公式汇总、

1. CML: $E(R_p) = R_f + \frac{R_m - R_f}{\sigma_m} \cdot \sigma_p$

2. SML: $E(R) = R_f + \beta \underbrace{[E(R_m) - R_f]}_{\text{系统性风险}}$

where $\beta = \frac{Cov(R_m, R_i)}{\sigma_m^2}$

3. $\alpha = R_p - E(R) = R_p - \{R_f + \beta[E(R_m) - R_f]\}$

4. $$\begin{cases} \mu_p = w_1\mu_1 + w_2\mu_2 \\ \sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\rho\sigma_1\sigma_2 \end{cases}$$

5. $\delta = \frac{\partial P}{\partial S}$ $\Gamma = \frac{\partial \delta}{\partial S}$ $\Rightarrow \Delta\delta = \Delta S \times \Gamma$ $\nu = \frac{\partial P}{\partial \sigma}$

$\theta = \frac{\partial P}{\partial t}$ $\rho = \frac{\partial P}{\partial r}$

6. $$\Delta P = \frac{\partial P}{\partial S}\Delta S + \frac{1}{2}\frac{\partial^2 P}{\partial S^2}(\Delta S)^2 + \frac{\partial P}{\partial \sigma}\Delta\sigma$$
$$= \delta \Delta S + \frac{1}{2}\Gamma(\Delta S)^2$$

7. $\delta_p = \sum w_i \delta_i$, $\Gamma_p = \sum w_i \Gamma_i$, $\nu = \sum w_i \nu_i$

8. $B = \sum C_i e^{-y t_i}$

9. $Dur \triangleq -\frac{1}{B}\frac{\partial B}{\partial y}$, ~~Cov~~ $Convex \triangleq \frac{1}{B}\frac{\partial^2 B}{\partial y^2}$ *

10. 连续复利: $Dur = \sum \cdot \frac{C_i e^{-y t_i}}{B} t_i$

$$Convex = \sum \frac{C_i e^{-y t_i}}{B} t_i^2$$

11. $\frac{\Delta B}{B} = -D\Delta y + \frac{1}{2}C(\Delta y)^2$

2. $\begin{cases} C_p = \sum w_i C_i \\ D_p = \sum w_i D_i \end{cases}$, $w_i = \frac{x_i}{P}$

3. 局部久期: $Dur_i = -\frac{1}{P}\frac{\Delta P_i}{\Delta y_i}$

4. 修正久期: 若离散复利. $Dur^* = \sum \frac{v_i}{B} t_i$, v_i 为每期折现

$\Rightarrow Dur_{Adj.} = \frac{D^*}{1+y/m}$, m 为每年复利次数

5. $\sigma_{year} = \sigma_{day} \times \sqrt{252}$

6. EWMA: $\sigma_n^2 = \lambda \sigma_{n-1}^2 + (1-\lambda) u_{n-1}^2$, $u_{n-1} = \frac{x_n - x_{n-1}}{x_{n-1}}$

GARCH(1,1): $\sigma_n^2 = \omega + \alpha u_{n-1}^2 + \beta \sigma_{n-1}^2$.

长期平均方差: $V_L = \frac{\omega}{1-\alpha-\beta}$ (长期平均波动率为 $\sqrt{V_L}$)

* 补充: $E(\underbrace{\sigma_{n+k}^2}_{\text{第}n+k\text{天}}) = V_L + (\alpha+\beta)^k(\sigma_n^2 - V_L)$

7. $\rho_n = \frac{cov_n(x,y)}{\sigma_{x,n}\sigma_{y,n}}$.

8. EWMA: $\cancel{\sigma_n^2}$ $cov_n = \lambda cov_{n-1} + (1-\lambda) x_n y_n$.

GARCH: $cov_n = \omega + \alpha x_n y_n + \beta cov_{n-1}$

$\begin{cases} x_n = \frac{x_n - x_{n-1}}{x_n} \\ y_n = \frac{y_n - y_{n-1}}{y_n} \end{cases}$

9. Normal 假定下

$VaR = \mu + \sigma N^{-1}(X)$ (常假定 $\mu = 0$)

$ES = \mu + \sigma \frac{e^{-\frac{Y^2}{2}}}{\sqrt{2\pi}(1-X)}$

10. $VaR_{T\text{-days}} = VaR_{1\text{-day}} \times \sqrt{T}$, ES 同理.

21. 边际VaR：$\frac{\partial VaR}{\partial x_i}$ $\sum_i \frac{\partial VaR_p}{\partial x_i} \cdot x_i$

22. 成份VaR：$\frac{\partial VaR}{\partial x_i} x_i$ $\Rightarrow$ 恒有 $\cancel{\sum \frac{\partial VaR}{\partial x_i}} x_i = VaR_p$.

23. 历史模拟法估计的 s.e.：

$$s.e. = \frac{1}{f(x)} \sqrt{\frac{q(1-q)}{n}}$$

n为 sample size
q为分位数，
f(x) 为 ~~[illegible]~~ q 对应的 pdf

24. 极值理论：

$$VaR = u + \frac{\beta}{\xi} \left\{ \left[\frac{n}{n_u} (1-q) \right]^{-\xi} - 1 \right\}$$

$$ES = \frac{VaR + \beta - \xi u}{1 - \xi}$$

25. 线性模型：$\Delta P = \sum_i \alpha_i \Delta x_i$

$\Rightarrow \sigma_p^2 = \alpha^T C \alpha$， α为各资产配比
C为协方差阵

26. 现金流映射 $\begin{cases} ① 利率插值 \\ ② 波动率插值 \end{cases}$

27. PCA：

因子载荷表：

	PC1	PC2	PC3	…
1year	p_{11}	p_{12}	p_{13}	
2year	p_{21}	p_{22}	p_{23}	
3year	p_{31}	p_{32}	p_{33}	
⋮				

风险暴露表：

year1	year2	year3
S1	S2	S3

因子得分表标准差

PC1	PC2	PC3
σ_{PC1}	σ_{PC2}	σ_{PC3}

~~若有~~

~~$\Rightarrow P = \alpha_1 f_1 + \alpha_2 f_2 + \alpha_3 f_3$~~

~~某组合~~ ~~因子得分~~

P而

① 风险暴露(敞口):

α = 组合持有的期限债券在某PC上的载荷 × 暴露金额

$\underbrace{\alpha}_{\text{因子得分}}$

②

$\Delta P = \alpha_1 f_1 + \alpha_2 f_2$ (这里以 2 PC 为例)

$\Rightarrow \sigma_{\Delta P}^2 = \alpha_1^2 \sigma_{PC1}^2 + \alpha_2^2 \sigma_{PC2}^2$ (由于 $cov(f_1, f_2) = 0$)

3. Cornish - Fisher 展开

$VaR = \mu_p + w_q \sigma_p$

$w_q = z_q + \frac{1}{6}(z_q^2 - 1)\xi_p$.

z_q: N(0,1) 下的 q 分位数

ξ_p: 会给出

9. $RWA = \sum_{i=1}^{N} w_i L_i + \sum_{j=1}^{M} w_j^* C_j$.

10. Basel I : Captial ≥ 8% RWA