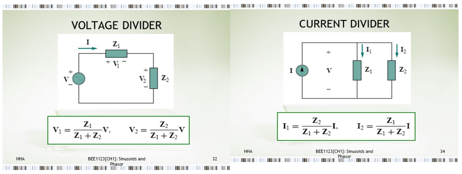
## DIVIDER FORMULAS:




## Chapter 11: AC Power Analysis

$$\boxed{X_{rms} = \sqrt{\frac{1}{T}\int_0^T x^2\,dt}}$$

$X_{rms} = \frac{X}{\sqrt{2}}$ **(only valid for sinusoidal functions)**

**If not a sinusoidal then:**

$$X_{rms} = \sqrt{\frac{1}{T}\left(\int_0^{T_1}(x_1^2)dt + \int_{T_1}^{T_2}(x_2)dt + ... + \int_{T_{n-1}}^{T_n}(x_n^2)dt\right)}$$

***COMPLEX POWER***

Complex power of an Impedance Z:

$$\mathbf{S} = I_{\rm rms}^2\mathbf{Z} = \frac{V_{\rm rms}^2}{\mathbf{Z}^*} = \mathbf{V}_{\rm rms}\mathbf{I}_{\rm rms}^*$$

**Angle of Impedance:**

$\theta_z = angle(\tilde{Z}) = angle(\tilde{S})$

**Complex Power (for any Circuit Element):**

$$\begin{aligned}
\text{Complex Power} &= \mathbf{S} = P + jQ = \mathbf{V}_{\rm rms}(\mathbf{I}_{\rm rms})^* \\
&= |\mathbf{V}_{\rm rms}|\,|\mathbf{I}_{\rm rms}|\underline{/\theta_v - \theta_i} \\
\text{Apparent Power} &= S = |\mathbf{S}| = |\mathbf{V}_{\rm rms}|\,|\mathbf{I}_{\rm rms}| = \sqrt{P^2 + Q^2} \\
\text{Real Power} &= P = \text{Re}(\mathbf{S}) = S\cos(\theta_v - \theta_i) \\
\text{Reactive Power} &= Q = \text{Im}(\mathbf{S}) = S\sin(\theta_v - \theta_i) \\
\text{Power Factor} &= \frac{P}{S} = \cos(\theta_v - \theta_i)
\end{aligned}$$

**Lagging power factor when** $\theta_v - \theta_i > 0$

**Leading power factor when** $\theta_v - \theta_i < 0$

1. $Q = 0$ for resistive loads (unity pf).
2. $Q < 0$ for capacitive loads (leading pf).
3. $Q > 0$ for inductive loads (lagging pf).

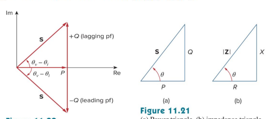



Figure 11.22
Power triangle.

Figure 11.21
(a) Power triangle, (b) impedance triangle.

**Reactive factor:** $rf = \sin(\theta_z)$

**Conservation Of Complex Power:**

$$\mathbf{S} = \mathbf{VI}^* = \mathbf{V}(\mathbf{I}_1^* + \mathbf{I}_2^*) = \mathbf{VI}_1^* + \mathbf{VI}_2^* = \mathbf{S}_1 + \mathbf{S}_2$$

**Maximum Average Power transfer(ALL MAX):**

$\tilde{Z_{Th}} = R_{Th} + jX_{Th}$

**After taking partial of** $P(\tilde{Z_L})$ **respect to** $R_L$ **and** $X_L$

$R_L(X_L) = \sqrt{R_{Th}^2 + (X_L + X_{Th})^2}$

$X_L = -X_{Th}$

**If (** $\tilde{Z_L} = R_L + jX_L$ **):**

$P_{max} = P(\tilde{Z_{Th}}^*) = \frac{|\tilde{V_{Th}}|^2}{8R_{Th}}$

**If else** $(\tilde{Z_L} = R_L)$**:**

$$P_{max} = P(|\tilde{Z_{Th}}|) = \frac{1}{2}\frac{|\tilde{V_{Th}}|^2|\tilde{Z_{Th}}|}{[[|\tilde{Z_{Th}}|+R_{Th}]^2+X_{Th}^2]}$$

**Power factor correction:**

$$\boxed{C = \frac{Q_C}{\omega V_{\rm rms}^2} = \frac{P(\tan\theta_1 - \tan\theta_2)}{\omega V_{\rm rms}^2}}$$

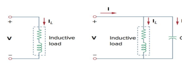



Figure 11.27
Power factor correction: (a) original inductive load, (b) inductive load with improved power factor.

## Chapter 12: Three - Phase Circuits(RMS Eqns)

**Balanced phase Voltages:** are equal in magnitude and are out of phase with each other by $120$

$\tilde{V_{an}} + \tilde{V_{bn}} + \tilde{V_{cn}} = 0$

$V_{an} = V_{bn} = V_{cn}$

**Phase Sequence:** is the time order in which the voltages pass through their respective maximum values.

Positive Sequence(abc):

$$\boxed{\begin{aligned}
\mathbf{V}_{an} &= V_p\underline{/0^\circ} \\
\mathbf{V}_{bn} &= V_p\underline{/-120^\circ} \\
\mathbf{V}_{cn} &= V_p\underline{/-240^\circ} = V_p\underline{/+120^\circ}
\end{aligned}}$$

