# 第二次作业

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## （1） 习题 4.11

```r
path1 <- "/Users/xinby/Desktop/Sufe/Multivariate-Stat-Analysis/hw2/exec4.11.xlsx"
data1 <- readxl::read_excel(path1)
g = factor(data1$g)
y = cbind(data1$x1, data1$x2)
fit = manova(y ~ g)
summary(fit, test = "Wilks")
```

```
##           Df   Wilks approx F num Df den Df Pr(>F)
## g          2 0.92539   1.1069      4    112 0.3569
## Residuals 57
```

从上述输出结果可见，Wilk 统计量的 p 值为 0.3569，大于 0.05，因此无法拒绝原假设。故在统计学上认为没有明显差距。

## （2） 例 4.4.2

### a. 检验轮廓的平行性

```r
path2 <- "/Users/xinby/Desktop/Sufe/Multivariate-Stat-Analysis/hw2/examp4.4.2.xlsx"
data2 <- readxl::read_excel(path2)

# separate the data into grps:
x <- filter(data2, g == 1) %>%
  select(-g)
y <- filter(data2, g == 2) %>%
  select(-g)
dat2 <- select(data2, -g)

# calculate average and variance of data:
x.bar <- apply(x, 2, mean)
y.bar <- apply(y, 2, mean)
S1 <- cov(x)
S2 <- cov(y)
```

```r
sp <- (29 * S1 + 29 * S2)/(58)
C <- matrix(c(-1, 0, 0, 1, -1, 0, 0, 1, -1, 0, 0, 1), ncol = 4)

# to test if parallel:
c1 <- C %*% (x.bar - y.bar)
c2 <- C %*% sp %*% t(C)
T2 <- 30 * 30/(30 + 30) * t(c1) %*% solve(c2, c1)
T.05 <- 3 * 58/56 * qf(0.95, 3, 56)

print(T2)
print(T.05)

##          [,1]
## [1,] 8.016171
## [1] 8.605018
```

由上述输出结果可知，T2 结果小于临界值，因此不能拒绝原平行假设。

**b. 检验两轮廓是否重合**

```r
s1 <- sum(x.bar - y.bar)
s2 <- sum(sp)
# calculate T_square
T2_b <- 30 * 30/(30 + 30) * s1^2/s2
print(T2_b)
# compare with F quantile
qf(0.95, 1, 58)

## [1] 1.53277
## [1] 4.006873
```

由上述输出结果可知，计算结果小于临界值，无法拒绝原假设，量轮廓重合。

**c. 检验两轮廓水平**

```r
# calculate T_square
Cz <- 0.5 * C %*% (x.bar + y.bar)
S <- cov(dat2)
csc <- C %*% S %*% t(C)
T2_c <- (30 + 30) * t(Cz) %*% solve(csc, Cz)
T.05_c <- 3 * 59/57 * qf(0.95, 3, 57)
```

```
print(T2_c)
print(T.05_c)

##          [,1]
## [1,] 24.82071
## [1] 8.590518
```

