Mathematical aspect of the combinatorial game "Mahjong"

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Abstract

We study some mathematical aspects of the Mahjong game. In particular, we use combinatorial theory and write a Python program to study some special features of the game. The results confirm some folklore concerning the game, and expose some unexpected results. Related results and possible future research in connection to artificial intelligence are mentioned.

AMS Classifications: 05A15, 05A05, 60C05.

Keywords. Combinatorial theory, probability, nine gate.

1 Introduction

Mahjong is a popular recreational game which originated in China a long time ago [1]. Nowadays, it is widely played in different countries, including the United States. It is a game of skill, strategy, calculation, and some luck. Some researchers suggested that Mahjong is a good mind game with positive impact for patients with Alzhemier's disease; see [2].

The purpose of this article is to explore some mathematical aspects of the Mahjong game. In particular, we apply elementary combinatorial theory and write a Python program to study some special hands of the game. Our study leads to affirmative answers on some folklore concerning the game, and some unexpected results. Our study is the first attempt to study the Mahjong game using mathematical and computational techniques. We will conclude the paper with some possible future research in connection to artificial intelligence, and indicate the difference between the Mahjong game, and other recreational games such as chess and Go game.

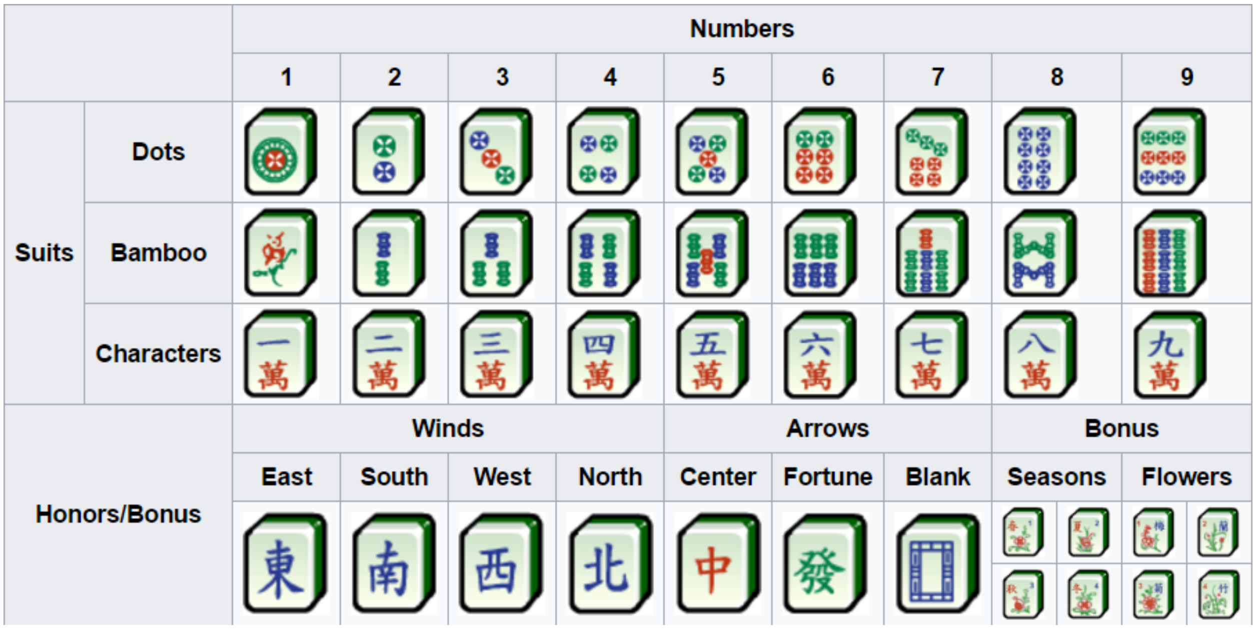
2 Basic rules and mathematics questions of Mahjong game

The rules of Mahjong are not hard to understand.[2] There are 144 tiles in total, consisting of 36 tiles of bamboo type, 36 tiles of dot type, 36 tiles of character type, and some special tiles. These

[1]There are other theories of the origin of Mahjong. Many believed it was introduced 150 years ago but some say it was invented by Confucius 2500 years ago. [6, 8]

[2]There are different variations of the game [7]. Here, we describe the basic version.

tiles are shown in the picture below; there are 4 different flowers and 4 different seasons, and 4 copies of each of the other tiles.



