CS201: Discrete Math for Computer Science
2022 Spring Semester   Written Assignment # 5
Due: May 20th, 2022, please submit one pdf file through Sakai
Please answer questions in English. Using any other language will lead to a zero point.

Plagiarism in an Assignment or a Quiz:

- **For the first time: the score of the assignment or quiz will be zero**
- **For the second time: the score of the course will be zero**
- **When two assignments are nearly identical, the policy will apply to BOTH students, unless one confesses having copied without the knowledge of the other.**

Any late submission will lead to a zero point with no exception.

Q. 1. (5 points) Show that a subset of an *antisymmetric* relation is also *antisymmetric*.

Q. 2. (5 points) Suppose that the relation R is symmetric. Show that R^* is symmetric.

Q. 3. (5 points) Let R be a reflexive relation on a set A. Show that $R \subseteq R^2$.

Q. 4. (5 points) Suppose that R is a *symmetric* relation on a set A. Is $\overline{R}$ also symmetric? Explain your answer.

Q. 5. (10 points) For two positive integers, we write $m \preceq n$ if the sum of the (distinct) prime factors of the first is less than or equal to the product of the (distinct) prime factors of the second. For instance $75 \preceq 14$, because $3 + 5 \leq 2 \cdot 7$.

(a) Is this relation reflexive? Explain.

(b) Is this relation antisymmetric? Explain.

(c) Is this relation transitive? Explain.

Q. 6. (10 points) Give an examples of a relation R such that its transitive closure R^* satisfies $R^* = R \cup R^2 \cup R^3$, but $R^* \neq R \cup R^2$.

Q. 7. (10 points) Which of the following are equivalence relations on the set of all people?

(1) $\{(x, y) | x$ and y have the same sign of the zodiac$\}$

(2) $\{(x, y) | x$ and y were born in the smae year$\}$

(3) $\{(x, y) | x$ and y have been in the same city$\}$

Q. 8. (10 points) Show that $\{(x, y) | x - y \in \mathbb{Q}\}$ is an equivalence relation on the set of real numbers, where $\mathbb{Q}$ denotes the set of rational numbers. What are $[1]$, $[\frac{1}{2}]$, and $[\pi]$?

Q. 9. (10 points) Consider a relation $\propto$ on the set of functions from $\mathbb{N}^+$ to $\mathbb{R}$, such that $f \propto g$ if and only if $f = O(g)$.

(a) Is $\propto$ an equivalence relation?

(b) Is $\propto$ a partial ordering?

(c) Is $\propto$ a total ordering?

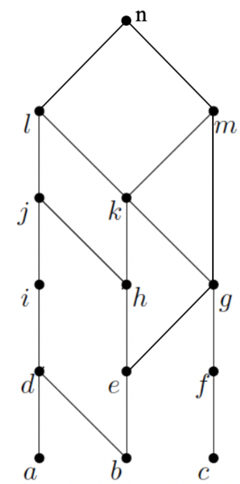
Q. 10. (10 points) Let $\mathbf{R}(S)$ be the set of all relations on a set S. Define the relation $\preceq$ on $\mathbf{R}(S)$ by $R_1 \preceq R_2$ if $R_1 \subseteq R_2$, where R_1 and R_2 are relations on S. Show that $\mathbf{R}(s), \preceq)$ is a poset.

Q. 11. (10 points) We consider partially ordered sets whose elements are sets of natural numbers, and for which the ordering is given by $\subseteq$. For each such partially ordered set, we can ask if it has a minimal or maximal element. For example, the set $\{\{0\}, \{0, 1\}, \{2\}\}$, has minimal elements $\{0\}, \{2\}$, and maximal elements $\{0, 1\}, \{2\}$.

(a) Prove or disprove: there exists a nonempty $R \subseteq \mathcal{P}(\mathbb{N})$ with no maximal element.

(b) Prove or disprove: there exists a nonempty $R \subseteq \mathcal{P}(\mathbb{N})$ with no minimal element.

(c) Prove or disprove: there exists a nonempty $T \subseteq \mathcal{P}(\mathbb{N})$ that has neither minimal nor maximal elements.



Q. 12. (10 points) Answer these questions for the partial order represented by this Hasse diagram.

(a) Find the maximal elements.

(b) Find the minimal elements.

(c) Is there a greatest element?

(d) Is there a least element?

(e) Find all upper bounds of $\{a, b, c\}$.

(f) Find the least upper bound of $\{a, b, c\}$, if it exists.

(g) Find all lower bounds of $\{f, g, h\}$.

(h) Find the greatest lower bound of $\{f, g, h\}$, if it exists.