$\Delta -> Y$ **Transformations:**

$\tilde{Z_Y} = \frac{1}{3}\tilde{Z_\Delta}$

$V_{\phi,Y} = \frac{1}{\sqrt{3}}V_{\phi,\Delta}$ **Line-Phase relationship**

TABLE 12.1
Summary of phase and line voltages/currents for balanced three-phase systems.[1]

| Connection | Phase voltages/currents | Line voltages/currents |
|---|---|---|
| Y-Y | $\mathbf{V}_{an} = V_p\underline{/0^\circ}$<br>$\mathbf{V}_{bn} = V_p\underline{/-120^\circ}$<br>$\mathbf{V}_{cn} = V_p\underline{/+120^\circ}$<br>Same as line currents | $\mathbf{V}_{ab} = \sqrt{3}V_p\underline{/30^\circ}$<br>$\mathbf{V}_{bc} = \mathbf{V}_{ab}\underline{/-120^\circ}$<br>$\mathbf{V}_{ca} = \mathbf{V}_{ab}\underline{/+120^\circ}$<br>$\mathbf{I}_a = \mathbf{V}_{an}/\mathbf{Z}_Y$<br>$\mathbf{I}_b = \mathbf{I}_a\underline{/-120^\circ}$<br>$\mathbf{I}_c = \mathbf{I}_a\underline{/+120^\circ}$ |
| Y-Δ | $\mathbf{V}_{an} = V_p\underline{/0^\circ}$<br>$\mathbf{V}_{bn} = V_p\underline{/-120^\circ}$<br>$\mathbf{V}_{cn} = V_p\underline{/+120^\circ}$<br>$\mathbf{I}_{AB} = \mathbf{V}_{AB}/\mathbf{Z}_\Delta$<br>$\mathbf{I}_{BC} = \mathbf{V}_{BC}/\mathbf{Z}_\Delta$<br>$\mathbf{I}_{CA} = \mathbf{V}_{CA}/\mathbf{Z}_\Delta$ | $\mathbf{V}_{ab} = \mathbf{V}_{AB} = \sqrt{3}V_p\underline{/30^\circ}$<br>$\mathbf{V}_{bc} = \mathbf{V}_{BC} = \mathbf{V}_{ab}\underline{/-120^\circ}$<br>$\mathbf{V}_{ca} = \mathbf{V}_{CA} = \mathbf{V}_{ab}\underline{/+120^\circ}$<br>$\mathbf{I}_a = \mathbf{I}_{AB}\sqrt{3}\underline{/-30^\circ}$<br>$\mathbf{I}_b = \mathbf{I}_a\underline{/-120^\circ}$<br>$\mathbf{I}_c = \mathbf{I}_a\underline{/+120^\circ}$ |
| Δ-Δ | $\mathbf{V}_{ab} = V_p\underline{/0^\circ}$<br>$\mathbf{V}_{bc} = V_p\underline{/-120^\circ}$<br>$\mathbf{V}_{ca} = V_p\underline{/+120^\circ}$<br>$\mathbf{I}_{AB} = \mathbf{V}_{ab}/\mathbf{Z}_\Delta$<br>$\mathbf{I}_{BC} = \mathbf{V}_{bc}/\mathbf{Z}_\Delta$<br>$\mathbf{I}_{CA} = \mathbf{V}_{ca}/\mathbf{Z}_\Delta$ | Same as phase voltages<br>$\mathbf{I}_a = \mathbf{I}_{AB}\sqrt{3}\underline{/-30^\circ}$<br>$\mathbf{I}_b = \mathbf{I}_a\underline{/-120^\circ}$<br>$\mathbf{I}_c = \mathbf{I}_a\underline{/+120^\circ}$ |
| Δ-Y | $\mathbf{V}_{ab} = V_p\underline{/0^\circ}$<br>$\mathbf{V}_{bc} = V_p\underline{/-120^\circ}$<br>$\mathbf{V}_{ca} = V_p\underline{/+120^\circ}$<br>Same as line currents | Same as phase voltages<br>$\mathbf{I}_a = \frac{V_p\underline{/-30^\circ}}{\sqrt{3}\mathbf{Z}_Y}$<br>$\mathbf{I}_b = \mathbf{I}_a\underline{/-120^\circ}$<br>$\mathbf{I}_c = \mathbf{I}_a\underline{/+120^\circ}$ |

[1]Positive or *abc* sequence is assumed.

**Power Three phase Systems:**

**Below is true for** true for either y or delta connected load

Per-phase Powers:

For Y-connected load: $I_L = I_\phi \; but \; V_L = \sqrt{3}V_\phi$

For $\Delta$ − connected load: $I_L = \sqrt{3}I_\phi \; but \; V_L = V_\phi$

And then total complex power of a 3 phase system:

$\tilde{S_{3\phi}} = 3\tilde{S_\phi} = 3\tilde{V_\phi}\tilde{I_\phi}^* = 3I_\phi{}^2\tilde{Z_\phi} = 3\frac{V_\phi{}^2}{\tilde{Z_\phi}^*}$

$\tilde{S_{3\phi}} = P_{3\phi} + jQ_{3\phi} = \sqrt{3}V_L I_L\angle\theta_\phi$

$\theta_\phi$ is the phase angle of that element or impedance angle

Don't make the mistake of:

$\tilde{S_{3\phi}} \neq \tilde{V_L}\tilde{I_L}^* \neq \sqrt{3}V_L I_L\angle\theta_\phi$

Power Calculation in 3-phase circuits: (TIPS)

**1.) Look what you are given**

**a.)Case 1: given just impedances or Currents/ Voltages:**

- Try to change everything to a Y - Y configuration

-> If (there are multiple balanced loads given){
When ( L1 and L2 are in parallel):
Transform L1 and L2 to Y forms and connect combine their individual impedances.
Then Make single phase equivalent circuit.

**b.)Case 2: given just the a Power quantity**

- Notice that $\tilde{S_{\phi,Y}} = \tilde{S_{\phi,\Delta}}$
- That means we can make a single equivalent directly of individual Loads.

