PointCNN

Yangyan Li, Rui Bu, Mingchao Sun, Baoquan Chen

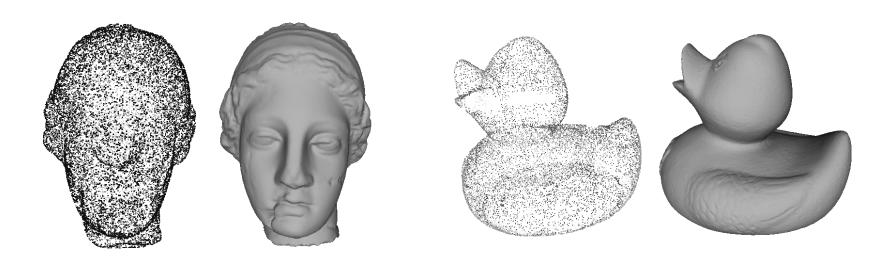
HANYANG UNIV. AI LAB

BYEONGJO KIM

Problem - Point Cloud

Point Cloud

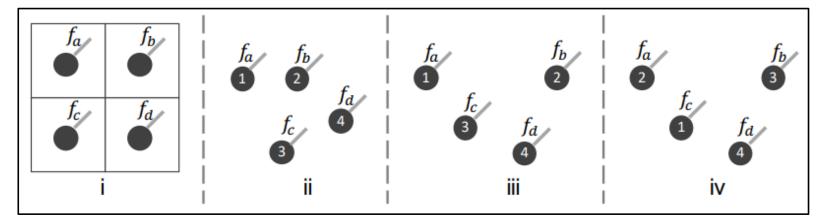
- more effective in 3D space, or line sketches in 2D (less dimension)
- irregular, unordered



Problem - Point Cloud

Point Cloud

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$$f_{ii} = Conv(\mathbf{K}, [f_a, f_b, f_c, f_d]^T),$$

