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# Computer Architecture and Operating Systems

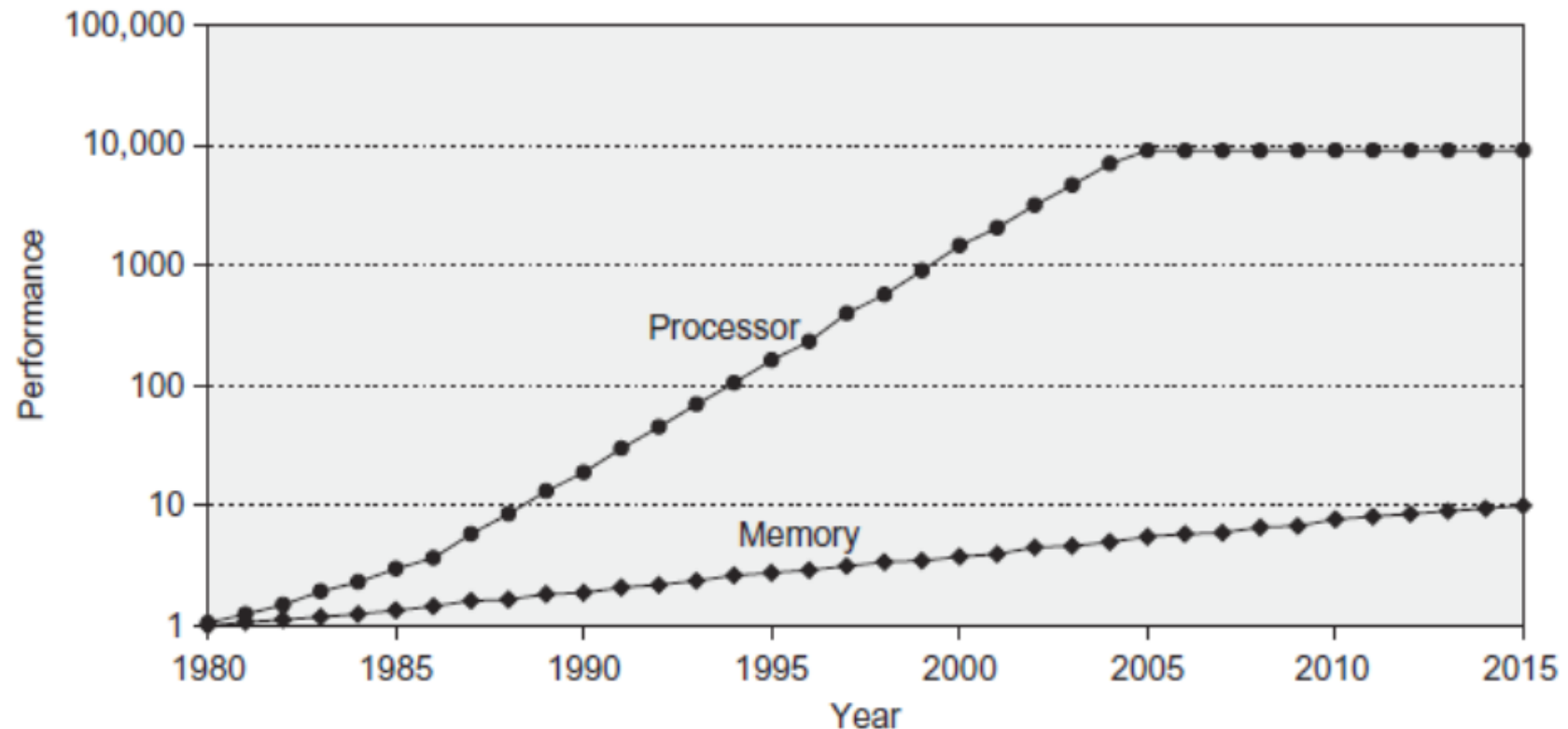
## Lecture 8: Memory and Caches

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# Processor-Memory Performance Gap

- Computer performance depends on:
  - Processor performance
  - Memory performance



# Memory Challenge

- Make memory appear as fast as processor
- Ideal memory:
  - Fast
  - Cheap (inexpensive)
  - Large (capacity)

