end{cases}
\end{aligned} \tag{7}
$$

This is a two-Objective nonlinear programming model. Since the trader has a maximum tolerance value of the risk ratio, labeled as L (Assumption 2.). We transfer these absolute computations into the square of the power of two. Rewrite Equ.7 as follows. It has a numerical solution $w_{0,d}, w_{1,d}, w_{2,d}, M_d$

$$
\begin{aligned}
&max: V_p = R_p - 0.02\sqrt{(w_{1,d}M_d - w_{1,d-1}M_{d-1}\frac{P_{1,d}}{P_{1,d-1}})^2} - 0.01\sqrt{(w_{2,d}M_d - w_{2,d-1}M_{d-1}\frac{P_{2,d}}{P_{2,d-1}})^2} \\
&s.t \begin{cases} w_{1,d} + w_{2,d} \le 1, \\ 0 \le w_{1,d}, \\ 0 \le w_{2,d}, \\ w_{0,d-1}M_{d-1} + w_{1,d-1}M_{d-1}\frac{P_{1,d}}{P_{1,d-1}} + w_{2,d-1}M_{d-1}\frac{P_{2,d}}{P_{2,d-1}} \\ = M_d + 0.02\sqrt{(w_{1,d}M_d - w_{1,d-1}M_{d-1}\frac{P_{1,d}}{P_{1,d-1}})^2} + 0.01\sqrt{(w_{2,d}M_d - w_{2,d-1}M_{d-1}\frac{P_{2,d}}{P_{2,d-1}})^2}, \\ q_1 w_{1,d} + q_2 w_{2,d} \le L, \end{cases}
\end{aligned} \tag{8}
$$

4.2 Situation II: metal market is closed

Under this situation, we need to make sure the that the amount of gold could not be changed after our buying or selling action, which asks the following equation to be hold.

$$
\frac{w_{2,d-1}M_{d-1}}{P_{2,d-z}} = \frac{w_{2,d}M_d}{P_{2,d-z}} \tag{9}
$$

where day $d - z$ is the last tradable day, and $P_{2,d-z}$ is the closing price on the last tradable day.

Then, Wealth balance Equ.1 should be

$$
w_{0,d-1}M_{d-1} + w_{1,d-1}M_{d-1}\frac{P_{1,d}}{P_{1,d-1}} + w_{2,d-1}M_{d-1} = M_d + 0.02|w_{1,d}M_d - w_{1,d-1}M_{d-1}\frac{P_{1,d}}{P_{1,d-1}}| \tag{10}
$$

And the expressions of gold return ratio and risk ratio, r_2, Q_2, change as well

$$\begin{aligned} r_2 &= \frac{P_{2,d-z} - P_{2,d-z-23}}{P_{2,d-z-27}} \\ Q_2 &= \frac{2 * \sigma_{2,(d-z-27,d-z)}}{P_{2,d-z}} \end{aligned} \tag{11}$$

Hence, the target optimization object is rewritten as follows

$$max : V_p = R_p - 0.02\sqrt{(w_{1,d}M_d - w_{1,d-1}M_{d-1}\frac{P_{1,d}}{P_{1,d-1}})^2} \tag{12}$$

In a summary, the one-objective optimization problem under situation two is

$$max : V_p = R_p - 0.02\sqrt{(w_{1,d}M_d - w_{1,d-1}M_{d-1}\frac{P_{1,d}}{P_{1,d-1}})^2}$$

$$s.t \begin{cases} w_{1,d} + w_{2,d} \leq 1, \\ 0 \leq w_{1,d}, \\ 0 \leq w_{2,d}, \\ w_{0,d-1}M_{d-1} + w_{1,d-1}M_{d-1}\frac{P_{1,d}}{P_{1,d-1}} + w_{2,d-1}M_{d-1} \\ = M_d + 0.02\sqrt{(w_{1,d}M_d - w_{1,d-1}M_{d-1}\frac{P_{1,d}}{P_{1,d-1}})^2} \\ q_1 w_{1,d} + q_2 w_{2,d} \leq L, \\ w_{2,d-1}M_{d-1}/P_{2,d-z} = w_{2,d}M_d/P_{2,d-z} \end{cases} \tag{13}$$

5 Case Result

It should be noticed that in the first few days, we refuse to take any adventurous action.Collecting enough historical data for analysis is the main topic and believed to help you avoid unexpected loss. In the discussion of section 3, a set of 26 days data is our minimum requirement for TAIs' computation.

