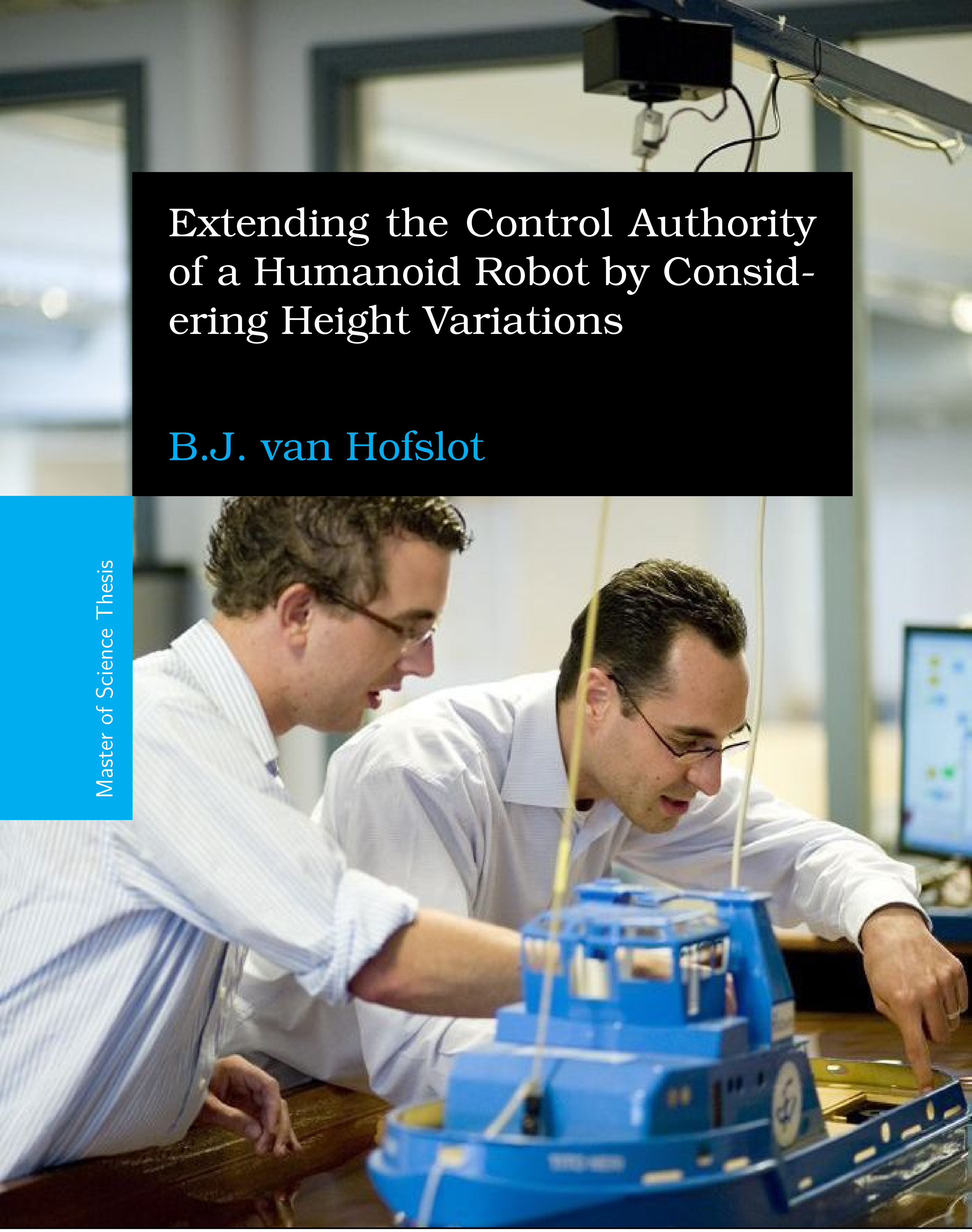
Extending the Control Authority of a Humanoid Robot by Considering Height Variations

B.J. van Hofslot

Master of Science Thesis



TU Delft Delft University of Technology

Cognitive Robotics

Extending the Control Authority of a Humanoid Robot by Considering Height Variations

Master of Science Thesis

For the degree of Master of Science in Systems and Control at Delft University of Technology

B.J. van Hofslot

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Abstract

This is an abstract.