**But can only choose two!**

# Memory Technology

- Static RAM (SRAM)
  - 0.5ns – 2.5ns, \$2000 – \$5000 per GB
- Dynamic RAM (DRAM)
  - 50ns – 70ns, \$20 – \$75 per GB
- Magnetic disk
  - 5ms – 20ms, \$0.20 – \$2 per GB
- Ideal memory
  - Access time of SRAM
  - Capacity and cost/GB of disk

# Locality

**No need for large memory to access it fast  
Just exploit locality**

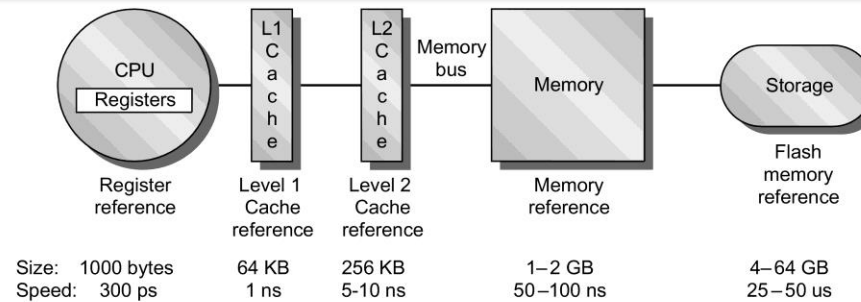
- **Temporal Locality:**
  - Locality in time
  - If data used recently, likely to use it again soon
  - How to exploit: keep recently accessed data in higher levels of memory hierarchy
- **Spatial Locality:**
  - Locality in space
  - If data used recently, likely to use nearby data soon
  - How to exploit: when access data, bring nearby data into higher levels of memory hierarchy too

# Taking Advantage of Locality

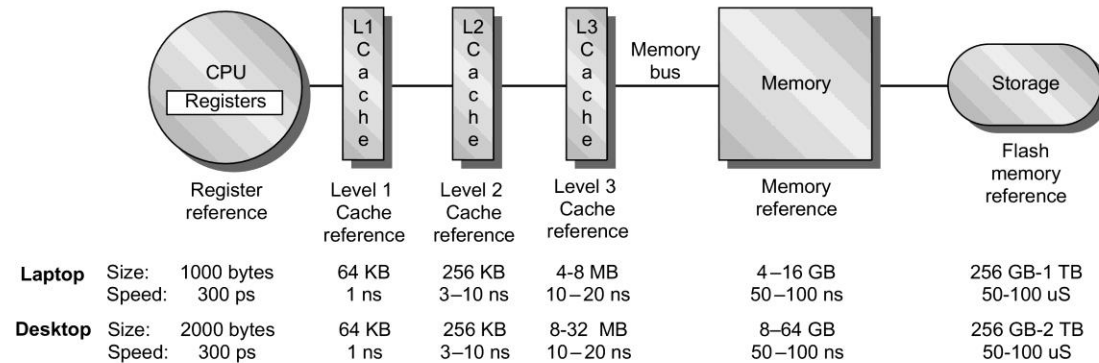
- Memory hierarchy
- Store everything on disk
- Copy recently accessed (and nearby) items from disk to smaller DRAM memory
  - Main memory
- Copy more recently accessed (and nearby) items from DRAM to smaller SRAM memory
  - Cache memory attached to CPU

