Measuring Baryon Acoustic Oscillations and Cosmological Parameters via Alcock–Paczynski Test

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April 24, 2025

1 Introduction

Baryon Acoustic Oscillations (BAO) imprint a characteristic scale in the late-time matter distribution, set by the comoving sound horizon at recombination. As a standard ruler, the BAO feature is a powerful probe of the cosmic expansion history. In this project we take a simulated linear matter power spectrum, generated with the `CLASS` Boltzmann code, and carry out two tasks:

1. Extract the BAO scale r_{peak} by isolating the oscillatory component of $P(k)$, Fourier-transforming to the two-point correlation function $\xi(r)$, and locating its peak.
2. Perform an Alcock–Paczynski (AP) test on that scale to infer cosmological parameters (Ω_m, Ω_k, w) via Markov-Chain Monte Carlo.

Since repeated issues with the computer simulating the power spectra meant that only the one-dimensional $P(k)$ could be successfully simulated, the implementation here uses the isotropic parameter D_V/r_d. Deprecated code which implements this pipeline for a two-dimensional power spectrum, theoretically yielding significantly tighter constraints, can be found on the `GitHub` for this project.

2 Methodology

2.1 Power spectrum and correlation function

Using `CLASS`, we begin by computing the matter power spectrum $P(k)$ at $z = 0$ under the fiducial cosmology $\{\omega_b, \omega_c, h, A_s, n_s, \tau\} = \{0.02238, 0.1160, 0.68, 2.1 \times 10^{-9}, 0.97, 0.054\}$. As detailed in [1], the high-frequency oscillations caused by the BAO can be isolated by fiting a cubic spline (done here with `LSQUnivariateSpline`) to the power spectrum and plotting the difference.

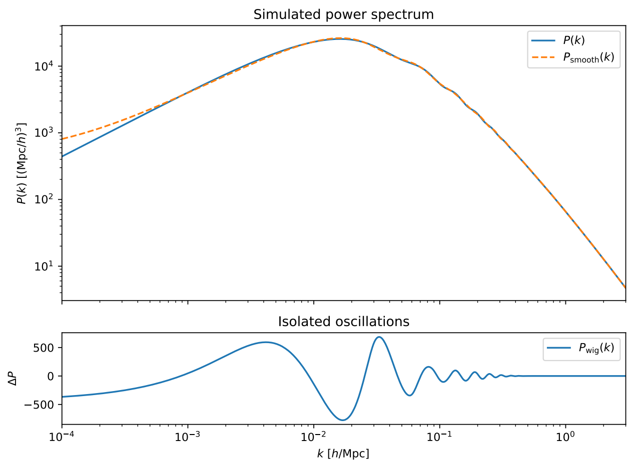

Figure 1: Top: simulated matter power spectrum $P(k)$ (solid) and its smooth fit $P_{\mathrm{smooth}}(k)$ (dashed). Bottom: isolated oscillatory component $P(k) - P_{\mathrm{smooth}}(k)$.

In theory, the correlation function is then the Fourier transform

$$\xi(r) = \frac{1}{2\pi^2} \int_0^\infty dk\, k^2\, P(k)\, \frac{\sin(kr)}{kr};$$

however, because only a finite portion of the power spectrum can be simulated, this invariably leads to spurious oscillations near the edges of the domain as the matter power spectrum drops instantaneously to zero. To suppress this, we multiply the integrand by a 'window', a double-sided tanh function, as done in [2]. The characteristic scale is then

$$s = \arg\max_r \xi_{\mathrm{wig}}(r)\,,$$

which we measured to be $s \sim 106.2h^{-1}\,\mathrm{Mpc}$. Figures 1–2 illustrate $P(k)$, $P_{\mathrm{wig}}(k)$, and the resulting $\xi(r)$.

2.2 Alcock–Paczynski test

When an astronomical object is observed, we measure its angular size $\theta = \frac{L}{(1+z)D_A(z)}$, where D_A is the angular diameter distance, and its size in redshift space, $\Delta z = L\,H(z)$. Since the BAO are, theoretically, isotropic, the radial and the tangential L should be the same, meaning that the Alcock-Paczynski parameter

$$F_{\mathrm{AP}} \equiv \frac{\Delta z}{\theta} = (1+z)D_A(z)H(z)$$

should depend only on the redshift and on the underlying cosmology. If the theoretical value for the AP parameter based on the fiducial cosmology, $(1+$

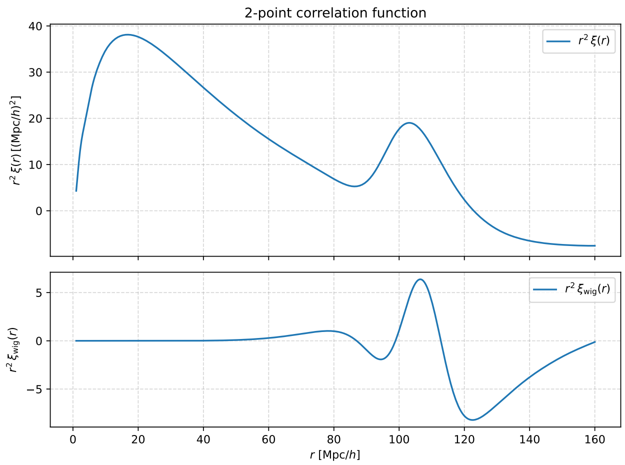

Figure 2: Top: 2-point correlation $\xi(r)$ scaled by r^2; bottom: Isolated 2-point correlation function due to BAO.

