Sea $\epsilon(x) = \frac{f'''(\xi)}{4!}(x-a)(x-b)(x-\frac{a+b}{2})$ entonces:

$$\int_a^b \epsilon(x)dx = \int_a^b \frac{f'''(\xi)}{4!}(x-a)(x-b)(x-\frac{a+b}{2})dx$$

$$\Rightarrow \frac{f'''(\xi)}{4!}\int_a^b (x-a)(x-b)(x-\frac{a+b}{2})dx$$

$$\Rightarrow \frac{1}{4}x(-2a(b-x)^2+(b-x)^2x+a^2(-2b+x))$$

$$\Rightarrow \frac{1}{4}b(-2a(b-b)^2+(b-b)^2b+a^2(-2b+b)) - \frac{1}{4}a(-2a(b-a)^2+(b-a)^2a+a^2(-2b+a))$$

$$\Rightarrow \frac{1}{4}a^2b^2 - \frac{1}{4}(a(-a(b-a)^2+a^2(-2b+a)))$$

$$\Rightarrow \frac{1}{4}a^2b^2 - \frac{1}{4}a(-ab^2+2a^2b-a^3-2ba^2+a^3)$$

$$\Rightarrow \frac{1}{4}a^2b^2 - \frac{1}{4}a^2b^2$$

$$\Rightarrow 0$$