# Memory Hierarchy

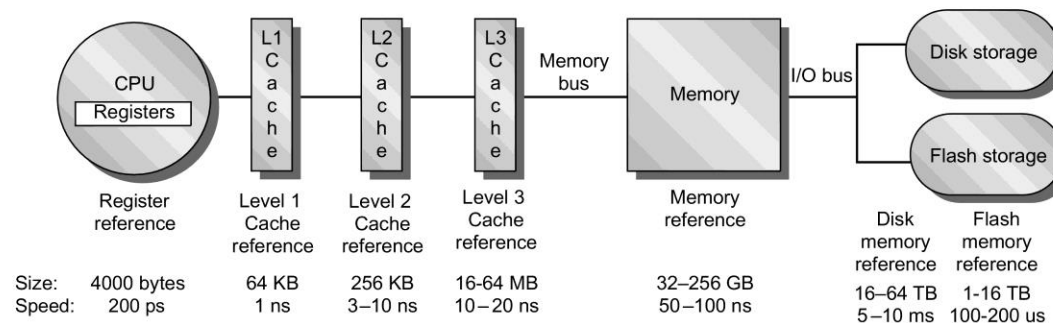
## ■ Personal mobile device



## ■ Laptop or desktop

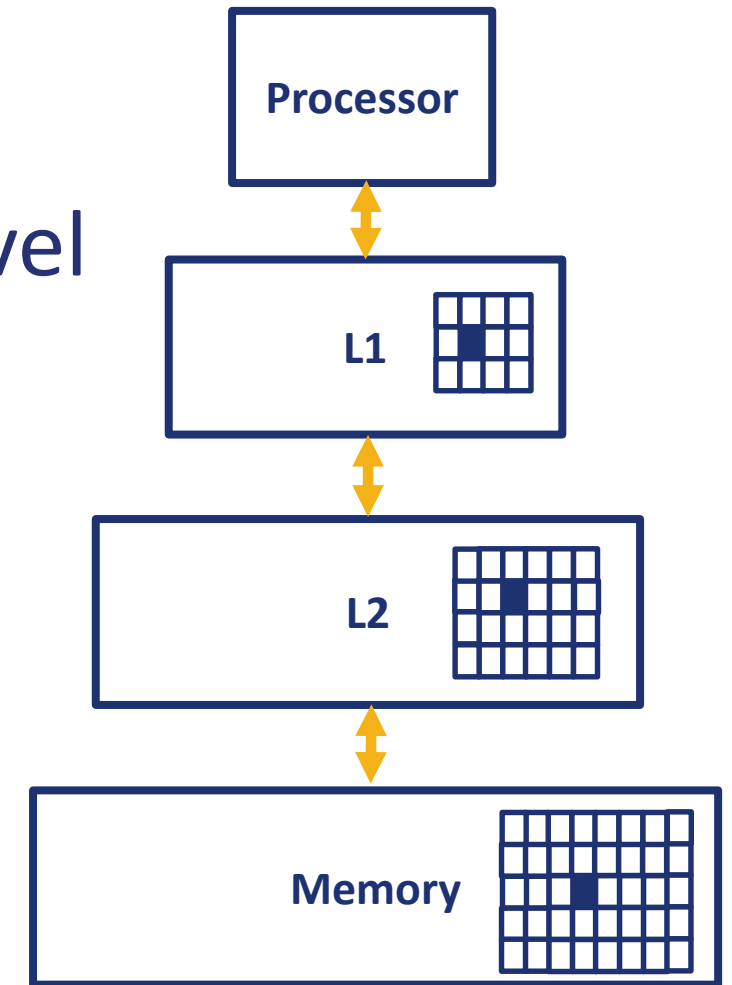


## ■ Server



# How It Works?

- **Block** (aka **line**): unit of copying
  - May be multiple words
- If accessed data is present in upper level
  - **Hit**: access satisfied by upper level
    - Hit ratio: hits/accesses
- If accessed data is absent
  - **Miss**: block copied from lower level
    - Time taken: miss penalty
    - Miss ratio: misses/accesses =  $1 - \text{hit ratio}$
  - Then accessed data supplied from upper level





# Hits and Misses

- On cache hit, CPU proceeds normally
- On cache miss
  - Stall the CPU pipeline
  - Fetch block from next level of hierarchy
  - Instruction cache miss
    - Restart instruction fetch
  - Data cache miss
    - Complete data access

# Miss Types

- **Compulsory**: first time data accessed
- **Capacity**: cache too small to hold all data of interest
- **Conflict**: data of interest maps to a location in cache mapped to different data

# Memory Performance

- **Hit:** data found in that level of memory hierarchy
- **Miss:** data not found (must go to next level)
  - **Hit Rate** = # hits / # memory accesses = 1 – Miss Rate
  - **Miss Rate** = # misses / # memory accesses = 1 – Hit Rate
- **Average memory access time (AMAT):** average time for processor to access data
  - **AMAT** =  $t_{\text{cache}} + MR_{\text{cache}}[t_{\text{MM}} + MR_{\text{MM}}(t_{\text{VM}})]$

# Cache Memory

- Cache memory
  - The level of the memory hierarchy closest to the CPU
- Given accesses  $X_1, \dots, X_{n-1}, X_n$