$z)D_A(z)H(z)$, matches with the observed value $\frac{F_{\rm AP}}{\theta}$, it indicates that the BAO scale has been fitted properly.

Given the measured $r_{\rm peak}$, we construct the isotropic AP parameter

$$F_{\rm meas}(z) = \frac{D_V(z)}{r_d}, \qquad D_V(z) \equiv \left[(1+z)^2 D_A^2(z)\,\tfrac{cz}{H(z)}\right]^{1/3},$$

using our extraction of $r_{\rm peak}$ at a set of redshift bins $z_i \in [0.7, 1.8]$. The theoretical counterpart is

$$F_{\rm th}(z;\Omega_m,\Omega_k,w) = (1+z)\,D_A(z;\Omega_i)\,H(z;\Omega_i)\,r_d^{\rm fid},$$

where D_A and H are numerically integrated for arbitrary (Ω_m, Ω_k, w).

To calculate this numerically, we sample the posterior

$$\mathcal{P}(\Omega_m,\Omega_k,w) \propto \exp\Big[-\tfrac{1}{2}\sum_i \frac{(F_{\rm meas}(z_i) - F_{\rm th}(z_i;\Omega_m,\Omega_k,w))^2}{\sigma_i^2}\Big] \times \Pi(\Omega_i),$$

using the flat priors $\Pi:\ \ 0 < \Omega_m < 1,\ -0.5 < \Omega_k < 0.5,\ -2 < w < 0$. We ran four chains of 10^6 steps each with a 10% burn in, and computed the Gelman-Rubin R statistic to be < 1.005 for all three parameters, indicating a good fit.

3 Results

Our MCMC returns

$$\Omega_m = 0.259 \pm 0.16, \quad \Omega_k = 0.072 \pm 0.30, \quad w = -1.095 \pm 0.24.$$

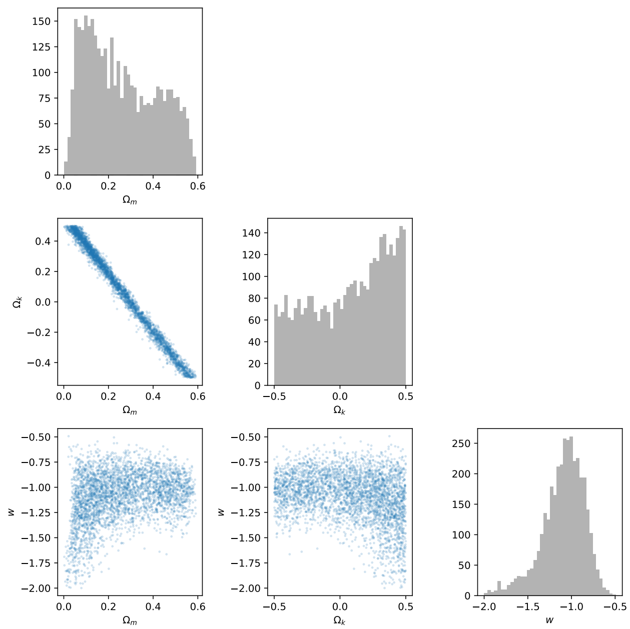

Figure 3: Corner plot of posterior samples for (Ω_m, Ω_k, w).

While the parameters are accurate, the high error results likely from their degenerate dependence on $H(z)$, which prevents all three parameters from being constained strongly; running an anisotropic AP test would likely improve the strength of the test significantly.

4 Further work

The intention behind this project was to perform an anisotropic AP test and include correction terms for redshift space distortions, but repeated issues with the computer simulating the power spectra meant that only the one-dimensional $P(k)$ could be simulated; therefore, the most obvious avenue for further work is to successfully simulate the two-dimensional power spectrum and account for redshift space distortions.

Another option is to run this pipeline on real survey data such as the recent DESI data, in effect replicating their analysis. Finally, by simulating power spectra under many different cosmologies and repeatedly testing whether the theoretical AP parameter matched the one computed from the BAO scale extraction, we could gain valuable insight into the effectiveness of the BAO fitting procedure.

5 References

1. S.-F. Chen *et. al. Baryon Acoustic Oscillation Theory and Modelling Systematics for the DESI 2024 results.* MNRAS 000, 1–30 (2024). https://arxiv.org/abs/2402.14070.

2. B. Bassett, R. Hlozek. *Baryon Acoustic Oscillations.* Dark Energy, Ed. P. Ruiz-Lapuente (2010). https://arxiv.org/abs/0910.5224v1.

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