Chapter 11: AC Power Analysis

(11.1)Effective or RMS VALUE:

$$X_{rms} = \sqrt{\frac{1}{T}\int_0^T x^2\,dt}$$

$X_{rms} = \frac{X}{\sqrt{2}}$ **(only valid for sinusoidal functions)**

If not a sinusoidal then:

$$X_{rms} = \sqrt{\frac{1}{T}(\int_0^{T_1}(x_1^2)dt + \int_{T_1}^{T_2}(x_2)dt + ... + \int_{T_{n-1}}^{T_n}(x_n^2)dt)}$$

The effective value: of a periodic current is the dc current that delivers the same average power to a resistor as the sinusoidal periodic current.

(11.2)COMPLEX POWER

Complex power of an Impedance Z:

$$\mathbf{S} = I_{rms}^2\mathbf{Z} = \frac{V_{rms}^2}{\mathbf{Z}^*} = \mathbf{V}_{rms}\mathbf{I}_{rms}^*$$

Angle of Impedance:

$$\theta_z = angle(\tilde{Z}) = angle(\tilde{S})$$

Complex Power (for any Circuit Element):

$$\text{Complex Power} = \mathbf{S} = P + jQ = \mathbf{V}_{rms}(\mathbf{I}_{rms})^*$$
$$= |\mathbf{V}_{rms}|\,|\mathbf{I}_{rms}|\,\underline{/\theta_v - \theta_i}$$
$$\text{Apparent Power} = S = |\mathbf{S}| = |\mathbf{V}_{rms}|\,|\mathbf{I}_{rms}| = \sqrt{P^2 + Q^2}$$
$$\text{Real Power} = P = \mathrm{Re}(\mathbf{S}) = S\cos(\theta_v - \theta_i)$$
$$\text{Reactive Power} = Q = \mathrm{Im}(\mathbf{S}) = S\sin(\theta_v - \theta_i)$$
$$\text{Power Factor} = \frac{P}{S} = \cos(\theta_v - \theta_i)$$

Lagging power factor when $\theta_v - \theta_i > 0$
Leading power factor when $\theta_v - \theta_i < 0$
Reactive factor: $rf = \sin(\theta_z)$

1. $Q = 0$ for resistive loads (unity pf).
2. $Q < 0$ for capacitive loads (leading pf).
3. $Q > 0$ for inductive loads (lagging pf).

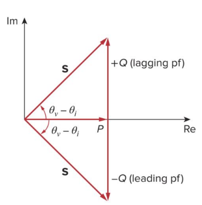


Figure 11.22
Power triangle.

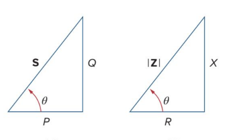


Figure 11.21
(a) Power triangle, (b) impedance triangle.

Conservation Of Complex Power:
(assumed using passive sign conv)
For Series:

$$\mathbf{V} = \mathbf{V}_1 + \mathbf{V}_2$$

$$\mathbf{S} = \mathbf{VI}^* = (\mathbf{V}_1 + \mathbf{V}_2)\mathbf{I}^* = \mathbf{V}_1\mathbf{I}^* + \mathbf{V}_2\mathbf{I}^* = \mathbf{S}_1 + \mathbf{S}_2$$

For parallel:

$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2$$

$$\mathbf{S} = \mathbf{VI}^* = \mathbf{V}(\mathbf{I}_1^* + \mathbf{I}_2^*) = \mathbf{VI}_1^* + \mathbf{VI}_2^* = \mathbf{S}_1 + \mathbf{S}_2$$

$$\tilde{S_1} + \tilde{S_2} + .. + \tilde{S_n} = 0$$

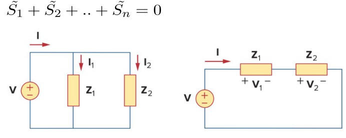


Figure 11.23
An ac voltage source supplied loads connected in: (a) parallel, (b) series.

