Assignment5

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Q1.

Suppose R is an antisymmetric relation, S is a subset of R

Because R is antisymmetric

Thus $(a, b) \in R \wedge (b, a) \in R \rightarrow (a = b)$

Therefore $(a, b) \in R \wedge (a \neq b) \rightarrow (b, a) \notin R$

Because $S \subseteq R$

Therefore $(a, b) \notin R \rightarrow (a, b) \notin S$

Then $(a, b) \in S \wedge (a \neq b) \rightarrow (b, a) \notin S$

This equals $(a, b) \in S \wedge (b, a) \in S \rightarrow (a = b)$

Therefore the subset of antisymmetric relation is also antisymmetric

Q2.

R is symmetric

$R^* = \cup_{k=1}^{\infty} R^k$

If $(a, b) \in R^*$, then there exist $(a, v_1) \in R, (v_1, v_2) \in R, \ \dots\ , (v_k, b) \in R$

Because R is symmetric

Therefore there also exist $(b, v_k) \in R, \ \dots\ , (v_2, v_1) \in R, (v1, a) \in R$

Therefore $(b, a) \in R$

Therefore R^* is also symmetric

Q3.

$R^2 = R \circ R$

For every $(a, b) \in R$

Because R is reflexive, there exist $(b, b) \in R$

Therefore $(a, b) \in R^2$

Therefore $R \subseteq R^2$

Q4.

Yes

Because R is a symmetric relation on a set A

Then $(a, b) \in R \rightarrow (b, a) \in R$

Therefore $(a, b) \notin R \rightarrow (b, a) \notin R$

Thus $(a,b) \in \overline{R} \rightarrow (b,a) \in \overline{R}$

Therefore the $\overline{R}$ is symmetric

Q5.

(a) Yes

Because for $n_i > 0$, $\prod_{k=0}^{i} n_i \geq \sum_{k=0}^{i} n_i$

Therefore $n \preccurlyeq n$, $(n,n) \in R$

Therefore this relation is reflective

(b)No

e.g.

$75 \preccurlyeq 14$ becasuse $3+5 \leq 2*7$

$14 \preccurlyeq 75$ because $2+7 \leq 3*5$

(c)No

e.g.

$26 \preccurlyeq 22$ because $2+13 \leq 2*11$

$22 \preccurlyeq 14$ because $2+11 \leq 2*7$

But $26 \not\preccurlyeq 14$ because $2+13 > 2*7$

Q6.

$$R = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$R^2 = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$R^3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R^* = R \cup R^2 \cup R^3$

Q7.

(1) R is reflective because everyone have the same sign of zodiac with himself $(x,x) \in R$

R is symmetric because x and y have the same sign of the zodiac then y and x have the same sign of the zodiac $(x,y) \in R \wedge (y,x) \in R$

R is transitive because x and y have the same sign of the zodiac and y and z have the same sign of the zodiac then x and z have the same sign of the zodiac $(x,y) \in R \wedge (y,z) \in R \rightarrow (x,z) \in R$

Therefore this is a equivalence relation

(2)R is reflective because everyone was born in the same year with himself $(x,x) \in R$

R is symmetric because x and y were born in the same year then y and x was born in the same year $(x,y) \in R \wedge (y,x) \in R$

R is transitive because x and y were born in the same year and y and z were born in the same year then x and z were born in the same year$(x,y) \in R \wedge (y,z) \in R \rightarrow (x,z) \in R$

Therefore this is a equivalence relation

(3) R is not transitive

e.g. x, y have been in Shanghai, y, z have been in Beijing, but x have not been in Beijing

In this case $(x,y) \in R, (y,z) \in R$ but $(x,z) \notin R$

Therefore R is not equivalence

Q8.

R is reflective $\forall x \in R, x - x = 0 \in Q$

R is symmetric $x - y \in Q \rightarrow (y - x) = -(x - y) \in Q$ because Q is close under negative

R is transitive $x - y \in Q, y - z \in Q$

Thus $(x - y) + (y - z) = x - z \in Q$

Therefore R is transitive

$[1] = k, k \in Q$

$[\frac{1}{2}] = k, k \in Q$

$[\pi] = k + \pi, k \in Q$

Q9.

(a)No

$\propto$ is not symmetric e.g. $n = O(n^2)$ but $n^2 \neq O(n)$

(b)Yes

$\propto$ is reflective $f \leq \frac{1}{2}f$, $f = O(f)$

$\propto$ is antisymmetric

If $f = O(g), f \neq g$, there exist $f \leq Cg$

There doesn't exist $g \leq C`f$

Therefore $\propto$ is antisymmetric

$\propto$ is transitive

If $f \leq C_1 g, g \leq C_2 h$

Then $f \leq C_1 C_2 h$

$f = O(h)$

Therefore $\propto$ is transitive

Thus $\propto$ is partial ordering

(c) Yes

Every function is either $f \propto g$ or $g \propto f$

Therefore $\propto$ is a total ordering

Q10.

$\preccurlyeq$ is reflective

Because $R \subseteq R$ for every $R \in R(S)$

$\preccurlyeq$ is antisymmetric

Because if $R_1 \subseteq R_2, R_1 \neq R_2$, then $R_2 \nsubseteq R_1$

$\preccurlyeq$ is transitive

Because if $R_1 \subseteq R_2, R_2 \subseteq R_3$

Then $R_1 \subseteq R_3$

Therefore $(R(S), \preccurlyeq)$ is a poset

Q11.

(a)There exists a nonempty R ⊆ P(N) with no maximal element.

Assume set $R = \{\{1\}, \{1, 2\}, \ldots, \{1, 2, \ldots k\}\}, n \in N$

$\{1, 2, \ldots, n\}$ is the maximal element in R

Because N is an infinite set

We can always find $\{1, 2, \ldots, n\} \subseteq \{1, 2, \ldots, n+1\}, n \in N$ in R

Which means we can not find a maximal value in R

Therefore there exists a nonempty R ⊆ P(N) with no maximal element

(b)There exists a nonempty R ⊆ P(N) with no minimal element.

Assume set $R = \{\{k, \ldots, n\}, \{k+1, \ldots, n\}, \ldots\}, k \in N, n \in N, k < n$

Because $n \in N, N$ is an ifinite set

We can always find $\{x+1, \ldots, n\} \subseteq \{x, \ldots, n\}$

Which means we can not find a minimal value in R

Therefore there exists a nonempty R ⊆ P(N) with no minimal element

(c)There exists a nonempty T ⊆ P(N) that has neither minimal nor maximal elements.

Set $\{p, \ldots, q\} \in T$ is the minimal element of T, $\{j, \ldots, k\} \in T$ is the maximal element of T and $j < p < q < k$

Because $p, q, j, k \in N$, N is an infinite set

Therefore we can always find $\{p+1, \ldots, q-1\} \subseteq \{p, \ldots, q\}$, $\{j, \ldots, k\} \subseteq \{j-1, \ldots, k+1\}$ as new minimal and maximal element

Thus, there exists a nonempty T ⊆ P(N) that has neither minimal nor maximal elements.

Q12.

(a)n

(b) a, b, c

(c) Yes n

(d) No

(e) l, n

(f) l

(g) There is no lower bound of $\{f, g, h\}$

(h) None