Punto 1.5

Sergio Montoya Ramírez

31 de agosto de 2022

Sea $f(x)$ una función con (1) y (2) sus expansiones de Taylor.

$$f(x+h) = f(x) + hf'(x) + \frac{h^2}{2}f''(x) + \frac{h^3}{3!}f'''(x) + \frac{h^4}{4!}D^4f(x) + ... \tag{1}$$

$$f(x-h) = f(x) - hf'(x) + \frac{h^2}{2}f''(x) - \frac{h^3}{3!}f'''(x) + \frac{h^4}{4!}D^4f(x) + ... \tag{2}$$

Con esto podemos despejar $D^4f(x)$ en ambas expansiones lo que nos dejaria con:

$$\begin{aligned}
&f(x+h) = f(x) + hf'(x) + \frac{h^2}{2}f''(x) + \frac{h^3}{3!}f'''(x) + \frac{h^4}{4!}D^4f(x) + ... \\
&-\frac{h^4}{4!}D^4f(x) = f(x) - f(x+h) + hf'(x) + \frac{h^2}{2}f''(x) + \frac{h^3}{3!}f'''(x) + ... \\
&D^4f(x) = \frac{4!}{h^4}\left(-f(x) + f(x+h) - hf'(x) - \frac{h^2}{2}f''(x) - \frac{h^3}{3!}f'''(x) - ...\right) \\
&D^4f(x) = 4!\frac{f(x+h)-f(x)}{h^4} - \frac{4!}{h^3}f'(x) - \frac{12}{h^2}f''(x) - \frac{4}{h}f'''(x) - ... \\
&f(x-h) = f(x) - hf'(x) + \frac{h^2}{2}f''(x) - \frac{h^3}{3!}f'''(x) + \frac{h^4}{4!}D^4f(x) + ... \\
&\frac{h^4}{4!}D^4f(x) = -f(x) + f(x-h) + hf'(x) - \frac{h^2}{2}f''(x) + \frac{h^3}{3!}f'''(x) - ... \\
&D^4f(x) = 4!\frac{f(x-h)-f(x)}{h^4} + \frac{4!}{h^3}f'(x) - \frac{12}{h^2}f''(x) + \frac{4}{h}f'''(x) - ...
\end{aligned}$$

y con esto podemos restarlas ambos para encontrar $D^4f(x)$ central

$$\begin{aligned}
&D^4f(x) = 4!\frac{f(x+h)-f(x)}{h^4} - \frac{4!}{h^3}f'(x) - \frac{12}{h^2}f''(x) - \frac{4}{h}f'''(x) - ... \\
&-4!\frac{f(x-h)-f(x)}{h^4} + \frac{4!}{h^3}f'(x) - \frac{12}{h^2}f''(x) + \frac{4}{h}f'''(x) - ... \\
&D^4f(x) = 2\frac{4!}{h^3}f'(x) - \frac{4}{h}f'''(x) \; \textit{Si reemplazamos} \\
&D^4f(x) \approx \frac{f(x_{j+2h}) - 4f(x_{j+h}) + 6f(x_j) - f(x_{j-2h}) - 4f(x_{j-h})}{h^4}
\end{aligned}$$