**c. ) case 3: Combination of case 1 and 2**

- Try to break this circuit in a single phase equivalent ( convert everything to Y)
- For the Load given a Power quantity with a black box use the fact that $\tilde{S_{\phi,Y}} = \tilde{S_{\phi,\Delta}}$

-> Next the circuit build the single phase circuit

**2.)Once you Configured your Circuit. If not given a reference set one:**

Example: You might be given a magnitude, and there is not way to even solve for a variable need with this magnitude. Therefore give the phasor a reference angle of 0.

If you are given a line voltage but need to use line to neutral voltage make it such that line to voltage phase is your reference.

**Y-Y Balanced circuit and** $\Delta$ **-** $\Delta$

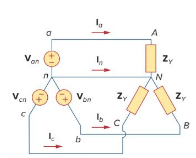



Figure 12.10
Balanced Y-Y connection.

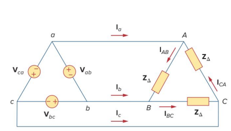



Figure 12.17
A balanced Δ-Δ connection.

**Alternative solving (** $\Delta$ **- Y), (** $\Delta$ **,** $\Delta$ **)**

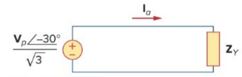
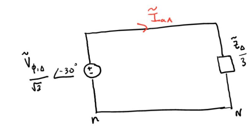
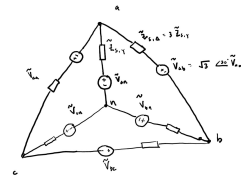



Figure 12.20
The single-phase equivalent circuit.




**Chapter 14: Frequency Response**

**Types of Transfer functions:**

$\tilde{H(\omega)} = Voltage\ gain = \frac{\tilde{V_o}}{\tilde{V_i}}$

**2nd Order Resonant Circuits(RLC):**

$B = \omega_2 - \omega_1$

$\omega_1 = \omega_o\left[\sqrt{1 + (\frac{1}{2Q})^2} - \frac{1}{2Q}\right]$

$\omega_2 = \omega_o\left[\sqrt{1 + (\frac{1}{2Q})^2} + \frac{1}{2Q}\right]$

$\omega_o = \frac{1}{\sqrt{LC}}$

**RLC Series - Characteristics :**

$\omega_1 = -\frac{R}{2L} + \sqrt{(\frac{R}{2L})^2 + \frac{1}{LC}}$

$\omega_2 = \frac{R}{2L} + \sqrt{(\frac{R}{2L})^2 + \frac{1}{LC}}$

$\omega_o = \sqrt{\omega_1 \cdot \omega_2}$

$Q = \frac{\omega_o L}{R} = \frac{1}{\omega_o RC}$

$B = \frac{L}{R} = \frac{\omega_o}{Q}$

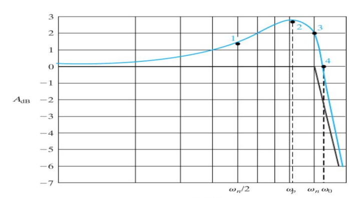



Figure E.12 ▲ Four points on the corrected amplitude plot for a pair of complex poles.

**RLC Parallel - Characteristics**

$\omega_1 = -\frac{1}{2RC} + \sqrt{(\frac{1}{2RC})^2 + \frac{1}{LC}}$

$\omega_2 = \frac{1}{2RC} + \sqrt{(\frac{1}{2RC})^2 + \frac{1}{LC}}$

$Q = \frac{\omega_o}{B} = \omega_o RC = \frac{R}{\omega_o L}$

$B = \frac{1}{RC}$

**Passive Filters:**

**Low Pass Filters:**

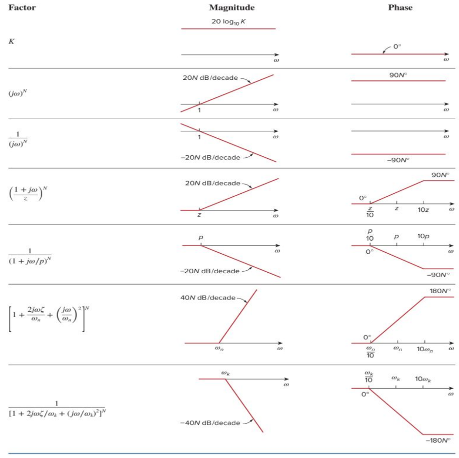
$H(\tilde{\omega}, \omega_c) = \frac{\tilde{V_o}}{\tilde{V_i}} = \frac{1}{1 + j\omega/\omega_c}$

**RC (Vo across C) and LR (Vo across R)**

$\omega_{c,RC} = \frac{1}{RC}$

$\omega_{c,LR} = \frac{R}{L}$

TABLE 14.3
Summary of Bode straight-line magnitude and phase plots.




## Chapter 19: Two port - networks

$T-\pi Transformations$

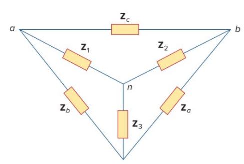


**Figure 9.22**
Superimposed $Y$ and $\Delta$ networks.

*Y-Δ Conversion:*

$$\mathbf{Z}_a=\frac{\mathbf{Z}_1\mathbf{Z}_2+\mathbf{Z}_2\mathbf{Z}_3+\mathbf{Z}_3\mathbf{Z}_1}{\mathbf{Z}_1}$$
$$\mathbf{Z}_b=\frac{\mathbf{Z}_1\mathbf{Z}_2+\mathbf{Z}_2\mathbf{Z}_3+\mathbf{Z}_3\mathbf{Z}_1}{\mathbf{Z}_2}$$
$$\mathbf{Z}_c=\frac{\mathbf{Z}_1\mathbf{Z}_2+\mathbf{Z}_2\mathbf{Z}_3+\mathbf{Z}_3\mathbf{Z}_1}{\mathbf{Z}_3}$$

