Singular Value Decomposition

SVD is based on a theorem from linear algebra which says that a rectangular matrix *M* can be split down into the product of three new matrices:

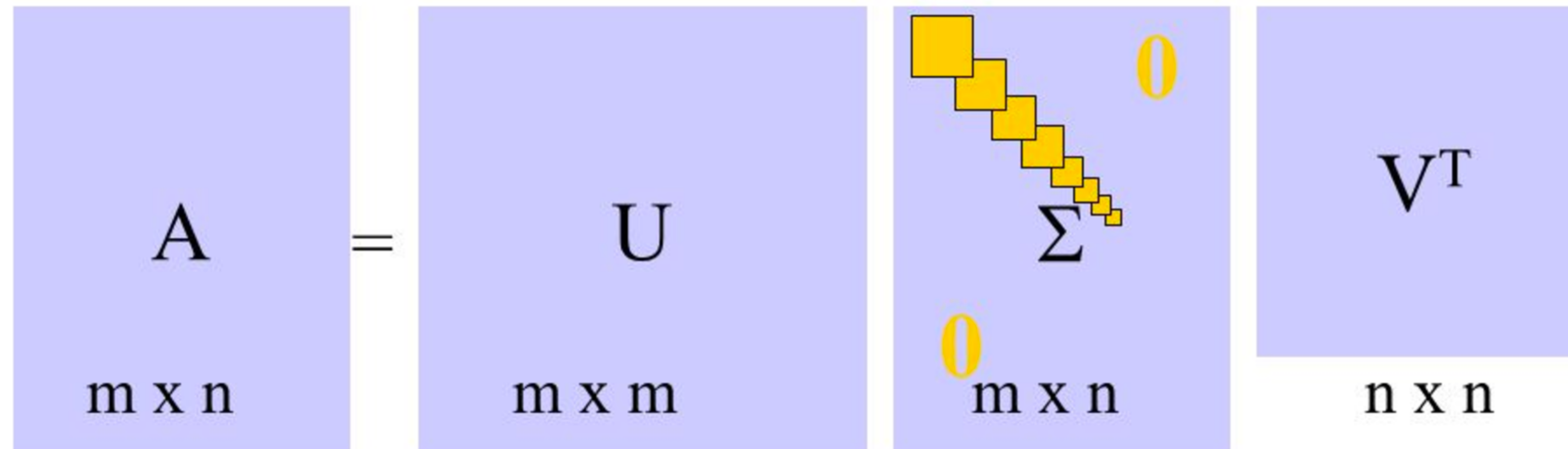
- An orthogonal matrix *U*;
- A diagonal matrix *S*;
- The transpose of an orthogonal matrix *V*.

$$\boxed{A_{mn} = U_{mm} S_{mn} V^{T}_{nn}}$$

Singular Value Decomposition

Usually written as follows:

$$A_{mn} = U_{mm} S_{mn} V^{T}_{nn}$$



where

- $U^T U = I$; $V^T V = I$;
- the columns of U are orthonormal eigenvectors of AA^T,
- the columns of V are orthonormal eigenvectors of $A^T A$, and
- S is a diagonal matrix containing the square roots of eigenvalues from U or V in decreasing order, called singular values. The singular values are always real numbers.
- If the matrix A is a real matrix, then U and V are also real.

Calculating the SVD

$$A_{mn} = U_{mm} S_{mn} V^T_{nn}$$

Calculating the SVD consists of finding the eigenvalues and eigenvectors of AA^T and A^TA.

- the eigenvectors of A^TA will produce the columns of V
- the eigenvectors of AA^T will produce the columns of U.

Calculating the SVD

$A_{mn} = U_{mm} S_{mn} V^T_{nn}$

1. Compute A^T, A^TA (or AA^T)

2. Compute eigenvalues of A^TA and sort them in descending order along its diagonal by resolving:
   $|| A^TA - \lambda I || = 0$

   - Characteristic equation above: its resolution gives eigenvalues of A^TA.
   - Square root the eigenvalues of A^TA to obtain the singular values of A.
   - Build a diagonal matrix S by placing singular values in descending order along its diagonal and compute S^{-1}
   - Re-use eignenvalues from step 2 in descending order and compute the eigenvectors of A^TA. Place these eigenvectors along the columns of V and compute its transpose V^T.