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Preface

According to WIKIPEDIA, a preface (pronounced "*preffus*") is an introduction to a book written by the author of the book. In this preface I can discuss the interesting story of how this thesis came into being.

This is document is a part of my Master of Science graduation thesis. The idea of doing my thesis on this subject came after a discussion with my good friends Tweedledum and Tweedledee…

Acknowledgements

I would like to thank my supervisor prof.dr.ir. M.Y. First Reader for his assistance during the writing of this thesis. . .

By the way, it might make sense to combine the Preface and the Acknowledgements. This is just a matter of taste, of course.

Delft, University of Technology
October 1, 2018

B.J. van Hofslot

“Playing football is very simple, but playing simple football is the hardest thing there is.”

— *Johan Cruyff*

Chapter 1

Introduction

1-1 Motivation

Distinguish different goals of publication: [1]

- Improve behavior over rough-terrain
- Minimize energy consumption or mimic natural behavior
- Analyse the effects of height variation
- Extend control authority by using height variations

1-2 Research Objective

In this thesis is focussed on the last two goals. Exploring the effects of height variation. Extend the control authority by using height variations

1-3 Contributions

1-4 Thesis Outline

Chapter 2

Background

2-1 Modeling of Walking

2-1-1 Inverted Pendulum

2-1-2 Linear Inverted Pendulum Model

In modeling of walking, one of the most important assumptions often made is the modeling of the stance leg as a linear inverted pendulum (LIP), as for example in [2]. Besides this, a not-linearized inverted pendulum is also widely used in the modeling of walking [3]. For planning and control however, a linearized description is desirable. In the two-dimensional space (2D) LIP equations of motion

$$\ddot{x} = \frac{g}{l}x \tag{2-1}$$

where l is the pendulum length and x the Cartesian x-coordinate of the pendulum tip, the motion of the tip along the x-axis does not affect l. At any position x, a local virtual straight pendulum can be considered, so this motion is at a constant height and $l = z_0$ holds. As in three-dimensional space (3D) by the linear model the system dynamics can be decoupled, the dynamics in y-direction read the same: $\ddot{y} = \frac{g}{l}y$. In Figure 2-1 this motion is visualized if the center of mass (CoM) is relatively far from from the base. The pendulum base lies in the origin and $\boldsymbol{x} = [x, y]^T$ is the 2D CoM projection on the horizontal plane. Because the LIP assumption holds, the vertical component of the leg force $\boldsymbol{f}$ has to cancel out gravity acceleration: $f_z = mg$.

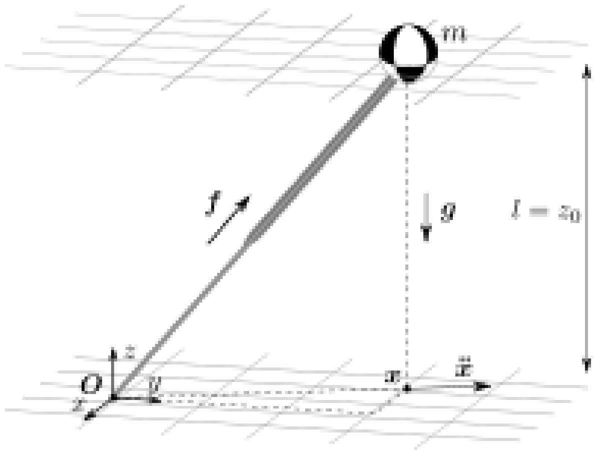

Figure 2-1: 3D motion of LIP model.

2-2 Ground Reference Points

2-2-1 Ground Reaction Forces

2-2-2 The center of pressure (CoP)

The feet attached to the LIP robot model increase the possibilities to control its motion. The ankles can apply a torque that would virtually move the position of the base of the inverted pendulum, so that the linear acceleration on the CoM as in Eq. (2-1) and the capture point as in Eq. (2-7) change. The new virtual base is called the CoP. By its definition, this point only lives within the support polygon [4]. In Figure 2-2 the definition of the CoP is visualized. If the point mass is restricted to move on a constant height, the vertical component of $\boldsymbol{f'}$ counteracts gravity: $f'_z = g$.