$$f_{iii} = Conv(\mathbf{K}, [f_a, f_b, f_c, f_d]^T),$$

$$f_{iv} = Conv(\mathbf{K}, [f_c, f_a, f_b, f_d]^T).$$

$$f_{ii} \equiv f_{iii}$$

$$f_{iii} \neq f_{iv}$$

$$f_{ii} = f_{iii}$$

$$f_{iii} = f_{iii}$$

$$f_{iii} = f_{ii}$$

$$f_{iii} = Conv(\mathbf{K}, X_{iii} \times [f_a, f_b, f_c, f_d]^T),$$

$$f_{iv} = Conv(\mathbf{K}, X_{iv} \times [f_c, f_a, f_b, f_c]^T),$$

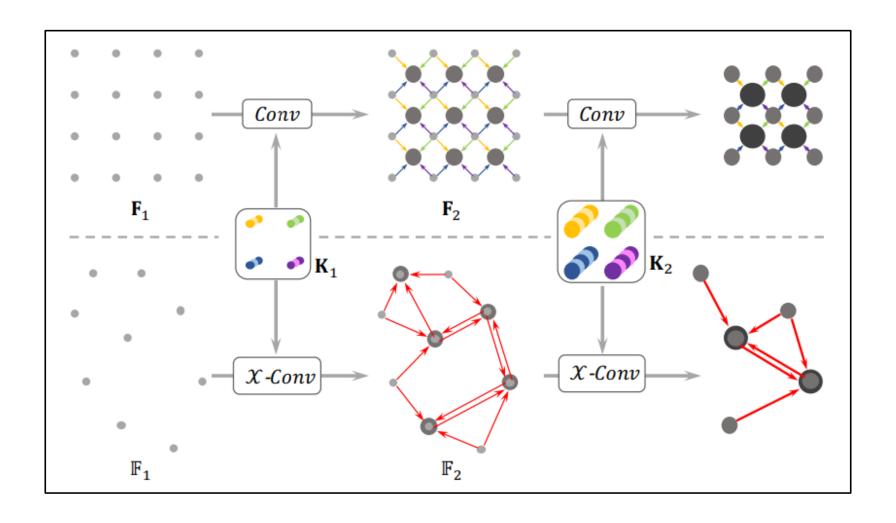
$$X = MLP(p_1, p_2, ..., p_K)$$

PointCNN

Hierarchical Convolution

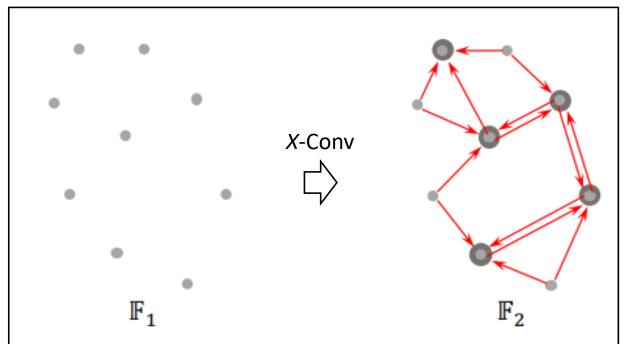
X-Conv Operator

Hierarchical Convolution



Hierarchical Convolution

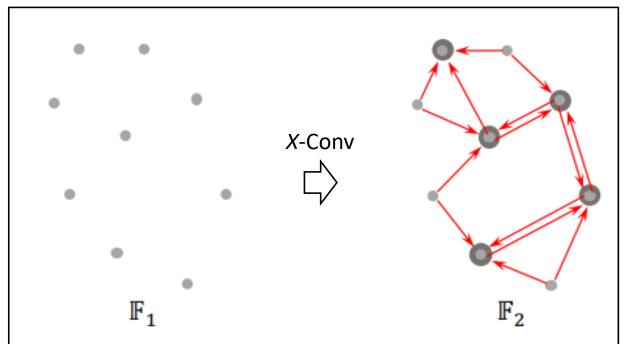
$$\mathbb{F}_1 = \{(p_{1,i},f_{1,i}): i=1,2,...,N_1\} \mid \{p_{1,i}:p_{1,i}\in\mathbb{R}^D\} \mid \{f_{1,i}:f_{1,i}\in\mathbb{R}^{C_1}\} \mid C: feature channel depth \}$$
 $\mathbb{F}_2 = \{(p_{2,i},f_{2,i}): f_{2,i}\in\mathbb{R}^{C_2}, i=1,2,...,N_2\}$



- $N_2 < N_1$, and $C_2 > C_1$
- $\{p_{2,i}\}$ is $\{p_{1,i}\}$'s representative points but not necessarily a subset of $\{p_{1,i}\}$
- $\{p_{2,i}\}$ is $\{p_{1,i}\}$'s random down-sampling for classification tasks farthest point sampling for segmentation tasks

Hierarchical Convolution

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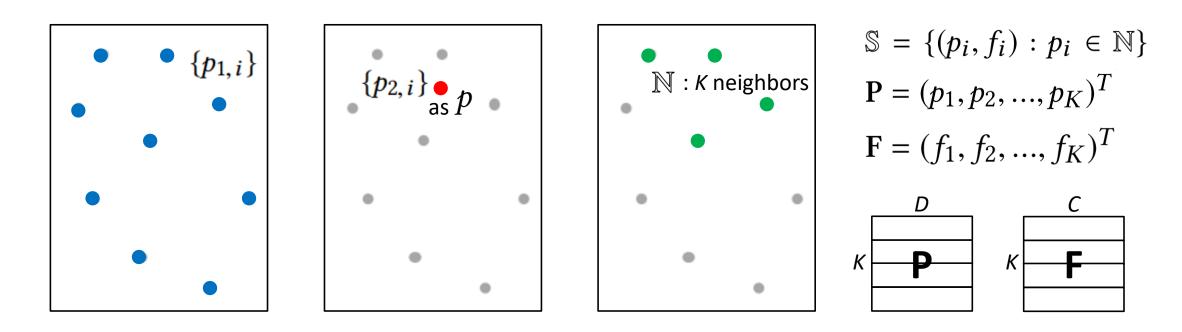
```
\mathbf{F}_{p} = \mathcal{X} - Conv(\mathbf{K}, p, \mathbf{P}, \mathbf{F})
= Conv(\mathbf{K}, MLP(\mathbf{P} - p) \times [MLP_{\delta}(\mathbf{P} - p), \mathbf{F}]),
```