由上述输出结果可知，计算结果大于临界值，拒绝原假设。 因此认为轮廓不是水平的，四个问题的回答有区别。

## （3） 习题 5.5

### a. 试给出判别规则，并预报明天是否会下雨，用回代法估计误判概率

```
# import data
path3.train <- "/Users/xinby/Desktop/Sufe/Multivariate-Stat-Analysis/hw2/exec5.5.xlsx"
path3.test <- "/Users/xinby/Desktop/Sufe/Multivariate-Stat-Analysis/hw2/exec5.5a.xlsx"
dat3 <- readxl::read_excel(path3.train)
dat3.test <- readxl::read_excel(path3.test)

# separate data by grps
rain <- filter(dat3, g == 1) %>%
   select(-g)
rain <- as.matrix(rain)
nrain <- filter(dat3, g == 2) %>%
   select(-g)
nrain <- as.matrix(nrain)
dat3.train <- dat3 %>%
   select(-g)
dat3.train <- as.matrix(dat3.train)
dat3.test <- t(as.matrix(dat3.test))

# test if same variance (the result shows not equal)
var1 <- cov(rain)
var2 <- cov(nrain)

# calculate mean and var for each grp
mu1 <- apply(rain, 2, mean)
mu2 <- apply(nrain, 2, mean)
```

```r
# calculate discriminator W(x) with new data
d1 <- t(dat3.test - mu1) %*% solve(var1, dat3.test - mu1)
d2 <- t(dat3.test - mu2) %*% solve(var2, dat3.test - mu2)
Wx <- d1 - d2
print(Wx)

# calculate misjudge rate
mis1.2 = 0
mis2.1 = 0
all = 0

# go through the train set to see if it is correctly judged
for (line in 1:20) {
    # calculate each obs' distance
    d1.test <- t(dat3.train[line, ] - mu1) %*% solve(var1, dat3.train[line, ] - mu1)
    d2.test <- t(dat3.train[line, ] - mu2) %*% solve(var2, dat3.train[line, ] - mu2)
    # judge which group it belongs to
    Wx.test <- 0
    if (d1.test - d2.test <= 0) {
        Wx.test[line] = 1
    } else {
        Wx.test[line] = 2
    }
    # count the correct rate
    if (Wx.test[line] == 1 && dat3$g[line] == 2) {
        mis1.2 = mis1.2 + 1
    }
    if (Wx.test[line] == 2 && dat3$g[line] == 1) {
        mis2.1 = mis2.1 + 1
    }
    all = all + 1
}

misjudge_rate_2.1 = mis2.1/all
misjudge_rate_1.2 = mis1.2/all
print(misjudge_rate_2.1)
print(misjudge_rate_1.2)
```

```
##           [,1]
## [1,] -1.950508
## [1] 0
## [1] 0.25
```

由于方差计算认为两组相差较大，故采用二次型判别规则。 令

$$W(x)=(x-\mu_1)'\Sigma_1^{-1}(x-\mu1)-(x-\mu_2)'\Sigma_2^{-1}(x-\mu2)$$

则有：

$$x\in\pi_1\sim\text{if}\sim W(x)\leq 0,\backslash x\in\pi_2,\sim\text{if}\sim W(x)>0.$$

由上述输出结果可知，W(x)<0，因此认为明天会下雨。误判概率如上输出所示。

**b. 给定先验概率，p1=0.3, p2=0.7，预报每天是否会下雨**

由 Bayes 准则的最大后验法，在正态分布条件下有：

$$P(\pi_i\mid\mathbf{x})=\frac{\exp\left\{-\frac{1}{2}[d^2(\mathbf{x},\pi_i)+\ln|\Sigma_i|-2\ln p_i]\right\}}{\sum_{j=1}^{k}\exp\left\{-\frac{1}{2}\left[d^2\left(\mathbf{x},\pi_j\right)+\ln\left|\Sigma_j\right|-2\ln p_j\right]\right\}}$$

```
p1 <- 0.3
p2 <- 0.7
Pr1 <- exp(-0.5 * (d1 + log(det(var1)) - 2 * log(p1)))
Pr2 <- exp(-0.5 * (d2 + log(det(var2)) - 2 * log(p2)))
Prob.pi1_x <- Pr1/(Pr1 + Pr2)
Prob.pi2_x <- Pr2/(Pr1 + Pr2)
```

由上述输出内容可知，由 Bayes 判别，认为明天不下雨。

**c. 在 b 的条件下考虑误判期望 c(2|1)=3c(1|2)，判断是否要在明天举行活动？**

此时加入考虑误判期望时，由于二次判别的判别效果严重依赖于数据的正态分布，而在本题中数据数量较少，故采用线性判别规则：

$$\mathbf{x}\in\pi_1\text{ , if }\mathbf{a}'(\mathbf{x}-\overline{\mathbf{\mu}})\geq\ln\left[\frac{c(1\mid 2)p_2}{c(2\mid 1)p_1}\right]$$

```
criteria <- log(3 * (p1)/(p2))
print(criteria)


a <- solve(cov(dat3.train), mu1 - mu2)
mu = 0.5 * (mu1 + mu2)
print(t(a) %*% (dat3.test - mu))
```

```
## [1] 0.2513144
##           [,1]
## [1,] 0.3622237
```