Maximum Average Power transfer(ALL MAX):

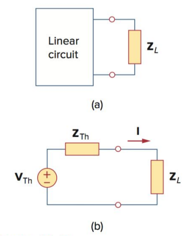


Figure 11.7
Finding the maximum average power transfer: (a) circuit with a load, (b) the Thevenin equivalent.

$$\tilde{Z_{Th}} = R_{Th} + jX_{Th}$$
$$\tilde{Z_L} = R_L + jX_L$$

$$P = \frac{1}{2}|\mathbf{I}|^2R_L = \frac{|\mathbf{V}_{Th}|^2R_L/2}{(R_{Th}+R_L)^2 + (X_{Th}+X_L)^2}$$

After taking the partial of $P(\tilde{Z_L})$
with respect to R_L **and** X_L **we get:**

$$R_L(X_L) = \sqrt{R_{Th}^2 + (X_L + X_{Th})^2}$$
$$X_L = -X_{Th}$$

If ($\tilde{Z_L} = R_L + jX_L$ **):**

$$P_{max} = P(\tilde{Z_{Th}}^*) = \frac{|\tilde{V_{Th}}|^2}{8R_{Th}}$$

If else $(\tilde{Z_L} = R_L)$**:**

$$P_{max} = P(|\tilde{Z_{Th}}|) = \frac{1}{2}\frac{|\tilde{V_{Th}}|^2|\tilde{Z_{Th}}|}{[(|\tilde{Z_{Th}}|+R_{Th}]^2 + X_{Th}^2]}$$

Chapter 12: Three - Phase Circuits(RMS Eqns)

Balanced phase Voltages: are equal in magnitude and are out of phase with each other by 120

$$\tilde{V_{an}} + \tilde{V_{bn}} + \tilde{V_{cn}} = 0$$
$$V_{an} = V_{bn} = V_{cn}$$

Phase Sequence: is the time order in which the voltages pass through their respective maximum values.
Positive Sequence(abc):

$$\mathbf{V}_{an} = V_p\underline{/0^\circ}$$
$$\mathbf{V}_{bn} = V_p\underline{/-120^\circ}$$
$$\mathbf{V}_{cn} = V_p\underline{/-240^\circ} = V_p\underline{/+120^\circ}$$

Negative Sequence(acb):

$$\mathbf{V}_{an} = V_p\underline{/0^\circ}$$
$$\mathbf{V}_{cn} = V_p\underline{/-120^\circ}$$
$$\mathbf{V}_{bn} = V_p\underline{/-240^\circ} = V_p\underline{/+120^\circ}$$

Balanced load: is one in which the phase impedances are equal in magnitude an in phase.

$\Delta -> Y$ Transformations:

$$\tilde{Z_Y} = \frac{1}{3}\tilde{Z_\Delta}$$

$V_{\phi,Y} = \frac{1}{\sqrt{3}}V_{\phi,\Delta}$ **Line-Phase relationship**

Phasor diagrams

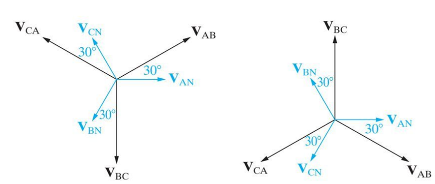


Figure 11.9 Phasor diagrams showing the relationship between line-to-line and line-to-neutral voltages in a balanced system. (a) The positive sequence. (b) The acb sequence.

For negative sequence:

$$\tilde{V_{AB}} = \sqrt{3}\tilde{V_{AN}}\angle -30$$

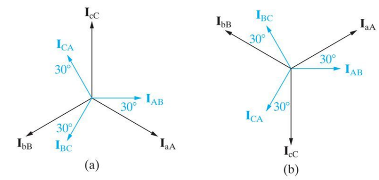


Figure 11.13 Phasor diagrams showing the relationship between line currents and phase currents in a Δ-connected load. (a) The positive sequence. (b) The negative sequence.