5.1 Data Process

Before our case analysis and strategy realization, there are 10 missing values found in gold closing price data. We choose to drop them, considering the imprecision of manual filings and the minor influence it actually brings.

5.2 Trading Time

Following our trading rules discussed in section 3, The trading time of gold is computed and showed in Fig.2. There are overall 45 timestamps that need the adjustment of the portfolio setting. And that of Bitcoin is showed in Fig.3. There are overall 149 timestamps that need the adjustment of the portfolio setting. Our principle is that we will exchange the portfolio setting of cash, gold and bitcoin as long as there is a signal in either gold or bitcoin. Then we get **184** timestamps in total.

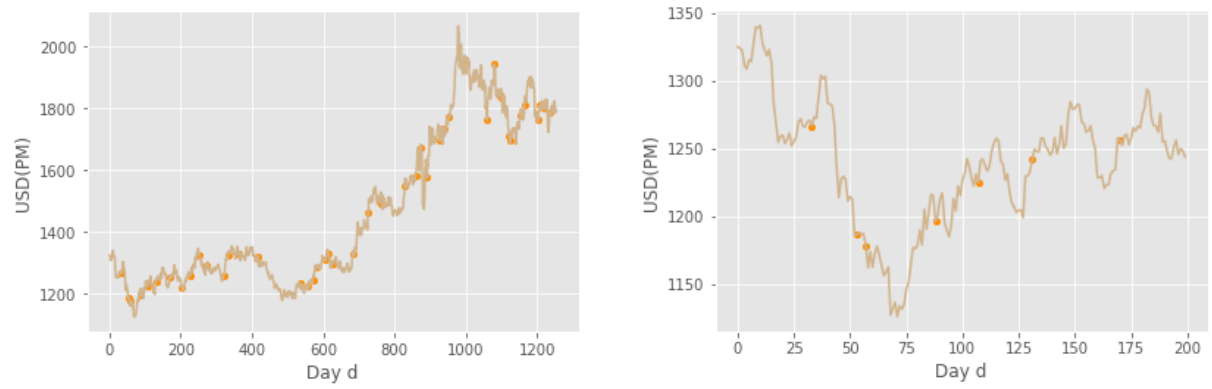

Figure 2: Gold daily prices, U.S. dollars per troy ounce. Data dotted are the trading time computed.

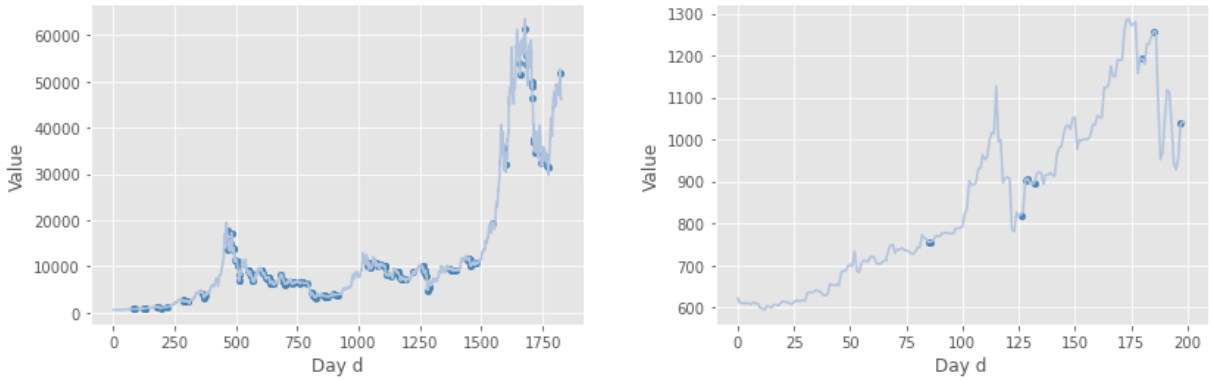

Figure 3: Bitcoin daily prices, U.S. dollars per bitcoin. Data dotted are the trading time computed.