When the game starts, each of the four players draws 13 tiles as a starting hand. Then, each player draws and then discards one tile in turns until one player forms a winning hand by using 13 tiles on hand and a newly drawn tile or a newly discarded tile of another player. A standard winning hand consists of an identical pair, and four sets of pungs or chows, where a pung is three identical tiles and a chow is three consecutive tiles from the same suit of dots, bamboo, or characters. The following picture shows two examples of a winning hand.

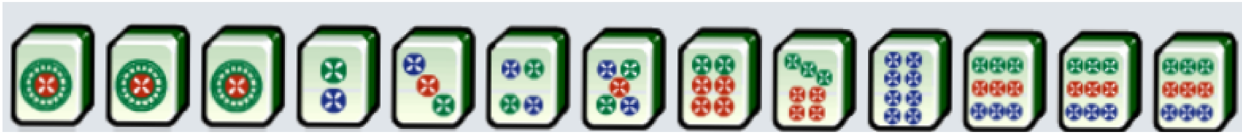
Example 1

Example 2

The flowers and seasons tiles can add additional points to the winning score. Whenever one draws a season or flower tile, one puts it face up and draw another tile. Different winning hands will determine different winning scores.[3] The score of the winner depends on how many seasons and flowers the player has and the rareness of the winning hand. For more details, one may see [7].

A motivation of our study is the following hand:

[3]People may gamble using the winning score.



We will express this special hand as $X_1X_1X_1X_2X_3X_4X_5X_6X_7X_8X_9X_9X_9$ for notational convenience. This hand is special because any additional dot tile would lead to a winning hand. For example, if we draw a one dot tile, then we get a winning hand:

$$X_1X_1X_1 \quad X_1X_2X_3 \quad X_4X_5X_6 \quad X_7X_8X_9 \quad X_9X_9;$$

if we draw a dot 2, then we get a wining hand:

$$X_1X_1X_1 \quad X_2X_2 \quad X_3X_4X_5 \quad X_6X_7X_8 \quad X_9X_9X_9.$$

One can check that each of the nine dot tiles can make this hand a winning hand. This hand is called the "Nine Gates".

It is believed that the "Nine Gates" is unique. In other words, if one has a hand of 13 tiles of dots, this is the only hand of dot tiles that yields a winning hand for any addition of a dot tile. However, there is no known mathematical proof of this folklore.

In connection to this, one can also get an "Eight Gates" hand of dot tiles a winning hand can be formed using eight of the nine dot tiles. It is unclear how many such hands are possible, and which is the exceptional dot tile which cannot form a winning hand when added to the "Eight Gates" hand. For instance, it is believed that there is no "Eight Gates" hand so that one can win with any dot tile but the 5 dot tile.

Of course, one can ask similar questions for "Seven Gates", "Six Gates", etc.

In the next two sections, we will use combinatorial theory and develop a Python program to answer these questions and explore related results. In Section 5, we describe the computational results and their implications; related problems and further research will be discussed in Section 6.

One can download the Python program from http://cklixx.people.wm.edu/mathlib/Mahjong.py. The computational results is contained in http://cklixx.people.wm.edu/mathlib/Mahjoing-results.txt.

