

# Characterization of general inverters VTC:

## Threshold voltages:

- \*  $V_{IL}$  - is the maximum allowable input voltage such that the gate is off
- \*  $V_{IH}$  - is the min allowable input voltage such that the gate is on
- \* The above threshold voltages are found by setting  $dV_o/dV_i = -1$
- \* undefined logic level between :  $V_{IL} < V_i < V_{IH}$
- \*  $V_{OH}$  = output when  $v_i = 0$
- \*  $V_{OL}$  = output when  $v_i = V_{DD}$
- \*  $V_{th} = V_{IH} - V_{IL}$
- \* A good logic gate has a small transition width
- \*  $V_{th} = V_{OH} - V_{OL}$
- \* A good logic gate has a large logic swing

**Noise Margin:** ability of a digital logic circuit to maintain its logic state under varying voltage levels

## Noise Margins:

$$NM_H = V_{OH} - V_{IH}$$

$$NM_L = V_{IL} - V_{OL}$$

$$NM = \min(NM_H, NM_L)$$

## Example : VTC of ideal inverter

$$V_{OH} = V_{CC}$$

$$V_{OL} = 0$$

$$V_{IH} = V_{IL} = V_{CC}/2$$

$$NM_H = \frac{V_{DD}}{2}$$

$$NM_L = \frac{V_{DD}}{2}$$

$$V_{th} = 0$$

$$\frac{V_{OH}}{V_{IL}} = \frac{V_{OH}}{V_{IL}} = 1$$

## Propagation Delays:

$$t_p = \frac{t_{PHL} + t_{PLH}}{2} = \text{Max pulse rate for transmission thru it}$$

$$t_{PHL} = \text{time at 50\% output } V_o \text{ falling - time at 50\% input rising } V_i$$

$$t_{PLH} = \text{time at 50\% output } V_o \text{ rising - time at 50\% input falling } V_i$$

$$t_r = \text{rise time (10\% -> 90\%)*} V_o \text{ (just look at output)}$$

$$t_f = \text{fall time (90\% -> 10\%)*} V_o \text{ (just look at output)}$$

$$\text{Circuit Bandwidth} = \frac{35}{t_r}$$

## Diode:

$$I_D = I_s(e^{V_D/(nV_T)} - 1)$$

## Models of diode:

- (1) ideal switch diode model
- (2) ideal switch with voltage offset
  - \* we use this on in this course
- (3) piecewise linear
  - \*  $I_{off} <= .01 I_{on}$
  - \*  $V_{off} = .6V, V_{on} = .7V$

## Limitation of Diode Digital Circuits

- \* Can't implement, NOT, NOR or NAND gates
- \* They have problems, voltage drop across them

## MOSFETS

**Enhancement MOS** - channel isn't already built in (normally off)

**Depletion MOS** - channel is already present (normally on)

## NMOS:

$$\text{Triode section : } V_{GS} > V_{Tn} \text{ and } V_{DS} <= V_{GS} - V_{Tn}$$

