Business Data Mining Semester 2, 2019

Lecture 8 Modeling with Linear Regression

Kim, Yang Sok Dept. of MIS, Keimyung University

- Introduction
- Linear Regression
- Exercises
 - Exercise 1: Linear Regression with Rapidminer
 - Exercise 2: Linear Regression after Removing Insignificant Attributes
 - Exercise 3: Linear Regression with Variance Filter
 - Exercise 4: Linear Regression with Correlation Filter
 - Exercise 5: Linear Regression with Weight Filter
 - Exercise 6: Linear Regression with Forward Selection
 - Exercise 7: Linear Regression with Backward Elimination
 - Exercise 8: Linear Regression with Model Explainer
 - Exercise 9: Linear Regression with Model Simulator
- Conclusion

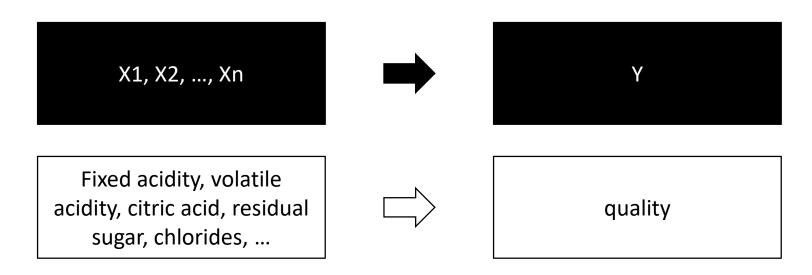
Introduction

- In this lecture, we also learn linear regression technique, a wellknown prediction technique, accompanying with above issues
- · In addition, we will focus on the following two issues
 - Feature selection: How to select subset of attributes that are appropriate for model learning?
 - Parameter settings: How to set appropriate parameters for model learning?

Linear Regression

Concept

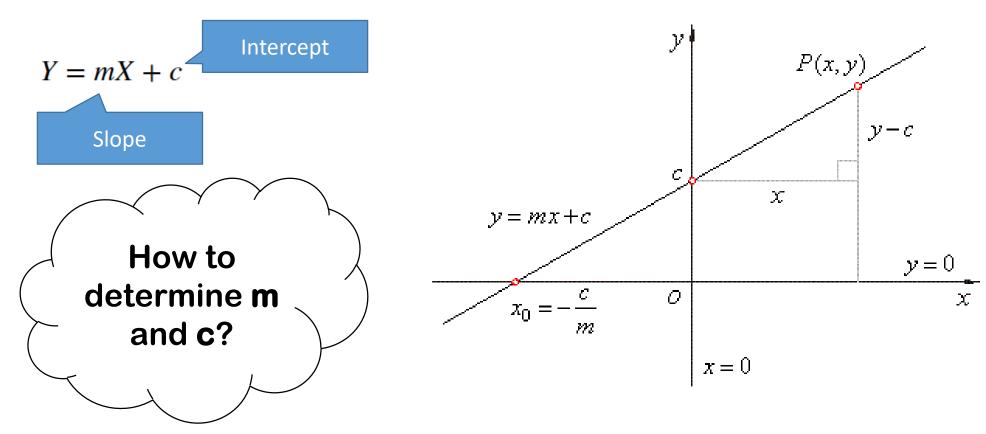
 Linear regression is a method for modeling the relationship between one or more independent variables and a dependent variable.



fixed acidity	volatile acidity	citric acid	residual sug	chlorides	free sulfur d	total sulfur d	density	pH	sulphates	alcohol	quality
7.400	0.700	0	1.900	0.076	11	34	0.998	3.510	0.560	9.400	5
7.800	0.880	0	2.600	0.098	25	67	0.997	3.200	0.680	9.800	5
7.800	0.760	0.040	2.300	0.092	15	54	0.997	3.260	0.650	9.800	5
11.200	0.280	0.560	1.900	0.075	17	60	0.998	3.160	0.580	9.800	6
7.400	0.700	0	1.900	0.076	11	34	0.998	3.510	0.560	9.400	5

Linear Regression

Let X be the independent variable and Y be the dependent variable.
 We will define a linear relationship between these two variables as follows:



Concept

- Linear regression was developed in the field of statistics, but has been borrowed by machine learning.
- Linear regression is a linear model, e.g. a model that assumes a linear relationship between the input variables (x) and the single output variable (y). More specifically, that y can be calculated from a linear combination of the input variables (x).
- When there is a single input variable (x), the method is referred to as simple linear regression. When there are multiple input variables, literature from statistics often refers to the method as multiple linear regression.

Loss Function

- The loss is the error in our predicted value of m and c. Our goal is to minimize this error to obtain the most accurate value of m and c.
- We will use the Mean Squared Error function to calculate the loss.
 There are three steps in this function:
- 1. Find the difference between the actual y and predicted y value(y = mx + c), for a given x.
- 2. Square this difference.
- 3. Find the mean of the squares for every value in X.

$$E = \frac{1}{n} \sum_{i=0}^{n} (y_i - \bar{y}_i)^2$$

How to Learn Linear Regression Model?

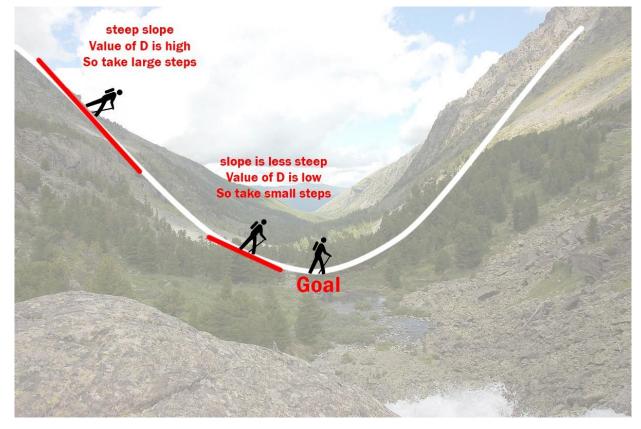
• Here y_i is the actual value and \bar{y}_i is the predicted value. Lets substitute the value of \bar{y}_i :

$$E = rac{1}{n} \sum_{i=0}^n (y_i - (mx_i + c))^2$$

 So we square the error and find the mean. hence the name Mean Squared Error. Now that we have defined the loss function, lets get into the interesting part — minimizing it and finding m and c.

The Gradient Descent Algorithm

- Gradient descent is an iterative optimization algorithm to find the minimum of a function. Here that function is our Loss Function.
- Understanding Gradient Descent



The Gradient Descent Algorithm

- 1. Initially let m = 0 and c = 0. Let L be our learning rate. This controls how much the value of m changes with each step. L could be a small value like 0.0001 for good accuracy.
- 2. Calculate the partial derivative of the loss function with respect to m, and plug in the current values of x, y, m and c in it to obtain the derivative value D

$$D_m = rac{1}{n} \sum_{i=0}^n 2(y_i - (mx_i + c))(-x_i) \ D_m = rac{-2}{n} \sum_{i=0}^n x_i (y_i - ar{y}_i)$$

 $D_{\rm m}$ is the value of the partial derivative with respect to m. Similarly lets find the partial derivative with respect to c, Dc :

$$D_c = rac{-2}{n} \sum_{i=0}^n (y_i - ar{y}_i)$$

The Gradient Descent Algorithm

3. Now we update the current value of **m** and **c** using the following equation:

$$m = m - L \times D_m$$

$$c = c - L \times D_c$$

4. We repeat this process until our loss function is a very small value or ideally 0 (which means 0 error or 100% accuracy). The value of **m** and **c** that we are left with now will be the optimum values.

Preparing Data For Linear Regression

Linear Assumption

• Linear regression assumes that the relationship between your input and output is linear. This may be obvious, but it is good to remember when you have a lot of attributes. You may need to transform data to make the relationship linear (e.g. log transform for an exponential relationship).

Remove Noise

Linear regression assumes that your input and output variables are not noisy.
 Consider using data cleaning operations that let you better expose and clarify the signal in your data. This is most important for the output variable and you want to remove outliers in the output variable (y) if possible.

Remove Collinearity

 Linear regression will over-fit your data when you have highly correlated input variables. Consider calculating pairwise correlations for your input data and removing the most correlated.

Linear Regression

Learning Approaches for Linear Regression Model

Gaussian Distributions

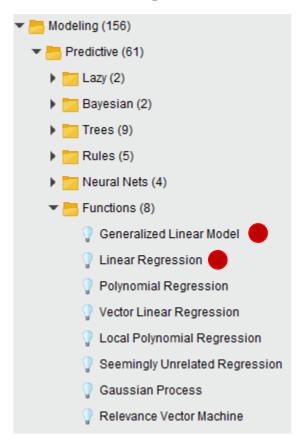
 Linear regression will make more reliable predictions if your input and output variables have a Gaussian distribution. You may get some benefit using transforms (e.g. log or BoxCox) on you variables to make their distribution more Gaussian looking.

Rescale Inputs

• Linear regression will often make more reliable predictions if you rescale input variables using standardization or normalization.

Linear Regression in Rapidminer

 Rapidminer provides <<Linear Regression>> in 'Modeling>Predictive>Function' package