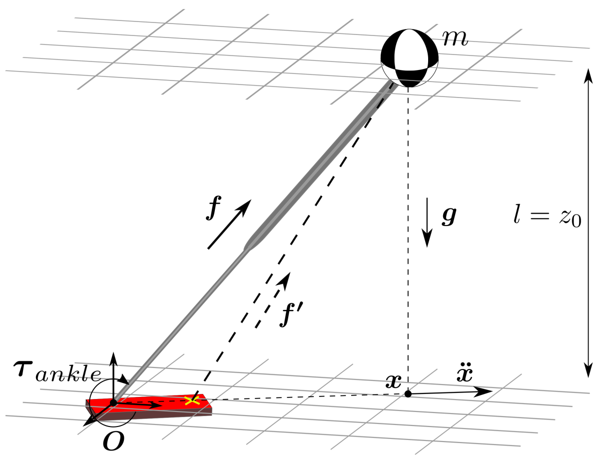

Figure 2-2: 3D motion of LIP model with foot. The yellow cross points out the CoP location.

2-2-3 The zero moment point (ZMP)

The ZMP coincides during stable walking with the CoP, like described in [4]. The two points however are not equal in unstable or more complicated cases, like falling over. The CoP is restricted to be in the support polygon, as this is a point that links to contact forces [5]. The ZMP however is not restricted to lie within the support polygon. The ZMP is the point on the ground where the tipping moment equals zero. The tipping moment is defined as the component of the moment that is tangential to the ground surface. The ZMP initially was introduced in [6].

2-2-4 The centroidal momentum pivot (CMP)

The earlier mentioned points give sufficient measure for a LIP model with point mass and finite-sized feet. However, any angular momentum applied by the body does not affect those points. In the case of the CoP for example, the model assumes the resulting reaction force acts from the CoP through the CoM. The CMP takes angular momentum into account, which can be used as a measure and for control [7]. This is defined as the point where a line passing through the CoM, parallel to the ground reaction force intersects with the ground surface. The CMP is defined as

$$x_{CMP} = x_{ZMP} + \frac{\tau_{y,CoM}}{F_{gr,z}} \tag{2-2}$$

$$y_{CMP} = y_{ZMP} - \frac{\tau_{x,CoM}}{F_{gr,z}} \tag{2-3}$$

where τ_{CoM} is the torque around the CoM, $[x_{ZMP}, y_{ZMP}]$ the ZMP location on the horizontal plane and $F_{gr,z}$ is the ground reaction force in z-direction in Cartesian space. In Figure 2-3 is displayed how body angular momentum affects the ground reaction force $\boldsymbol{f'}$ from the CoP and how the CMP can be determined with the intersection of a parallel line through the CoM and the ground plane. For clarity the point in the image lies on the line from $\boldsymbol{O}$ to $\boldsymbol{x}$. This has not to be the case however, as the body can exert angular momentum along all axes.

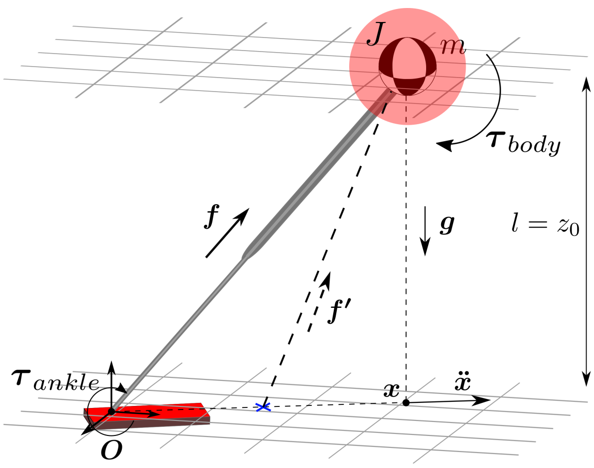

Figure 2-3: 3D motion of LIP model with foot and body inertia. The blue cross points out the CMP location.

2-2-5 Other Points

2-3 Energy of Walking

2-3-1 LIP Orbital Energy

A crucial finding in an extended use of LIP models can be found in [8]. Because force is mass times acceleration: $F = ma$, impuse momentum is force times velocity: $I = Fv$ and the energy or work done by a force is the force times the distance, and thus the impulse integrated over the time interval: $E = Fs = \int Fvdt$, there can be reasoned that if one takes the time integral of the product of the second and the first derivative of a state, an expression for a normalized energy can be achieved: $\frac{E}{m} = \int avdt$. In the mentioned publication that same action is applied on Eq. (2-1):

$$\int (\ddot{x} - \frac{g}{l}x)\dot{x}dt = \frac{1}{2}\dot{x}^2 - \frac{g}{2z_0}x + C = 0 \tag{2-4}$$

with C the integration constant. The LIP Orbital Energy is defined as $E_{LIP} = -C$. If $E_{LIP} > 0$, the point mass will cross the x position of the pendulum base with its current velocity. If $E_{LIP} < 0$, the point mass will not cross the pendulum base and will have a turning point where the velocity becomes zero.