3 Mathematical Analysis

In this section, we focus on Mahjong hands of 13 tiles chosen from the 36 dot tiles to study the questions of "Nine Gates", "Eight Gates", etc. We will continue to use the notation

$$X_1, \ldots, X_9$$

to represent the 1-dot, ..., 9-dot tiles each with 4 copies, and denote a hand by a "product" of 13 terms such as

$$X_1X_1X_1X_2X_3X_4X_5X_6X_7X_8X_9X_9X_9, \text{ which may further simplify to } X_1^3X_2X_3X_4X_5X_7X_8X_9^3.$$

We have the following facts about these 36 tiles based on basic combinatorial theory; for example, see [1].

Proposition 3.1. *Consider 36 dot tiles with 4 copies of $X_1, \ldots, X_9$. Suppose a 13-dot hand is represented as $X_1^{n_1} \cdots X_9^{n_9}$ and associated with a sequence $(n_1, \ldots, n_9)$ with $0 \leq n_j \leq 4$ for all j such that $n_1 + \cdots + n_9 = 13$.*

(a) *The number of ways to choose 13 random tiles from 36 tiles (allowing repeated patterns):*

$$\binom{36}{13} = 2310789600.$$

(b) *The number of ways of getting a certain 13 dot tiles hand with n_j copies of j-dot tiles for $j = 1, \ldots, 9$, so that $n_1, \ldots, n_9$ are integers in $\{0, 1, 2, 3, 4\}$ adding up to 13:*

$$\binom{4}{n_1} \cdots \binom{4}{n_9}.$$

(c) *The probability of getting a 13 dot tiles hand with n_j copies of j-dot tiles:*

$$\frac{\binom{4}{n_1} \cdots \binom{4}{n_9}}{\binom{36}{13}}.$$

So, the probability of getting the hand of nine gates $X_1X_1X_1X_2X_3X_4X_5X_6X_7X_8X_9X_9X_9$ out of the 36 dot tiles is

$$\frac{262144}{2310789600} = 0.00011344347.$$

(d) *All possible selections of m tiles out of the 36 tiles for $m = 0, \ldots, 36$, correspond to the degree m terms in the expansion:*

$$\begin{aligned}&(1 + X_1 + X_1^2 + X_1^3 + X_1^4)(1 + X_2 + X_2^2 + X_2^3 + X_2^4) \cdots (1 + X_9 + X_9^2 + X_9^3 + X_9^4)\\ &= \sum_{0 \leq n_1 + \cdots + n_9 \leq 36} X_1^{n_1} X_2^{n_2} \ldots X_9^{n_9}\end{aligned}$$

In particular, the term with lowest degree $1 = X_1^0 \cdots X_9^0$ corresponds to the selection of none of the tiles $X_1 \ldots X_9$, and the term with highest degree $X_1^4 \cdots X_9^4$ corresponds to the selection of all the 36 tiles.

(e) *The number of different 13-dot hands equals to the total number of summands $X_1^{n_1} X_2^{n_2} \cdots X_9^{n_9}$ with $n_1 + n_2 + ... + n_9 = 13$ in the expansion in* (d), *and equal to coefficient of X^{13} in the expansion:*

$$(1 + X + X^2 + X^3 + X^4)^9 = \sum_{i=0}^{36} \alpha_i X^i.$$

We have $\alpha_{13} = 93600$.

(f) *There is a 13-dot hand $X_1^{n_1} X_2^{n_2} \cdots X_9^{n_9}$ if and only if there is a 13-dot hand $X_9^{n_1} X_8^{n_2} \cdots X_1^{n_9}$. Consequently, the adding X_j forms a winning hand with the former 13-dot hand if and only if adding X_{10-j} will form a winning hand with the latter 13-dot hand.*

The properties (a) – (e) allow us to set up the Python program to determine the hands of nine gates, eight gates, etc. and compute their probability. Property (f) is of theoretical interest that every single hand of 13 tiles has a dual hand if we replace X_k by X_{10-k}. If the original hand can win with an ℓ-dot tile, then its dual hand can win with $(10-\ell)$-dot tile. So two hands that are dual to each other can win by same number of tiles. For example, $X_1X_1X_1X_1X_2X_2X_2X_2X_2X_3X_3X_3X_4X_5$ and $X_5X_6X_7X_7X_7X_8X_8X_8X_8X_9X_9X_9X_9$ are dual to each other. Adding X_3, X_4 or X_6 to the first hand will yield a winning hand. Accordingly, adding X_7, X_6 or X_4 to the second hand will yield a winning hand. Evidently, the dual hand of the "Nine Gates" is itself.