### Impedance Parameters

$$\mathbf{z}_{11}=\frac{\mathbf{V}_1}{\mathbf{I}_1}\bigg|_{\mathbf{I}_2=0},\quad \mathbf{z}_{12}=\frac{\mathbf{V}_1}{\mathbf{I}_2}\bigg|_{\mathbf{I}_1=0}$$
$$\mathbf{z}_{21}=\frac{\mathbf{V}_2}{\mathbf{I}_1}\bigg|_{\mathbf{I}_2=0},\quad \mathbf{z}_{22}=\frac{\mathbf{V}_2}{\mathbf{I}_2}\bigg|_{\mathbf{I}_1=0}$$

**Reciprocal Network:**
**When two port network is linear (entirely resistor, caps, inductors )and has no dependent**

$$\mathbf{V}_1=\mathbf{z}_{11}\mathbf{I}_1+\mathbf{z}_{12}\mathbf{I}_2$$
$$\mathbf{V}_2=\mathbf{z}_{21}\mathbf{I}_1+\mathbf{z}_{22}\mathbf{I}_2$$

**sources, z12 = z21**

**EQUIVALENT CIRCUITS**

**Admittance Parameters:**

$$\mathbf{y}_{11}=\frac{\mathbf{I}_1}{\mathbf{V}_1}\bigg|_{\mathbf{V}_2=0},\quad \mathbf{y}_{12}=\frac{\mathbf{I}_1}{\mathbf{V}_2}\bigg|_{\mathbf{V}_1=0}$$
$$\mathbf{y}_{21}=\frac{\mathbf{I}_2}{\mathbf{V}_1}\bigg|_{\mathbf{V}_2=0},\quad \mathbf{y}_{22}=\frac{\mathbf{I}_2}{\mathbf{V}_2}\bigg|_{\mathbf{V}_1=0}$$

$$\mathbf{I}_1=\mathbf{y}_{11}\mathbf{V}_1+\mathbf{y}_{12}\mathbf{V}_2$$
$$\mathbf{I}_2=\mathbf{y}_{21}\mathbf{V}_1+\mathbf{y}_{22}\mathbf{V}_2$$

**EQUIVALENT CIRCUITS**

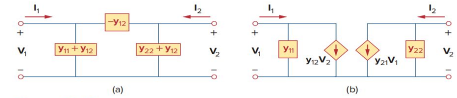


**Figure 19.13**
(a) Π-equivalent circuit (for reciprocal case only), (b) general equivalent circuit.

### Hybrid Parameters

$$\mathbf{V}_1=\mathbf{h}_{11}\mathbf{I}_1+\mathbf{h}_{12}\mathbf{V}_2$$
$$\mathbf{I}_2=\mathbf{h}_{21}\mathbf{I}_1+\mathbf{h}_{22}\mathbf{V}_2$$

$$\mathbf{h}_{11}=\frac{\mathbf{V}_1}{\mathbf{I}_1}\bigg|_{\mathbf{V}_2=0},\quad \mathbf{h}_{12}=\frac{\mathbf{V}_1}{\mathbf{V}_2}\bigg|_{\mathbf{I}_1=0}$$
$$\mathbf{h}_{21}=\frac{\mathbf{I}_2}{\mathbf{I}_1}\bigg|_{\mathbf{V}_2=0},\quad \mathbf{h}_{22}=\frac{\mathbf{I}_2}{\mathbf{V}_2}\bigg|_{\mathbf{I}_1=0}$$

**EQUIVALENT CIRCUIT**

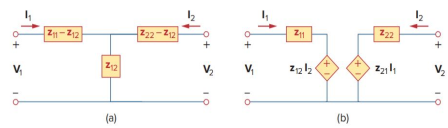


**Figure 19.5**
(a) T-equivalent circuit (for reciprocal case only), (b) general equivalent circuit.

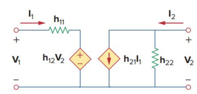


**Figure 19.20**
The $h$-parameter equivalent network of a two-port network.

F or reciprocal networks, h12 = −h21.

**Inverse Hybrid parameters:**

$$\mathbf{I}_1=\mathbf{g}_{11}\mathbf{V}_1+\mathbf{g}_{12}\mathbf{I}_2$$
$$\mathbf{V}_2=\mathbf{g}_{21}\mathbf{V}_1+\mathbf{g}_{22}\mathbf{I}_2$$

$$\mathbf{g}_{11}=\frac{\mathbf{I}_1}{\mathbf{V}_1}\bigg|_{\mathbf{I}_2=0},\quad \mathbf{g}_{12}=\frac{\mathbf{I}_1}{\mathbf{I}_2}\bigg|_{\mathbf{V}_1=0}$$
$$\mathbf{g}_{21}=\frac{\mathbf{V}_2}{\mathbf{V}_1}\bigg|_{\mathbf{I}_2=0},\quad \mathbf{g}_{22}=\frac{\mathbf{V}_2}{\mathbf{I}_2}\bigg|_{\mathbf{V}_1=0}$$

**EQUIVALENT CIRCUIT**

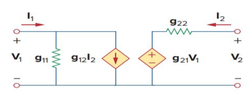


**Figure 19.21**
The $g$-parameter model of a two-port network.

**For Reciprocal:**

For reciprocal networks, g12 = −g21.

