Punto 6

Sergio David López Becerra
Sergio Montoya

Sea $Px^2(a_0, a_1) = \left[\dfrac{\partial x^2}{\partial a_0}, \dfrac{\partial x^2}{\partial a_1}\right]$. Luego, podemos decir que:

$$Px^2(a_0, a_1) = \left[-2\sum_{i=1}^{n}(y_i - (a_0 + a_i x_i)), -2\sum_{i=1}^{n}(y_i - (a_0 + a_i x_i))(x_i)\right] = [0, 0].$$

Entonces, $Px^2(a_0, a_1) = \displaystyle\sum_{i=1}^{n} y_i - a_0 n - \sum_{i=1}^{n} a_i x_i = 0 \longrightarrow a_0 = \frac{\sum_{i=1}^{n} y_i - a_0 n - \sum_{i=1}^{n} a_i x_i}{n}$,
$a_0 = \overline{y} - a_1\overline{x}$.

Ahora bien, teniendo en cuenta que $\displaystyle\sum_{i=1}^{n} y_i x_i - a_0\sum_{i=1}^{n} x_i - a_1\sum_{i=1}^{n} {x_i}^2 = 0$.

$$a_1\sum_{i=1}^{n} {x_i}^2 = \sum_{i=1}^{n} y_i x_i + \left(\sum_{i=1}^{n} x_i\right)\left[a_1\frac{1}{n}\sum_{i=1}^{n} x_i - \frac{1}{n}\sum_{i=1}^{n} y_i\right],$$

$$a_1\sum_{i=1}^{n} {x_i}^2 - a_1\frac{1}{n}\sum_{i=1}^{n} x_i = \sum_{i=1}^{n} y_i x_i - \frac{1}{n}\left(\sum_{i=1}^{n} y_i\right)\left(\sum_{i=1}^{n} x_i\right).$$

$$\therefore\ a_1 = \frac{\sum_{i=1}^{n} y_i x_i - \frac{1}{n}\left(\sum_{i=1}^{n} y_i\right)\left(\sum_{i=1}^{n} x_i\right)}{\sum_{i=1}^{n} {x_i}^2 - \frac{1}{n}\sum_{i=1}^{n} x_i}.$$

Sea $Dx^2(a_0, a_1, a_2) = \left[\dfrac{\partial x^2}{\partial a_0}, \dfrac{\partial x^2}{\partial a_1}, \dfrac{\partial x^2}{\partial a_2}\right] = [0, 0, 0]$. Luego, podemos decir que:

$$\frac{\partial x^2}{\partial a_0} = -2\sum_{i=1}^{n}(y_i - a_0 - a_1 x_i - a_2 {x_i}^2) = 0 \longrightarrow \sum_{i=1}^{n} y_i = \sum_{i=1}^{n}(a_0 - a_1 x_i - a_2 {x_i}^2).$$

$$\frac{\partial x^2}{\partial a_1} = -2\sum_{i=1}^{n} x_i(y_i - a_0 - a_1 x_i - a_2 {x_i}^2) = 0 \longrightarrow \sum_{i=1}^{n} y_i x_i = \sum_{i=1}^{n}(a_0 x_i - a_1 {x_i}^2 - a_2 {x_i}^3).$$

$$\frac{\partial x^2}{\partial a_2} = -2\sum_{i=1}^{n} {x_i}^2(y_i - a_0 - a_1 x_i - a_2 {x_i}^2) = 0 \longrightarrow \sum_{i=1}^{n} y_i {x_i}^2 = \sum_{i=1}^{n}(a_0 {x_i}^2 - a_1 {x_i}^3 - a_2 {x_i}^4).$$