4 Programming

Based on the mathematical results in the previous section, we write a Python program to study different hands of 13 dot tiles. The program is available at

http://cklixx.people.wm.edu/mathlib/Mahjong.py.

One can also see the listing of the results at

http://cklixx.people.wm.edu/mathlib/Mahjong-results.txt.

The basic idea of this program is to generate all 93600 of such hands calculated in Proposition 3.1 (e), and test each of them to see how many different tiles would complete a winning hand. It is worth pointing that in our Algorithm 1, we associate each hand of 13 dot tiles $X_1^{n_1}\cdots X_9^{n_9}$ with the sequence $(n_1,\ldots,n_9)$ such that $0\le n_j\le 4$ for every j and $n_1+\cdots+n_9=13$. This allows us to modify the program easily to check what are needed to form a winning pattern for a reduced hand after some "pungs" or "chows" were performed in a game.

For each 13-tile hand, we add a new tile from 1-9 to it, to create a 14-tile hand. To determine if this is a winning hand, we must identify a pair and four sets of pungs and chows. For each of the tiles that appeared at least twice in the hand, we take two of them out as the pair. If the remaining 12-tile hand $\{j_1,\ldots,j_{12}\}$ can be divided into four sets of pungs and chows, then this is a winning hand.

Claim *If $j_1=j_2=j_3$, we may always assume that they form a pung and check whether the remaining pieces $\{j_4,\ldots,j_{12}\}$ form three sets of pungs and chows.*

To see that, assume that $j_1=j_2=j_3$, and we can divide $\{j_1,\ldots,j_{12}\}$ into three sets of pungs and chows without using $\{j_1,j_2,j_3\}$ as a pung. Then j_1,j_2,j_3 will be in three sets of chows of the form $\{j_1,j_1+1,j_2+2\}$ plus one other set of pung or chow. But then then we can rearrange the twelve tiles as three sets of chows into three set of pungs $\{j_1,j_1,j_1\},\{j_1+1,j_1+1,j_1+1\},\{j_1+2,j_1+2,j_1+2\}$ together with the remaining set of pung or chow. Thus, our claim is proved. $\square$

Clearly, after removing $\{j_1,j_2,j_3\}$ from $\{j_1,\ldots,j_{12}\}$, if the three smallest number in the remaining set are the same, we can again assume that they form a pung. Else, we will extract a set of chow and proceed in a similar manner. We will use this idea in Algorithm 3 below.

Below is the pseudocode of our algorithms:

Algorithm 1: Pseudocode for generating every possible hand.
Goal: Form every possible hand of 13 tiles.

Output: All possible 13-tile hands

```
hand ← array of size 9                          ▷ hand[i] represents number of i-dot tiles, 0-4
for each hand where hand[i] from 0-4 do
    if sum(hand) = 13 then
        add hand to allPossibleHands
    end if
end for
return allPossibleHands
```

Algorithm 2: Pseudocode for checking for number of gates.

Goal: Check whether a 13-tile hand can form a winning pattern with one more tile.

Input: A 9-element list called *hand*, representing 13 tiles.

Output: Which tiles does *hand* need to win.

```
isWinningTile ← array of size 9 initialized to false
for i = 1, . . . , 9 do
    if hand[i] ≠ 4 then
        hand[i] ← hand[i] + 1                   ▷ Add an i-tile to get a 14-tile hand
        for j = 1, . . . , 9, if hand[j] ≥ 2, do
            hand[j] ← hand[j] − 2               ▷ first select j as a pair
            if checkFourSets(hand) then         ▷ check if remaining 12 tiles has 4 sets
                isWinningTile[i] ← True
            end if
        end for
    end if
end for
return isWinningTile
```

Algorithm 3: Pseudocode for checking four sets of pungs or chows

Goal: Check whether a 12-tile hand consists of four sets of 3-tile.