**Transmission Parameters**

$$\mathbf{V}_2=\mathbf{a}\mathbf{V}_1-\mathbf{b}\mathbf{I}_1$$
$$\mathbf{I}_2=\mathbf{c}\mathbf{V}_1-\mathbf{d}\mathbf{I}_1$$

$$\mathbf{a}=\frac{\mathbf{V}_2}{\mathbf{V}_1}\bigg|_{\mathbf{I}_1}\quad \mathbf{c}=\frac{\mathbf{I}_2}{\mathbf{V}_1}\bigg|_{\mathbf{I}_1}$$

$$\mathbf{A}=\frac{\mathbf{V}_1}{\mathbf{V}_2}\bigg|_{\mathbf{I}_2=0},\quad \mathbf{B}=-\frac{\mathbf{V}_1}{\mathbf{I}_2}\bigg|_{\mathbf{V}_2=0}$$
$$\mathbf{C}=\frac{\mathbf{I}_1}{\mathbf{V}_2}\bigg|_{\mathbf{I}_2=0},\quad \mathbf{D}=-\frac{\mathbf{I}_1}{\mathbf{I}_2}\bigg|_{\mathbf{V}_2=0}$$

**INVERSE TRANSMISSION PARAMETERS:**
**a transmission network is reciprocal if:**

$$\mathbf{AD}-\mathbf{BC}=1,\qquad \mathbf{ad}-\mathbf{bc}=1$$

**TABLE FOR PARAMETER CONVERSION:**

**High Pass Filters:**

$$H(\tilde{\omega},\omega_c)=\frac{\tilde{V_o}}{\tilde{V_i}}=\frac{j\omega/\omega_c}{1+j\omega/\omega_c}$$

**RC(Vo across R) and LR (Vo across L):**

$$\omega_{c,RC}=\frac{1}{RC}$$
$$\omega_{c,LR}=\frac{R}{L}$$

**Band Pass Filter - RLC series (Vo Across R)**

$$H(\tilde{\omega})=\frac{\tilde{V_o}}{\tilde{V_i}}=\frac{R}{R+j(\omega L-\frac{1}{\omega C})}$$
$$\omega_o=\frac{1}{\sqrt{LC}}$$

**Band Stop Filter - RLC series (Vo across L and C)**

$$H(\tilde{\omega})=\frac{\tilde{V_o}}{\tilde{V_i}}=\frac{j(\omega L-\frac{1}{\omega C})}{R+j(\omega L-\frac{1}{\omega C})}$$
$$\omega_o=\frac{1}{\sqrt{LC}}$$

**Bode Plots**

TABLE 14.2
Specific gain and their decibel values.*

| Magnitude $H$ | $20\log_{10}H$ (dB) |
|---|---|
| 0.001 | −60 |
| 0.01 | −40 |
| 0.1 | −20 |
| 0.5 | −6 |
| $1/\sqrt{2}$ | −3 |
| 1 | 0 |
| $\sqrt{2}$ | 3 |
| 2 | 6 |
| 10 | 20 |
| 20 | 26 |
| 100 | 40 |
| 1000 | 60 |

* Some of these values are approximate.

$$H_{dB}=20\log_{10}H$$

The origin is where $\omega=1$ or $\log\omega=0$ and the gain is zero.

**STANDARD FORM:**

$$\mathbf{H}(\omega)=\frac{K(j\omega)^{\pm1}(1+j\omega/z_1)[1+j2\zeta_1\omega/\omega_k+(j\omega/\omega_k)^2]\cdots}{(1+j\omega/p_1)[1+j2\zeta_2\omega/\omega_n+(j\omega/\omega_n)^2]\cdots}$$

**Corner frequency for quadratic term:**

$$\zeta\omega_n=\alpha.\qquad \omega_n^2=\alpha^2+\beta^2$$

$$Q_{dB}=\frac{1}{2\zeta}$$

**If** $\zeta$ **< 1, the roots of the quadratic are complex,**

**If** $If\ \zeta\geq1$ **we factor then (s+p1)(s+p2)**

**And graph accordingly**

**Peaking Frequency:**

$$\omega_p=\omega_n\sqrt{1-2\zeta^2},$$

**TABLE 19.1**

Conversion of two-port parameters.