The instantaneous capture point (ICP)

Although the finding of the LIP Orbital Energy was very important for future robot motion modeling, more than a decade later [9] introduced the capture point (CP). Taking $E_{LIP} = 0$ and taking the square root of Eq. (2-4) gives

$$x_{CP} = \sqrt{\frac{z_0}{g}}\dot{x} \tag{2-5}$$

where x_{CP} is the CP, measured from the current pendulum tip position, based on the current tip velocity $\dot{x}$. This is the point where the velocity is exactly driven to zero and the pendulum is upright, where neither crossing of the pendulum base ocurred nor turning of body velocity. In Figure 2-4 a 2D visual explanation is given of this point. Later, the ICP was introduced [10], which gives a slightly different discription of the point:

$$x_{ICP} = x + \sqrt{\frac{z_0}{g}}\dot{x} \tag{2-6}$$

where x_{ICP} is the ICP. In this way, the point can be described in the environment coordinates. The x- and y-coordinate can be decoupled as in the equations of motion of Eq. (2-1). However, in the 2D horizontal plane, convergence to the capture point in one direction does not include convergence to the capture point in the other. In other words: the direction of motion is not restricted to move towards the pendulum base as in the sideview case.

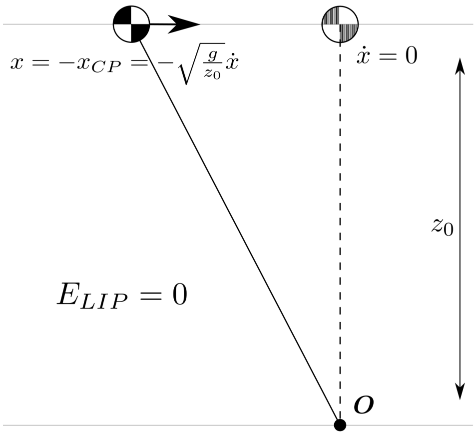

Figure 2-4: Visualization of path and states by the capture of the point mass according ICP theory.

ICP dynamics

Because the ankle is not always located at the same location as the ICP for the current horizontal velocity, for modeling and planning the time derivative is taken of the ICP, which is named the ICP dynamics [10]. This time derivative can be written as a function of the current ICP location:

$$\dot{\boldsymbol{x}}_{ICP} = \sqrt{\frac{g}{z_0}}\boldsymbol{x}_{ICP} \tag{2-7}$$

where $\boldsymbol{x}_{ICP}$ is the xy-vector of the ICP location and assuming that the pendulum base is the origin.

2-3-2 Nonlinear Orbital Energy

E_{orbit}[11]

2-3-3 Boundedness Condition

2-4 CoM Height Variation

2-5 Control Framework IHMC

Chapter 3

Theoretic Limits on Capture

3-1 Unconstrained Capture Region

$$x_{balistic} = \sqrt{2} x_{cp} \tag{3-1}$$

[1]

3-2 Height Constrained Capture

$$x_{cp,height} = (\frac{\sqrt{2g\delta z_{max}}}{g} + \sqrt{\frac{z_o + \delta z_{max}}{g}})(\dot{x}_0 - \frac{x_0}{z_0}\sqrt{2g\delta z_{max}}). \tag{3-2}$$

3-3 Impact Influenced Capture

[3]