Exercise 1: Linear Regression with Rapidminer

Task & Process

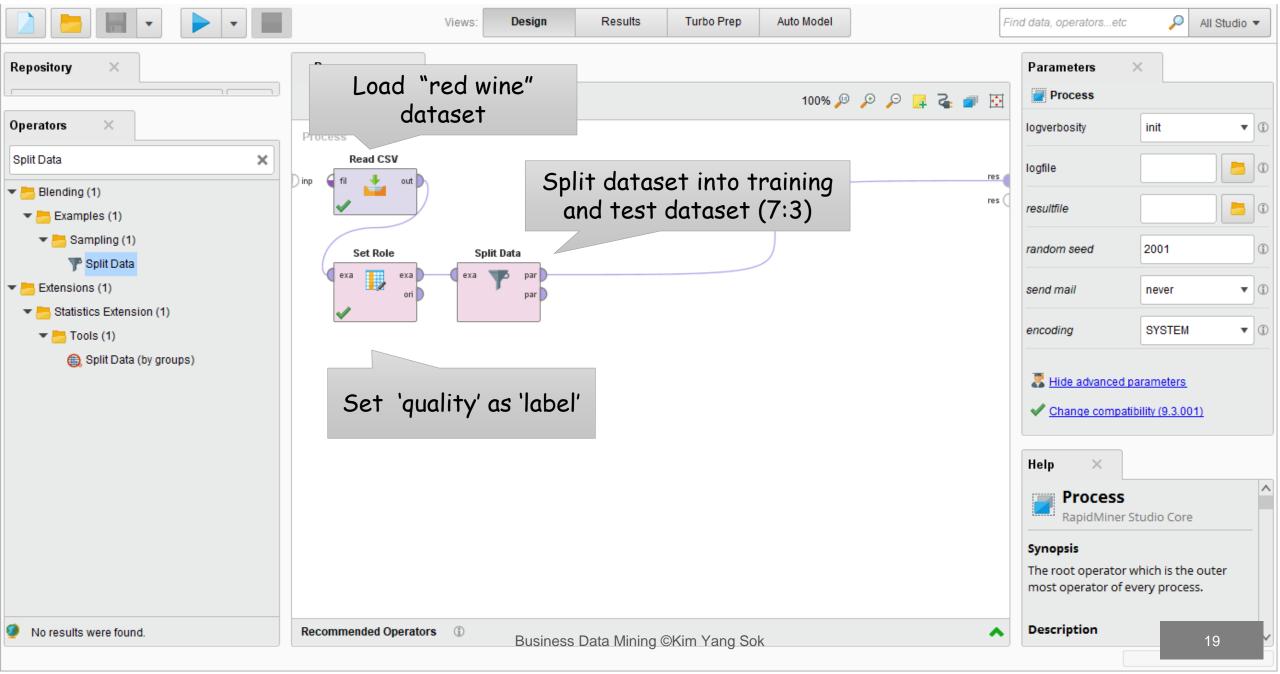
Task

· After loading data, build a linear regression model using split test design

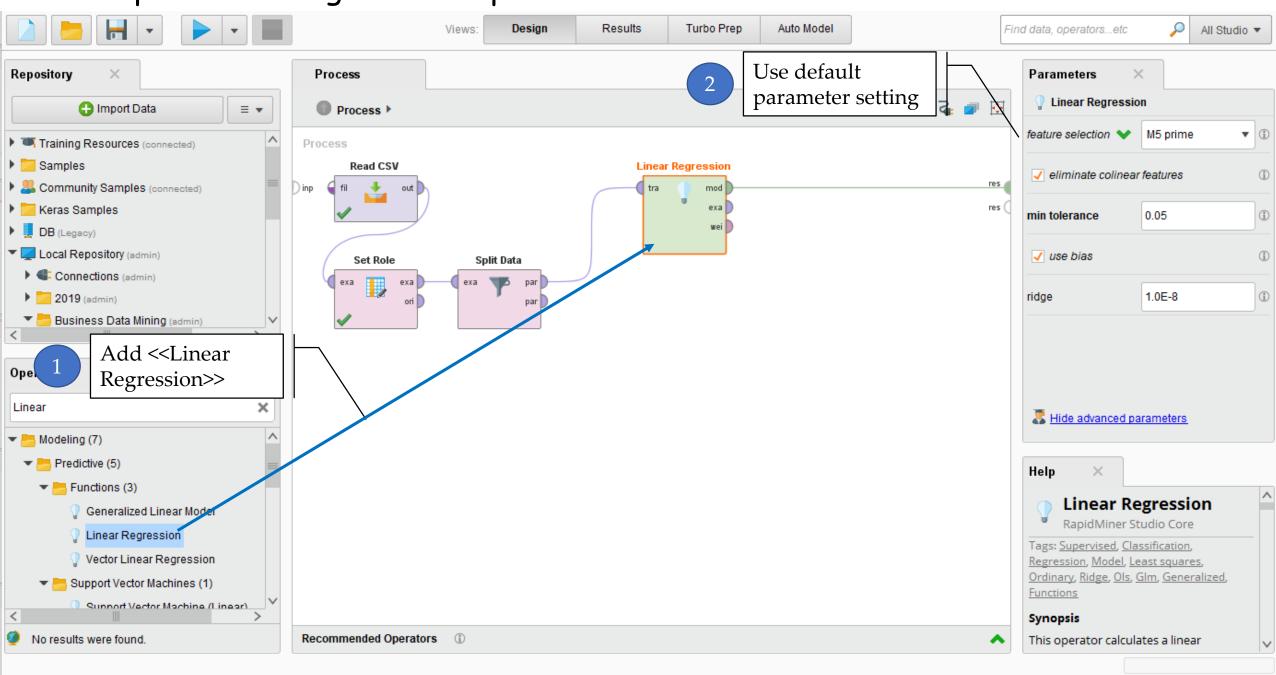
Process

- Load "red wine" dataset
- Set "quality" as label
- Create split test design with <<Split Data>>
- Create a linear regression model with the train dataset using linear regression algorithm
- Apply the model to the test dataset
- · Set regression performance measures
- Run the analysis process and check the performance results

Create split test design with «Split Data»



Create split test design with «Split Data»



Exercise 1: Linear Regression with Rapidminer

Parameters of <<Linear Regression>>

feature selection

- Not all attributes are useful for linear regression.
- Traditionally, forward selection (FS) or backward elimination (BE) are used to select features.
- If you decide not to use these wrapper operators, you may use the ones which are bundled with multiple linear regression operator.
 - M5 prime, greedy, T-Test, iterative T-Test

eliminate collinear features

- This parameter indicates if the algorithm should try to delete collinear features during the regression or not.
- min tolerance: This parameter is only available when the eliminate collinear features parameter is set to true. It specifies the minimum tolerance for eliminating collinear features.

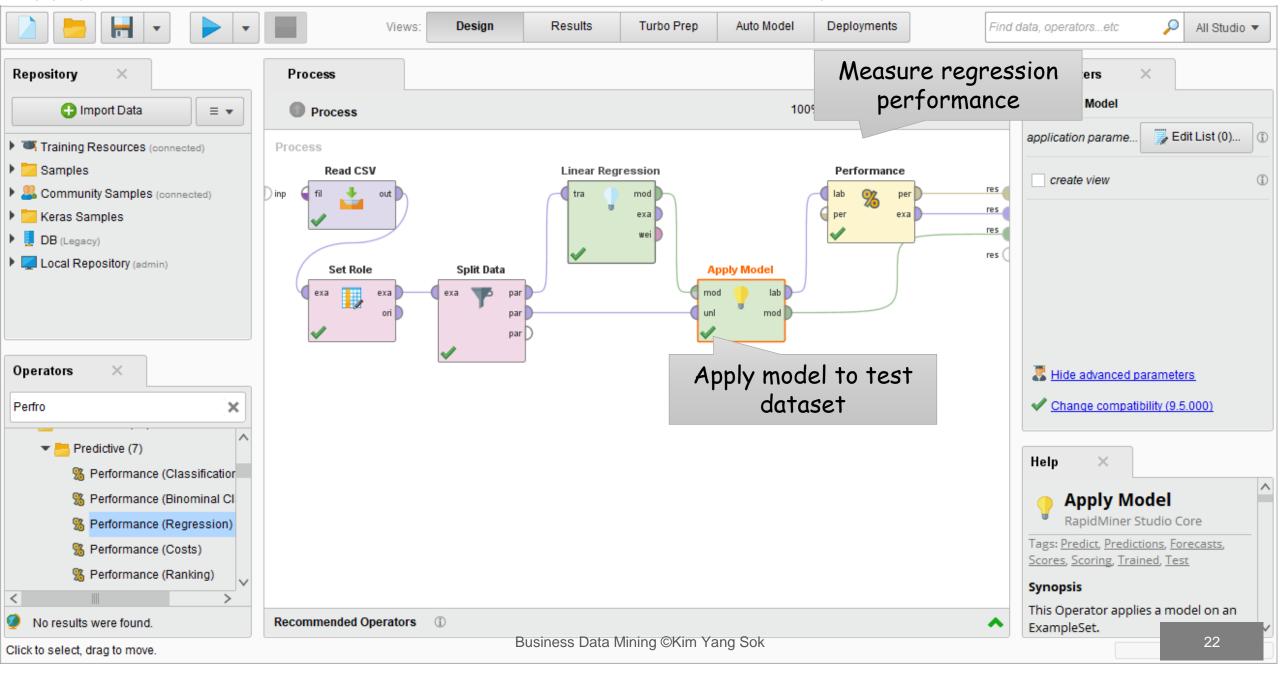
· use bias

This parameter indicates if an intercept value should be calculated or not.

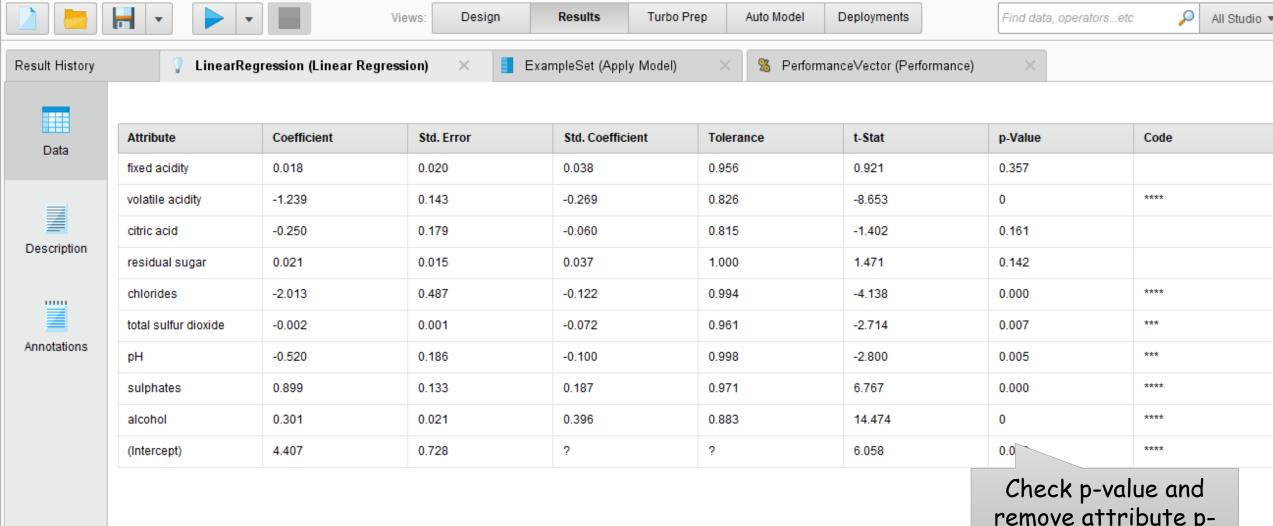
· ridge

This parameter specifies the ridge parameter for using in ridge regression.