| | z | | y | | h | | g | | T | | t | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| z | $\mathbf{z}_{11}$ | $\mathbf{z}_{12}$ | $\frac{\mathbf{y}_{22}}{\Delta_y}$ | $-\frac{\mathbf{y}_{12}}{\Delta_y}$ | $\frac{\Delta_h}{\mathbf{h}_{22}}$ | $\frac{\mathbf{h}_{12}}{\mathbf{h}_{22}}$ | $\frac{1}{\mathbf{g}_{11}}$ | $-\frac{\mathbf{g}_{12}}{\mathbf{g}_{11}}$ | $\frac{\mathbf{A}}{\mathbf{C}}$ | $\frac{\Delta_T}{\mathbf{C}}$ | $\frac{\mathbf{d}}{\mathbf{c}}$ | $\frac{1}{\mathbf{c}}$ |
| | $\mathbf{z}_{21}$ | $\mathbf{z}_{22}$ | $-\frac{\mathbf{y}_{21}}{\Delta_y}$ | $\frac{\mathbf{y}_{11}}{\Delta_y}$ | $-\frac{\mathbf{h}_{21}}{\mathbf{h}_{22}}$ | $\frac{1}{\mathbf{h}_{22}}$ | $\frac{\mathbf{g}_{21}}{\mathbf{g}_{11}}$ | $\frac{\Delta_g}{\mathbf{g}_{11}}$ | $\frac{1}{\mathbf{C}}$ | $\frac{\mathbf{D}}{\mathbf{C}}$ | $\frac{\Delta_t}{\mathbf{c}}$ | $\frac{\mathbf{a}}{\mathbf{c}}$ |
| y | $\frac{\mathbf{z}_{22}}{\Delta_z}$ | $-\frac{\mathbf{z}_{12}}{\Delta_z}$ | $\mathbf{y}_{11}$ | $\mathbf{y}_{12}$ | $\frac{1}{\mathbf{h}_{11}}$ | $-\frac{\mathbf{h}_{12}}{\mathbf{h}_{11}}$ | $\frac{\Delta_g}{\mathbf{g}_{22}}$ | $\frac{\mathbf{g}_{12}}{\mathbf{g}_{22}}$ | $\frac{\mathbf{D}}{\mathbf{B}}$ | $-\frac{\Delta_T}{\mathbf{B}}$ | $\frac{\mathbf{a}}{\mathbf{b}}$ | $-\frac{1}{\mathbf{b}}$ |
| | $-\frac{\mathbf{z}_{21}}{\Delta_z}$ | $\frac{\mathbf{z}_{11}}{\Delta_z}$ | $\mathbf{y}_{21}$ | $\mathbf{y}_{22}$ | $\frac{\mathbf{h}_{21}}{\mathbf{h}_{11}}$ | $\frac{\Delta_h}{\mathbf{h}_{11}}$ | $-\frac{\mathbf{g}_{21}}{\mathbf{g}_{22}}$ | $\frac{1}{\mathbf{g}_{22}}$ | $-\frac{1}{\mathbf{B}}$ | $\frac{\mathbf{A}}{\mathbf{B}}$ | $-\frac{\Delta_t}{\mathbf{b}}$ | $\frac{\mathbf{d}}{\mathbf{b}}$ |
| h | $\frac{\Delta_z}{\mathbf{z}_{22}}$ | $\frac{\mathbf{z}_{12}}{\mathbf{z}_{22}}$ | $\frac{1}{\mathbf{y}_{11}}$ | $-\frac{\mathbf{y}_{12}}{\mathbf{y}_{11}}$ | $\mathbf{h}_{11}$ | $\mathbf{h}_{12}$ | $\frac{\mathbf{g}_{22}}{\Delta_g}$ | $-\frac{\mathbf{g}_{12}}{\Delta_g}$ | $\frac{\mathbf{B}}{\mathbf{D}}$ | $\frac{\Delta_T}{\mathbf{D}}$ | $\frac{\mathbf{b}}{\mathbf{a}}$ | $\frac{1}{\mathbf{a}}$ |
| | $-\frac{\mathbf{z}_{21}}{\mathbf{z}_{22}}$ | $\frac{1}{\mathbf{z}_{22}}$ | $\frac{\mathbf{y}_{21}}{\mathbf{y}_{11}}$ | $\frac{\Delta_y}{\mathbf{y}_{11}}$ | $\mathbf{h}_{21}$ | $\mathbf{h}_{22}$ | $-\frac{\mathbf{g}_{21}}{\Delta_g}$ | $\frac{\mathbf{g}_{11}}{\Delta_g}$ | $-\frac{1}{\mathbf{D}}$ | $\frac{\mathbf{C}}{\mathbf{D}}$ | $\frac{\Delta_t}{\mathbf{a}}$ | $\frac{\mathbf{c}}{\mathbf{a}}$ |
| g | $\frac{1}{\mathbf{z}_{11}}$ | $-\frac{\mathbf{z}_{12}}{\mathbf{z}_{11}}$ | $\frac{\Delta_y}{\mathbf{y}_{22}}$ | $\frac{\mathbf{y}_{12}}{\mathbf{y}_{22}}$ | $\frac{\mathbf{h}_{22}}{\Delta_h}$ | $-\frac{\mathbf{h}_{12}}{\Delta_h}$ | $\mathbf{g}_{11}$ | $\mathbf{g}_{12}$ | $\frac{\mathbf{C}}{\mathbf{A}}$ | $-\frac{\Delta_T}{\mathbf{A}}$ | $\frac{\mathbf{c}}{\mathbf{d}}$ | $-\frac{1}{\mathbf{d}}$ |
| | $\frac{\mathbf{z}_{21}}{\mathbf{z}_{11}}$ | $\frac{\Delta_z}{\mathbf{z}_{11}}$ | $-\frac{\mathbf{y}_{21}}{\mathbf{y}_{22}}$ | $\frac{1}{\mathbf{y}_{22}}$ | $-\frac{\mathbf{h}_{21}}{\Delta_h}$ | $\frac{\mathbf{h}_{11}}{\Delta_h}$ | $\mathbf{g}_{21}$ | $\mathbf{g}_{22}$ | $\frac{1}{\mathbf{A}}$ | $\frac{\mathbf{B}}{\mathbf{A}}$ | $\frac{\Delta_t}{\mathbf{d}}$ | $-\frac{\mathbf{b}}{\mathbf{d}}$ |
| T | $\frac{\mathbf{z}_{11}}{\mathbf{z}_{21}}$ | $\frac{\Delta_z}{\mathbf{z}_{21}}$ | $-\frac{\mathbf{y}_{22}}{\mathbf{y}_{21}}$ | $-\frac{1}{\mathbf{y}_{21}}$ | $-\frac{\Delta_h}{\mathbf{h}_{21}}$ | $-\frac{\mathbf{h}_{11}}{\mathbf{h}_{21}}$ | $\frac{1}{\mathbf{g}_{21}}$ | $\frac{\mathbf{g}_{22}}{\mathbf{g}_{21}}$ | $\mathbf{A}$ | $\mathbf{B}$ | $\frac{\mathbf{d}}{\Delta_t}$ | $\frac{\mathbf{b}}{\Delta_t}$ |
| | $\frac{1}{\mathbf{z}_{21}}$ | $\frac{\mathbf{z}_{22}}{\mathbf{z}_{21}}$ | $-\frac{\Delta_y}{\mathbf{y}_{21}}$ | $-\frac{\mathbf{y}_{11}}{\mathbf{y}_{21}}$ | $-\frac{\mathbf{h}_{22}}{\mathbf{h}_{21}}$ | $-\frac{1}{\mathbf{h}_{21}}$ | $\frac{\mathbf{g}_{11}}{\mathbf{g}_{21}}$ | $\frac{\Delta_g}{\mathbf{g}_{21}}$ | $\mathbf{C}$ | $\mathbf{D}$ | $\frac{\mathbf{c}}{\Delta_t}$ | $\frac{\mathbf{a}}{\Delta_t}$ |
| t | $\frac{\mathbf{z}_{22}}{\mathbf{z}_{12}}$ | $\frac{\Delta_z}{\mathbf{z}_{12}}$ | $-\frac{\mathbf{y}_{11}}{\mathbf{y}_{12}}$ | $-\frac{1}{\mathbf{y}_{12}}$ | $\frac{1}{\mathbf{h}_{12}}$ | $\frac{\mathbf{h}_{11}}{\mathbf{h}_{12}}$ | $-\frac{\Delta_g}{\mathbf{g}_{12}}$ | $-\frac{\mathbf{g}_{22}}{\mathbf{g}_{12}}$ | $\frac{\mathbf{D}}{\Delta_T}$ | $\frac{\mathbf{B}}{\Delta_T}$ | $\mathbf{a}$ | $\mathbf{b}$ |
| | $\frac{1}{\mathbf{z}_{12}}$ | $\frac{\mathbf{z}_{11}}{\mathbf{z}_{12}}$ | $-\frac{\Delta_y}{\mathbf{y}_{12}}$ | $-\frac{\mathbf{y}_{22}}{\mathbf{y}_{12}}$ | $\frac{\mathbf{h}_{22}}{\mathbf{h}_{12}}$ | $\frac{\Delta_h}{\mathbf{h}_{12}}$ | $-\frac{\mathbf{g}_{11}}{\mathbf{g}_{12}}$ | $-\frac{1}{\mathbf{g}_{12}}$ | $\frac{\mathbf{C}}{\Delta_T}$ | $\frac{\mathbf{A}}{\Delta_T}$ | $\mathbf{c}$ | $\mathbf{d}$ |