ALGORITHM 1: X-Conv Operator

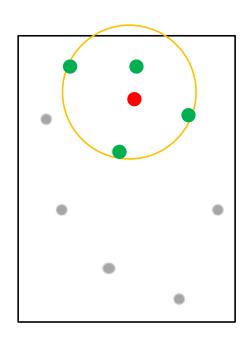
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Input : K, p, P, FOutput: F_p> Features "projected", or "aggregated", to p1: P' \leftarrow P - p> Move P to local coordinate system of p2: F_{\delta} \leftarrow MLP_{\delta}(P')> Individually lift each point into C_{\delta} dim. space3: F_* \leftarrow [F_{\delta}, F]> Concatenate F_{\delta} and F_*, F_* is a K \times (C_{\delta} + C_1) matrix4: X \leftarrow MLP(P')> Learn the K \times K X-transformation matrix5: F_{\mathcal{X}} \leftarrow X \times F_*> Weight and permute F_* with the learnt X6: F_p \leftarrow Conv(K, F_X)> Finally, typical convolution between K and F_X
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```

ALGORITHM 1: X-Conv Operator



 $\mathbf{F}_{m{p}}$: projected or aggregated into the representative point



K nearest neighbor search for extracting the K neighboring points

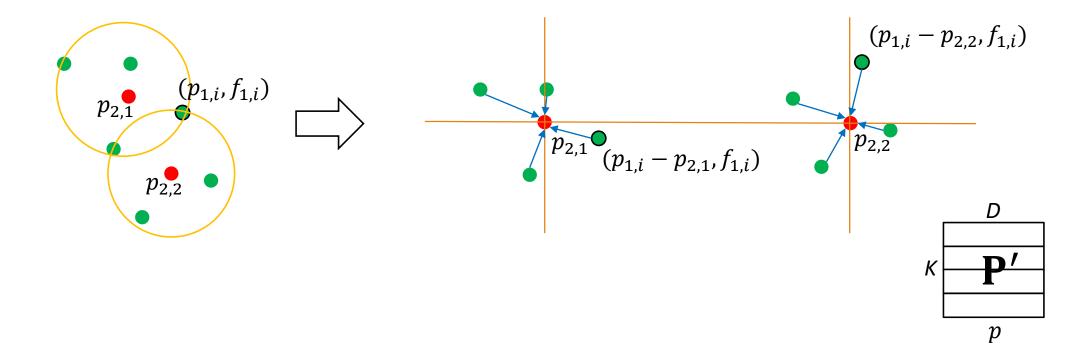
if point cloud with non-uniform point distribution radius search randomly sample K points out of the radius search results

```
\mathbf{F}_{p} = \mathcal{X} - Conv(\mathbf{K}, p, \mathbf{P}, \mathbf{F})
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Relative positions

Build local coordinate systems at the representative points



```
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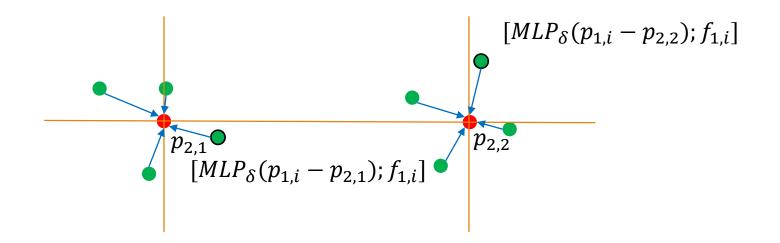
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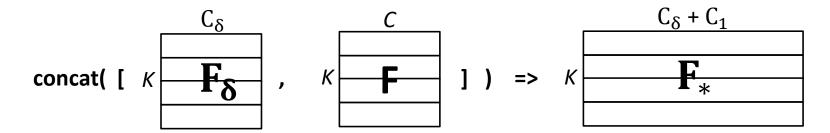
word, besides the associated features, that defines the output features. In other word, besides the associated features, the local coordinates themselves are part of the input features as well. However, the local coordinates are of quite different dimensionality and representation than the associated features. We first lift the coordinates into an higher dimensional and more abstract representation ($\mathbf{F}_{\delta} \leftarrow \mathit{MLP}_{\delta}(\mathbf{P'})$,