5.3 Overall Profit

Using our strategy, the worth of the initial **$1000** on 09/10/2021 is **$11619.9937**.The total profit on 09/10/2021 is **$10619.9937**, which experimentally shows the superiority of our strategy. It can help make a profit of **more than ten times** the initial capital. The daily overall wealth during this five years is also computed in U.S dollars, and showed in Fig.4. It gradually grows and experience a sudden drop in the last half year. Through our analysis to gold and bitcoin market price, it is believed to be caused by the huge fluctuation of bitcoin.

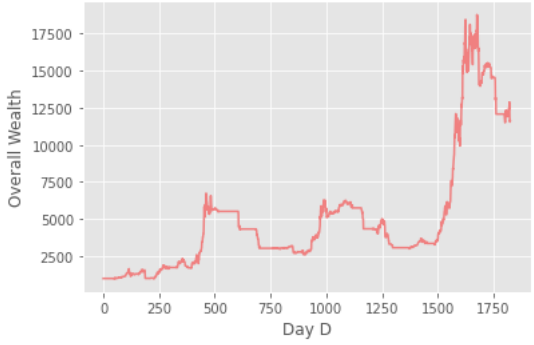

Figure 4: Overall Daily wealth, U.S. dollars. Total daily assets of cash, gold and bitcoin hold.

6 Model Proof

6.1 prediction

We have considered using LSTM to predict some future data, trying to make a better strategy. The result is below (Fig.5).

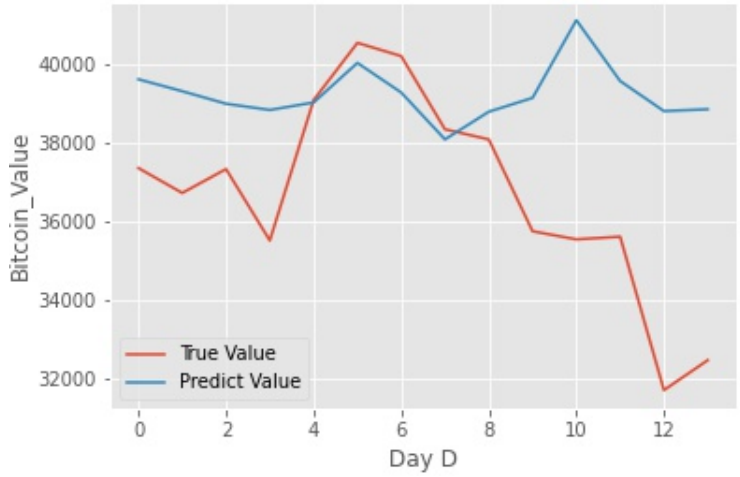

Figure 5: Comparison of predicted and true values for bitcoin. The first 14 days

It's obvious that the result isn't satisfying. One of the reasons is that both gold and bitcoin market is unstable, influenced by various factors, however the previous prices of gold and bitcoin is the only information we have. Another reason is because LSTM requires abundant data to fit the model in order to get more accurate results, but it will need future price data, which isn't consistent with our requirement. We also tried to use Incremental Learning to fit the model, but it still doesn't fit the trend well. These lead our prediction highly inaccurate, causing more errors when calculating the weight of Gold, Bitcoin and cash. As a result, We chose to discard the predictive data.

6.2 Trading rule

In our trading rule, we integrated the use of MACD and RSI, leading they complement with each other. To prove our trading rule's superiority, we calculate how much is the initial $1000 investment worth on 9/10/2021 use only MACD or RSI:

According to Fig.6 and Fig.7,

- If only use MACD, we can get **$9335.5692** on 9/10/2021.
- If only RSI, we can get **$1901.1162** on 9/10/2021.

Compared with the result **$11619.9937** we get using both MACD and RSI, Earnings are significantly smaller. What's more, as section 3 mentioned, MACD and RSI have more certified judgement standard, and the better abilities of describing financial products. So it can be said that our trading rule is the best strategy we can work out.