$X_4$
$X_1$
$X_{n-2}$
$X_{n-1}$
$X_2$
$X_3$

a. Before the reference to  $X_n$

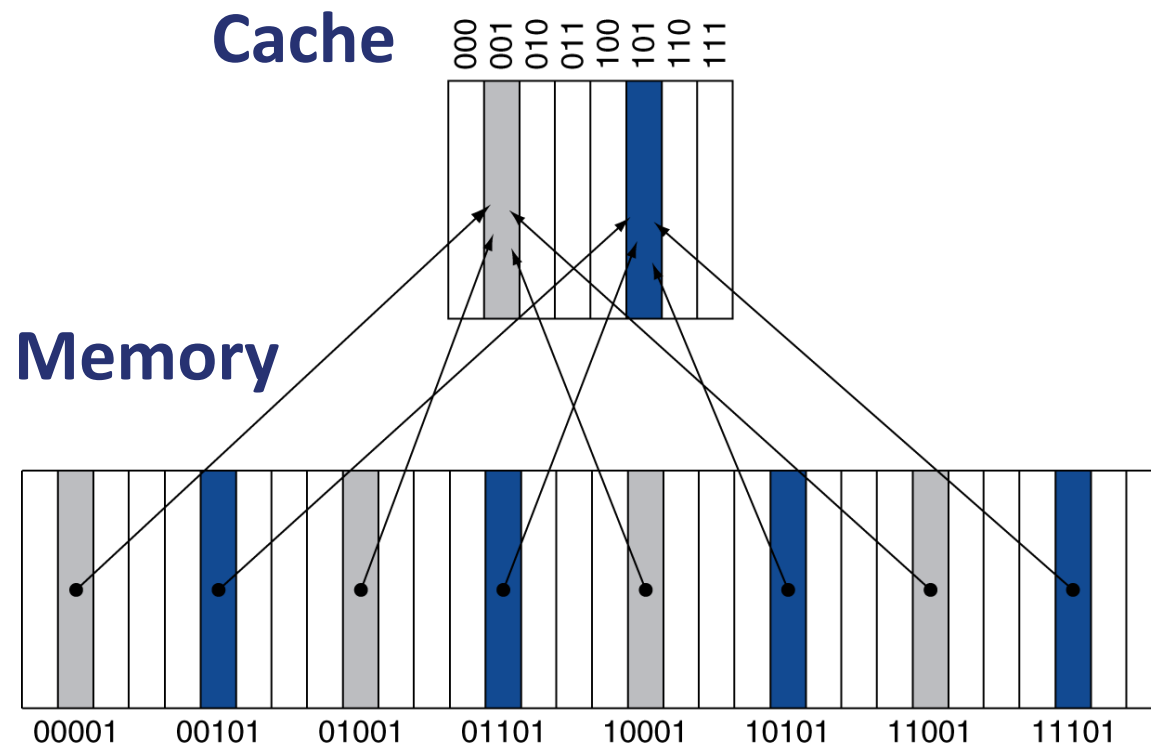
$X_4$
$X_1$
$X_{n-2}$
$X_{n-1}$
$X_2$
$X_n$
$X_3$

b. After the reference to  $X_n$

- How do we know if the data is present?
- Where do we look?

# Direct Mapped Cache

- Location determined by address
- Direct mapped: only one choice
  - (Block address) modulo (#Blocks in cache)



- #Blocks is a power of 2
- Use low-order address bits

# Tags and Valid Bits

- How do we know which particular block is stored in a cache location?
  - Store block address as well as the data
  - Actually, only need the high-order bits
  - Called the tag
- What if there is no data in a location?
  - Valid bit: 1 = present, 0 = not present
  - Initially 0

# Direct Mapped Cache Example

- 8-blocks, 1 word/block, direct mapped
- Initial state

Index	V	Tag	Data
000	N		
001	N		
010	N		
011	N		
100	N		
101	N		
110	N		
111	N		

# Direct Mapped Cache Example

Word addr	Binary addr	Hit/miss	Cache block
22	10 110	Miss	110

Index	V	Tag	Data
000	N		
001	N		
010	N		
011	N		
100	N		
101	N		
<b>110</b>	<b>Y</b>	<b>10</b>	<b>Mem[10110]</b>
111	N		