$$K = \sum_{k=0}^{D} \sum_{k=0}^{C_{\delta}} \sum_{k=0}^{C_{$$

```
\mathbf{F}_{p} = \mathcal{X} - Conv(\mathbf{K}, p, \mathbf{P}, \mathbf{F})
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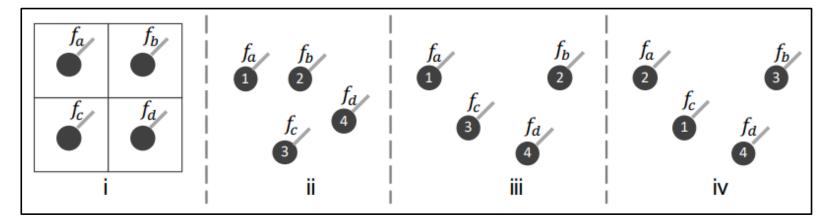


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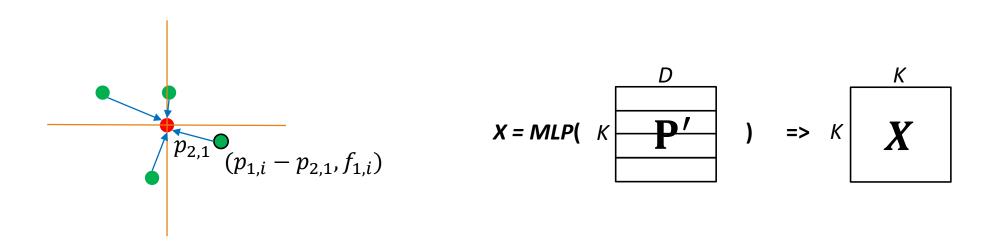
$$f_{iv} = Conv(\mathbf{K}, X_{iv} \times [f_c, f_a, f_b, f_d]^T),$$

$$X = MLP(p_1, p_2, ..., p_K)$$

The lifted features are weighted and permuted by X-transformation with the associated features

Not individually, but together(be applied on the entire neighboring points)

X is dependent on the order of the points.



```
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X is supposed to permute F_* according to the input points.

It has to be aware of the specific input order.

$$\mathbf{F}_{X} = \kappa \boxed{\begin{array}{c} \kappa \\ \mathbf{X} \end{array}} \times \kappa \boxed{\begin{array}{c} C_{\delta} + C_{1} \\ \hline \mathbf{F}_{*} \end{array}} = > \kappa \boxed{\begin{array}{c} C_{\delta} + C_{1} \\ \hline \mathbf{F}_{\mathbf{X}} \end{array}}$$

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$$F_p = Conv(\mathbf{K}, F_X)$$

K => trainable kernel

$$\mathbf{F_p} = \mathbf{Conv}(\ \kappa)$$
 \mathbf{K}
 $\mathbf{F_X}$
 $\mathbf{F_X}$
 $\mathbf{F_X}$
 $\mathbf{F_X}$