Apply the model to the test dataset & Set regression performance measures

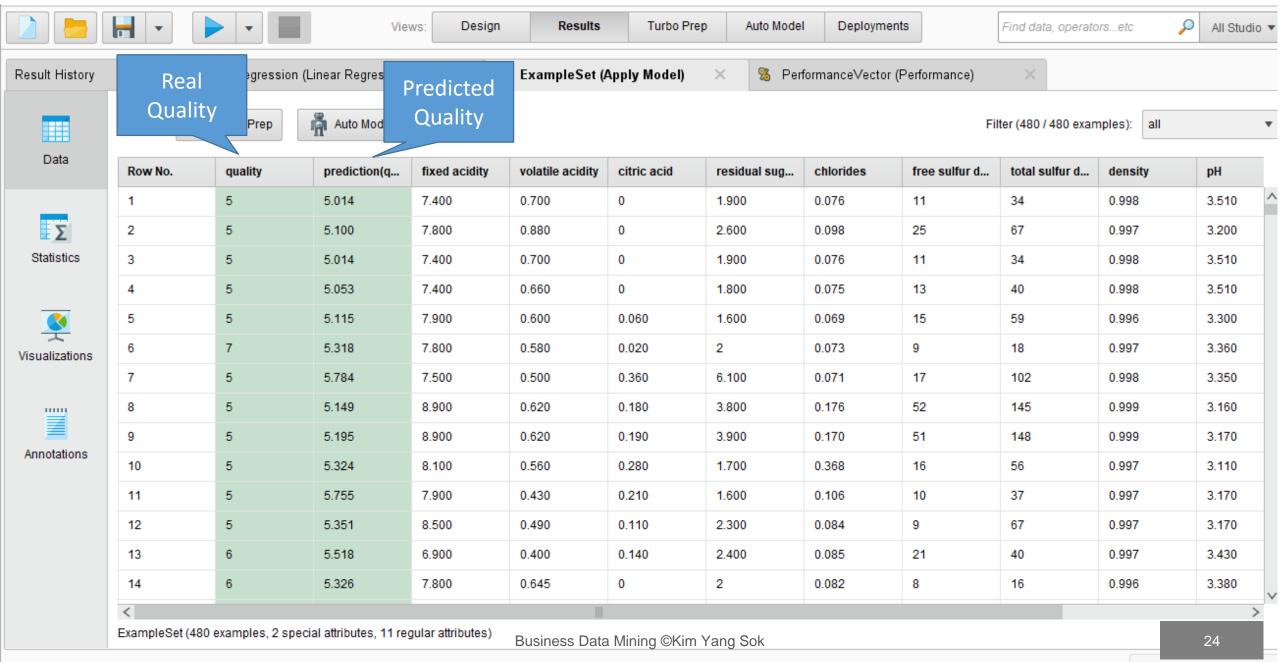


Run the analysis process and check the results - Model

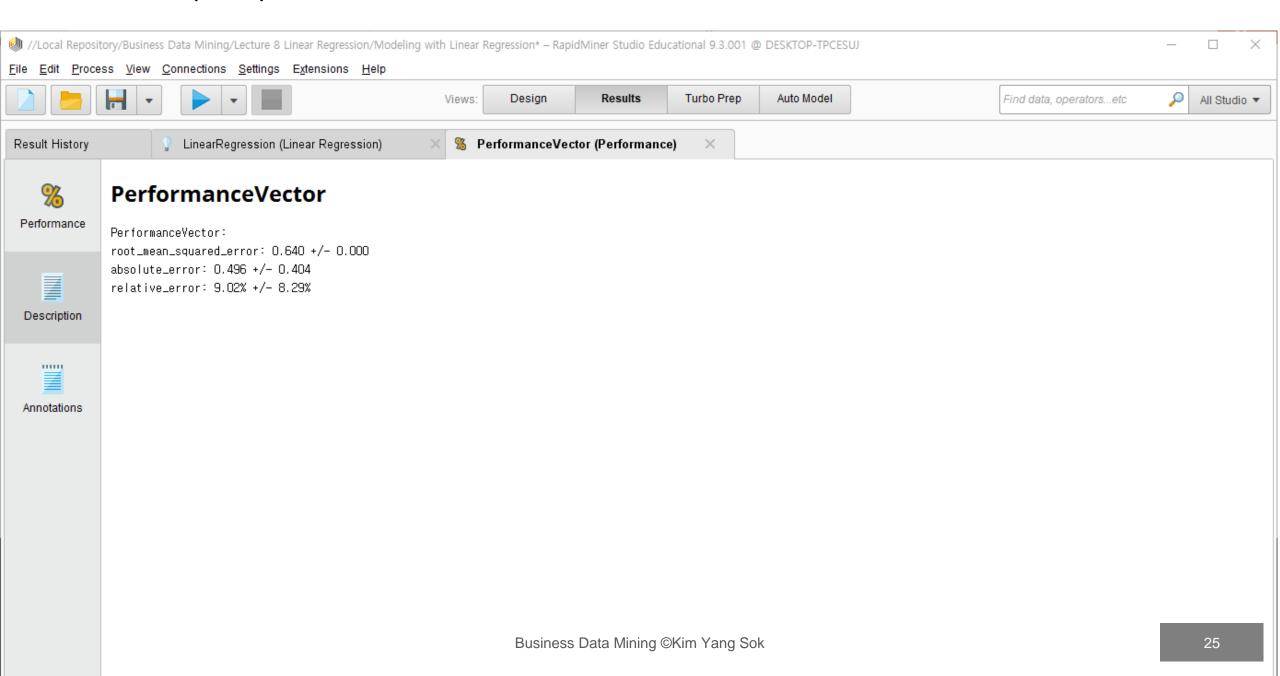


remove attribute pvalue > 0.05

Run the analysis process and check the results - Examples



Run the analysis process and check the results - Performance



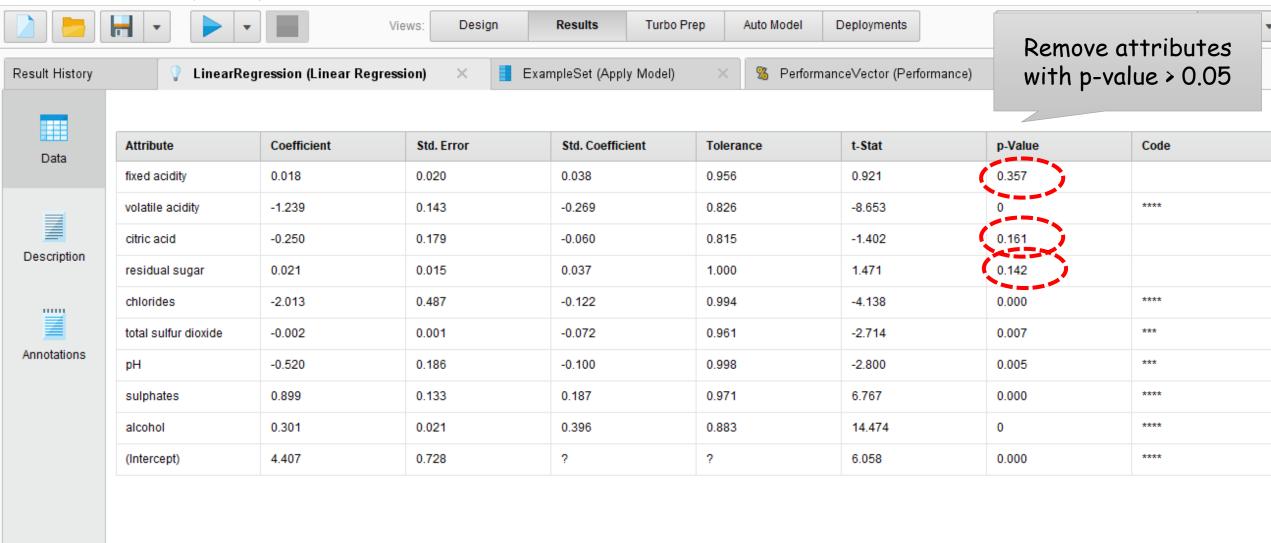
Exercise 2: Linear Regression after Removing Insignificant Attributes

Exercise 2: Linear Regression after Removing Insignificant Attributes

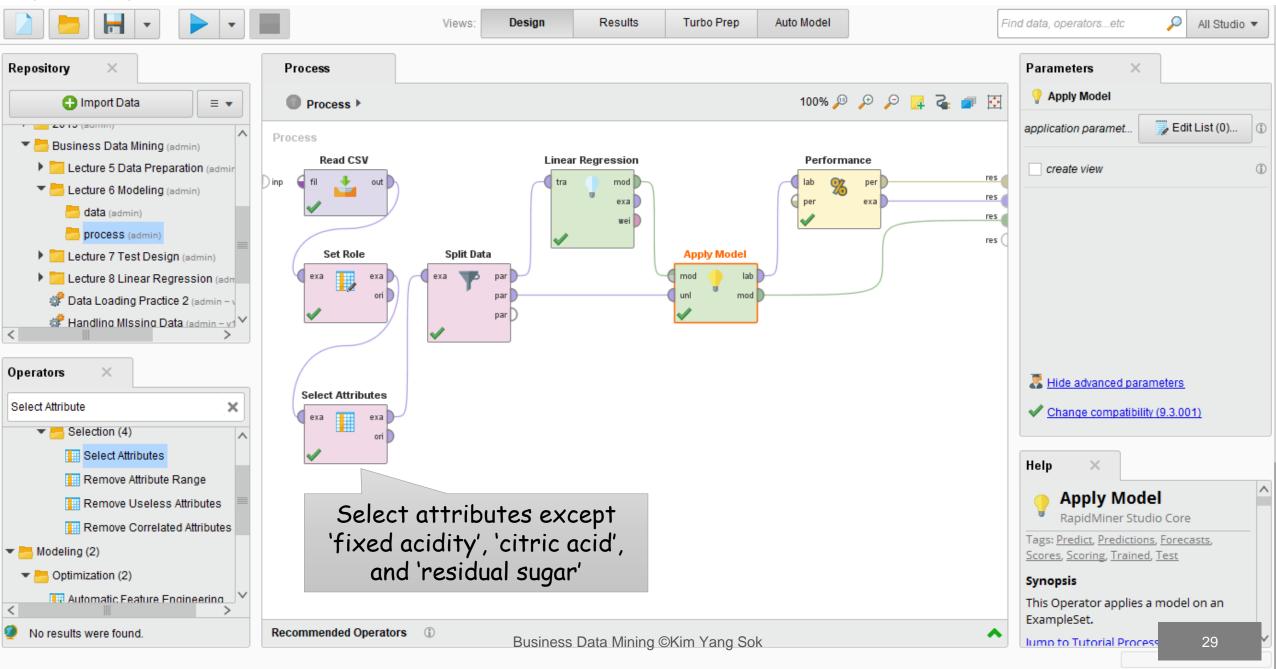
Task & Process

- Task
 - Build a model after removing insignificant attributes
- Process
 - Load "red wine" dataset
 - Add Select Attributes operator and remove insignificant attributes manually
 - · Learn a model with the training datasets and apply it to the testing dataset
 - Run the analysis process and check the performance results

Run the analysis process and check the results - Model



Update process and run again!



Run the analysis process and check the results - Model

Attribute	Coefficient	Std. Error	Std. Coefficient	Tolerance	t-Stat	p-Value		Code		
fixed acidity	0.018	0.020	0.038	0.956	0.921	0.357	PerformanceVector: root_mean_squared_error: 0.640 +/- 0.000 absolute_error: 0.496 +/- 0.404 relative_error: 9.02% +/- 8.29%			
volatile acidity	-1.239	0.143	-0.269	0.826	-8.653	0				
citric acid	-0.250	0.179	-0.060	0.815	-1.402	0.161				
residual sugar	0.021	0.015	0.037	1.000	1.471	0.142				
chlorides	-2.013	0.487	-0.122	0.994	-4.138	0.000	***			
otal sulfur dioxide	-0.002	0.001	-0.072	0.961	-2.714	0.007		***		
Н	-0.520	0.186	-0.100	0.998	-2.800	0.005		***		
sulphates	0.899	0.133	0.187	0.971	6.767	0.000	****			
alcohol	0.301	0.021	0.396	0.883	14.474	0	PerformanceVector: root_mean_squared_error: 0.638 +/- 0.000			
									0,000 7 0,000	





0.728

4.407

Attribute	Coefficient	Std. Error	Std. Coefficient	Tolerance	t-Stat	p-Value	Code
volatile acidity	-1.127	0.121	-0.244	0.880	-9.289	0	***
chlorides	-2.159	0.462	-0.130	0.997	-4.673	0.000	***
total sulfur dioxide	-0.002	0.001	-0.073	0.968	-2.977	0.003	***
pH	-0.548	0.140	-0.105	1.000	-3.924	0.000	***
sulphates	0.885	0.132	0.184	0.971	6.715	0.000	***
alcohol	0.298	0.020	0.392	0.902	14.790	0	***
(Intercept)	4.635	0.476	?	?	9.745	0	***

6.058

0.000

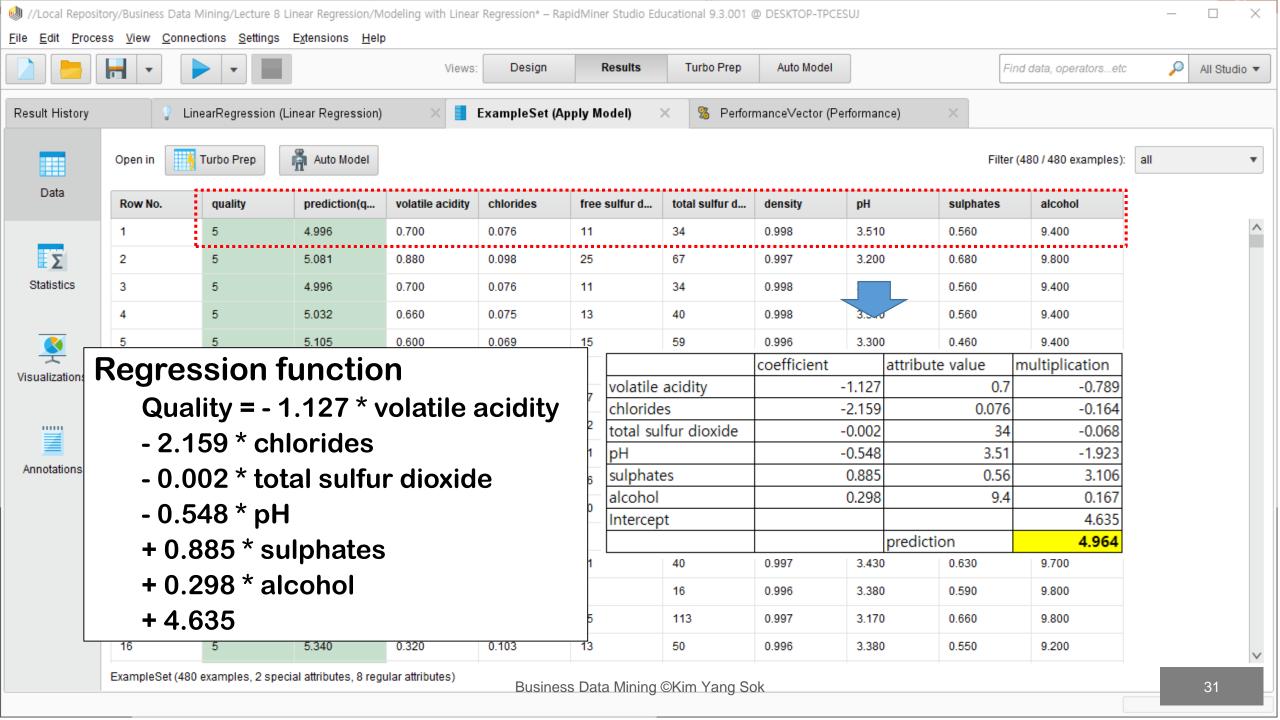
absolute_error: 0.496 +/- 0.402 relative_error: 9.00% +/- 8.25%



Data

(Intercept)





Exercise 3: Linear Regression with Variance Filter

Feature Selection with Variance Thresholds

- Variance thresholds remove features whose values don't change much from observation to observation (i.e. their variance falls below a threshold). These features provide little value.
- For example, if you had a public health dataset where 96% of observations were for 35-year-old men, then the 'Age' and 'Gender' features can be eliminated without a major loss in information.
- Because variance is dependent on scale, you should always normalize your features first.

