Assignment2

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Q1.

(a) If $A-B=A$, then $B\nsubseteq A$ or $B=\emptyset$

When $A-B=A$ is true, then $B\subset A$ Is false

Therefore the statement is false

(b)$\because (A\cap B\cap C)=\{x|(x\in A)\wedge(x\in B)\wedge(x\in C)\}$

$((x\in B)\vee(x\in A))\rightarrow(x\in(A\cup B))$

$\therefore (x\in(A\cap B\cap C))\rightarrow(x\in(A\cup B))$

The statement is true

(c)$\because \overline{(A-B)}=\overline{(A\cap\overline{B})}=\overline{A}\cup B$

$B-A=B\cap\overline{A}$

$\therefore \overline{(A-B)}\cap(B-A)=(\overline{A}\cup B)\cap(B\cap\overline{A})=B\cap\overline{A}$

The statement is not true

Q2.

(a)The symmetric difference is associative.

$A\oplus(B\oplus C)=A\oplus((B\wedge\neg C)\vee(\neg B\wedge C))=(A\wedge B\wedge C)\vee(\neg A\wedge\neg B\wedge C)\vee(\neg A\wedge B\wedge\neg C)\vee(A\wedge\neg B\wedge\neg C)$

$(A\oplus B)\oplus C=(A\wedge B\wedge C)\vee(\neg A\wedge\neg B\wedge C)\vee(\neg A\wedge B\wedge\neg C)\vee(A\wedge\neg B\wedge\neg C)$

Therefore $A\oplus(B\oplus C)=(A\oplus B)\oplus C$

(b)Yes

$A\oplus C=A\cup C-A\cap C=(A-(A\cap C))+(C-(A\cap C))$

$B\oplus C=B\cup C-B\cap C=(B-(B\cap C))+(C-(B\cap C))$

$\because A\oplus C=B\oplus C$

$A\cap C=B\cap C$

$\therefore A=B$

Q3.

(1)$(B-A)\cup(C-A)=(B\cap\overline{A})\cup(C\cap\overline{A})=(B\cup C)\cap\overline{A}$

$(B\cup C)-A=(B\cup C)\cap\overline{A}$

$\therefore (B-A)\cup(C-A)=(B\cup C)-A=(B\cup C)\cap\overline{A}$

(2)$(A\cap B)\cap\overline{(B\cap C)}\cap(A\cap C)=A\cap(B\cap C)\cap\overline{(B\cap C)}=\emptyset$

Q4.

If $A\subseteq B$

$\forall a\in A(a\in B)$

So $(P(a)\in P(A))\wedge(P(a)\in P(B))$

$\forall a\in A(P(a)\in P(B))$

$\therefore P(A)\subseteq P(B)$

If $P(A)\subseteq P(B)$

$\forall a\in A(P(a)\in P(B))$

Which means $\exists b(P(a)=P(b))$

$\because P\ is\ injective$

$a=b$

$\forall a\in A\exists b\in B(a=b)$

$\therefore A\subseteq B$

Q5.

(a)A function is onto but not one to one

$f(x)=f(-x)$, it's not one ot one

$\forall(y\geq 0)\exists x\in R(f(x)=y)$, it's onto

(b)A function which is neither one to one nor not onto

$f(3)=6$ is not in $\{2,4\}$,so it's not one to one

4 has no x in $\{1,3\}$ that f(x) = 4

(c)A function which is both one to one and onto

When $x\neq y$, $8-2x\neq 8-2y$, therefore f(x) is one to one

There exist f(x) = y if and only if 8-2x = y and x = 4-0.5y, it's onto

(d)A function which is onto but bot one to one

f(1) = f(1.5), it's not one to one

$\forall y\in Z\exists x\in R(f(x)=\lfloor x+1\rfloor)$, it's onto

(e)A function which is onto but not one to one

$f(0.5)\notin R^{+}$, it's not one to one

$\forall(y\in R^{+})\exists(x\in R^{+})f(x)=y$, it's onto

(f)A function which is one to one but not onto

When $x\neq y, x+1\neq y+1$, therefore f(x) is one to one

$\forall x\in Z^{+}f(x)\neq 1$, it's not onto

Q6.

(c)Because f(x) is both onto and one to one, it has $f^{-1}(x)=4-0.5x$

Q7.

If $x\%1\in[0,\frac{1}{3}]$

$\lfloor x\rfloor=\lfloor x+\frac{1}{3}\rfloor=\lfloor x+\frac{2}{3}\rfloor=x$

$\lfloor x\rfloor+\lfloor x+\frac{1}{3}\rfloor+\lfloor x+\frac{2}{3}\rfloor=\lfloor 3x\rfloor=3x$

If $x\%1\in[\frac{1}{3},\frac{2}{3}]$

$\lfloor x\rfloor=\lfloor x+\frac{1}{3}\rfloor=x$

$\lfloor x+\frac{2}{3}\rfloor=x+1$

$3\lfloor x\rfloor=3\lfloor x/1\rfloor+\lfloor 3(x\%1)\rfloor=3x+1$

$\lfloor 3x\rfloor =\lfloor x\rfloor + \lfloor x+\frac{1}{3}\rfloor +\lfloor x+ \frac{2}{3}\rfloor =3x+1 $

If $x\%1\in[\frac{2}{3},1)$

$\lfloor x\rfloor=x$

$\lfloor x+\frac{1}{3}\rfloor=\lfloor x+\frac{2}{3}\rfloor=x+1$

$3\lfloor x\rfloor=3\lfloor x/1\rfloor+\lfloor 3(x\%1)\rfloor=3x+2$

$\lfloor 3x\rfloor=\lfloor x\rfloor+\lfloor x+\frac{1}{3}\rfloor+\lfloor x+\frac{2}{3}\rfloor=3x+2$

Q8.

