Assignment1

12011517 李子南

Q1.

(a) $\neg p$

(b) $p \wedge \neg q$

(c) $p \rightarrow q$

(d) $\neg p \rightarrow \neg q$

(e) $p \rightarrow q$

(f) $\neg p \wedge q$

(g) $q \rightarrow p$

Q2.

(a)

p	q	$p \oplus q$	$p \wedge q$	$(p \oplus q) \rightarrow (p \wedge q)$
F	F	F	F	T
T	F	T	F	F
F	T	T	F	F
T	T	F	T	T

(b)

p	q	$p \leftrightarrow q$	$\neg p \leftrightarrow q$	$(p \leftrightarrow q) \oplus (\neg p \leftrightarrow q)$
F	F	T	F	T
T	F	F	T	T
F	T	F	T	T
T	T	T	F	T

(c)

p	q	$p \oplus q$	$p \oplus \neg q$	$(p \oplus q) \rightarrow (p \oplus \neg q)$
F	F	F	T	T
T	F	T	F	F
F	T	T	F	F
T	T	F	T	T

Q3.

(a)

p	q	$p \to q$
F	F	T
T	F	F
F	T	T
T	T	T

p	q	$\neg p \vee q$
F	F	T
T	F	F
F	T	T
T	T	T

The two propositions are equivalent

(b)

p	q	$p \oplus q$
F	F	F
T	F	T
F	T	T
T	T	F

p	q	$\neg p \vee \neg q$
F	F	T
T	F	T
F	T	T
T	T	F

The two propositions are not equivalent

(c)

p	q	r	$(p \to q) \to r$
F	F	F	F
T	F	F	T
F	T	F	F
F	F	T	T
T	T	F	F
T	F	T	T
F	T	T	T
T	T	T	T

p	q	r	$p \to (q \to r)$
F	F	F	T
T	F	F	T
F	T	F	T
F	F	T	T
T	T	F	F
T	F	T	T
F	T	T	T
T	T	T	T

The two propositions are not equivalent

(d)

p	q	r	$\neg p \to \neg(q \to r)$
F	F	F	F
T	F	F	T
F	T	F	T
F	F	T	F
T	T	F	T
T	F	T	T
F	T	T	F
T	T	T	T

p	$\neg p$
F	T
T	T

The two propositions are not equivalent

(e)

p	q	r	$(p \vee q) \to r$
F	F	F	T
T	F	F	F
F	T	F	F
F	F	T	T
T	T	F	F
T	F	T	T
F	T	T	T
T	T	T	T

p	q	r	$(p \to r) \wedge (q \to r)$
F	F	F	T
T	F	F	F
F	T	F	F
F	F	T	T
T	T	F	F

p	q	r	$(p \to r) \land (q \to r)$
T	F	T	T
F	T	T	T
T	T	T	T

The two propositions are equivalent

Q4.

(a)

$\neg(p \oplus q)$

$\equiv (\neg p \land q) \land \neg(p \land \neg q)$

$\equiv (p \lor \neg q) \land (\neg p \lor q)$

$\equiv (q \to p) \land (p \to q)$

$\equiv p \leftrightarrow q$

(b)

$\neg(p \to q) \to \neg q$

$\equiv (p \to q) \lor \neg q$

$\equiv (\neg p \lor q) \lor \neg q$

$\equiv \neg p \lor (q \lor \neg q)$

$\equiv True$

(c)

$(p \to q) \to ((r \to p) \to (r \to q))$

$\equiv (\neg p \lor q) \to ((\neg r \lor p) \to (\neg r \lor q))$

$\equiv (\neg p \lor q) \to (\neg p \lor \neg r \lor q)$

$\equiv (p \land \neg q) \lor (\neg p \lor \neg r \lor q)$

$\equiv \neg q \lor q \lor \neg r$

$\equiv True$

Q5.

$(p \lor \neg q) \land (q \lor \neg r) \land (r \lor \neg p)$

$\equiv ((p \lor \neg q) \land (q \lor \neg r) \land r) \lor ((p \lor \neg q) \land (q \lor \neg r) \land p)$

$\equiv (p \land q \land r) \lor (\neg p \land \neg q \land \neg r)$

If $(p \land q \land r) \lor (\neg p \land \neg q \land \neg r)$ is true, then $(p \land q \land r)$ or $(\neg p \land \neg q \land \neg r)$ Is true.

Therefore, only when p, q, r have the same truth value, $(p \lor \neg q) \land (q \lor \neg r) \land (r \lor \neg p)$ Is true.

Q6.