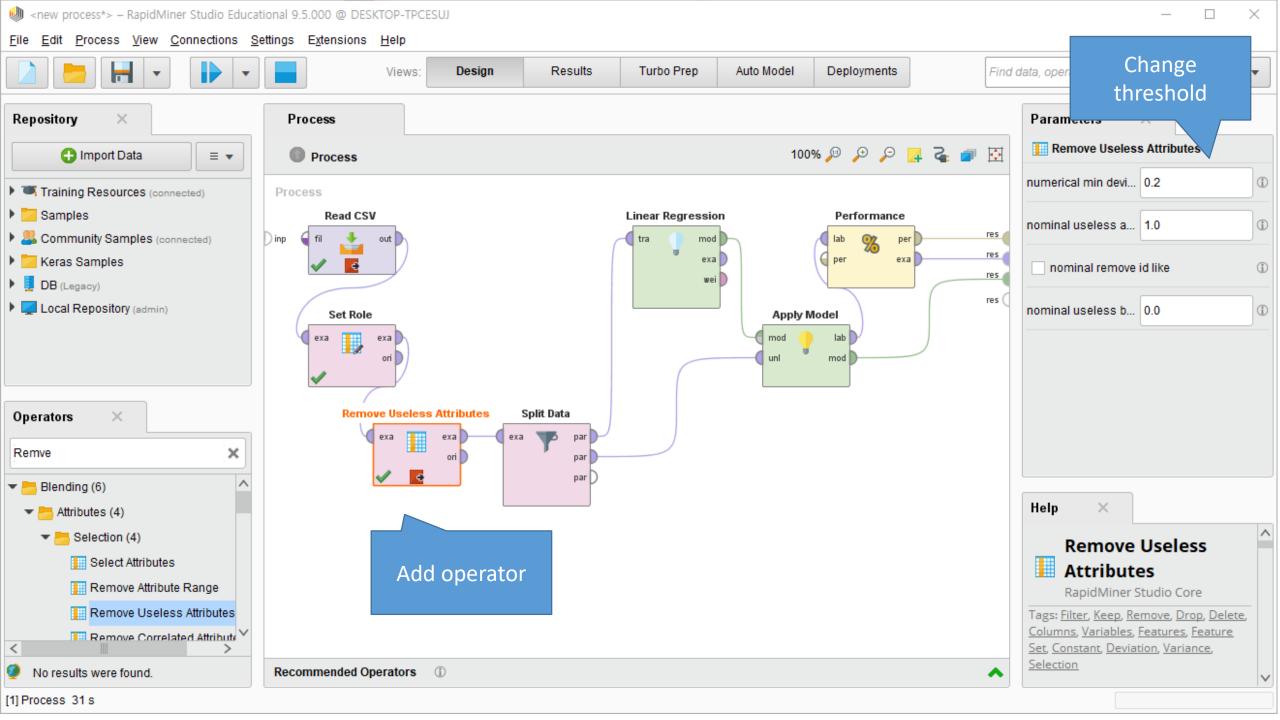
Feature Selection with Variance Thresholds

Strengths

- Applying variance thresholds is based on solid intuition: features that don't change much also don't add much information.
- This is an easy and relatively safe way to reduce dimensionality at the start of your modeling process.

Weaknesses

- If your problem does require dimensionality reduction, applying variance thresholds is rarely sufficient.
- Furthermore, you must manually set or tune a variance threshold, which could be tricky.
- It is better to start with a conservative (i.e. lower) threshold.



Exercise 4: Linear Regression with Correlation Filter

Feature Selection with Correlation Thresholds

- Correlation thresholds remove features that are highly correlated with others (i.e. its values change very similarly to another's). These features provide redundant information.
- For example, if you had a real-estate dataset with 'Floor Area (Sq. Ft.)' and 'Floor Area (Sq. Meters)' as separate features, you can safely remove one of them.
- Which one should you remove? Well, you'd first calculate all pairwise correlations. Then, if the correlation between a pair of features is above a given threshold, you'd remove the one that has larger mean absolute correlation with other features.

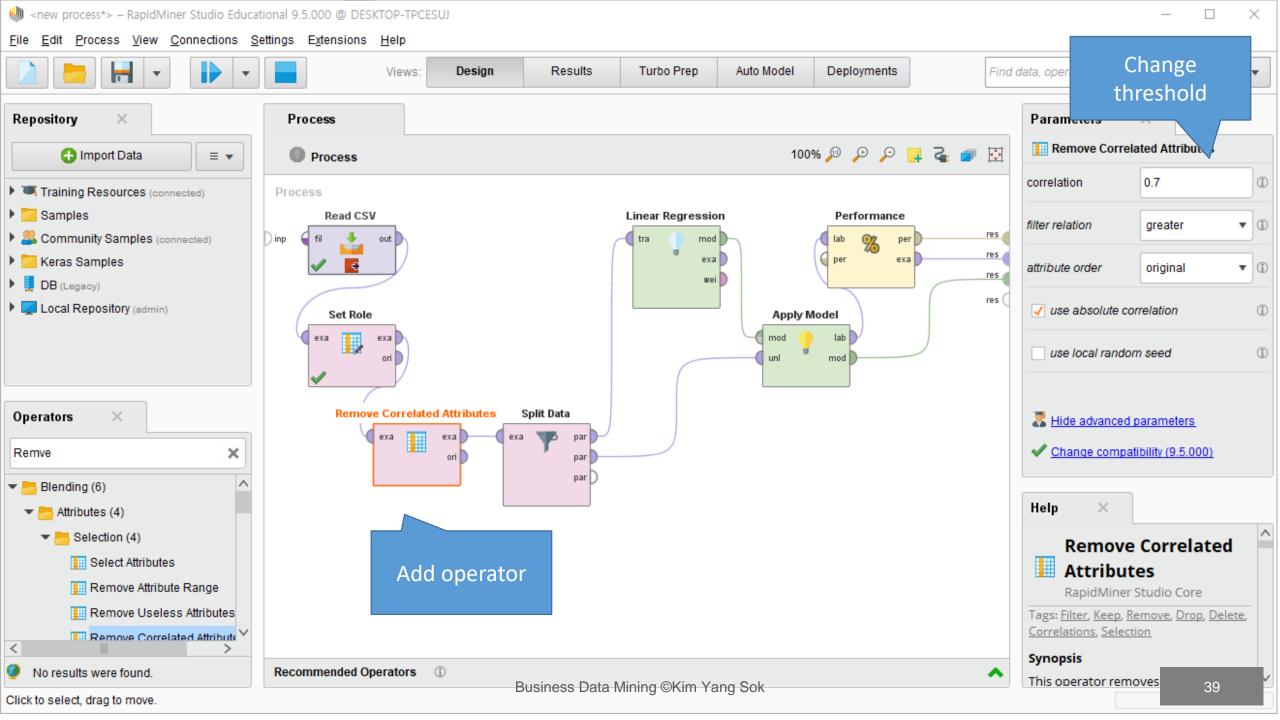
Feature Selection with Correlation Thresholds

Strengths

- Applying correlation thresholds is also based on solid intuition: similar features provide redundant information.
- Some algorithms are not robust to correlated features, so removing them can boost performance.

Weaknesses:

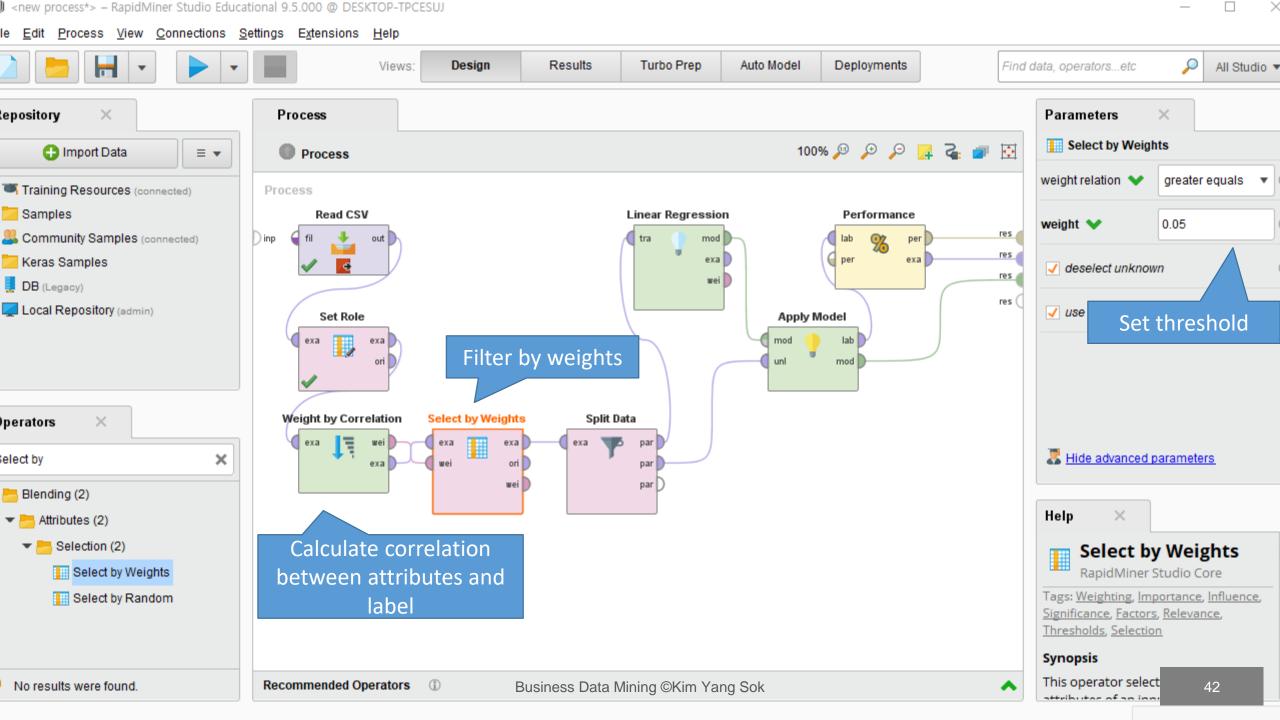
- You must manually set or tune a correlation threshold, which can be tricky to do.
- In addition, if you set your threshold too low, you risk dropping useful information.
- Whenever possible, we prefer algorithms with built-in feature selection over correlation thresholds.
- Even for algorithms without built-in feature selection, Principal Component Analysis (PCA) is often a better alternative.



Exercise 5: Linear Regression with Weight Filter

Feature Selection with Filter Methods

- Filter feature selection methods apply a statistical measure to assign a scoring to each feature.
- The features are ranked by the score and either selected to be kept or removed from the dataset.
- The methods are often univariate and consider the feature independently, or with regard to the dependent variable.
- Some examples of some filter methods include the Chi squared test, information gain and correlation coefficient scores.



Exercise 6: Linear Regression with Forward Selection using Rapidminer

Feature Selection with Stepwise Search

- Stepwise search is a supervised feature selection method based on sequential search, and it has two flavors: forward and backward.
- Forward stepwise search
 - Start without any features.
 - Then, you'd train a 1-feature model using each of your candidate features and keep the version with the best performance.
 - You'd continue adding features, one at a time, until your performance improvements stall.
- Backward stepwise search
 - Start with all features in your model and then remove one at a time until performance starts to drop substantially.
- Despite many textbooks listing stepwise search as a valid option, it almost always underperforms other supervised methods such as regularization.

Exercise 6: Linear Regression with Feature Selection using Rapidminer

Task & Process

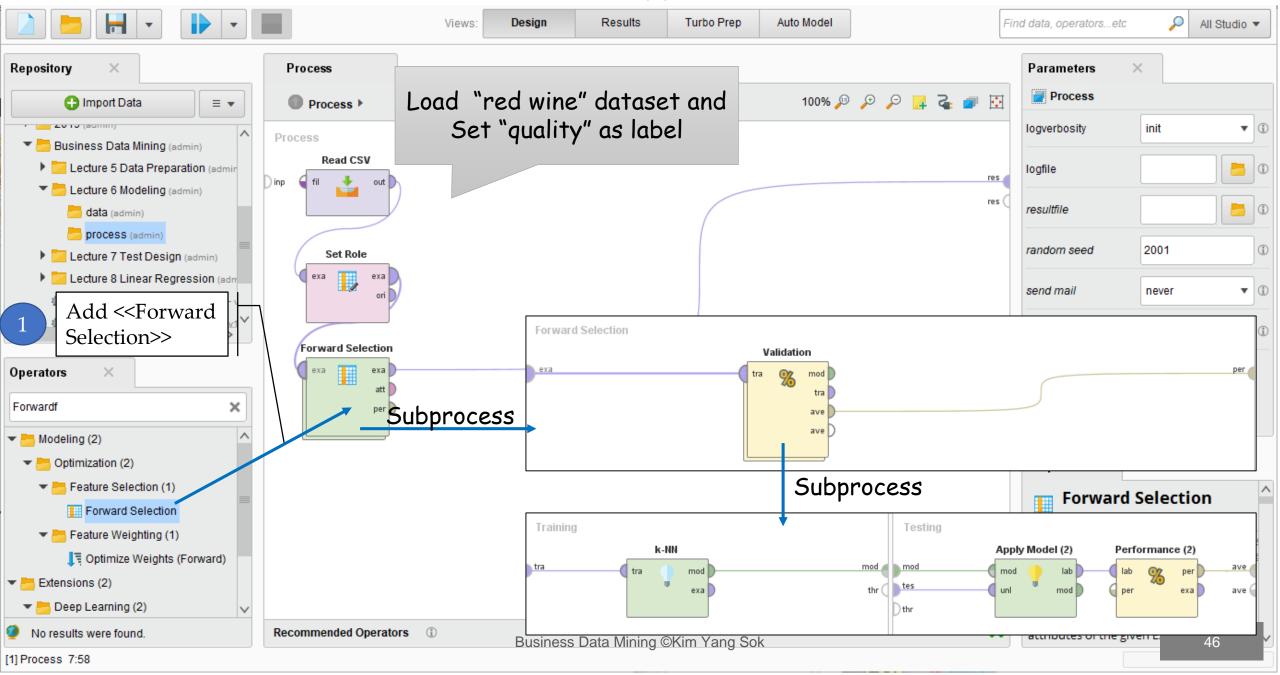
Task

After loading data, build a linear regression model with feature selection.