ALGORITHM 1: X-Conv Operator

4: $X \leftarrow MLP(\mathbf{P'})$

5: $\mathbf{F}_{\mathcal{X}} \leftarrow \mathcal{X} \times \mathbf{F}_*$

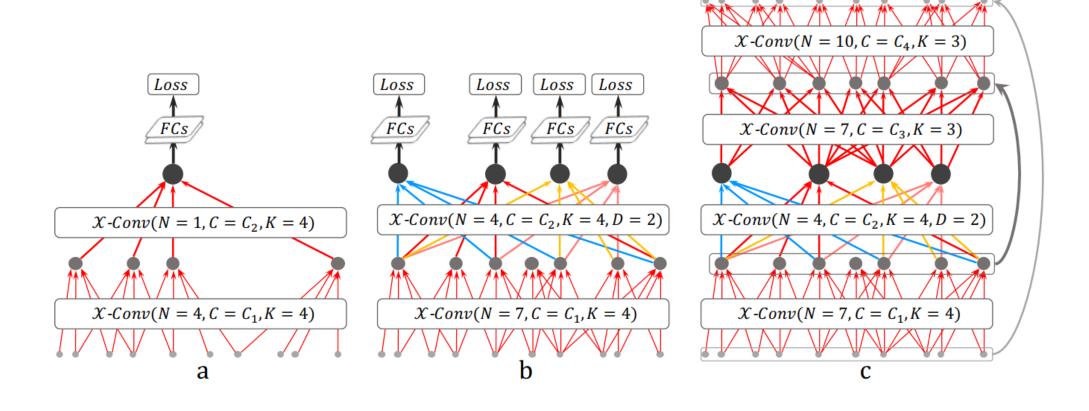
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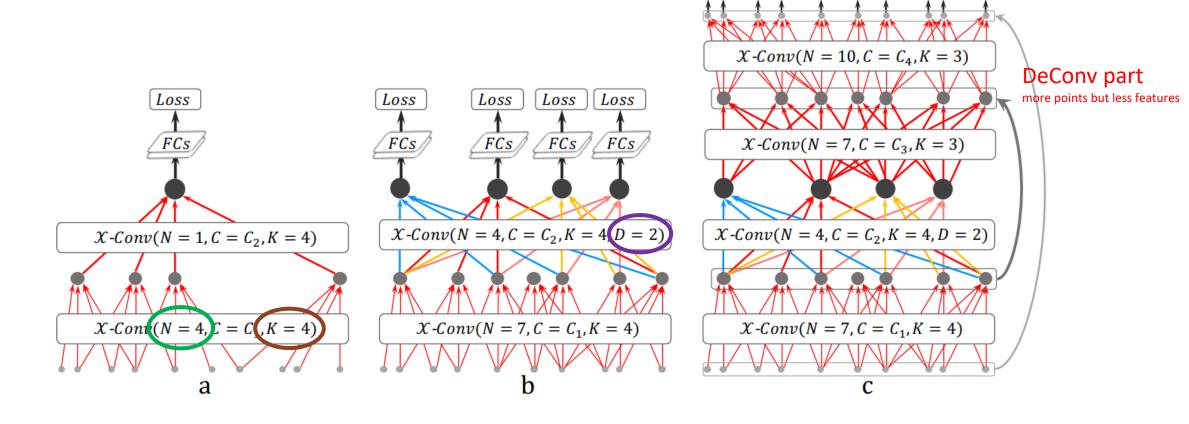
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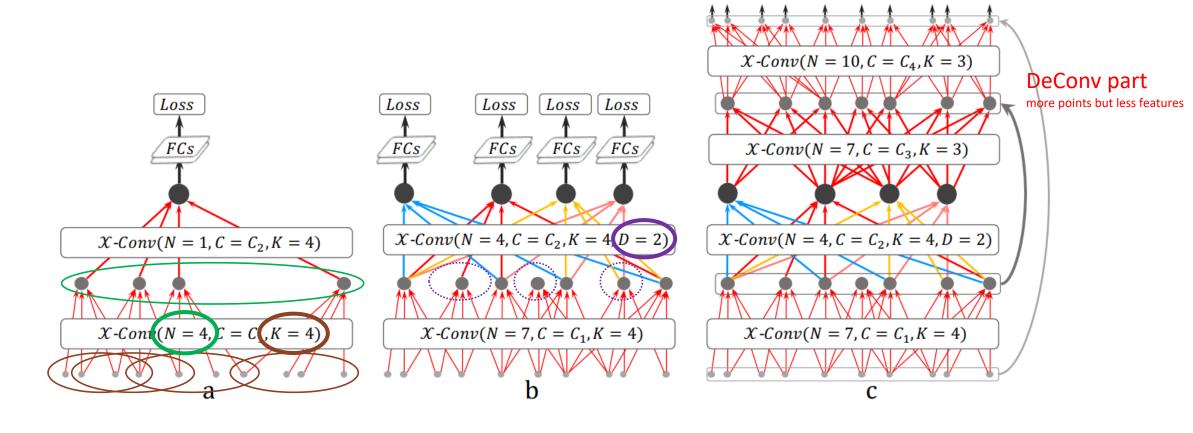
6: \mathbf{F}_{p} ← Conv(\mathbf{K} , \mathbf{F}_{χ}) \triangleright Finally, typical convolution between \mathbf{K} and \mathbf{F}_{χ}