# Direct Mapped Cache Example

Word addr	Binary addr	Hit/miss	Cache block
26	11 010	Miss	010

Index	V	Tag	Data
000	N		
001	N		
<b>010</b>	<b>Y</b>	<b>11</b>	<b>Mem[11010]</b>
011	N		
100	N		
101	N		
110	Y	10	Mem[10110]
111	N		

# Direct Mapped Cache Example

Word addr	Binary addr	Hit/miss	Cache block
22	10 110	Hit	110
26	11 010	Hit	010

Index	V	Tag	Data
000	N		
001	N		
<b>010</b>	<b>Y</b>	<b>11</b>	<b>Mem[11010]</b>
011	N		
100	N		
101	N		
<b>110</b>	<b>Y</b>	<b>10</b>	<b>Mem[10110]</b>
111	N		

# Direct Mapped Cache Example

Word addr	Binary addr	Hit/miss	Cache block
16	10 000	Miss	000
3	00 011	Miss	011
16	10 000	Hit	000

Index	V	Tag	Data
<b>000</b>	<b>Y</b>	<b>10</b>	<b>Mem[10000]</b>
001	N		
010	Y	11	Mem[11010]
<b>011</b>	<b>Y</b>	<b>00</b>	<b>Mem[00011]</b>
100	N		
101	N		
<b>110</b>	<b>Y</b>	<b>10</b>	<b>Mem[10110]</b>
111	N		

# Direct Mapped Cache Example

Word addr	Binary addr	Hit/miss	Cache block
18	10 010	Miss	010

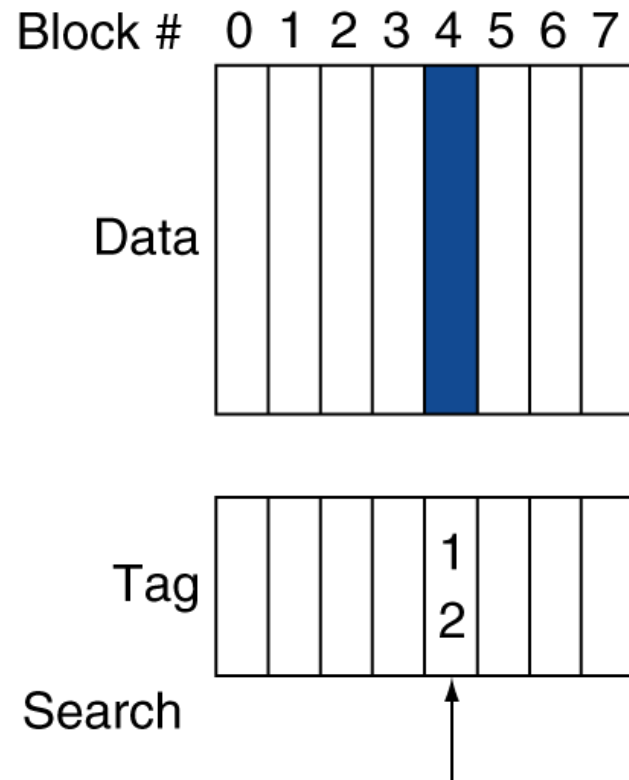
Index	V	Tag	Data
000	Y	10	Mem[10000]
001	N		
<b>010</b>	<b>Y</b>	<b>10</b>	<b>Mem[10010]</b>
011	Y	00	Mem[00011]
100	N		
101	N		
110	Y	10	Mem[10110]
111	N		

# Associative Caches

- Fully associative
  - Allow a given block to go in any cache entry
  - Requires all entries to be searched at once
  - Comparator per entry (expensive)
- $n$ -way set associative
  - Each set contains  $n$  entries
  - Block number determines which set
    - (Block number) modulo (#Sets in cache)
  - Search all entries in a given set at once
  - $n$  comparators (less expensive)