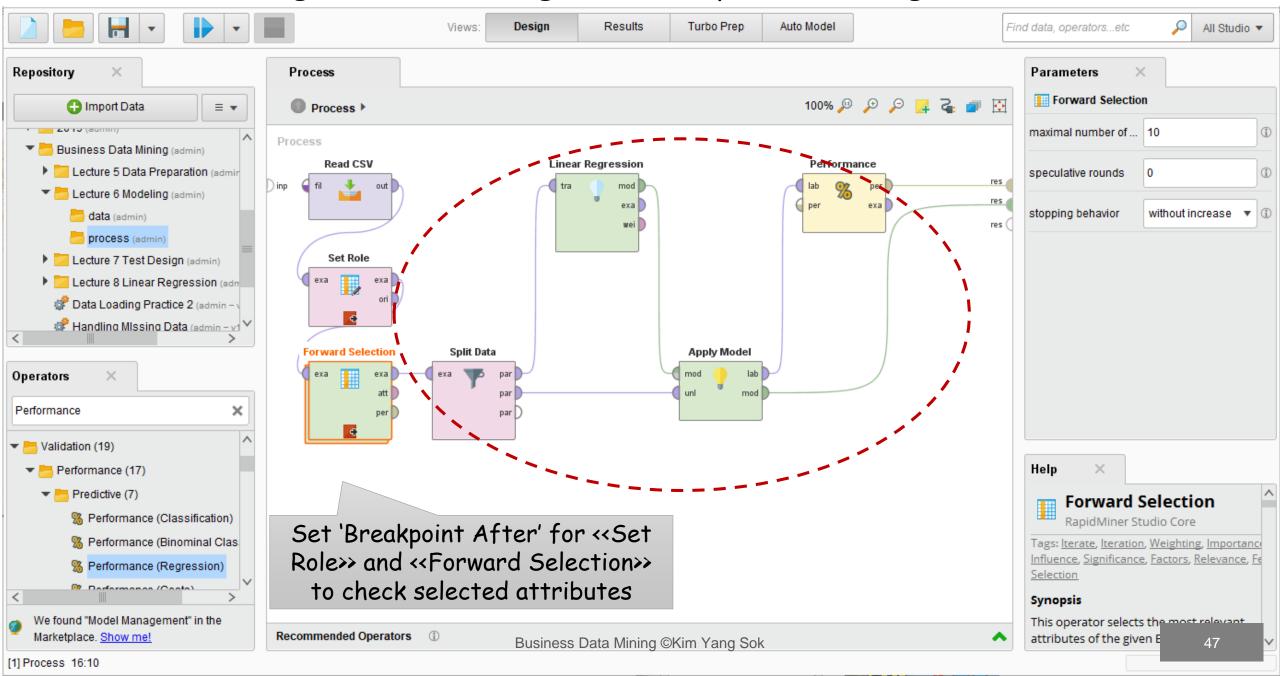
Process

- Load "red wine" dataset and Set "quality" as label
- Select attributes with forward selection approach
- Perform a linear regression modeling with the split test design
- Run the analysis process and evaluate analysis results

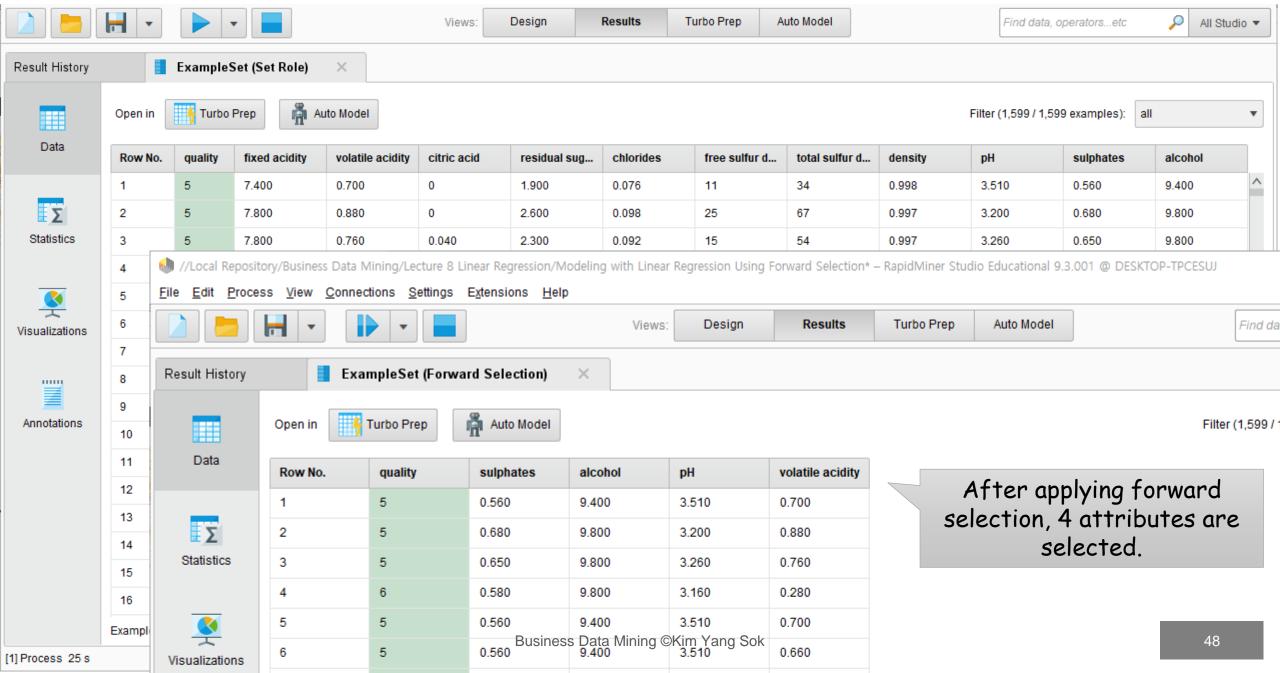
Select attributes with forward selection approach



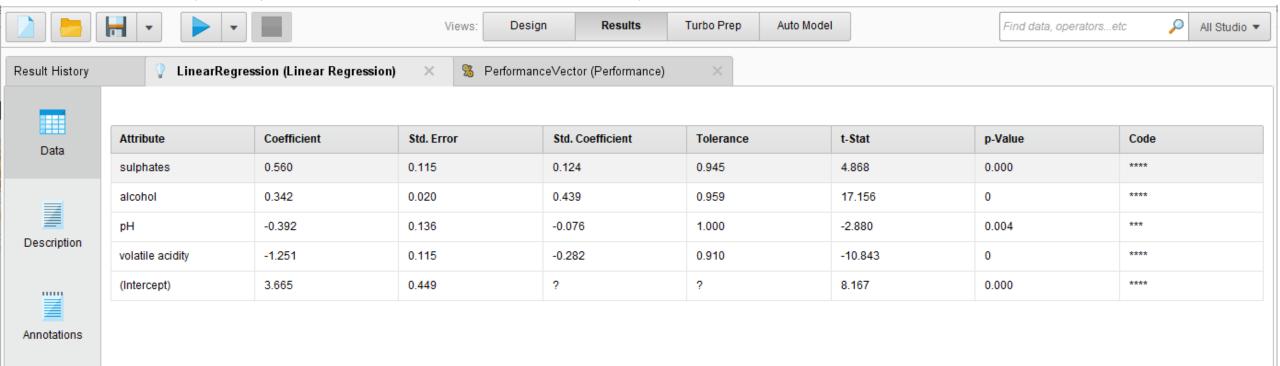
Perform a linear regression modeling with the split test design



Run the analysis process and evaluate analysis results



Run the analysis process and evaluate analysis results



PerformanceVector:

root_mean_squared_error: 0.693 +/- 0.000

absolute_error: 0.540 +/- 0.434

relative_error: 9.93% +/- 9.50%

Exercise 7: Linear Regression with Backward Elimination using Rapidminer

Exercise 7: Linear Regression with Feature Selection using Rapidminer

Task & Process

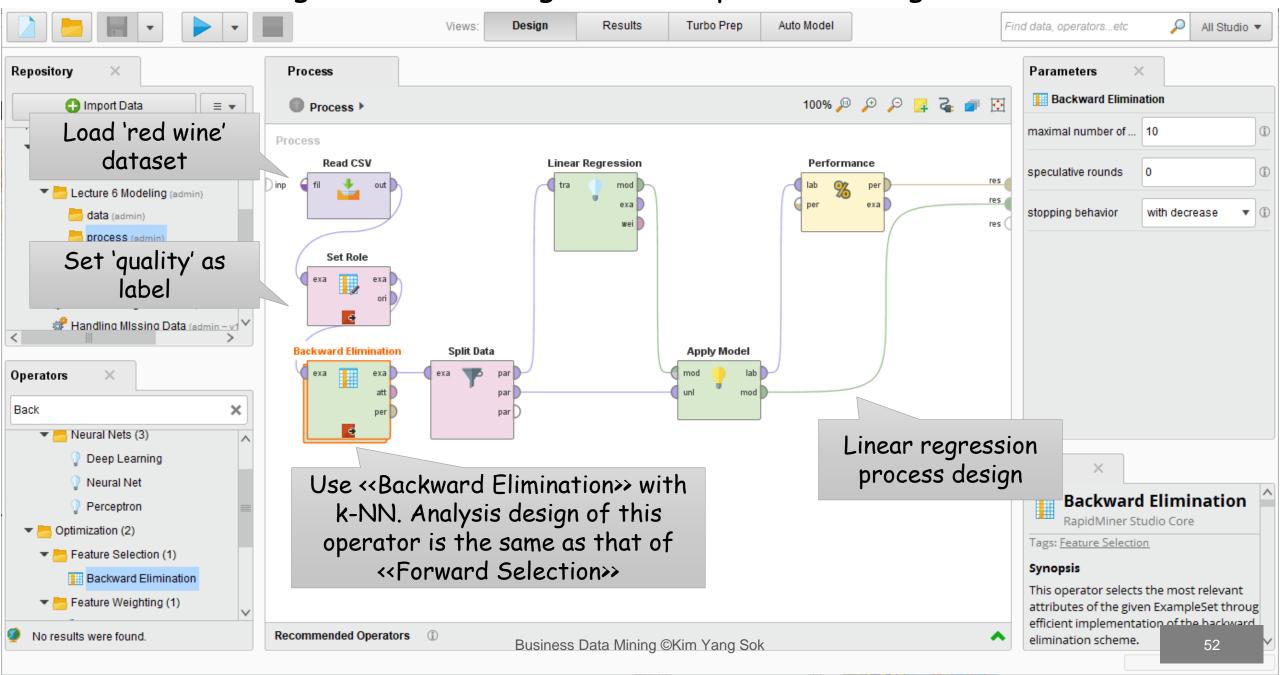
Task

After loading data, build a linear regression model with backward elimination.