(a)yes

Assume $f(g(a)) = f(g(b))$

Because $f \circ g$ is one to one

Therefore $a = b$

Because g is one to one

Then $g(a) = g(b)$

Therefore f is one to one

(b)yes

(c)yes

proof:(b)&(c)

If $f \circ g$ is one to one

Assume $g(a) = g(b)$

Then $f(g(a)) = f(g(b))$

Because $f \circ g$ is one to one

Therefore $a = b$, g is one to one, f is one to one

(d)Let $c \in C$

Because $f \circ g$ is onto, there is $a \in A$ that $c = f \circ g(a)$

Let $b = g(a), b \in B$

Therefore for every $c \in C$, there is $b \in B$ such that $c = f(b)$

Therefore f is onto

(e)No

For example $A = \{0\}, B = \{0, 1\}, C = \{0\}$ with $g(0) = 0, f(0) = 0, f(1) = 1$

In this case f and $f \circ g$ is onto but g is not onto

Q9.

$(n+1)^3 - n^3 = 3n^2 + 3n + 1$

$n^3 - (n-1)^3 = 3(n-1)^2 + 3(n-1) + 1$

$(n+1)^3 - n^3 + n^3 - (n-1)^3 + ... + 2^3 - 1^3 = (n+1)^3 - 1^3 = 3(n^2 + (n-1)^2 + ... + 2^2 + 1^2) + 3(n + (n-1) + ... + 2 + 1) +$

$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$

Q10.

(a)$A\{x|x \in N\},\ B\{x \in Z^+\}$

(b)$A\{x|x = 2n, n \in N\},\ B\{x|x = 4n, n \in N\}$

(c)$A\{x|x \in R\},\ B\{x|x \in R^+\}$

Q11.

(a)The set is countable. Assume that CS201 have n students, there exist C_n^k subsets of size k. Therefore the set of all subsets of students in CS201 have $\sum_{k=1}^{n} C_n^k$ objects in it. It's countable.

(b)The set is countable. We can list all object as follow:

$$\begin{matrix} (0,0) & (0,1) & (0,2) & \cdots & (0,n) \\ (1,0) & (1,1) & (1,2) & \cdots & (1,n) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ (n,0) & (n,1) & (n,2) & \cdots & (n,n) \end{matrix}$$

(c)The set is uncountable. Assume that the set is countable, then we list it as:

$$\begin{matrix} (0,b_1) & (0,b_2) & (0,b_3) & \cdots & (0,b_n) \\ (1,b_1) & (1,b_2) & (1,b_3) & \cdots & (1,b_n) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ (n,b_1) & (n,b_2) & (n,b_3) & \cdots & (n,b_n) \end{matrix}$$

Because set R is uncountable, then sequence $b_1,\ b_2 ... b_n$ can't have all numbers in R, so this set can't be listed.

Therefore this set is also uncountable.

Q12.

Suppose that $A - B$ is a countable set.

B is a countable set.

Then $(A - B) \cup B$ is countable.

$A \subseteq ((A - B) \cup B)$

Therefore A is contained in a countable set.

But A is an uncountable set.

It's contradictory.

Therefore $A - B$ is an uncountable set

Q13.

$\because m \in Z^+, n \in Z^+$

$\frac{\partial f}{\partial m} = m + n - \frac{1}{2} > 0$

$\frac{\partial f}{\partial n} = n + m - \frac{3}{2} > 0$

Therefore f is strictly increasing, f is one to one.

Suppose $f(m-1,1) = k, k \in Z^+$, then we can get $f(m-1,1) + 1 = f(1,m)$

$f(1,1) = 1$

Therefore $\forall y \in Z^+ \exists x \in Z^+((f(1,x) = y) \vee (f(x,1) = y))$, f is onto

Therefore set $Z^+ * Z^+$ is countable

Q14.

There exist $f(x) = x$ is a injective function from $(0,1)$ to $[0,1]$

There also exist $f(x) = \frac{1}{2}x$ is a injective function from $[0,1]$ to $(0,1)$

By Schrö der-Bernstein theorem we have$|(0,1)| = |[0,1]|$

Q15.

If $|A| = |B|$ and $|B| = |C|$

Then there exist an injective function $f : A \to B$ and $g : B \to C$ and $p : B \to A$ and $q : C \to B$

So $h = g \circ f$ is an injective function from $A \to C$ and $r = p \circ q$ is an injective function from $C \to A$

By Schrö der-Bernstein theorem we have $|A| = |C|$

Q16.

$\because f(x)\ is\ \Theta(g(x)), g(x)\ is\ \Theta(h(x))$

There exist $x > k_1, k_1 \in Z^+$, which $C_1|g(x)| \le |f(x)| \le C_2|g(x)|, C_1 \in R, C_2 \in R$

There exist $x > k_2, k_2 \in Z^+$, which $C_3|h(x)| \le |g(x)| \le C_4|h(x)|, C_3 \in R, C_4 \in R$

Therefore there exist$x > max(k_1, k_2)$, $C_1C_3|h(x)| \le |f(x)| \le C_2C_4|h(x)|$

$\therefore f(x)\ is\ \Theta(h(x))$

Q17.

(a) $2n$ multiplications and n additions are used

(b) $\Theta(n)$