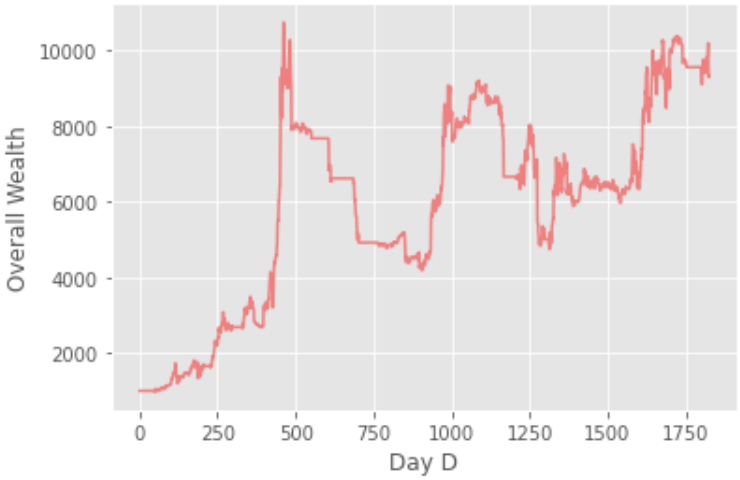

Figure 6: Overall Daily wealth, U.S. dollars. It is computed as the daily sum value of cash, gold and bitcoin. Only use MACD

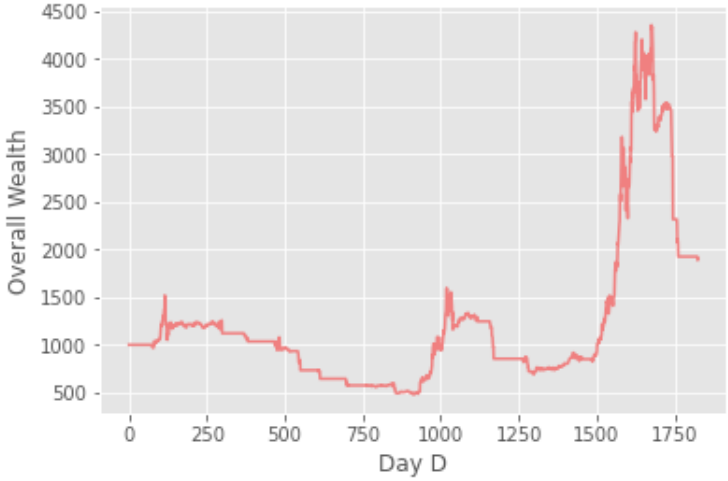

Figure 7: Overall Daily wealth, U.S. dollars. It is computed as the daily sum value of cash, gold and bitcoin. Only use RSI

6.3 portfolio strategy

Since our nonlinear optimization model has exact numerical solution, the validation part should focus on the construction process of our model. In these part, we will further into our philosophy behind the definition of return ratio r_i and risk ratio Q_i.

Restricted by the data available and bad prediction result, the definition of r_i and Q_i should reflect as much as information hidden in the past stream price data. Historical changes of price and its volatility in the past is highly summarised the information we needed. So we decided to start from these two dimensional information. And to follow the logic of TAIs computation in the trading rules, a period of four weeks —- 28 days, is chosen. Furthermore, the selection of L as 15% is mainly following the rule of thumb[10].

7 Sensitivity Analysis

To test the sensitivity of our strategy to the transaction cost, we computed the final result of our model under 10 set of transaction cost pair, showed in Table.3. And objective optimization function could be generalized as

$$max : V_p = R_p - \alpha_1 \sqrt{(w_{1,d}M_d - w_{1,d-1}M_{d-1}\frac{P_{1,d}}{P_{1,d-1}})^2} - \alpha_2 \sqrt{(w_{2,d}M_d - w_{2,d-1}M_{d-1}\frac{P_{2,d}}{P_{2,d-1}})^2} \tag{14}$$

According to the table, we can obviously notice that without any transaction cost, the overall wealth reaches the highest values, $14820.1596. And as the increasing of bitcoin transaction cost, the overall wealth shows a decreasing trend as gold transaction cost fixed. The results greatly reflect that our model is very sensitive to transaction cost which will significantly influence the amount gold and bitcoin we will trade.