(a)They are logically equivalent

$(p \to q) \vee (p \to r)$

$\equiv (\neg p \vee q) \vee (\neg p \vee r)$

$\equiv \neg p \vee q \vee r$

$p \to (q \vee r)$

$\equiv \neg p \vee q \vee r$

(b)They are logically equivalent

$\neg p \to (q \to r)$

$\equiv p \vee \neg q \vee r$

$q \to (p \vee r)$

$\equiv \neg q \vee p \vee r$

(c)They are logically equivalent

$(p \to q) \wedge (p \to r)$

$\equiv (\neg p \vee q) \wedge (\neg p \vee r)$

$\equiv \neg p \vee (q \wedge r)$

$p \to (q \wedge r)$

$\equiv \neg p \wedge (q \vee r)$

(d)They are not logically equivalent

$(p \vee q) \to r$

$\equiv (\neg p \wedge \neg q) \vee r$

$(p \to r) \vee (q \to r)$

$\equiv (\neg p \vee r) \vee (\neg q \vee r)$

$\equiv r \vee \neg p \vee \neg q$

Q7.

(a)$\exists x(C(x) \wedge D(x) \wedge F(x))$

(b)$\forall x(C(x) \wedge D(x)) \vee F(x)$

(c)$\exists x(C(x) \wedge F(x) \wedge \neg D(x))$

(d)$\neg \exists x(C(x) \wedge D(x) \wedge F(x))$

(e)$\exists x C(x) \wedge \exists x D(x) \wedge \exists x F(x)$

Q8.

(a)$\forall x F(x, Fred)$

(b)$\forall y F(Evelyn, y)$

(c)$\neg \exists x \forall y F(x, y)$

(d)$\exists p \exists q((p \neq q) \wedge F(Nancy, p) \wedge F(Nancy, q) \wedge \forall k(F(Nancy, k) \rightarrow ((k = q) \vee (k = p))))$

(e)$\exists y \forall x(F(x, y) \wedge \forall z(F(x, z) \rightarrow z = y))$

(f)$\exists x(\exists y(F(x, y) \wedge \forall z(F(x, z) \rightarrow (z = y)) \wedge (y \neq x)))$

Q9.

(a)$\exists y \forall x \forall z(\neg T(x, y, z) \wedge \neg Q(x, y))$

(b)$\forall x \forall y \neg P(x, y) \vee \exists x \exists y \neg Q(x, y)$

(c)$\forall x \forall y((Q(x, y) \wedge \neg Q(y, x)) \vee (Q(y, x) \wedge \neg Q(x, y)))$

(d)$\exists x \forall y \exists z \neg T(x, y, z)$

Q10.

$\because$

$(p \wedge q) \wedge (p \rightarrow \neg(q \wedge r)) \wedge (s \rightarrow r)$

$\equiv (p \wedge q) \wedge (\neg p \vee \neg q \vee \neg r) \wedge (\neg s \vee r)$

$\equiv (p \wedge q \wedge \neg r) \wedge (\neg s \vee r)$

$\equiv p \wedge q \wedge \neg r \wedge \neg s$

$\therefore$

$(p \wedge q) \wedge (p \rightarrow \neg(q \wedge r)) \wedge (s \rightarrow r) \rightarrow \neg s$

$\equiv \neg p \vee \neg q \vee r \vee s \vee \neg s$

$\because$ $(s \vee \neg s)$ is always true

$\therefore$ The statement is true.

Q11.

(a)Domain of x: all the people

P(x): x is in the class

Q(x): x enjoys whale watching

R(x): x cares about ocean pollution

Somebody in this class enjoys whale watching: $\exists x(P(x) \wedge Q(x))$

Every person who enjoys whale watching cares about ocean pollution: $\forall x(Q(x) \rightarrow R(x))$

Therefore, there is a person in this class who cares about ocean pollution: $\exists x(p(x) \wedge R(x))$

	Step	Reason
1	$\exists x(P(x) \wedge Q(x))$	Premise
2	$P(a) \wedge Q(a)$	Existential instantiation from (1)
3	$Q(a)$	Simplification from (2)
4	$\forall x(Q(x) \rightarrow R(x))$	Premise
5	$Q(a) \rightarrow R(a)$	Universal instantiation from (4)
6	$R(a)$	Modus pond from (3) and (5)
7	$P(a)$	Simplificationfrom (2)
8	$P(a) \wedge R(a)$	Conjunction from (6) and (7)
9	$\exists x(p(x) \wedge R(x))$	Existential generalization from (8)

(b)Domain of x: all people

a: Zeke

P(x): x owns a computer

Q(x): x can use a word processing

C(x): x is in this class

Each of the 93 students in this class owns a personal computer: $\forall x(C(x) \rightarrow P(x))$

Everyone who owns a personal computer can use a word processing program: $\forall x(P(x) \rightarrow Q(x))$