由上述输出结果，根据判别规则，当考虑误判代价时，明天不应该举行活动。

## (4) 习题 5.6

**a. 对于 14 名运动员，分别在方差相等和不等的假设下进行 Bayes 判别**

在正态分布假设下，若考虑误判概率相等，先验概率相等，当同方差时有：

$$P(\pi_i \mid \mathbf{x}) = \frac{\exp\left[-\frac{1}{2}d^2(\mathbf{x}, \pi_i)\right]}{\sum_{j=1}^{k}\exp\left[-\frac{1}{2}d^2(\mathbf{x}, \pi_j)\right]}$$

在异方差时有：

$$P(\pi_i \mid \mathbf{x}) = \frac{\exp\left\{-\frac{1}{2}[d^2(\mathbf{x}, \pi_i) + \ln|\Sigma_i|]\right\}}{\sum_{j=1}^{k}\exp\left\{-\frac{1}{2}\left[d^2(\mathbf{x}, \pi_j) + \ln|\Sigma_j|\right]\right\}}$$

其中

$$d^2(\mathbf{x}, \boldsymbol{\pi}_i) = (\mathbf{x} - \boldsymbol{\mu}_i)'\boldsymbol{\Sigma}_i^{-1}(\mathbf{x} - \boldsymbol{\mu}_i)$$

```r
# import the data
path4.train <- "/Users/xinby/Desktop/Sufe/Multivariate-Stat-Analysis/hw2/exec5.6.xlsx"
path4.test <- "/Users/xinby/Desktop/Sufe/Multivariate-Stat-Analysis/hw2/exec5.6a.xlsx"
dat4.train <- readxl::read_excel(path4.train)
dat4.test <- readxl::read_excel(path4.test)

# Same Variance Assumption: USING `lda`
prd_1 <- lda(g ~ x1 + x2 + x3 + x4 + x5 + x6, prior = c(0.5, 0.5), data = dat4.train)

# Different Variance Assumption: USING `qda`
prd_2 <- qda(g ~ x1 + x2 + x3 + x4 + x5 + x6, prior = c(0.5, 0.5), data = dat4.train)

# Output the classification result
id <- c(1:14)
class_1 <- predict(prd_1, dat4.test)$class
class_2 <- predict(prd_2, dat4.test)$class
result <- cbind(id, class_1, class_2)
```

```
print("classification :")
print(result)

## [1] "classification :"
##       id class_1 class_2
##  [1,]  1       1       1
##  [2,]  2       1       1
##  [3,]  3       1       1
##  [4,]  4       1       1
##  [5,]  5       1       1
##  [6,]  6       1       1
##  [7,]  7       1       1
##  [8,]  8       2       2
##  [9,]  9       2       2
## [10,] 10       1       1
## [11,] 11       2       2
## [12,] 12       2       2
## [13,] 13       1       2
## [14,] 14       2       2
```

由上述输出结果可见，同方差假设下，一级运动员有1～7、10、13，健将级运动员有：8、9、11、12、14；异方差假设下，一级运动员有1～7、10，健将级运动员有：8、9、11～14.

**b. 试按照回代法和交叉验证法分别对（1）的误判概率进行估计**

```
# Same Var in-sample validation
class_train.eqvar <- predict(prd_1, dat4.train)$class
real_train <- dat4.train$g
print("same var - in_sample vald.")
print(prop.table(table(g = real_train, class_train.eqvar), 1))

# cross validation
prd_1.cv <- lda(g ~ x1 + x2 + x3 + x4 + x5 + x6, prior = c(0.5, 0.5), CV = TRUE,
    data = dat4.train)
print("same var - cross vald.")
print(prop.table(table(g = real_train, prd_1.cv$class), 1))

# Diff Var in-sample validation
class_train.dfvar <- predict(prd_2, dat4.train)$class
```

```r
print("diff. var - in_sample vald.")
print(prop.table(table(g = real_train, class_train.dfvar), 1))

# cross validation
prd_2.cv <- qda(g ~ x1 + x2 + x3 + x4 + x5 + x6, prior = c(0.5, 0.5), CV = TRUE,
    data = dat4.train)
print("diff var - cross vald.")
print(prop.table(table(g = real_train, prd_2.cv$class), 1))
```

```
## [1] "same var - in_sample vald."
##    class_train.eqvar
## g   1 2
##   1 1 0
##   2 0 1
## [1] "same var - cross vald."
##
## g      1    2
##   1 1.00 0.00
##   2 0.08 0.92
## [1] "diff. var - in_sample vald."
##    class_train.dfvar
## g   1 2
##   1 1 0
##   2 0 1
## [1] "diff var - cross vald."
##
## g   1 2
##   1 1 0
##   2 0 1
```

