CS201: Discrete Math for Computer Science
2022 Spring Semester Quiz

Q. 1. (25 points) Nanshan distinct, Shenzhen, has more than 1,200,000 inhabitants. It was believed that each person has no more than 200,000 hairs on head. Suppose each person has at least one hair on his or her head. [Note: pay attention to the underlined words.]

(a) Fill in the blank: There had to be at least _____ people in Nanshan distinct who have the same number of hairs on their heads.

(b) Explain your answer.

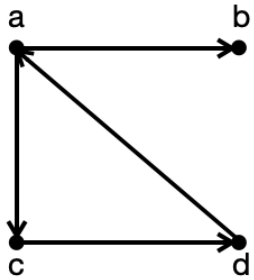
Q. 2. (25 points) Consider a box with infinitely many red, yellow, blue balls. Suppose at least two red balls must be selected. How many ways are there to select 100 balls from the box?

Note: Assume that the order does not matter, and the balls of the same color are indistinguishable. For example, we regard (red, red, blue) and (red, blue, red) as the same.

(a) Fill in the blank: There are _____ ways to select. Please leave the answer as an equation, e.g., $C(1,1)$ instead of 1.

(a) Explain how to derive the above answer using generating function. To derive the answer, please use the useful generating functions (in Lecture note 14 or the Table 1 on page 542 on textbook). Please do not enumerate all possibilities to obtain the answer.

Q. 3. (25 points) Consider relation R represented by the following graph.



(a) Fill in the blanks with "Yes" or "No" to indicate whether relation R satisfies the properties or not, respectively: Reflexive _____; Symmetric _____; Antisymmetric _____; Transitive _____.

(b) Draw the reflexive closure of relation R.

(c) Use Warshall's algorithm to derive binary matrix of the transitive closure of relation R.

Q. 4. (25 points) Consider a relation R defined on the set of functions from $\mathbf{Z}^+$ to $\mathbf{Z}^+$. Consider any of these functions $f : \mathbf{Z}^+$ to $\mathbf{Z}^+$ and $g : \mathbf{Z}^+$ to $\mathbf{Z}^+$, $(f, g) \in R$ if and only if $f(n)$ is $O(g(n))$. Recall the definition of 'Big-Oh' notation:

> Let f and g be functions from the set of integers or the set of real numbers to the set of real numbers. We say that $f(x)$ is $O(g(x))$ if there are constants C and k such that
>
> $$|f(x)| \leq C|g(x)|$$
>
> whenever $x > k$. [This is read as "$f(x)$ is big-oh of $g(x)$."]

(a) Is R reflexive? Explain your answer.

(b) Is R transitive? Explain your answer.

(c) Prove or disprove R is an equivalence relation.

(d) Prove or disprove R is a partial ordering.