# Associative Cache Examples

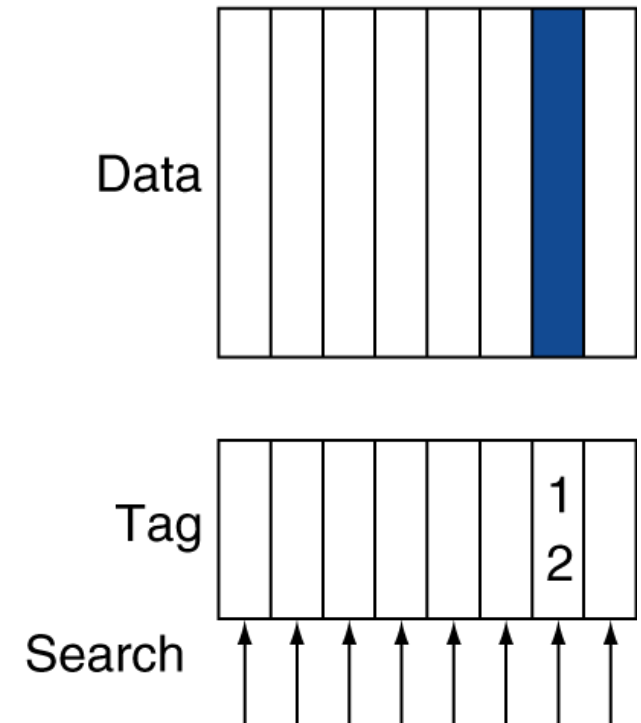
**Direct mapped**



**Set associative**



**Fully associative**



# Spectrum of Associativity

- For a cache with 8 entries

**One-way set associative  
(direct mapped)**

Block	Tag	Data
0		
1		
2		
3		
4		
5		
6		
7		

**Two-way set associative**

Set	Tag	Data	Tag	Data
0				
1				
2				
3				

**Four-way set associative**

Set	Tag	Data	Tag	Data	Tag	Data	Tag	Data
0								
1								

**Eight-way set associative (fully associative)**

Tag	Data	Tag	Data	Tag	Data	Tag	Data	Tag	Data	Tag	Data	Tag	Data	Tag	Data

# Associativity Example

- Compare 4-block caches
  - Direct mapped, 2-way set associative, fully associative
  - Block access sequence: 0, 8, 0, 6, 8
- Direct mapped

Block address	Cache index	Hit/miss	Cache content after access			
			0	1	2	3
0	0	miss	Mem[0]			
8	0	miss	Mem[8]			
0	0	miss	Mem[0]			
6	2	miss	Mem[0]		Mem[6]	
8	0	miss	Mem[8]		Mem[6]	



# Associativity Example

## ■ 2-way set associative

Block address	Cache index	Hit/miss	Cache content after access			
			Set 0		Set 1	
0	0	miss	Mem[0]			
8	0	miss	Mem[0]	Mem[8]		
0	0	hit	Mem[0]	Mem[8]		
6	0	miss	Mem[0]	Mem[6]		
8	0	miss	Mem[8]	Mem[6]		

## ■ Fully associative

Block address		Hit/miss	Cache content after access			
0		miss	Mem[0]			
8		miss	Mem[0]	Mem[8]		
0		hit	Mem[0]	Mem[8]		
6		miss	Mem[0]	Mem[8]	Mem[6]	
8		hit	Mem[0]	Mem[8]	Mem[6]	

# How Much Associativity

- Increased associativity decreases miss rate
  - But with diminishing returns
- Simulation of a system with 64KB D-cache, 16-word blocks, SPEC2000
  - 1-way: 10.3%
  - 2-way: 8.6%
  - 4-way: 8.3%
  - 8-way: 8.1%

# Replacement Policy

- Direct mapped
  - No choice
- Set associative
  - Prefer non-valid entry, if there is one
  - Otherwise, choose among entries in the set
- Least-recently used (LRU)
  - Choose the one unused for the longest time
    - Simple for 2-way, manageable for 4-way, too hard beyond that
- Random
  - Gives approximately the same performance as LRU for high associativity

# Write-Through

- On data-write hit, could just update the block in cache
  - But then cache and memory would be inconsistent
- Write through: also update memory
- But makes writes take longer
  - e.g., if base CPI = 1, 10% of instructions are stores, write to memory takes 100 cycles
    - Effective CPI =  $1 + 0.1 \times 100 = 11$
- Solution: write buffer
  - Holds data waiting to be written to memory
  - CPU continues immediately
    - Only stalls on write if write buffer is already full