检验结果如上输出所示。 由此可知，对于同方差假设，回代法检验的误判概率都为 0，交叉验证的误判概率为 P1|2=0.08, P2|1=0； 对于异方差假设，回代法和交叉验证法得到的误判概率均为 0.

### c. 假设同方差，p1=0.8, p2=0.2，试着对这 14 名运动员进行 Bayes 判别

```r
prd_3 <- lda(g ~ x1 + x2 + x3 + x4 + x5 + x6, prior = c(0.8, 0.2), data = dat4.train)
result <- predict(prd_3, dat4.test)$class
cbind(id, result)
```

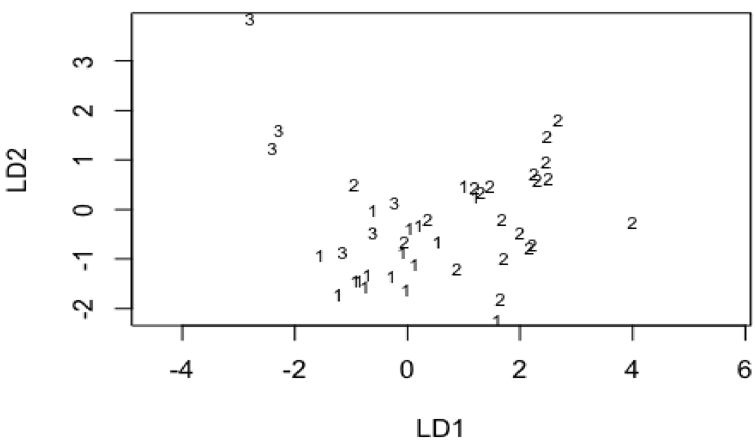
```
##      id result
##  [1,]  1      1
##  [2,]  2      1
##  [3,]  3      1
##  [4,]  4      1
##  [5,]  5      1
##  [6,]  6      1
##  [7,]  7      1
##  [8,]  8      1
##  [9,]  9      2
## [10,] 10      1
## [11,] 11      2
## [12,] 12      2
## [13,] 13      1
## [14,] 14      2
```

由上述输出可知，在考虑先验概率的条件下，一级运动员有 1～8、10、13；健将级运动员有 9、11、12、14.

## (5) 习题 5.8

**试给出 Fisher 判别函数，将所有品牌的两个判别函数得分画成散点图，用不同符号表示不同厂商**

```
path5 <- "/Users/xinby/Desktop/Sufe/Multivariate-Stat-Analysis/hw2/exec5.8.xlsx"
dat5 <- readxl::read_excel(path5)
fisher <- lda(dat5$g ~ ., dat5)
print(fisher$scaling)
plot(fisher)
```




```
##            LD1          LD2
## x1  0.022344104  0.045417606
## x2  0.369109646 -0.332405063
## x3 -0.837675738 -0.386499597
## x4 -0.000763493 -0.006017311
## x5  1.420281838  1.039957871
## x6  0.202200109 -0.203863959
## x7  0.195246015 -0.235306430
## x8 -0.030687468 -0.026966644
```

散点图如上图所示。第 i 判别函数的形式为：

$$y_i = t_i^T x$$

其系数如上输出所示。在本题中，由于$k = 3$，故可以参考第一、二判别函数。