$$I_{DS} = k[2(V_{GS} - V_{Tn})V_{DS} - V_{DS}^2]$$

## Saturation region -

$$V_{GS} > V_{Tn} \text{ and } V_{DS} >= V_{GS} - V_{Tn}$$

$$I_{DS} = k(V_{GS} - V_{Tn})^2$$

$$k = (1/2)\mu_n C_{ox} * (W/L) \quad [A/V^2]$$

$$k = (1/2)k'(W/L)$$

## Body Effect

$$V_T = V_{T0} + \gamma * (\sqrt{2|\phi_F|} * V_{SB} - \sqrt{2|\phi_F|})$$

## PMOS - like NMOS but S (VDD) and D (GND)

## Triode section :

$$V_{GS} > |V_T| \text{ and } V_{SD} <= V_{GS} - |V_T|$$

$$I_{SD} = k[2(V_{GS} - |V_T|)V_{SD} - V_{SD}^2]$$

## Saturation region :

$$V_{GS} > |V_T| \text{ and } V_{SD} >= V_{GS} - |V_T|$$

$$I_{SD} = k(V_{GS} - |V_T|)^2$$

## MOS Equiv circuits:

**Transconductance: Q = saturation**

$$g_m = \frac{\Delta I_D}{\Delta V_{GS}} |_{\Delta V_{DS}=0}$$

$$\Rightarrow g_m = 2k * (V_{GS} - V_T)$$

$$\Rightarrow g_m = 2\sqrt{kI_D}$$

**Dynamic Resistance (ac): Q = Saturation**

$$\Rightarrow r_d = \frac{1}{k(V_{GS} - V_T)^2}$$

**Static Resistance (dc): Q= any**

$$R_{DS} = V_{DS}/I_{DS}$$

## MOS inverters (body -source connected)

(Note) -  $V_o$  is taken across  $M_L(S)$  and  $M_D(D)$

## Resistive E-NMOS inverter

\* The Resistor (load) is connected to  $V_{DD}$ , this is then connected to the drain of the E-NMOS (driver)

\* It takes up too much space

## Saturated Enhancement load E-NMOS

\* The E-NMOS ( $M_L$ ) its drain con.to  $V_{DD}$ , its gate connected  $V_{GL}$  to  $V_{SL}$ , this is then connected to the drain of the E-NMOS ( $M_D$ ),  $V_i = V_{GD}$

\* lower  $V_{oh}$ , lower  $NM$ , small  $V_{th}$

\* large  $V_{th}$ , decr exp graph

## Linear Enhancement Load

\* two batteries

## Depletion Load NMOS inverter

\* The D-NMOS ( $M_L$ ), its drain- $\rightarrow V_{DD}$ ,  $M_L$ 's gate is connected to its source, That source terminal connected to drain of driver ( $M_D$ ).

\* Only uses one batt.

Example Problems :

$$k_D(V_i - V_{TD})^2 = k_L[2|V_{TL}|(V_{DD} - V_o) - (V_{DD} - V_o)^2]$$

$$K_R = \text{geometricratio} = \frac{k_D}{k_L}$$

## Design NOR and NAND - using NMOS inverter (M\_L config like inv)

**NAND** -  $M_{D1} \dots M_{Dn}$  in series

\* drivers are the same trans

**NOR** -  $M_{D1} \dots M_{Dn}$  in parallel

\* driver are the same trans

## CMOS inverter

\*  $I_{D,max} = I_D(V_i = V_o)$

$$V_M = V_i = \frac{V_{TN} + \sqrt{K_R(V_{DD} - V_{TN})}}{1 + \sqrt{K_R}}$$

## Symmetrical CMOS inverters

\*  $V_{TN} = |V_{TP}|, k_p = k_n, \text{ Coxsame}, V_M = V_{DD}/2$

$$\Rightarrow K_R = k_p/k_n = 1 \Rightarrow \frac{\mu_n}{\mu_p} = \frac{(W/L)_p}{(W/L)_n}$$

$$\begin{aligned} & \text{Usually } \mu_n/\mu_p = 2.5 \Rightarrow (W/L)_p = 2.5 * (W/L)_n \\ & 2.5 * (W/L)_p = 2.5 * (W/L)_n \Rightarrow (W/L)_p = 2.5 * (W/L)_n \\ & 2.5 * (W/L)_p = 2.5 * (W/L)_n \Rightarrow (W/L)_p = 2.5 * (W/L)_n \\ & 2.5 * (W/L)_p = 2.5 * (W/L)_n \Rightarrow (W/L)_p = 2.5 * (W/L)_n \\ & 2.5 * (W/L)_p = 2.5 * (W/L)_n \Rightarrow (W/L)_p = 2.5 * (W/L)_n \\ & 2.5 * (W/L)_p = 2.5 * (W/L)_n \Rightarrow (W/L)_p = 2.5 * (W/L)_n \\ & 2.5 * (W/L)_p = 2.5 * (W/L)_n \Rightarrow (W/L)_p = 2.5 * (W/L)_n \end{aligned}$$

## Design NOR and NAND - using CMOS inverter

**Two input NOR gate (2 Mps and 2 MNs)**

\*  $M_{p1}$ 's connected in series (top near  $V_{DD}$ ), ( $G_N$ )  $M_{N1}$ 's

connected in parallel,  $M_{p1}$  and  $M_{N1}$

source and drain connected

Gates of  $M_{p1}$  and  $M_{N1}$  are connected,

$N_{N2}$  and  $M_{p2}$  gates are connected.