$\Delta_z = \mathbf{z}_{11}\mathbf{z}_{22} - \mathbf{z}_{12}\mathbf{z}_{21}$, $\Delta_h = \mathbf{h}_{11}\mathbf{h}_{22} - \mathbf{h}_{12}\mathbf{h}_{21}$, $\Delta_T = \mathbf{AD} - \mathbf{BC}$
$\Delta_y = \mathbf{y}_{11}\mathbf{y}_{22} - \mathbf{y}_{12}\mathbf{y}_{21}$, $\Delta_g = \mathbf{g}_{11}\mathbf{g}_{22} - \mathbf{g}_{12}\mathbf{g}_{21}$, $\Delta_t = \mathbf{ad} - \mathbf{bc}$

## Interconnection of Networks

**TRANSMISSION TWO PORT CASCADE:**

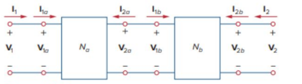



**Figure 19.41**
Cascade connection of two two-port networks.

$$\boxed{[\mathbf{T}] = [\mathbf{T}_a][\mathbf{T}_b]}$$

## Chapter 13: Mutual Inductance and Transformers

$L = N\frac{d\phi}{di}$ **(L is self inductance)**

**Let's consider multiple inductors in a circuit**

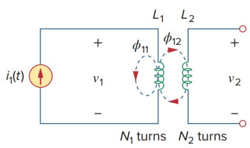



**Figure 13.2**
Mutual inductance $M_{21}$ of coil 2 with respect to coil 1.

$$v_2 = N_2\frac{d\phi_{1,2}}{di_1} * \frac{di_1}{dt} = M_{2,1}\frac{di_1}{dt}$$

$$M_{2,1} = N_2\frac{d\phi_{1,2}}{di_1}$$

**Dot Convention**

If a current **enters** the dotted terminal of one coil, the reference polarity of the mutual voltage in the second coil is **positive** at the dotted terminal of the second coil.

If a current **leaves** the dotted terminal of one coil, the reference polarity of the mutual voltage in the second coil is **negative** at the dotted terminal of the second coil.

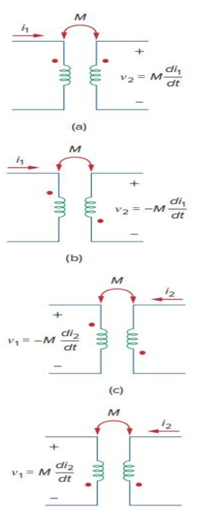



**Figure 13.5**
Examples illustrating how to apply the dot convention.

**SERIES DOT-Convention**

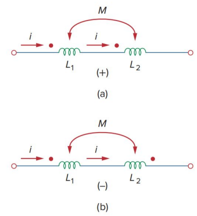



**Figure 13.6**
Dot convention for coils in series; the sign indicates the polarity of the mutual voltage: (a) series-aiding connection, (b) series-opposing connection.

$$\boxed{L = L_1 + L_2 + 2M \qquad \text{(Series-aiding connection)}}$$

For the coils in Fig. 13.6(b),

$$\boxed{L = L_1 + L_2 - 2M \qquad \text{(Series-opposing connection)}}$$

**Modeling Mutual inductance as dependent source:**

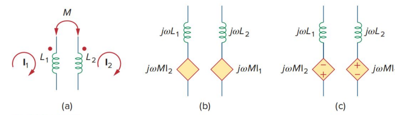



**Figure 13.8**
Model that makes analysis of mutually coupled easier to solve.

**13.3 Energy in a Coupled Circuit**

$$w(t) = \frac{1}{2}L_1 i_1(t)^2 + \frac{1}{2}L_2 i_2(t)^2 \pm M * i_1(t) * i_2(t)$$