Overall wealth	α_1	α_2
14810.1596	**0.000**	**0.000**
12357.2558	0.005	0.005
11425.2452	0.010	0.010
9811.6148	0.015	0.010
11619.9937	**0.020**	**0.010**
7154.6889	0.020	0.020
9296.9223	0.025	0.015
4168.5452	**0.030**	0.020
10052.7319	0.025	0.010
5012.4554	0.030	0.010
4833.1812	0.030	0.030

Table 3: Strategy results under different pairs of transaction cost

8 Strengths and Weaknesses

8.1 Strengths

- We have discussed various situations on different dates according to the actual situation of gold and bitcoin.
- Instead of trading everyday, we choose to date our transactions by using technical analysis indicators, to avoid frequent operations causing more loss.
- The equations in the portfolio strategy can be solved by strict numerical solutions, and we have achieve return maximization and risk minimization relatively.

8.2 Weaknesses

- Due to the market volatility, the future data are unpredictable, leading to some lag in our strategy.
- Although we have set the trading date through MACD and RSI, this strategy is not perfect and the selected date may still be flawed.
- The portfolio strategy do not maximize return and minimize risk absolutely.

9 Conclusion and discussion

In our work, we combine MACD and RSI to decide the trading days of gold and bitcoin, then taking the union of them. After determining when there will be a transaction, we use our portfolio strategy. Breaking the trading day into two scenarios of whether metal market is open or not, we define an appropriate return ratio and a risk ratio. Then by solving our mathematical model, we gain the overall wealth value and portfolio setting based on the principle of maximizing return and minimizing risk. Therefore. we get the best results. According to our trading model and strategy, the initial $1000 investment will worth $11619.9937 on 9/10/2021.

In real analysis, if we considered more aspects of information, like the social event, government attitude, we can take advanced actions instead of waiting for signals transmitted from historical data.

10 Memorandum

DATE: February 20, 2022

TO: Trader

FROM: Team 2222938

SUBJECT: A five year trading strategy for cash, gold and bitcoin

To maximize the overall return of a portfolio consisted of cash, gold and bitcoin, we construct a daily trading strategy based on the historical data. The strategy is consisted of two part analysis, a trading rule and a portfolio strategy. Using our strategy, the initial $1000 will finally turn out to be $11619.9937. Furthermore, goodness of the strategy have been proved and the result of sensitivity test to transaction cost is also quite well. The details will be discussed below.

In the first few days, we refuse to take any adventurous action. Collecting enough historical data for analysis is the main topic and is believed to help avoid unwanted loss. A set of 24 days price data is our minimum requirement for Technical Analysis Indicators (TAIs) computation. In our strategy, Moving average Convergence/Divergence (MACD) and Relative Strength Index (RSI) are used as TAIs to provide an insight to the right time of buying and selling. Only both of them give the same signal within 3 days, it will become a valid signal for gold or bitcoin. And no matter the date is valid for gold or bitcoin, it will become one of our trading day.

Once the right time for buying or selling is detected, our portfolio strategy will start to play on the stage. We compute the expected return ratio and risk ratio based on the historical data, and build a two-Objective nonlinear programming model, which will maximize return and minimize risk. We assume that 15% is the maximum risk ratio you could accept and use small tricks to transfer the optimization problem into an one-objective nonlinear model which has a numerical solution. Hence, we can obtain the optimal portfolio setting.

During the case analysis, we also tried several times series forecasting models, like Long Short-Term Memory (LSTM). The computed M.S.E is large and computation cost of incremental learning is quite expensive. Also, considered the price fluctuation is a process similar to Brownian motion, we think it is unworthy to do a price prediction. In addition to forecasting, it has been verified that the combined utilization of MACD and RSI will bring a greater profit.

Finally, we tested the sensitivity of our strategy to transaction cost. The result shows that our strategy is sensitive to transaction cost. It will reach $14820.1596 when there is not transaction cost, and drop to $4168.5452 when transaction costs of bitcoin and gold are 0.030 and 0.020.

Overall, we strongly suggested you to take our strategy into consideration. This rational and reasonable solution can definitely earn you a considerable return.

References

[1] Gerald Appel. *The moving average convergence-divergence trading method: advanced version*. Scientific Investment Systems, 1985.

[2] Francesco Bertoluzzo and Marco Corazza. "Reinforcement learning for automated financial trading: Basics and applications". In: *Recent advances of neural network models and applications*. Springer, 2014, pp. 197–213.

