

Sea $f(x)$ una función y por el metodo del trapezio simple nos quedaria $f(x) = p(x) + \epsilon(x)$, tal que,

$$\epsilon(x) = \frac{f'''(\xi)}{2}(x-a)(x-b), \quad a \leq \xi \leq b$$

Entonces el error de la integral de $f(x)$ es

$$\begin{aligned} E &= \int_a^b \epsilon(x) dx = \frac{f'''(\xi)}{2}(x-a)(x-b) = \frac{f'''(\xi)}{2}(x^2 - (a+b)x + ab) \\ &= \frac{f'''(\xi)}{2} \left(\frac{x^3}{3} - \frac{(a+b)x^2}{2} + abx \right) \Big|_a^b \\ &= \frac{f'''(\xi)}{2} \left(\left(\frac{b^3}{3} - \frac{(a+b)b^2}{2} + ab^2 \right) - \left(\frac{a^3}{3} - \frac{(a+b)a^2}{2} + a^2b \right) \right) \\ &= \frac{f'''(\xi)}{2} \left(\left(\frac{b^3}{3} - \frac{(ab^2 + b^3)b^2}{2} + ab^2 \right) - \left(\frac{a^3}{3} - \frac{a^3 + a^2b}{2} + a^2b \right) \right) \\ &= \frac{f'''(\xi)}{2} \left(\frac{-b^3}{6} + \frac{a^2b}{2} + \frac{a^3}{6} - \frac{ab^2}{2} \right) \\ &= \frac{f'''(\xi)}{2} \left(\frac{-2b^3 + 6a^2b - 6ab^2 + 2a^3}{12} \right) \\ &= \frac{f'''(\xi)}{2} \cdot \frac{h^3}{12} \end{aligned}$$