**The positive sign is selected for the mutual term if both currents enter or leave the dotted terminals of the coils; the negative sign is selected otherwise**

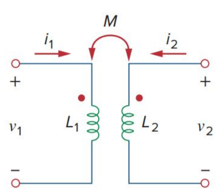



**Figure 13.14**
The circuit for deriving energy stored in a coupled circuit.

**M12 = M21 = M**
**Coupling coefficient:**

$$k = \frac{M}{\sqrt{L_1 * L_2}}$$

$$M = k\sqrt{L_1 * L_2}$$

**Ideal Transformers**

**An ideal transformer is one with perfect coupling ( k =1).**

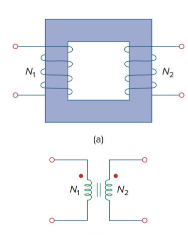



**Figure 13.30**
(a) Ideal transformer, (b) circuit symbol for an ideal transformer.

**A transformer is said to be ideal if it has the following properties: 1. Coils have very large reactances (L1, L2, M → ∞). 2. Coupling coefficient is equal to unity (k = 1). 3. Primary and secondary coils are lossless (R1 = 0 = R2).**

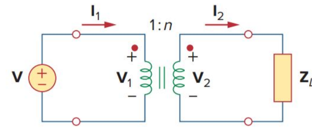



**Figure 13.31**
Relating primary and secondary quantities in an ideal transformer.

**n* is turns ratio or transformation ratio.**

$$\frac{\mathbf{I}_1}{\mathbf{I}_2} = \frac{\mathbf{V}_2}{\mathbf{V}_1} = n$$

1. If $\mathbf{V}_1$ and $\mathbf{V}_2$ are *both* positive or both negative at the dotted terminals, use $+n$ in Eq. (13.52). Otherwise, use $-n$.
2. If $\mathbf{I}_1$ and $\mathbf{I}_2$ *both* enter into or both leave the dotted terminals, use $-n$ in Eq. (13.55). Otherwise, use $+n$.

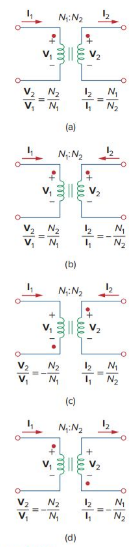



**Figure 13.32**
Typical circuits illustrating proper voltage polarities and current directions in an ideal transformer.

$$\boxed{\mathbf{Z}_{\text{in}} = \frac{\mathbf{Z}_L}{n^2}}$$

**Eliminating ideal transformer and Reflecting one circuit on the other side:**

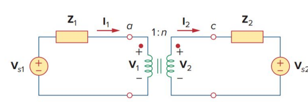



**Figure 13.33**
Ideal transformer circuit whose equivalent circuits are to be found.

Reflecting secondary circuit on primary side:

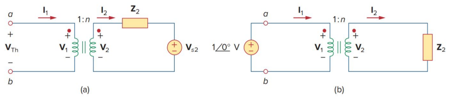


**Figure 13.34**
(a) Obtaining $\mathbf{V}_{Th}$ for the circuit in Fig. 13.33, (b) obtaining $\mathbf{Z}_{Th}$ for the circuit in Fig. 13.33.

$$\mathbf{V}_{Th} = \mathbf{V}_1 = \frac{\mathbf{V}_2}{n} = \frac{\mathbf{V}_{s2}}{n}$$

$$\mathbf{Z}_{Th} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = \frac{\mathbf{V}_2/n}{n\mathbf{I}_2} = \frac{\mathbf{Z}_2}{n^2}, \qquad \mathbf{V}_2 = \mathbf{Z}_2\mathbf{I}_2$$

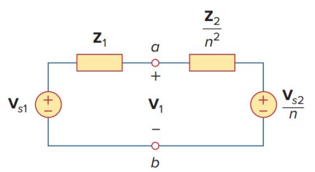



**Figure 13.35**
Equivalent circuit for Fig. 13.33 obtained by reflecting the secondary circuit to the primary side.

The general rule for eliminating the transformer and reflecting the secondary circuit to the primary side is: Divide the secondary impedance by $n^2$, divide the secondary voltage by $n$, and multiply the secondary current by $n$.

**Reflecting primary circuit on secondary side:**

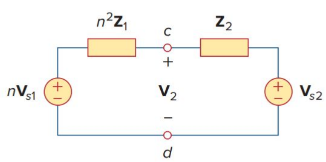



**Figure 13.36**
Equivalent circuit for Fig. 13.33 obtained by reflecting the primary circuit to the secondary side.

The rule for eliminating the transformer and reflecting the primary circuit to the secondary side is: Multiply the primary impedance by $n^2$, multiply the primary voltage by $n$, and divide the primary current by $n$.

**If there is an external connection we cant use the reflection, we have to use mesh or nodal analysis although we can use the main eqns v2/v1 = n = I1/ I2**

**The only reason why current flows into the primary winding because there is a load on the secondary.**

**Z_{primary} -> infinity**

**Because I1 = 0 -> I_2 = 0;**

**The primary is only an open circuit if the secondary is open circuit.**

**Permeance of space:**

$v = \frac{d\lambda}{dt}$

$\lambda = N\phi$

$\lambda$ **= flux linkage measured in [Wb*turns]**

$\phi = pNi$

$L_j = N_j^2 p_j$

**For Linear transformers:(p12 = p21 non-mag materials)**

$M = N_1 N_2 p$

$L_1 = N_1^2 p_1$

$L_2 = N_2^2 p_2$

**Dividing above equations if** $p_1 1 = P_2 2$

$\frac{N_1}{N_2} = \sqrt{\frac{L_1}{L_2}} = \frac{1}{n}$