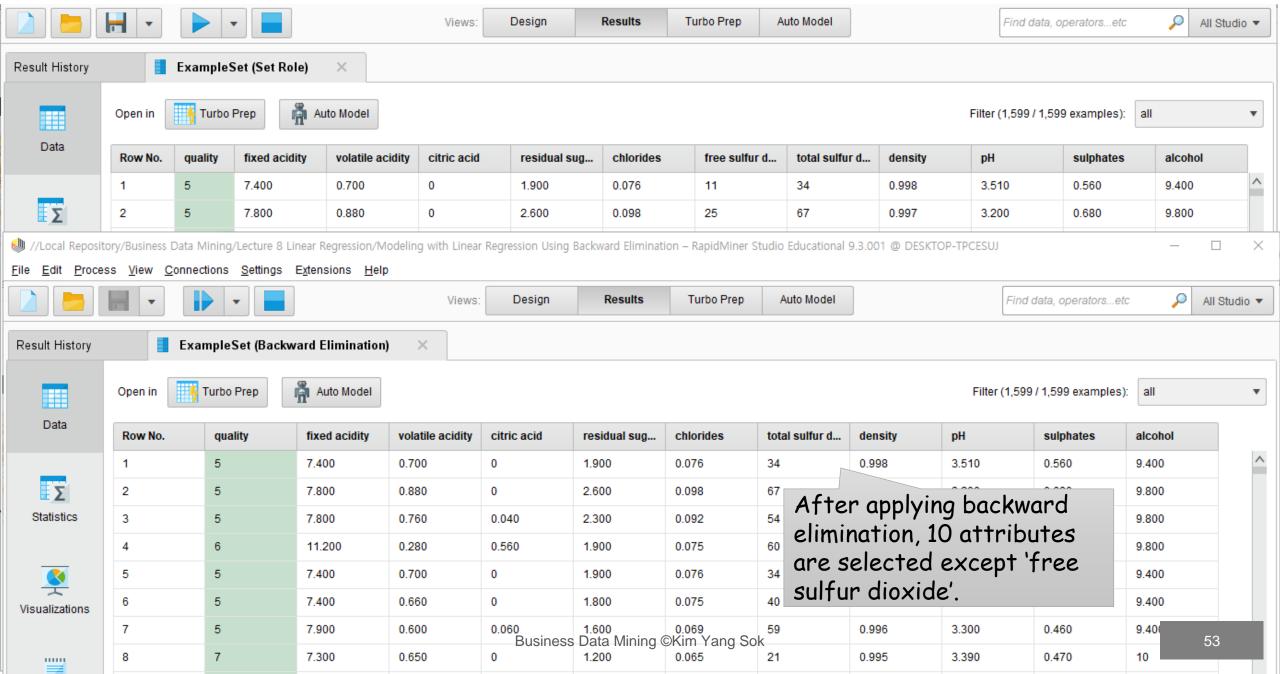
Process

- Load "red wine" dataset and Set "quality" as label
- · Select attributes with backward elimination approach
- Perform a linear regression modeling with the split test design
- Run the analysis process and evaluate analysis results

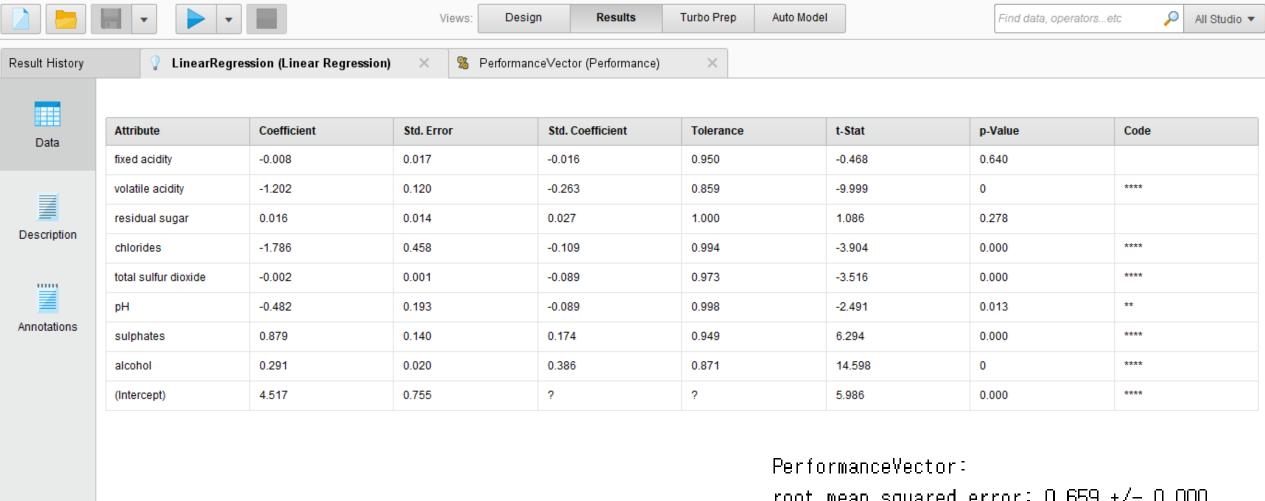
Perform a linear regression modeling with the split test design



Run the analysis process and evaluate analysis results



Run the analysis process and evaluate analysis results



root_mean_squared_error: 0.659 +/- 0.000

absolute_error: 0.509 +/- 0.419

relative_error: 8.99% +/- 7.92%

Exercise 8: Linear Regression with Model Explainer

Task & Process

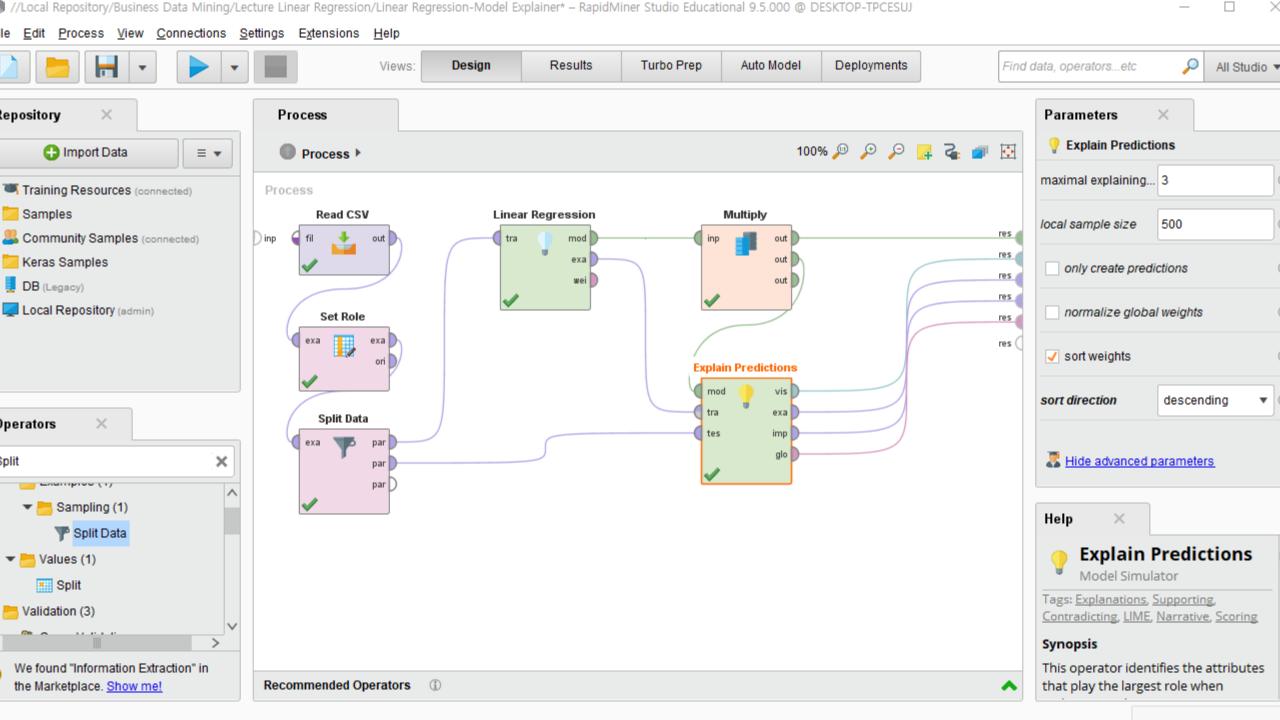
Task

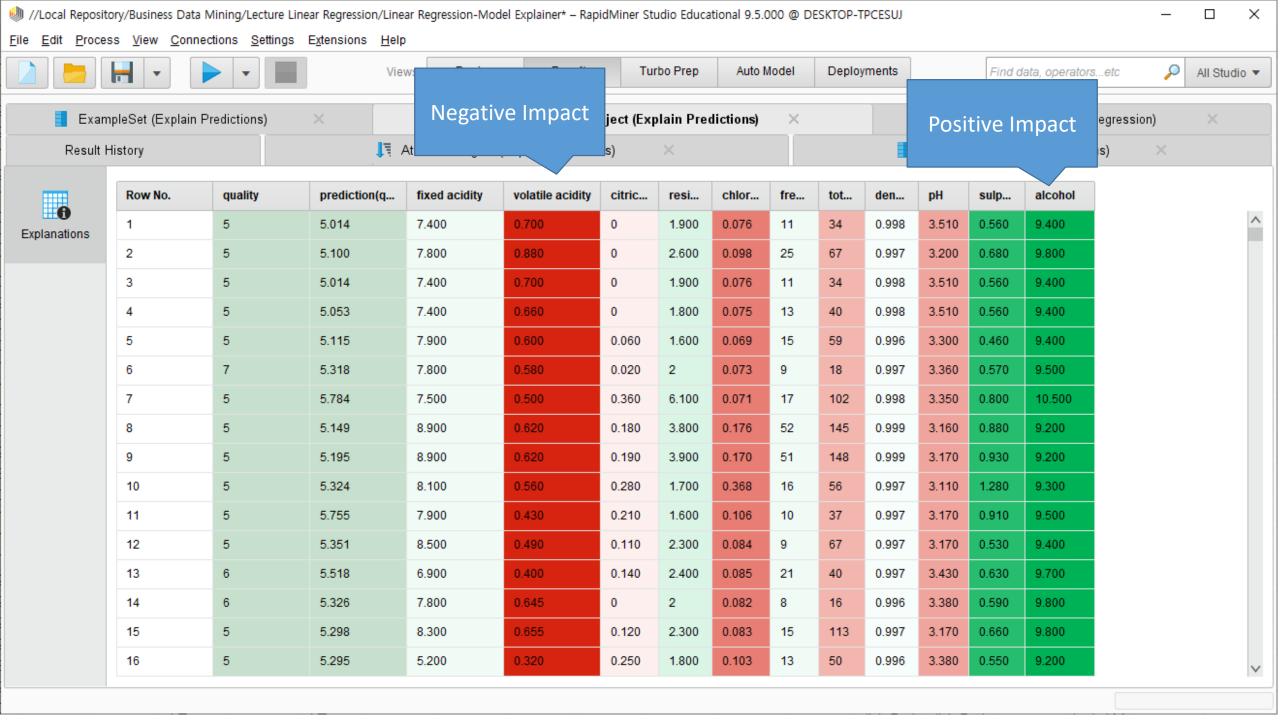
Which attributes play the largest role when making a prediction