# Write-Back

- Alternative: On data-write hit, just update the block in cache
  - Keep track of whether each block is dirty
- When a dirty block is replaced
  - Write it back to memory
  - Can use a write buffer to allow replacing block to be read first

# Write Allocation

- What should happen on a write miss?
- Alternatives for write-through
  - Allocate on miss: fetch the block
  - Write around: don't fetch the block
    - Since programs often write a whole block before reading it (e.g., initialization)
- For write-back
  - Usually fetch the block

# Multilevel Caches

- Primary cache attached to CPU
  - Small, but fast
- Level-2 cache services misses from primary cache
  - Larger, slower, but still faster than main memory
- Main memory services L-2 cache misses
- Some high-end systems include L-3 cache

# Measuring Cache Performance

- Components of CPU time
  - Program execution cycles
    - Includes cache hit time
  - Memory stall cycles
    - Mainly from cache misses
- With simplifying assumptions:

$$\begin{aligned}\text{Memory Stall Cycles} &= \frac{\text{Memory Accesses}}{\text{Program}} \times \text{Miss Rate} \times \text{Miss Penalty} \\ &= \frac{\text{Instructions}}{\text{Program}} \times \frac{\text{Misses}}{\text{Instructions}} \times \text{Miss Penalty}\end{aligned}$$



# Cache Performance Example

- Given
  - I-cache miss rate = 2%
  - D-cache miss rate = 4%
  - Miss penalty = 100 cycles
  - Base CPI (ideal cache) = 2
  - Load & stores are 36% of instructions
- Miss cycles per instruction
  - I-cache:  $0.02 \times 100 = 2$
  - D-cache:  $0.36 \times 0.04 \times 100 = 1.44$
- Actual CPI =  $2 + 2 + 1.44 = 5.44$ 
  - Ideal CPU is  $5.44/2 = 2.72$  times faster

# Average Access Time

- Hit time is also important for performance
- Average memory access time (AMAT)
  - $AMAT = \text{Hit time} + \text{Miss rate} \times \text{Miss penalty}$
- Example
  - CPU with 1ns clock, hit time = 1 cycle, miss penalty = 20 cycles, l-cache miss rate = 5%
  - $AMAT = 1 + 0.05 \times 20 = 2\text{ns}$ 
    - 2 cycles per instruction

# Overall Performance Summary

- When CPU performance increased
  - Miss penalty becomes more significant
- Decreasing base CPI
  - Greater proportion of time spent on memory stalls
- Increasing clock rate
  - Memory stalls account for more CPU cycles
- Can't neglect cache behavior when evaluating system performance

# Example: How Caches Affect Performance

## Matrix Multiplication

### Loop order: i, j, k

```
for (int i= 0; i < n; i++) {  
    for (int j= 0; j < n; j++) {  
        for (int k= 0; k < n; k++) {  
            C[i][j]+= A[i][k]*B[k][j];  
        }  
    }  
}
```

Running time:

13.714264 sec.

Performance:

~ 153 MFLOPS

### Loop order: i, k, j

```
for (int i= 0; i < n; i++) {  
    for (int k= 0; k < n; k++) {  
        for (int j= 0; j < n; j++) {  
            C[i][j]+= A[i][k]*B[k][j];  
        }  
    }  
}
```

Running time:

2.739385 sec.

Performance:

~ 795 MFLOPS

### Loop order: j, k, i

```
for (int j= 0; j < n; j++) {  
    for (int k= 0; k < n; k++) {  
        for (int i= 0; i < n; i++) {  
            C[i][j]+= A[i][k]*B[k][j];  
        }  
    }  
}
```

Running time:

19.074106 sec.