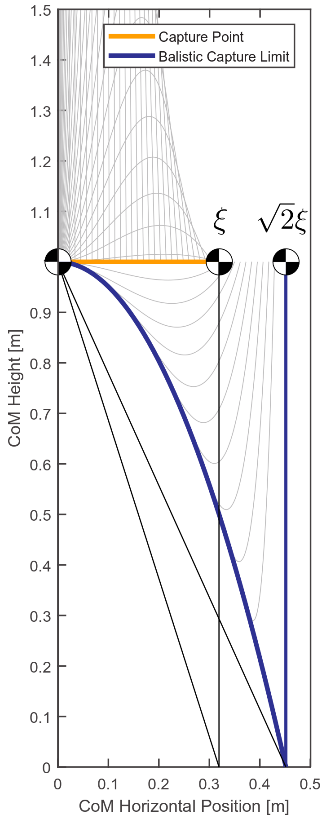
Capture Point
Balistic Capture Limit
ξ
$\sqrt{2}\xi$
CoM Height [m]
CoM Horizontal Position [m]
1.5
1.4
1.3
1.2
1.1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.1
0.2
0.3
0.4
0.5

Figure 3-1

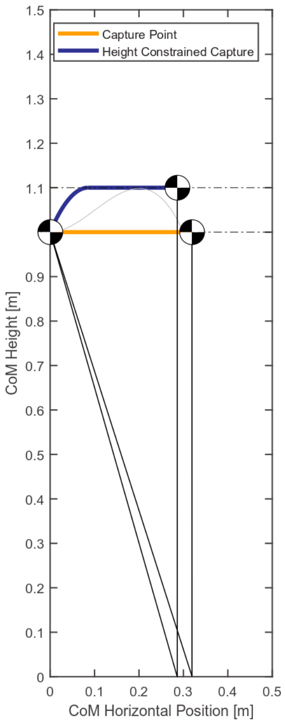
Capture Point
Height Constrained Capture
CoM Height [m]
CoM Horizontal Position [m]
1.5
1.4
1.3
1.2
1.1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.1
0.2
0.3
0.4
0.5

Figure 3-2

Chapter 4

Orbital Energy MPC

4-1 2D Polynomial

$$\frac{1}{2}\dot{x}^2\bar{f}^2(x) + gx^2 f(x) - 3g\int_{x_0}^{x} f(\xi)\xi d\xi = \frac{1}{2}\dot{x}_0^2\bar{f}^2(x_0) + gx_0^2 f(x_0). \tag{4-1}$$

$$u = \frac{g + f''(x)\dot{x}^2}{\bar{f}(x)} \tag{4-2}$$

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & x_0 & x_0^2 & x_0^3 \\ 0 & 1 & 2x_0 & 3x_0^2 \\ \frac{3}{2}gx_0^2 & gx_0^3 & \frac{3}{4}gx_0^4 & \frac{3}{5}gx_0^5 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \end{bmatrix}}_{\boldsymbol{c}} = \underbrace{\begin{bmatrix} z_f \\ z_0 \\ \frac{\dot{z}_0}{\dot{x}_0} \\ k \end{bmatrix}}_{\boldsymbol{b}} \tag{4-3}$$

where $k = \frac{1}{2}(\dot{x}_0 z_0 - \dot{x}_0 x_0)^2 + gx_0^2 z_0 - \frac{1}{2}z_f^2\dot{x}_f^2$.

4-1-1 Height Constraint

4-1-2 Leg Length Constraint

4-2 Challenges 3D Orbital Energy

4-3 Results

4-4 Discussion

Algorithm 1 Find cubic polynomial constants under height constraint

procedure FindPolZ($A^{-1}, \boldsymbol{b}(\dot{x}_f)$)
  $\dot{x}_f \leftarrow 0$ ▷ Initial guess
  repeat
    $\boldsymbol{c} \leftarrow A^{-1}\boldsymbol{b}(\dot{x}_f)$ ▷ Find polynomial constants
    $x_{zmax} \leftarrow max(\frac{-2c_2 \pm \sqrt{4c_2^2 - 12c_3c_1}}{6c_3})$ ▷ Traj. peak lies on highest x
    $z_{max} \leftarrow c_0 + c_1 x_{zmax}^2 + c_2 x_{zmax}^3 + c_3 x_{zmax}^3$ ▷ Corresponding height
    $\dot{x}_f \leftarrow \dot{x}_f + \alpha$ ▷ Some smart increment
  until $z_{max} < z_{const}$
return $\boldsymbol{c}$
end procedure