Input: A 9-element list called *hand*, representing 12 tiles.

Output: Whether the hand forms four sets of pungs or chows

```
setsFound = 0
for i = 1, . . . , 9 do
    if hand[i] ≥ 3 then                         ▷ Check for pung
        hand[i] ← hand[i] − 3
        setsFound = setsFound + 1
    end if
    if i + 2 < len(hand) then                   ▷ Check for chow
        minThree = min(hand[i], hand[i + 1], hand[i + 2])
        hand[i] = hand[i] − minThree
        hand[i + 1] = hand[i + 1] − minThree
        hand[i + 2] = hand[i + 2] − minThree
        setsFound = setsFound + minThree
    end if
end for
if setsFound = 4 then
```

```
    return True
else
    return False
end if
```

5 Computational Results and Implications

The program gives us the following results.

Proposition 5.1. *Consider the 93600 different 13 dot hands.*

(a) *The "Nine Gates"* $X_1X_1X_1X_2X_3X_4X_5X_6X_7X_8X_9X_9X_9$ *is the unique hand winning all 9 pieces with the probability of 0.0113% for a 13 dot hands as shown in Proposition 3.1 (c).*

(b) *There are 16 hands winning 8 pieces with a combined probability 0.0100% of drawing.*
$X_3X_3X_3X_4X_5X_6X_7X_8X_8X_8X_9X_9X_9$ [winning except for the 1 dot tile]
$X_3X_3X_3X_4X_5X_5X_6X_6X_7X_7X_8X_8X_8$ [winning except for the 1 dot tile]
$X_3X_3X_3X_4X_4X_5X_5X_6X_6X_7X_8X_8X_8$ [winning except for the 1 dot tile]
$X_2X_3X_4X_4X_4X_4X_5X_6X_7X_8X_9X_9X_9$ [winning except for the 4 dot tile]
$X_2X_3X_3X_3X_3X_4X_4X_5X_6X_7X_8X_8X_8$ [winning except for the 3 dot tile]
$X_2X_2X_2X_3X_4X_5X_6X_7X_7X_7X_9X_9X_9$ [winning except for the 9 dot tile]
$X_2X_2X_2X_3X_4X_5X_6X_7X_7X_7X_8X_8X_8$ [winning except for the 9 dot tile]
$X_2X_2X_2X_3X_4X_5X_6X_7X_7X_7X_7X_8X_9$ [winning except for the 7 dot tile]
$X_2X_2X_2X_3X_4X_5X_6X_6X_7X_7X_7X_7X_8$ [winning except for the 7 dot tile]
$X_2X_2X_2X_3X_4X_4X_5X_5X_6X_6X_7X_7X_7$ [winning except for the 9 dot tile]
$X_2X_2X_2X_3X_3X_4X_4X_5X_5X_6X_7X_7X_7$ [winning except for the 9 dot tile]
$X_2X_2X_2X_3X_3X_3X_4X_5X_6X_7X_8X_8X_8$ [winning except for the 1 dot tile]
$X_1X_2X_3X_3X_3X_3X_4X_5X_6X_7X_8X_8X_8$ [winning except for the 3 dot tile]
$X_1X_1X_1X_3X_3X_3X_4X_5X_6X_7X_8X_8X_8$ [winning except for the 1 dot tile]
$X_1X_1X_1X_2X_3X_4X_5X_6X_6X_6X_6X_7X_8$ [winning except for the 6 dot tile]
$X_1X_1X_1X_2X_2X_2X_3X_4X_5X_6X_7X_7X_7$ [winning except for the 9 dot tile]