Zeke, a student in this class: $C(a)$

Zeke can use a word processing program: $Q(a)$

	Step	Reason
1	$\forall x(C(x) \rightarrow P(x))$	Premise
2	$C(a) \rightarrow P(a)$	Universal instantiation from (1)
3	$C(a)$	Premise
4	$P(a)$	Modus pond from (2) and (3)
5	$\forall x(P(x) \rightarrow Q(x))$	Premise
6	$P(a) \rightarrow Q(a)$	Universal instantiation from (5)

	Step	Reason
7	$Q(a)$	Modus pond from (4) and (6)

(c)Domain: all students

P(x): x has taken a course in discrete mathematics

Q(x): x can take a course in algorithms

Each of five roommates, A, B, C, D, and E, has taken a course in discrete mathematics: $P(A),\ P(B),\ P(C),\ P(D),\ P(E)$

Every student who has taken a course in discrete mathematics can take a course in algorithms: $\forall x(P(x) \to Q(x))$

All five roommates can take a course in algorithms next year: $Q(A) \wedge Q(B) \wedge Q(C) \wedge Q(D) \wedge Q(E)$

	Step	Reason
1	$P(A)$	Premise
2	$\forall x(P(x) \to Q(x))$	Premise
3	$P(A) \to Q(A)$	Existential instantiation
4	$Q(A)$	Modus pond from (1) and (3)
5	$P(B),\ P(C),\ P(D),\ P(E)$	Premise
6	$Q(B),\ Q(C),\ Q(D),\ Q(E)$	Same reasons from (1) to (4) and (5)
7	$Q(A) \wedge Q(B) \wedge Q(C) \wedge Q(D) \wedge Q(E)$	Conjunction from (4) and (6)

Q12.

(a)$\exists n \in N(n^3 + 6n + 5\ is\ odd \wedge n\ is\ odd)$

(b)The statement in (a) is true

If $n^3 + 6n + 5$ Is odd, then $\exists k \in N(n^3 + 6n + 5 = 2k + 1)$

Then $n^3 = 2k - 6n - 4$, $(2k - 6n - 4)\%2 = 0$

Therefore n^3 is even

Assume that n is odd, then$\exists q \in N(n = 2q + 1)$

Then $n^3 = 8q^3 + 12q^2 + 6q + 1$

Because $(8q^3 + 12q^2 + 6q + 1)\%2 = 1$

Therefore n^3 is odd, it's contradict with previous conclusion, the assumption is wrong

Therefore n is even

Q13.

$\because a^2 + b^2\ is\ even$

$Assume\ that\ \exists k \in N(a^2 + b^2 = 2k)$

$Then\ a^2 + b^2 + 2ab = 2k + 2ab$

$(a+b)^2 = 2(k+ab), a\ and\ b\ are\ integer$

$(a+b)^2 = (a+b)(a+b)\ is\ even$

$\therefore a+b\ \ is\ even$

Q14.

Set $a = 2, b = \frac{1}{2}$

Then $\sqrt{2}$ is an irrational number

The statement is wrong.

Q15.

Assume $\sqrt[3]{2}$ is rational, then $\exists p \in N \exists q \in N(\frac{p}{q} = \sqrt[3]{2} \wedge (p, q\ are\ relatively\ prime))$

Therefore $\frac{p^3}{q^3} = 2$

$p^3 = 2q^3$, $\exists k \in N(p = 2k)$

$p^3 = 8k^3 = 2q^3$

$q^3 = 4k^3$

Therefore p and q are both even, they are not relatively prime. It's contradict with the assumption, the assumption is wrong.

Therefore $\sqrt[3]{2}$ is irrational

Q16.

$(\sqrt{2} + \sqrt{3})^2 = 5 + 2\sqrt{6}$

$9 < 5 + 2\sqrt{6} < 16, 5 + 2\sqrt{6}$ is not a perfect square.

Therefore $(\sqrt{2} + \sqrt{3}) = \sqrt{(5 + 2\sqrt{6})}$ is an irrational number.

Q17.

Case1: We win and our bid is the highest.

Then our payment is the second highest bid b_{h_2}

If $b_{h_1} = v_n > b_{h_2}$: our payoff is $v_n - b_{h_2} > 0$

If $b_{h_1} > v_n = b_{h_2}$: our payoff is $v_n - b_{h_2} = 0$

If $b_{h_1} > b_{h_2} > v_n$: our payoff is $v_n - b_{h_2} < 0$

If $b_{h_1} > v_n > b_{h_2}$: our payoff is $v_n - b_{h_2} > 0$

If $v_n > b_{h_1} > b_{h_2}$: our payoff is $v_n - b_{h_2} > 0$

Case2: We win and our bid is one of the highest.

Then our payment is the highest bid b_h

If $v_n = b_h$: our payment is $v_n - b_h = 0$

If $b_h > v_n$: our payment is $v_n - b_h < 0$

Case3: We lose.

The payoff is always 0

Overall, we can see in all cases $b_n = v_n$ will always lead to a payoff that is no small than $b_n \neq v_n$