For negative sequence:

$$\tilde{I_{AB}} = \sqrt{3}\tilde{I_{aA}}\angle -30$$

EQUATIONS FOR SOLVING 3 - phase

TABLE 12.1

Summary of phase and line voltages/currents for balanced three-phase systems.[1]

Connection	Phase voltages/currents	Line voltages/currents
Y-Y	$\mathbf{V}_{an} = V_p\underline{/0^\circ}$	$\mathbf{V}_{ab} = \sqrt{3}V_p\underline{/30^\circ}$
	$\mathbf{V}_{bn} = V_p\underline{/-120^\circ}$	$\mathbf{V}_{bc} = \mathbf{V}_{ab}\underline{/-120^\circ}$
	$\mathbf{V}_{cn} = V_p\underline{/+120^\circ}$	$\mathbf{V}_{ca} = \mathbf{V}_{ab}\underline{/+120^\circ}$
	Same as line currents	$\mathbf{I}_a = \mathbf{V}_{an}/\mathbf{Z}_Y$
		$\mathbf{I}_b = \mathbf{I}_a\underline{/-120^\circ}$
		$\mathbf{I}_c = \mathbf{I}_a\underline{/+120^\circ}$
Y-Δ	$\mathbf{V}_{an} = V_p\underline{/0^\circ}$	$\mathbf{V}_{ab} = \mathbf{V}_{AB} = \sqrt{3}V_p\underline{/30^\circ}$
	$\mathbf{V}_{bn} = V_p\underline{/-120^\circ}$	$\mathbf{V}_{bc} = \mathbf{V}_{BC} = \mathbf{V}_{ab}\underline{/-120^\circ}$
	$\mathbf{V}_{cn} = V_p\underline{/+120^\circ}$	$\mathbf{V}_{ca} = \mathbf{V}_{CA} = \mathbf{V}_{ab}\underline{/+120^\circ}$
	$\mathbf{I}_{AB} = \mathbf{V}_{AB}/\mathbf{Z}_\Delta$	$\mathbf{I}_a = \mathbf{I}_{AB}\sqrt{3}\underline{/-30^\circ}$
	$\mathbf{I}_{BC} = \mathbf{V}_{BC}/\mathbf{Z}_\Delta$	$\mathbf{I}_b = \mathbf{I}_a\underline{/-120^\circ}$
	$\mathbf{I}_{CA} = \mathbf{V}_{CA}/\mathbf{Z}_\Delta$	$\mathbf{I}_c = \mathbf{I}_a\underline{/+120^\circ}$
Δ-Δ	$\mathbf{V}_{ab} = V_p\underline{/0^\circ}$	Same as phase voltages
	$\mathbf{V}_{bc} = V_p\underline{/-120^\circ}$	
	$\mathbf{V}_{ca} = V_p\underline{/+120^\circ}$	
	$\mathbf{I}_{AB} = \mathbf{V}_{ab}/\mathbf{Z}_\Delta$	$\mathbf{I}_a = \mathbf{I}_{AB}\sqrt{3}\underline{/-30^\circ}$
	$\mathbf{I}_{BC} = \mathbf{V}_{bc}/\mathbf{Z}_\Delta$	$\mathbf{I}_b = \mathbf{I}_a\underline{/-120^\circ}$
	$\mathbf{I}_{CA} = \mathbf{V}_{ca}/\mathbf{Z}_\Delta$	$\mathbf{I}_c = \mathbf{I}_a\underline{/+120^\circ}$
Δ-Y	$\mathbf{V}_{ab} = V_p\underline{/0^\circ}$	Same as phase voltages
	$\mathbf{V}_{bc} = V_p\underline{/-120^\circ}$	
	$\mathbf{V}_{ca} = V_p\underline{/+120^\circ}$	
	Same as line currents	$\mathbf{I}_a = \dfrac{V_p\underline{/-30^\circ}}{\sqrt{3}\mathbf{Z}_Y}$
		$\mathbf{I}_b = \mathbf{I}_a\underline{/-120^\circ}$
		$\mathbf{I}_c = \mathbf{I}_a\underline{/+120^\circ}$

[1]Positive or *abc* sequence is assumed.