(c) *There are 79 hands of "Seven Gates" with a combined probability 0.0942% of drawing.*

(d) *There are 392 hands of "Six Gates" with a combined probability 0.5408% of drawing.*

- *There are 1335 hands of "Five Gates" with a combined probability 1.4215% of drawing.*
- *There are 2948 hands of "Four Gates" with a combined probability 2.9812% of drawing.*
- *There are 6739 hands of "Three Gates" with a combined probability 9.7559% of drawing.*
- *There are 14493 hands of "Two Gates" with a combined probability 17.8968% of drawing.*
- *There are 14067 hands of "One Gate" with a combined probability 14.8473% of drawing.*
- *There are 53530 hands which cannot win with any additional piece, with a combined probability 52.4409% chance of drawing.*

Several remarks are in order in connection to Proposition 4.1.

1. It is confirmed in (a) that the "Nine Gates" is the unique hand of 13 dots that can form a winning hand with the addition of any dot tile, and the probability of getting such a hand is

$$4^9/\binom{36}{13} = 0.0113\%.$$

2. In (b), one can compute the numbers of ways to get each of the hand of 13 dot tiles using Proposition 3.1 (b).

   For example, the number of ways to get $X_3X_3X_3X_4X_5X_6X_7X_8X_8X_8X_9X_9X_9$ equals

$$\binom{4}{0}^2\binom{4}{3}\binom{4}{1}^4\binom{4}{3}^2 = 4^7,$$

   and the number of ways to get $X_3X_3X_3X_4X_5X_5X_6X_6X_7X_7X_8X_8X_8$ equals

$$\binom{4}{0}^2\binom{4}{3}\binom{4}{1}\binom{4}{2}^3\binom{4}{1}\binom{4}{0} = 4^36^3.$$

   Adding the number of ways to get the 16 "Eight Gates" hands, we have

   $4^7+4^36^3+4^36^3+4^7+4^56+4^7+4^7+4^7+4^56+4^36^3+4^36^3+4^7+4^7+4^7+4^7+4^7 = 231424.$

   Dividing the sum by $\binom{36}{13}$, we see that the probability of 0.0100% of drawing these hands.

3. It is somewhat interesting (and counter intuitive) that if one draws 13 tiles out of the 36 dot tiles, there is a higher probability of getting the "Nine Gates" (0.0113%) than that of getting one of the "Eight Gates" (0.0100%).

4. Note that the 8 of the 16 hands in (b) are dual to the other 8 hands as described in Proposition 3.1 (f).

5. It is also interesting to note that the only way to get an "Eight Gates" hand so that the 4-dot tile cannot be added to form a winning hand is: $X_2X_3X_4X_4X_4X_4X_5X_6X_7X_8X_9X_9X_9$, where all the 4-dot tiles are used.

6. Similar comment applies to the hand where the k-dot tile cannot be added to form a winning hand for $k = 3, 6, 7$. In particular, when $k = 3$ there are two such "Eight Gates" hand. The same is true for $k = 7$.

7. There are three tiles that every "Eight Gates" can win with. They are $2, 5, 8$.

8. By (b), we see that there is no "Eight Gates" hand that win every dot tiles except the k-dot for $k = 2, 5$ or 8.

9. There are too many hands corresponding to "Seven Gates", "Six Gates", "Five Gates", etc. We do not list them in the proposition. Nevertheless, in the results output from our program in the Appendix, we put the statistics of the number of hands corresponding to each of the n-Gates and indicate the tiles whose addition will form winning hands. One can run the Python program available at http://cklixx.people.wm.edu/mathlib/Mahjong.py. to see all the possible outcomes as shown in http://cklixx.people.wm.edu/mathlib/Mahjong-results.txt.