[3] Shangkun Deng and Akito Sakurai. "Foreign Exchange Trading Rules Using a Single Technical Indicator from Multiple Timeframes". In: *2013 27th International Conference on Advanced Information Networking and Applications Workshops* (2013), pp. 207–212.

[4] Erkam Guresen, Gulgun Kayakutlu, and Tugrul U Daim. "Using artificial neural network models in stock market index prediction". In: *Expert Systems with Applications* 38.8 (2011), pp. 10389–10397.

[5] JianPin and Financial Administration. *Basic principles and practice of investment science*. Peking University Press, 2004.

[6] Charles D Kirkpatrick II and Julie A Dahlquist. *Technical analysis: the complete resource for financial market technicians*. FT press, 2010.

[7] Iyiola Omisore, Munirat Yusuf, and Nwufo Christopher. "The modern portfolio theory as an investment decision tool". In: *Journal of Accounting and Taxation* 4.2 (2011), pp. 19–28.

[8] Zhan Ruilan and Liang Jianying. "Optimization model for improving the value of financial asset portfolio". In: *Value Engineering* 3 (1998).

[9] J Welles Wilder. *New concepts in technical trading systems*. Trend Research, 1978.

[10] Qing Zhang. "Stock trading: An optimal selling rule". In: *SIAM Journal on Control and Optimization* 40.1 (2001), pp. 64–87.

[11] Li Zirui. "Quantitative investment trading strategy research". PhD thesis. Tianjin University, 2013.

Appendices

Appendix A Source Codes for Technical indicators Calculation

```
import numpy

class Indicators:
    def __init__(self, f1, f2):
        # input dataframe
        self.bit = numpy.array(f1)
        self.gold = numpy.array(f2)
        # EMA
        self.EMA12_bit = self.GET_EMA(self.bit, 12)
        self.EMA26_bit = self.GET_EMA(self.bit, 26)
```

```
        self.EMA12_gold = self.GET_EMA(self.gold, 12)
        self.EMA26_gold = self.GET_EMA(self.gold, 26)
        #DIF & DEA
        self.DIF_bit = self.EMA12_bit - self.EMA26_bit
        self.DIF_gold = self.EMA12_gold - self.EMA26_gold
        self.DEA_bit = self.GET_EMA(self.DIF_bit, 9)
        self.DEA_gold = self.GET_EMA(self.DIF_gold, 9)
        # RSI
        self.RSI_bit = self.GET_RSI(self.bit)
        self.RSI_gold = self.GET_RSI(self.gold)
        # BIAS
        self.BIAS_bit = self.GET_BIAS(self.MA_bit_short, self.MA_bit_long)
        self.BIAS_gold = self.GET_BIAS(self.MA_gold_short, self.MA_gold_long)

    def CAL_UD(self, array):
        tmp = 0
        U, D = [], []
        for i in array:
            if tmp == 0:
                tmp = i
                U.append(0)
                D.append(0)
            else:
                if(i > tmp):
                    U.append(abs(i-tmp))
                    D.append(0)
                elif(i < tmp):
                    U.append(0)
                    D.append(abs(i-tmp))
                else:
                    U.append(0)
                    D.append(0)
                tmp = i
        return numpy.array(U), numpy.array(D)

    def CAL_EMA(self, array, N, index):
        for i in range(N):
            if i == 0:
                EMA = array[index-N+i+1]
            else:
                EMA = (((2 * array[index-N+i+1])+(N-1)*EMA)/N+1)
        return EMA

    def GET_RSI(self, array):
        U, D = self.CAL_UD(array)
        EMA_U = self.GET_EMA(U, 14)
        EMA_D = self.GET_EMA(D, 14)
        RSI = (EMA_U/(EMA_U+EMA_D))
        return RSI

    def GET_EMA(self, array, N):
        EMA_N = []
        for i in range(len(array)):
            if i < N-1:
```

```
            EMA_N.append(0)
        else:
            EMA_ = self.CAL_EMA(array, N, i)
            EMA_N.append(EMA_)
    return numpy.array(EMA_N)

def GET_BIAS(self, MA_short, MA_long):
    BIAS = ((MA_short-MA_long)/MA_long)
    return numpy.array(BIAS)

######################type:'bit'/'gold'############################
def EMA_12(self, type, day):
    if type == 'bit':
        return self.EMA12_bit[:day]
    else:
        return self.EMA12_gold[:day]

def EMA_26(self, type, day):
    if type == 'bit':
        return self.EMA26_bit[:day]
    else:
        return self.EMA26_gold[:day]

def MA_short(self, type, day):
    if type == 'bit':
        return self.MA_bit_short[:day]
    else:
        return self.MA_gold_short[:day]

def MA_long(self, type, day):
    if type == 'bit':
        return self.MA_bit_long[:day]
    else:
        return self.MA_gold_long[:day]

def DIF(self, type, day):
    if type == 'bit':
        return self.DIF_bit[:day]
    else:
        return self.DIF_gold[:day]

def DEA(self, type, day):
    if type == 'bit':
        return self.DEA_bit[:day]
    else:
        return self.DEA_gold[:day]

def RSI(self, type, day):
    if type == 'bit':
        return self.RSI_bit[:day]
    else:
        return self.RSI_gold[:day]

def BIAS(self, type, day):
```

```
        if type == 'bit':
            return self.BIAS_bit[:day]
        else:
            return self.BIAS_gold[:day]
```