Power in a Balanced Load:

Instantaneous power in a three phase balanced system:

$$p_{3\phi} = 3V_\phi I_\phi cos\theta_z$$

This result is true for either y or delta connected load
Per-phase Powers:

$$P_\phi = p/3 = V_{1\phi}I_\phi cos\theta_z$$
$$Q_\phi = V_\phi I_\phi sin\theta_z$$
$$\tilde{S_\phi} = P_{1\phi} + j * Q_{1\phi} = \tilde{V_{1\phi}}\tilde{I_{1\phi}}^*$$

The total average power is the sum of the average powers in the phases:

$$P_{3\phi} = P_a + P_b + P_c = 3P_\phi = 3V_\phi * I_\phi cos\theta_z = \sqrt{3}V_L I_L cos\theta_z$$

For Y-connected load: $I_L = I_\phi \; but \; V_L = \sqrt{3}V_\phi$

For Δ **− connected load:** $I_L = \sqrt{3}I_\phi \; but \; V_L = V_\phi$

$$Q_{3\phi} = 3Q_{3\phi} = \sqrt{3}V_L * I_L sin(\theta_\phi)$$

And then total complex power of a 3 phase system:

$$\tilde{S_{3\phi}} = 3\tilde{S_\phi} = 3\tilde{V_\phi}\tilde{I_\phi}^* = 3{I_\phi}^2\tilde{Z_\phi} = 3\frac{{V_\phi}^2}{\tilde{Z_\phi}^*}$$

OR

$$\tilde{S_{3\phi}} = P_{3\phi} + jQ_{3\phi} = \sqrt{3}V_L I_L \angle\theta_\phi$$

Where :
θ_ϕ **is the phase angle of that element or impedance angle**

Don't make the mistake of:

$$\tilde{S_{3\phi}} \neq \vec{V_L}\tilde{I_L}^* \neq \sqrt{3}V_L I_L \angle\theta_\phi$$

Power Calculation in 3-phase circuits: (TIPS)

See what is given:

1.) Look what you are given

a.)Case 1: given just impedances or Currents/ Voltages:

- Try to change everything to a Y - Y configuration

-> If (there are multiple balanced loads given){
When (L1 and L2 are in parallel):

Transform L1 and L2 to Y forms and connect combine their individual impedances.

Then Make single phase equivalent circuit.

b.)Case 2: given just the a Power quantity

- Notice that $\tilde{S_{\phi,Y}} = \tilde{S_{\phi,\Delta}}$
- That means we can make a single equivalent directly of individual Loads.

c.) case 3: Combination of case 1 and 2

- Try to break this circuit in a single phase equivalent (convert everything to Y)
- For the Load given a Power quantity with a black box use the fact that $\tilde{S_{\phi,Y}} = \tilde{S_{\phi,\Delta}}$

-> Next the circuit build the single phase circuit

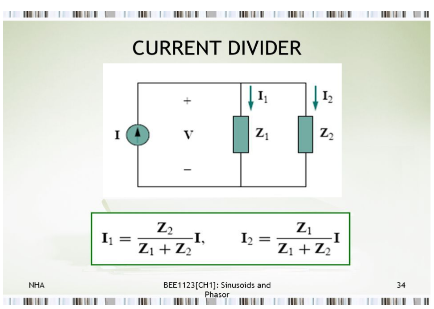
2.)Once you Configured your Circuit. If not given a reference set one:

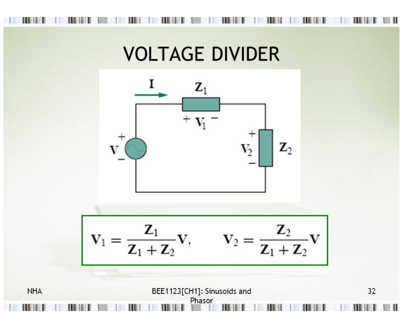
Example: You might be given a magnitude, and there is not way to even solve for a variable need with this magnitude. Therefore give the phasor a reference angle of 0.