Algorithm 2 Find cubic polynomial constants under leg length constraint

procedure FindPolL($A^{-1}, \boldsymbol{b}(\dot{x}_f)$)
  $\dot{x}_f \leftarrow 0$ ▷ Initial guess
  repeat
    $\boldsymbol{c} \leftarrow A^{-1}\boldsymbol{b}(\dot{x}_f)$ ▷ Find polynomial constants
    $x_{l^2max,1} \leftarrow \frac{-4c_2^2 + \sqrt{16c_2^4 - 24c_3^2(2+2c_1^2)}}{12c_3^2}$ ▷ $d(f(x)^2 + x^2)/dx = 0$
    $x_{l^2max,2} \leftarrow \frac{-4c_2^2 - \sqrt{16c_2^4 - 24c_3^2(2+2c_1^2)}}{12c_3^2}$ ▷ Complex solutions
    $x_{lmax} \leftarrow -|\sqrt{max(x_{l^2max,1}, x_{l^2max,2)}}|$
    $l_{max}^2 \leftarrow x_{lmax}^2 + (c_0 + c_1 x_{zmax}^2 + c_2 x_{zmax}^3 + c_3 x_{zmax}^3)^2$
    $\dot{x}_f \leftarrow \dot{x}_f + \alpha$ ▷ Some smart increment
  until $l_{max}^2 < l_{const}^2$
return $\boldsymbol{c}$
end procedure

Chapter 5

Towards Application

5-1 Challenges

5-1-1 Angular Momentum and Height Variation

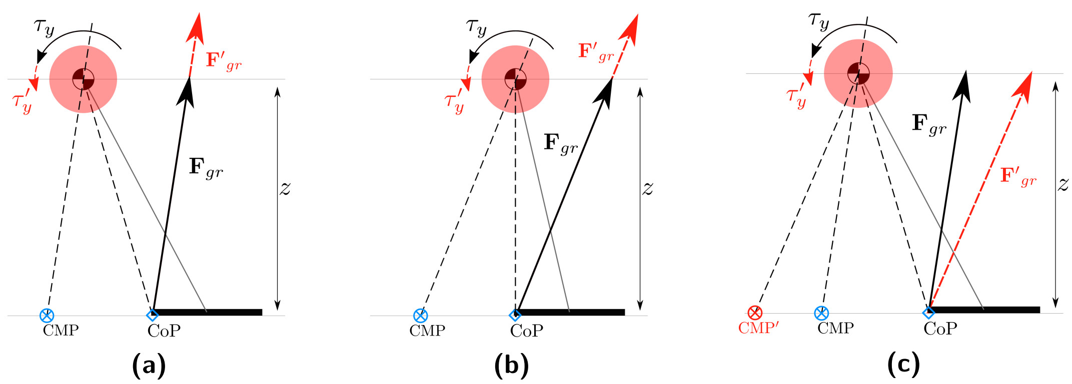

Figure 5-1

5-1-2 From 2D to 3D

5-1-3 Predictability of Dynamics

5-1-4 Singularity Avoidance

5-2 Methods

5-2-1 Adjust Quadratic Program Structure

5-2-2 Adjust Quadratic Program Inputs

$$\mathrm{i} = \frac{\mathbf{c}_{xy} - \mathbf{r}_{cmp,d}}{z} F_z \tag{5-1}$$

$$\mathrm{i} = \frac{\mathbf{c}_{xy} - \mathbf{r}_{cop,d}}{z} m(g + \ddot{z}_d) + \frac{\tau_y}{z} \tag{5-2}$$

$$\mathrm{i} = \underbrace{\frac{\mathbf{c}_{xy} - \mathbf{r}_{cmp,d}}{z} mg}_{\mathrm{i}_{d,lip}} + \underbrace{\frac{\mathbf{c}_{xy} - \mathbf{r}_{cop,d}}{z} m\ddot{z}_d}_{\mathrm{i}_{d,heightcontrol}} \tag{5-3}$$

5-3 Experimental Setup

5-4 Results

5-4-1 Simulation

5-4-2 Hardware

5-5 Discussion

Chapter 6

Conclusion

6-1 Recommendations

Appendix A

Yet Another Appendix

A-1 Test Section (Again?)

Ok, all is well.

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Glossary

List of Acronyms

ICP instantaneous capture point

CP capture point

ZMP zero moment point

CoP center of pressure

CoM center of mass

CMP centroidal momentum pivot

LIP linear inverted pendulum

2D two-dimensional space

3D three-dimensional space

List of Symbols

E_{LIP} LIP orbital energy

E_{orbit} Nonlinear orbital energy