10. For the five gates hand, there are 8 hands such that $1, 5, 9$ cannot be the winning pieces, namely,

$$X_3X_4X_5X_5X_5X_5X_6X_7X_7X_7X_9X_9X_9, \quad X_3X_4X_5X_5X_5X_5X_6X_7X_7X_7X_8X_8X_8,$$

$$X_3X_3X_3X_4X_5X_5X_5X_5X_6X_7X_7X_8X_9, \quad X_3X_3X_3X_4X_5X_5X_5X_5X_6X_7X_9X_9X_9,$$

$$X_2X_2X_2X_3X_3X_3X_4X_5X_5X_5X_5X_6X_7, \quad X_1X_2X_3X_3X_4X_5X_5X_5X_5X_6X_7X_7X_7,$$

$$X_1X_1X_1X_3X_4X_5X_5X_5X_5X_6X_7X_7X_7, \quad X_1X_1X_1X_3X_3X_3X_4X_5X_5X_5X_5X_6X_7.$$

In each case, X_5 appears four times so that X_5 cannot be the additional tile to form a winning hand.

11. In the results output from our program in the Appendix, we also list the statistics for "Four Gates" and "Three Gates". Here we list the tiles whose addition will lead to a winning hand.

12. There are many known "Three Gates" hands. For every triple (i, j, k) with $1 \leq i < j < k \leq 9$, one may ask whether there is a hand of 13 dot tiles such that one can get an winning hand by adding a tile from $\{X_i, X_j, X_k\}$. Our results show that out of the $\binom{9}{3} = 84$ possible choices of $\{X_i, X_j, X_k\}$ one can get "Three Gates" hands with these sets of winning tiles with the following 11 exceptions:

$$\{X_1, X_2, X_9\}, \{X_1, X_3, X_8\}, \{X_1, X_5, X_7\}, \{X_1, X_5, X_9\}, \{X_1, X_6, X_8\}, \{X_1, X_8, X_9\},$$

$$\{X_2, X_4, X_8\}, \{X_2, X_4, X_9\}, \{X_2, X_6, X_8\}, \{X_2, X_7, X_9\}, \{X_3, X_5, X_9\}.$$

13. It is easy to see that for any $1 \leq i, j \leq 9$, one can create a hand of 13 dot tiles so that a winning hand will be formed by adding the i-dot or the j-dot tile. For example, one may have a pair of i-dot tiles, and a pair of j-dot tiles, and three set of "pung" so that only the addition of the i-dot or j-dot will lead to a winning hand.

14. Similarly, one can have a hand of 4 sets of "pung" with a single k-dot tile so that one can only use an additional k-dot to form a winning hand.

6 Related problems and Further Research

We can also use our program to answer other problems. For example, if one randomly draws 14 tiles from the 36 dot tiles, what is the probability of getting a winning hand?

A calculation in our computer program shows that there are 118800 possible 14 tile dot hands, of which 13259 are winning. As a result, the probability of getting 14 tiles that form a winning hand is: 0.11161, which is larger than $\frac{1}{9}$. This result is higher than many Mahjong players would expect.

Of course, one can consider the full set of Mahjong with 144 tiles. Computing the probability of getting certain special hands will be more complicated.

In fact, a more challenging project is to develop a computing Mahjong-playing system. There has been great progress in research in artificial intelligence and machine learning. Computer systems have been built that beat the best chess player and Go player; see [3, 4, 9, 10]. It would be interesting to develop a Mahjong playing machine to beat the best Mahjong player in the world.

Note that Mahjong is different from chess and Go because the players do not have the full information of other players during the game. One needs to anticipate what other players are hiding in their hands and create their own game plan. Also, skillful players would be able to anticipate other players' strategies by observing their discarded tiles. Also, it is possible for two or three players to form a coalition to play against the other players. So, playing Mahjong well would require good use of combinatorial theory, probability theory, game theory, psychology, etc. To develop a good Mahjong playing machine will require another level of intelligence.

Acknowledgment

This research of the first two authors was partially supported by the NSF EXTREEMS-QED grant 1331021, and the Cissy Patterson Fund of the College of William and Mary.

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