 \triangleright Learn the $K \times K$ X-transformation matrix

 \triangleright Weight and permute \mathbf{F}_* with the learnt \mathcal{X}

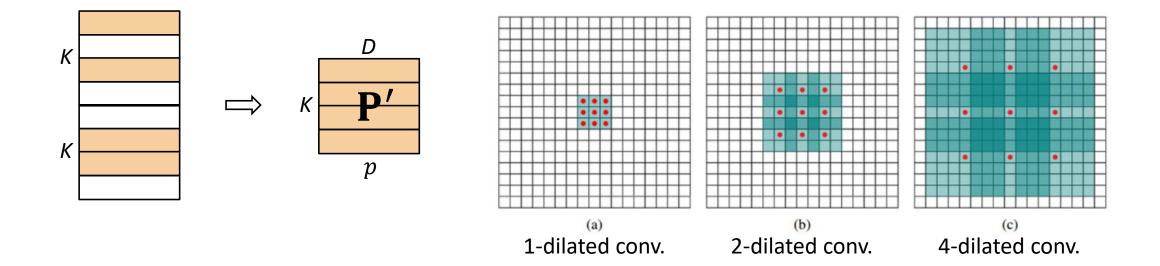






maintain the depth of the network && keeping the receptive field growth rate

=> Dilation rate (D)



Other important things

using **ELU** nonlinear activation function

batch normalization is applied on P', F_p not F_* , X

using ADAM optimizer

0.01 learning rate

dropout before the last fully connected layer for reducing over-fitting

not beneficial if the neighboring points are always the same set in the same order

- randomly sample and shuffle the input points
- neighboring point sets and order can be different from batch to batch => Data augmentation
- N points as input => $Gaussian \ distribution \ N(N, \left(\frac{N}{8}\right)^2)$ points are used for training

PointCNN model zoo

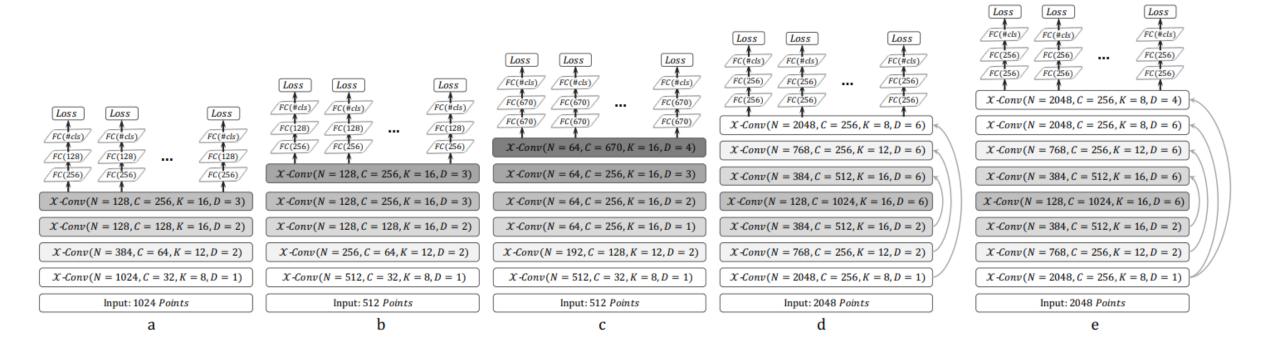


Figure 8: PointCNN model zoo, where (a) is used for ModelNet40 and ScanNet classification, (b) is used for TU-Berlin sketch classification, (c) is used for Quick Draw sketch classification, (d) is used for ScanNet and S3DIS segmentation, and (e) is used for ShapeNet Parts segmentation.

Better Than State-of-the-art

cloud, thus we call it *PointCNN*. Experiments show that PointCNN achieves on par or better performance than state-of-the-art methods on multiple challenging benchmark datasets and tasks.

Discussions and Future Work

- > Strong over-fitting on small datasets.
- > Is there some structural constraints in X?
- > In PointCNN the learnt features are "projected" or "aggregated" at the specific representative points
- outperform at tasks where locations matter more.
- > PointCNN performs quite well in learning the digits' shape information.
 - the sparser the data is, the more prominent the advantage of PointCNN can be observed.
- > Study the principle criteria for making the choice (CNN+dense or PointCNN+point cloud)
- > PointCNN + CNN for 3D model.