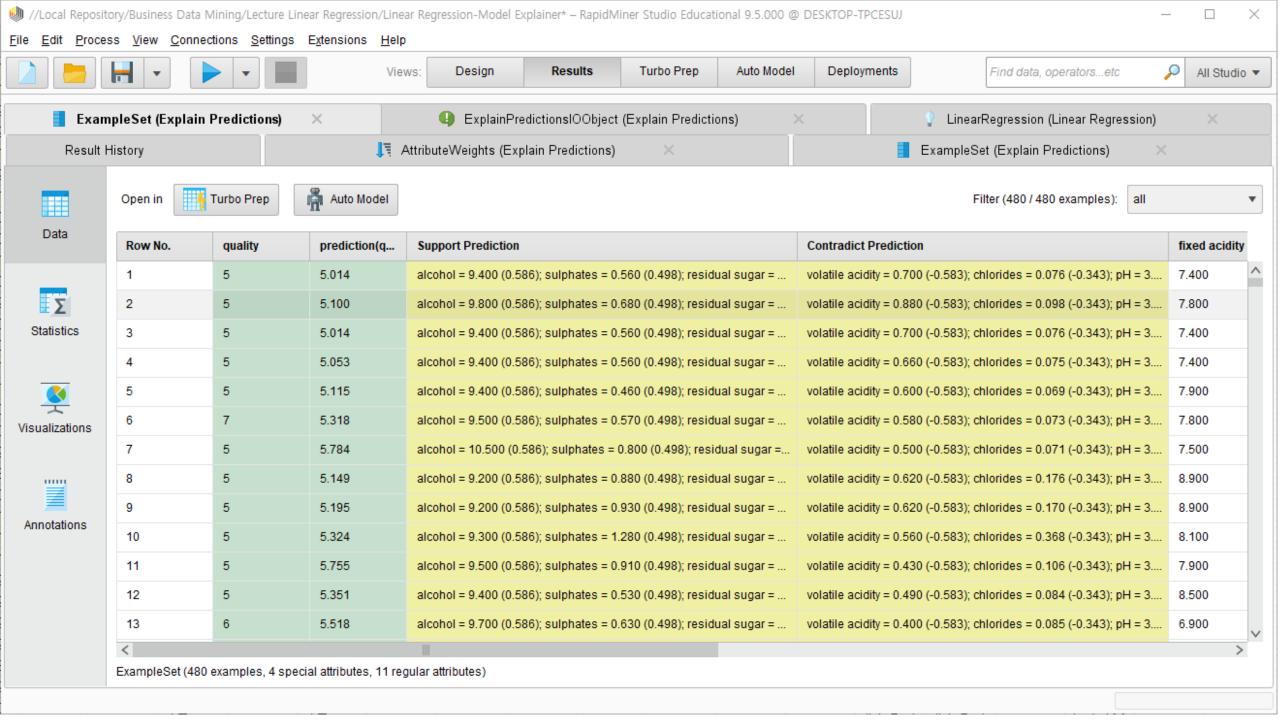
Process

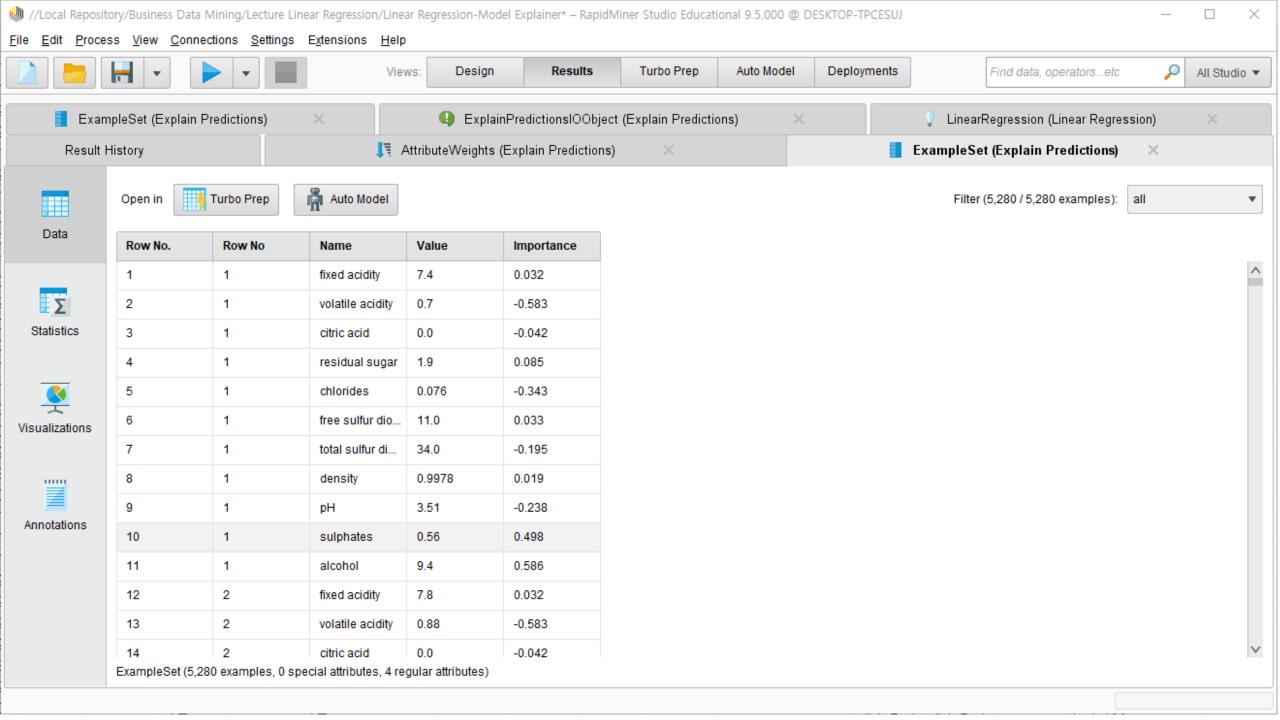
- Load "red wine" dataset and Set "quality" as label
- Split data into training and testing datasets
- · Learn a linear regression model with training dataset
- Add Explain Predictions operator and provide the model learned and testing dataset
- Run the analysis process and examine prediction results

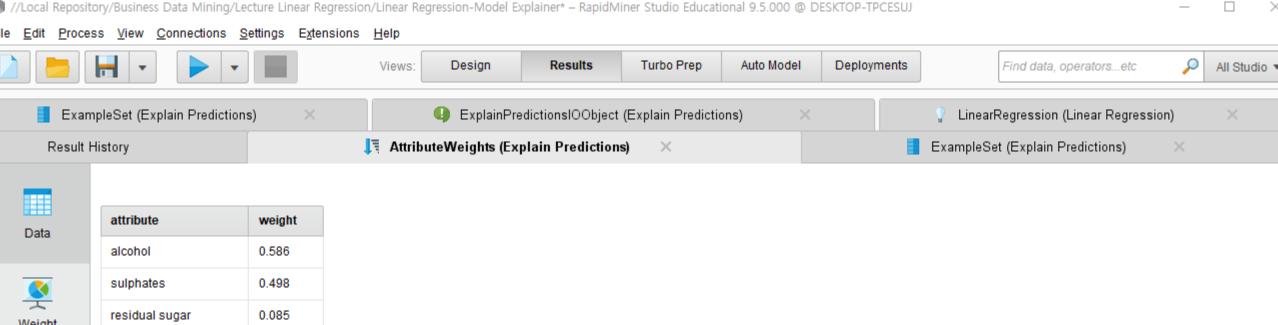
•











Weight Visualizations

Annotations

attribute	weight
alcohol	0.586
sulphates	0.498
residual sugar	0.085
free sulfur dioxide	0.033
fixed acidity	0.032
density	0.019
volatile acidity	0
citric acid	0
chlorides	0
total sulfur dioxide	0
рН	0

Exercise 9: Linear Regression with Model Simulator

Task & Process

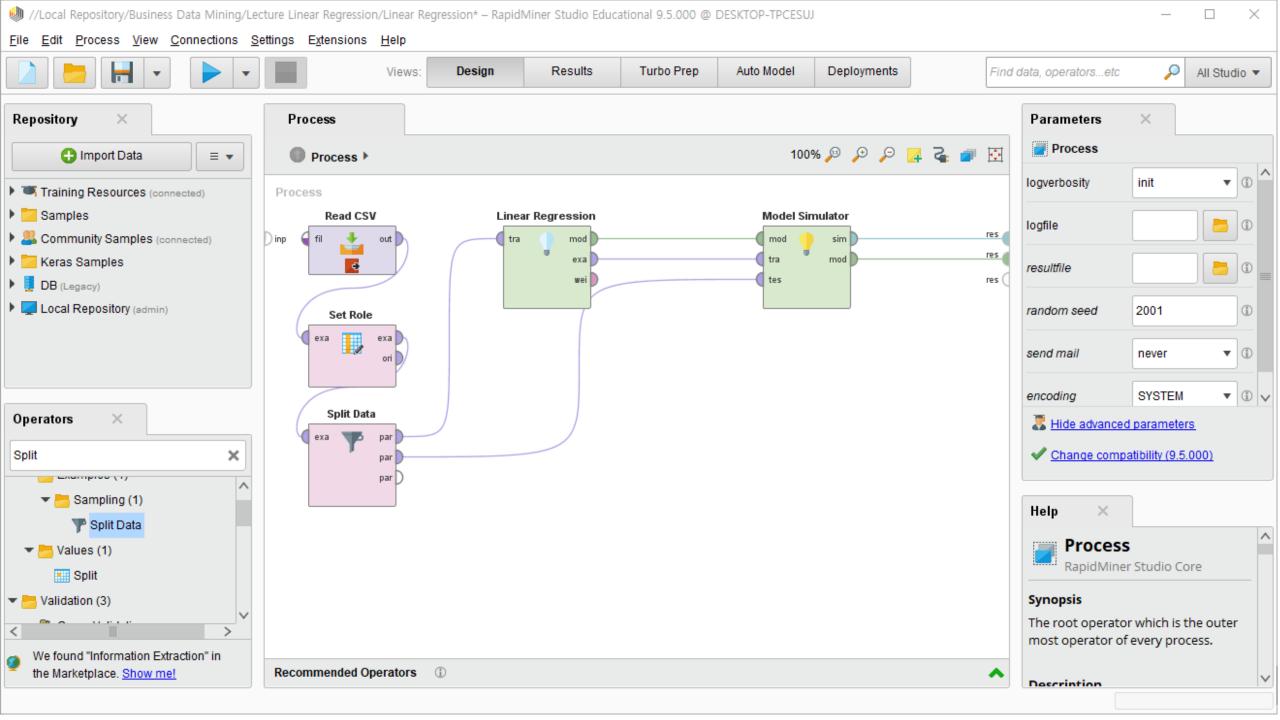
Task

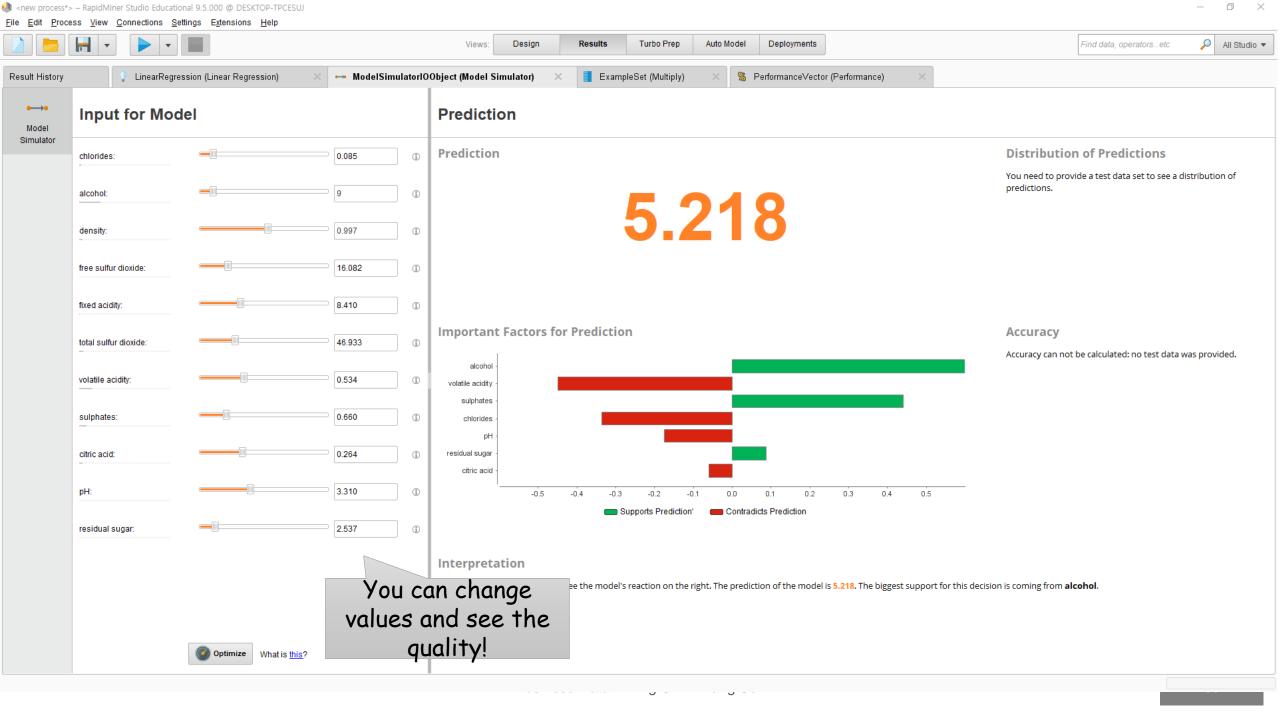
Simulate the target value with the model