Appendix B Source Codes for Solving Nonlinear Programming Problem

```
import numpy
from scipy.optimize import minimize

def solve1(array1, array2, w0_, w1_, w2_, M_, i, j, L=0.15):
    r1 = (array1[i] - array1[i-27])/(array1[i-27])
    r2 = (array2[j] - array2[j-27])/(array2[j-27])
    s1 = numpy.std(array1[i-27:i+1])
    s2 = numpy.std(array2[j-27:j+1])
    q1 = (2 * s1)/array1[i]
    q2 = (2 * s2)/array2[j]
    P1 = array1[i]/array1[i-1]
    P2 = array2[j]/array2[j-1]
    c1, c2 = 0.02, 0.01

    V = lambda X: -((r1*X[0]*X[2]+r2*X[1]*X[2])-(c1*(((X[0]*X[2]-w1_*M_*P1)**(2))**(0.5)))

    cons1 = dict(type='ineq', fun=lambda X: X[0])
    cons2 = dict(type='ineq', fun=lambda X: X[1])
    cons3 = dict(type='ineq', fun=lambda X: 1-X[0]-X[1])
    cons4 = dict(type='ineq', fun=lambda X: L-q1*X[0]-q2*X[1])
    cons5 = dict(type='eq', fun=lambda X: X[2]+(c1*(((X[0]*X[2]-w1_*M_*P1)**(2))**(0.5))) -
    cons = [cons1, cons2, cons3, cons4, cons5]
    result = minimize(
        V, numpy.array((0.5,0.5,1000)), constraints=cons)
    return result



def solve2(array1, array2, w0_, w1_, w2_, M_, i, j, L=0.15):
    r1 = (array1[i] - array1[i-27])/(array1[i-27])
    r2 = (array2[j-1] - array2[j-28])/(array2[j-28])
    s1 = numpy.std(array1[i-27:i+1])
    s2 = numpy.std(array2[j-28:j])
    q1 = (2 * s1)/array1[i]
    q2 = (2 * s2)/array2[j-1]
    P1 = array1[i]/array1[i-1]
    P2 = 1
    c1, c2 = 0.02, 0.01

    V = lambda X: -((r1*X[0]*X[2]+r2*X[1]*X[2])-(c1*(((X[0]*X[2]-w1_*M_*P1)**(2))**(0.5)))

    cons1 = dict(type='ineq', fun=lambda X: X[0])
    cons2 = dict(type='ineq', fun=lambda X: X[1])
    cons3 = dict(type='ineq', fun=lambda X: 1-X[0]-X[1])
```

```
    cons4 = dict(type='ineq', fun=lambda X: L-q1*X[0]-q2*X[1])
    cons5 = dict(type='eq', fun=lambda X: X[2]+(c1*(((X[0]*X[2]-w1_*M_*P1)**(2))**(0.5))) +
    cons = [cons1, cons2, cons3, cons4, cons5]
    result = minimize(
        V, numpy.array((0.5,0.5,1000)), constraints=cons)
    return result
```