Output taken

across  $M_{p1}$  and  $M_{N1}$

$$1/G_p = 1/g + 1/g \Rightarrow g = 2G_p$$

$$(W/L)_{p,tot} = 2(W/L)_p$$

$$(W/L)_{n,tot} = (W/L)_n$$

**Two input NAND gate (2 Mps and 2 MNs)**

\*  $M_{N1}$ 's connected in series near GND,

Output is taken across the intersection

of  $M_{N1}$  and  $M_{P1}$ ,  $M_{P1}$  and  $M_{P2}$

are connected in parallel, gates are

connected

from each  $M_{Ni}$  to  $M_{Pi}$  like inverter.

$$1/G_N = 1/g + 1/g \Rightarrow g = 2G_N$$

$$(W/L)_{n,tot} = 2(W/L)_n$$

$$(W/L)_{p,tot} = (W/L)_p$$

## CMOS VTC Regions

$V_i$	$0 < V_i < V_{TN}$	$V_{TN} < V_i < V_M$	$V_M$	$V_M < V_i < V_{DD} -  V_{TP} $	$< V_i$
$M_P$	Ohmic	ohmic	sat	sat	off
$M_N$	OFF	sat	sat	ohmic	ohmic

## Examples(NMOS - CMOS gates):

**EE 307 Solved Examples 1: Design of NMOS and CMOS Gates**

\* Gate designs are based on good inverter designs.

- For a depletion NMOS inverter and a depletion NMOS 2-input NAND gate with equal logic levels,  $K_D = 45 \mu A/V^2$ ,  $K_L = 40 \mu A/V^2$ ,  $V_{TN0} = 1.5V$ ,  $V_{TD0} = -3V$ . For the inverter,  $(\frac{W}{L})_D = 2$ . The body effect is to be ignored and  $V_{os} = 0.25V$  for both gates when  $V_{DD} = 5V$ .
- Complete the design of the inverter by finding its  $(\frac{W}{L})_L$ .
- Complete the design of the 2-input NMOS NAND gate by finding its  $(\frac{W}{L})_D$  &  $(\frac{W}{L})_L$ .

**Solution**

For inverter:  $V_{DD} = 5V$ ,  $V_{TD} = V_{TN0} = 1.5V$ ,  $V_{TL} = V_{TN0} = 1.5V$

For driver:  $V_{DS0} = V_{OL} = 0.25V < (V_{GS0} - V_{TD}) = (5 - 1.5)V \Rightarrow$  ohmic

$$I_{DD0} = \frac{1}{2} K_D (\frac{W}{L})_D (V_{GS0} - V_{TD})^2 = \frac{1}{2} K_D (\frac{W}{L})_D (5 - 1.5)^2$$

$$I_{DD0} = \frac{1}{2} K_D (\frac{W}{L})_D (3.5)^2 = 1.6875 K_D$$

For Load:  $V_{GS0} = V_{DD} - V_{OL} = 4.75V > (V_{GS0} - V_{TL}) = (5 - 1.5)V \Rightarrow$  saturation

$$I_{DL} = \frac{1}{2} K_L (\frac{W}{L})_L (V_{GS0} - V_{TL})^2 = 4.5 K_L (\frac{W}{L})_L$$

$$I_{DD0} = I_{DL} \Rightarrow 1.6875 K_D = 4.5 K_L (\frac{W}{L})_L$$

$$(\frac{W}{L})_L = 0.422$$

- Load for inverter and NAND are identical  $\Rightarrow (\frac{W}{L})_L = 0.422$
- $(\frac{W}{L})_D$  increases proportionately with # of inputs
- $(\frac{W}{L})_D = 4 * (\frac{W}{L})_D \text{ for inverter}$

**CMOS GATE DESIGN**

Example 3: For a symmetrical CMOS inverter and a symmetrical 3-input CMOS NOR gate with a  $5\mu m$  process,  $\mu_n/\mu_p = 2.5$  and  $(\frac{W}{L})_n = \frac{20\mu m}{5\mu m}$  for the inverter.