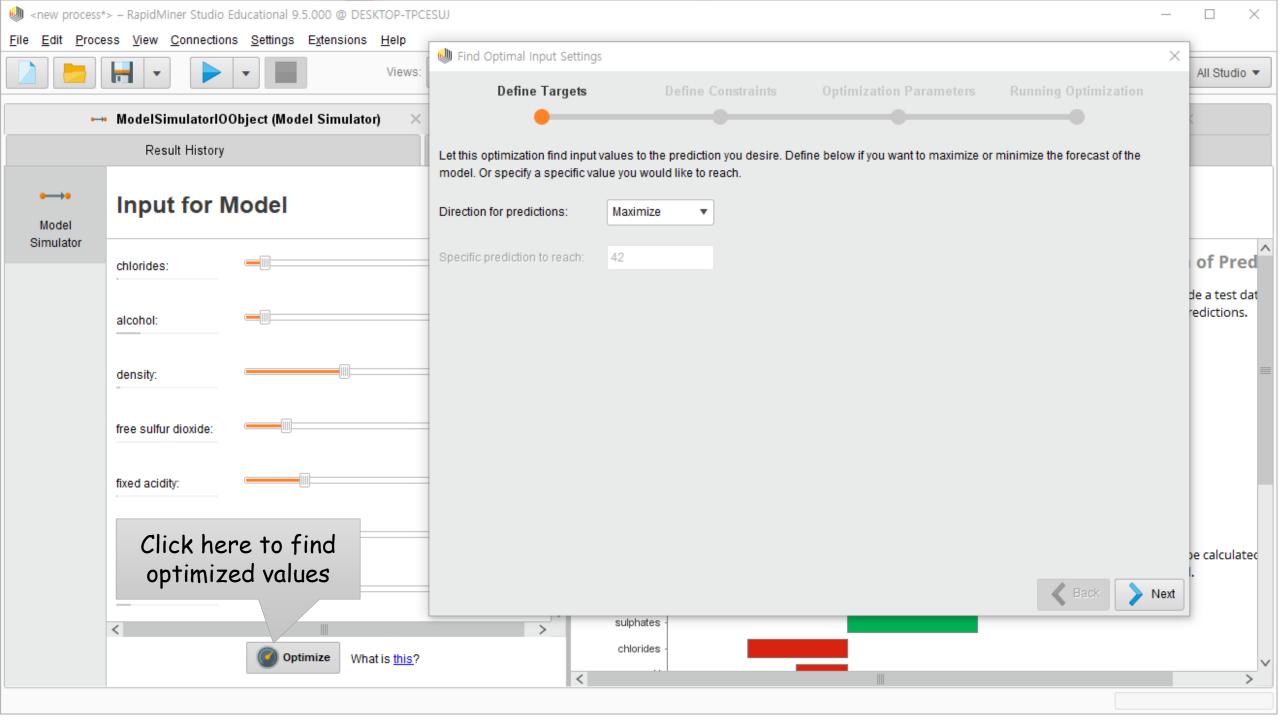
Process

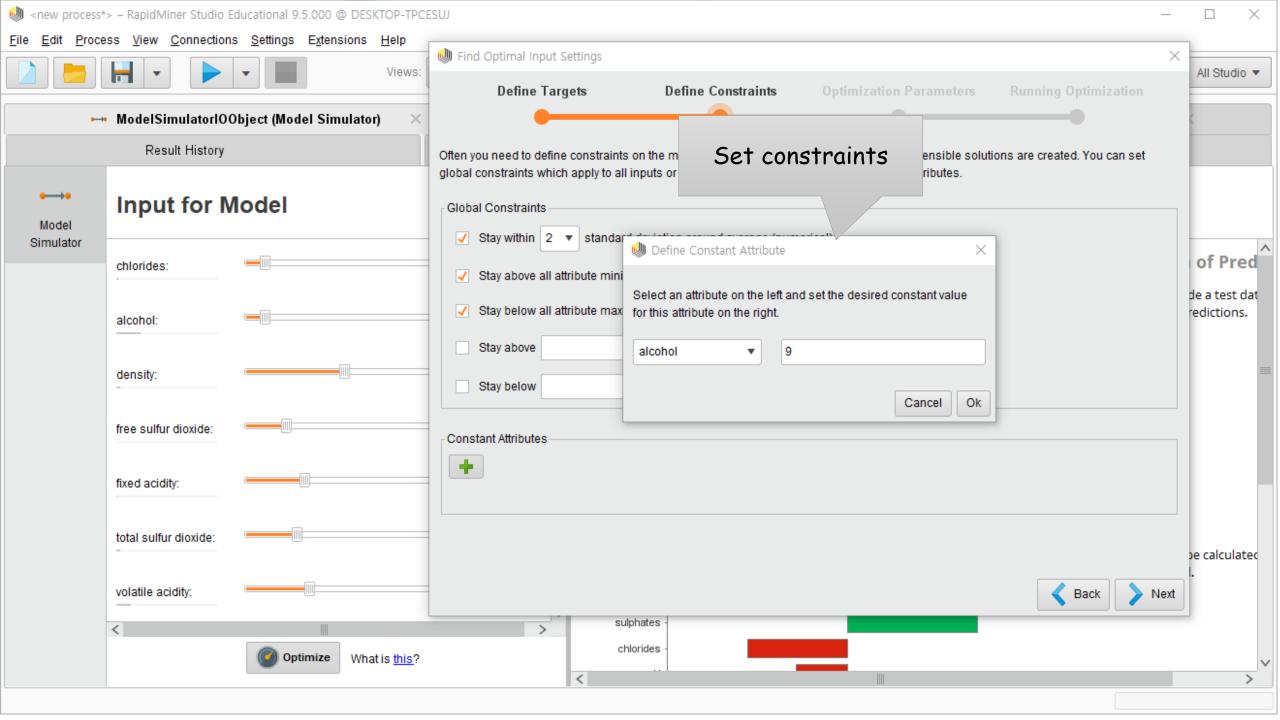
- Load "red wine" dataset and Set "quality" as label
- Split data into training and testing datasets
- Learn a linear regression model with training dataset
- Add Model Simulator operator and provide the model learned and testing dataset
- Run the analysis process and examine simulation results

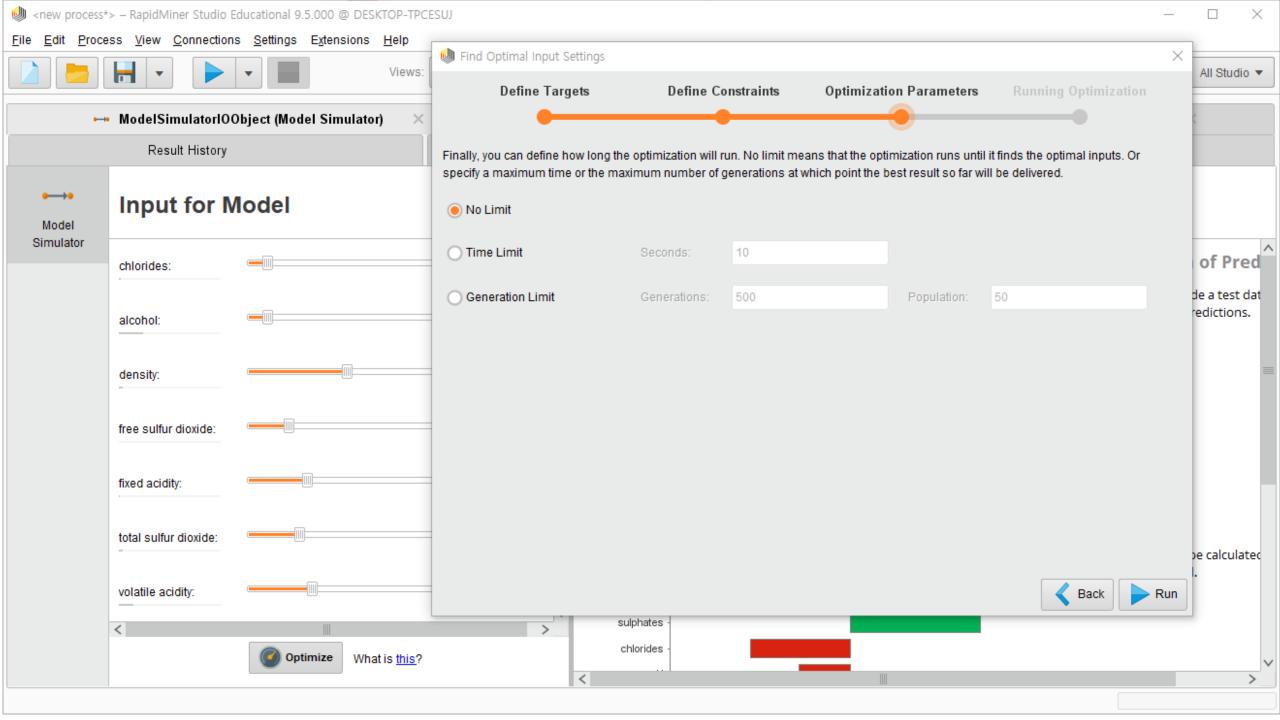
•

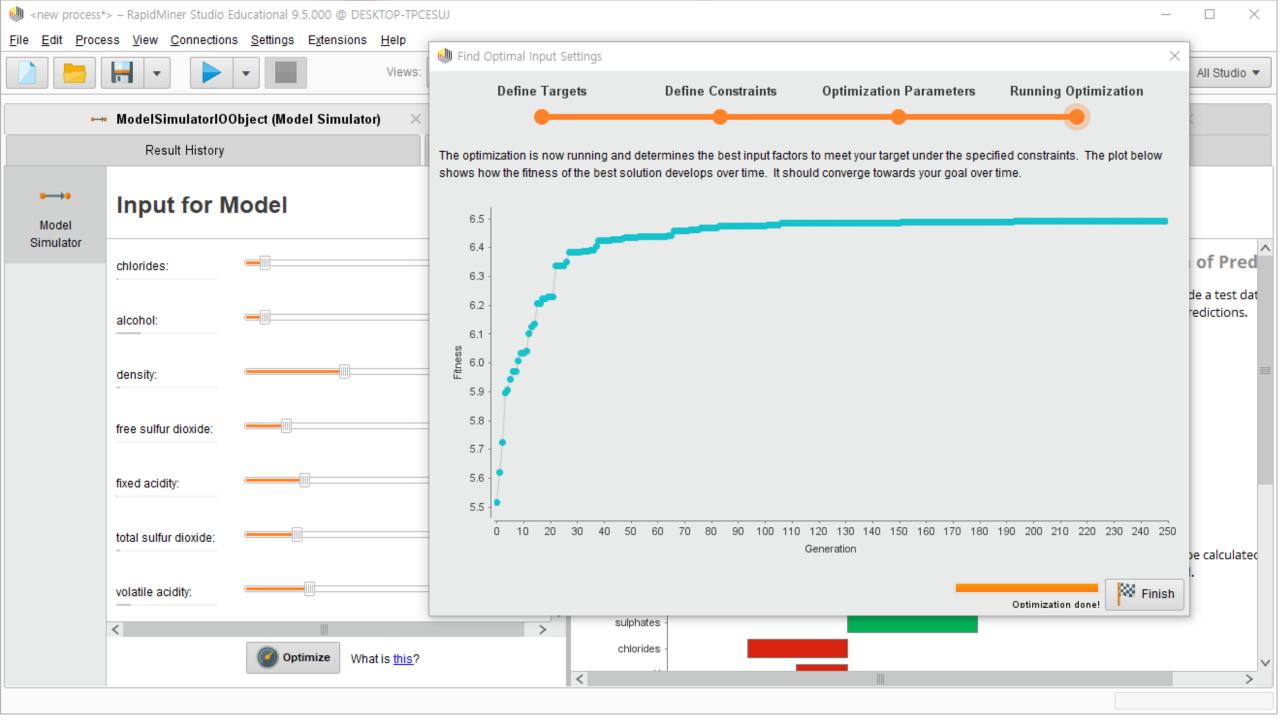


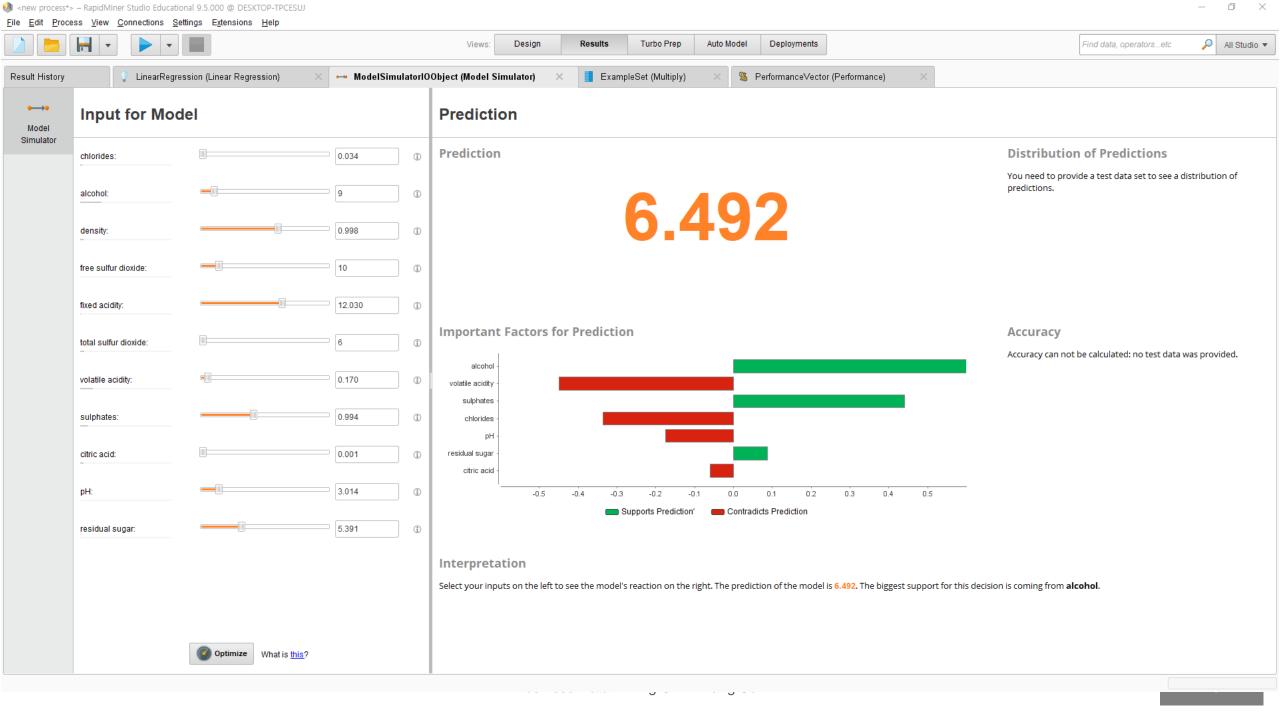












Conclusion

- In this lecture, we focused on Linear Regression algorithm.
 - This algorithm derives a linear equation that represents the data
- · We also learn also various feature selection algorithms.
- In the next lectures, you will learn optimization in model building with feature extraction algorithms.



QUESTIONS?