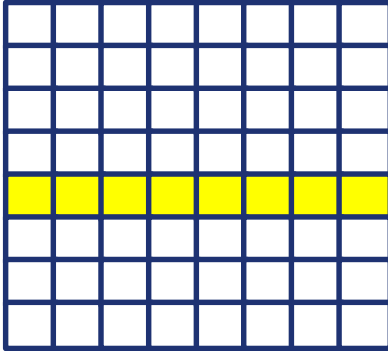
Performance:

~ 113 MFLOPS

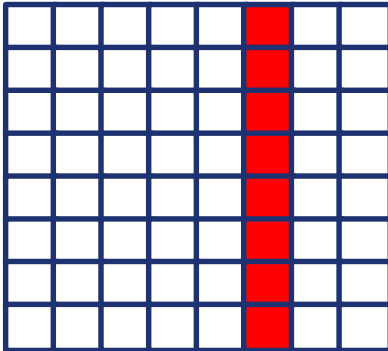
# Memory Access Patterns

Loop order: i, j, k

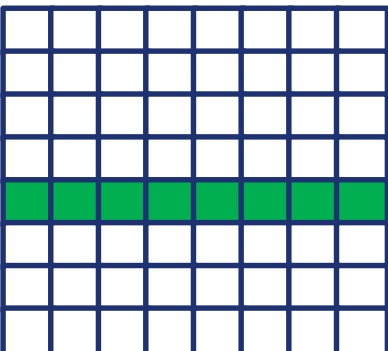
A



B

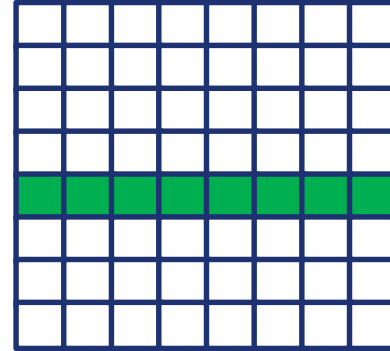


C

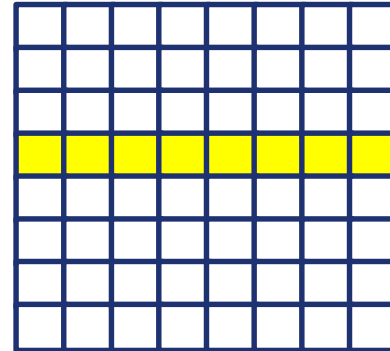


Loop order: i, k, j

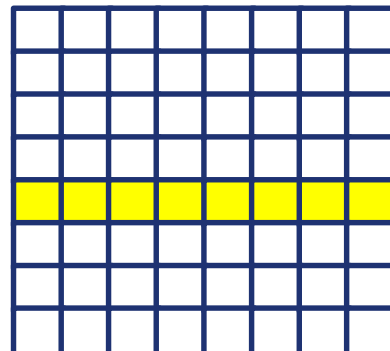
A



B

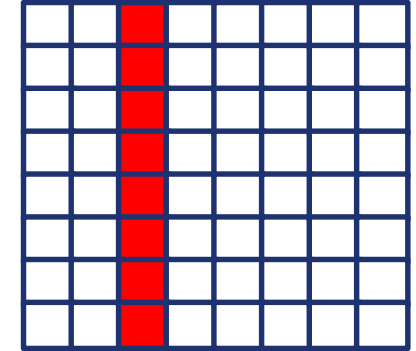


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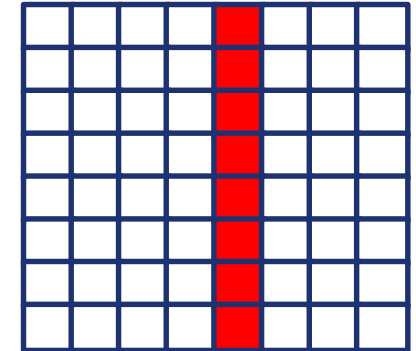


Loop order: j, k, i

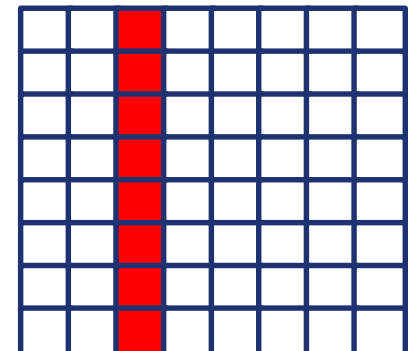
A



B



C



# Any Questions?

```
                .text
__start:      addi t1, zero, 0x18
                addi t2, zero, 0x21
cycle:        beq t1, t2, done
                slt t0, t1, t2
                bne t0, zero, if_less
                nop
                sub t1, t1, t2
                j cycle
                nop
if_less:      sub t2, t2, t1
                j cycle
done:         add t3, t1, zero
```