- Find  $W_p$  &  $L_p$  for the PMOS transistor in the inverter
- Sketch the 3-input CMOS NOR gate and find  $W_n$  &  $L_n$  for its NMOS transistors
- Find  $W_p$  &  $L_p$  for the PMOS transistors in the 3-input NOR gate.

**Solution**

(a) For the inverter,  $(\frac{W}{L})_p = \frac{\mu_n}{\mu_p} (\frac{W}{L})_n = 2.5 (\frac{20\mu m}{5\mu m})$

$$(\frac{W}{L})_p = \frac{50\mu m}{5\mu m} \Rightarrow W_p = 50\mu m, L_p = 5\mu m$$

$(\frac{W}{L})_n$  is same for inverter & each NMOS in the NOR gate

$$(\frac{W}{L})_n = \frac{20\mu m}{5\mu m}$$

$$W_n = 20\mu m, L_n = 5\mu m$$

- For the 3-input MOS NOR,  $(\frac{W}{L})_p = 3(\frac{W}{L})_{inverter} = \frac{150\mu m}{5\mu m}$
- $$W_p = 150\mu m, L_p = 5\mu m$$

**Example 3: CMOS NOR & NAND Channel Areas on a Chip**

For a symmetrical CMOS inverter with a  $5\mu m$  process,  $\mu_n/\mu_p = 2.5$ ,  $(\frac{W}{L})_n = \frac{20\mu m}{5\mu m}$ . A 1-input CMOS NOR & CMOS NAND are designed based on the inverter design such that at the same  $V_{DD}$ , they all have same logic levels.

- Sketch the CMOS NOR & the CMOS NAND
- Determine the PMOS & NMOS areas on the chip for the inverter
- Determine the PMOS & NMOS areas on the chip for the 1-input CMOS NOR
- Determine the PMOS & NMOS areas on the chip for the 1-input CMOS NAND

**Solution**

(a) Sketch of 1-input CMOS NOR: Extend sketch of 3-input CMOS NOR of example 2 by one more input.

(b) PMOS & NMOS Areas on chip for inverter

$\mu_n/\mu_p = 2.5$ ;  $(\frac{W}{L})_n = \frac{20\mu m}{5\mu m}$  (given)

$$(\frac{W}{L})_p = \frac{\mu_n}{\mu_p} (\frac{W}{L})_n = 2.5 (\frac{20\mu m}{5\mu m}) = \frac{50\mu m}{5\mu m}$$

$$A_n = 20 \times 5 \mu m^2 = 100 \mu m^2$$

$$A_p = 50 \times 5 \mu m^2 = 250 \mu m^2$$

(c) PMOS & NMOS areas on chip for 1-input CMOS NOR

$G_{N,gate} = G_{N,inverter} \Rightarrow (\frac{W}{L})_{N,gate} = (\frac{W}{L})_{N,inverter} = \frac{20\mu m}{5\mu m}$

$G_{P,gate} = G_{P,inverter} \Rightarrow (\frac{W}{L})_{P,gate} = (\frac{W}{L})_{P,inverter} = \frac{50\mu m}{5\mu m}$

$$A_n = 20 \times 5 \mu m^2 = 100 \mu m^2$$

$$A_p = 200 \times 5 \mu m^2 = 1000 \mu m^2$$

(d) PMOS & NMOS Areas on Chip for 1-input CMOS NAND

$G_{N,gate} = G_{N,inverter} \Rightarrow (\frac{W}{L})_{N,gate} = (\frac{W}{L})_{N,inverter} = \frac{20\mu m}{5\mu m}$

$G_{P,gate} = G_{P,inverter} \Rightarrow (\frac{W}{L})_{P,gate} = (\frac{W}{L})_{P,inverter} = \frac{50\mu m}{5\mu m}$

$$A_n = 20 \times 5 = 100 \mu m^2$$

$$A_p = 50 \times 5 = 250 \mu m^2$$

\* **Comparison**

Gate type	Area in $\mu m^2$	$A_p$	$A_n$
Inverter	100	250	100
1-input NOR	100	1000	100
1-input NAND	400	250	100

\* Areas of both CMOS NOR & CMOS NAND increase with fan-in (# of inputs), but areas increase less for NOR. Thus NAND is preferred in CMOS; NOR is standard gate in CMOS.