If you are given a line voltage but need to use line to neutral voltage make it such that line to voltage phase is your reference.

3.) Solve using or method :

On Calculator function for parallel impedances:
z1(x,y) = x || y





Source Transformations:

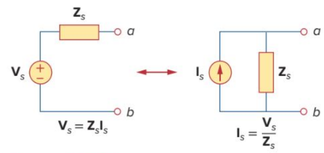


Figure 10.16
Source transformation.

$$\mathbf{V}_s = \mathbf{Z}_s\mathbf{I}_s \quad \Leftrightarrow \quad \mathbf{I}_s = \frac{\mathbf{V}_s}{\mathbf{Z}_s}$$

Three phase - Circuit visuals (assuming Positive sequence):

Y-Y Balanced circuit:

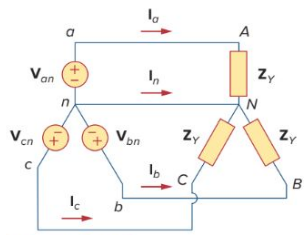


Figure 12.10
Balanced Y-Y connection.

Alternative way of analyzing a balanced Y-Y system:

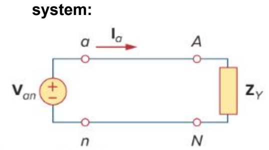


Figure 12.12
A single-phase equivalent circuit.

Balanced Y - Δ :

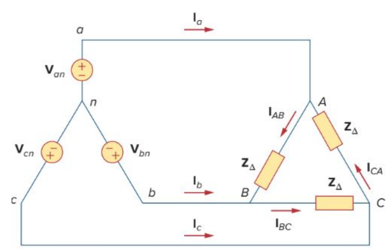


Figure 12.14
Balanced Y-Δ connection.

Alternative way of analyzing a balanced Y- Δ :

Balanced Δ - Y:

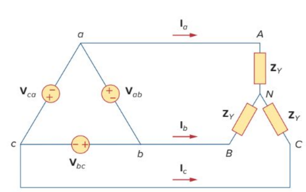


Figure 12.18
A balanced Δ-Y connection.

Alternative way of analyzing Δ - Y balanced:

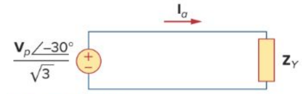


Figure 12.20
The single-phase equivalent circuit.

Balanced Δ - Δ :

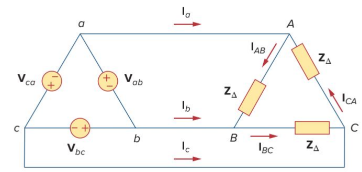
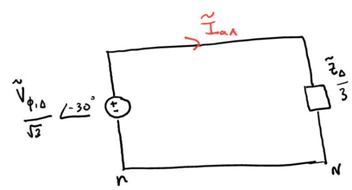
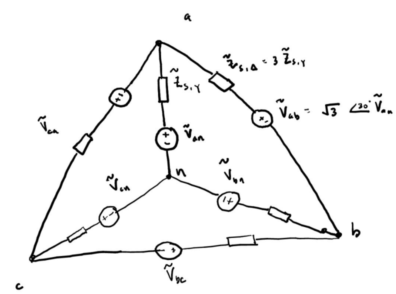


Figure 12.17
A balanced Δ-Δ connection.

Alternative way of analyzing Δ - Y balanced:



A Three phase source with winding impedance:



Help: (POWER LOSS)

What percentage of the average power at the sending end of the line is delivered to the loads?

$$\textbf{\% diff} = \frac{P_{s,\phi} - P_{\ell,\phi}}{P_{s,\phi}} \cdot 100 \ \%$$