3. Compute U as $U = AS^{-1}V$

4. Compute the true scores T as $T = US$

http://scikit-learn.org/stable/modules/generated/sklearn.decomposition.TruncatedSVD.html

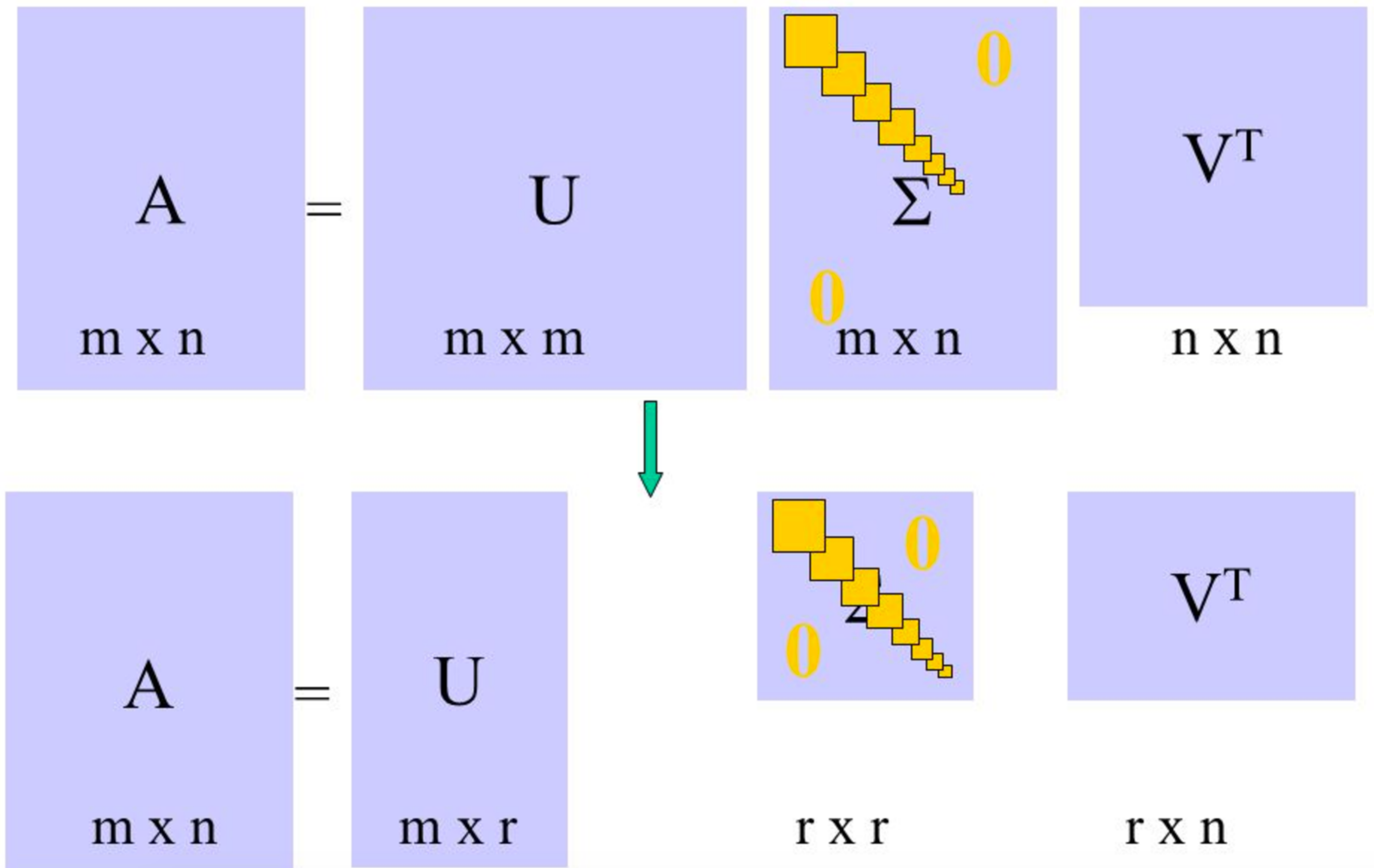
Truncated SVD

SVD	Truncated SVD
$A_{mn} = U_{mm} S_{mn} V^T_{nn}$	$A_{mn} = U_{mt} S_{tt} V^T_{tn}$

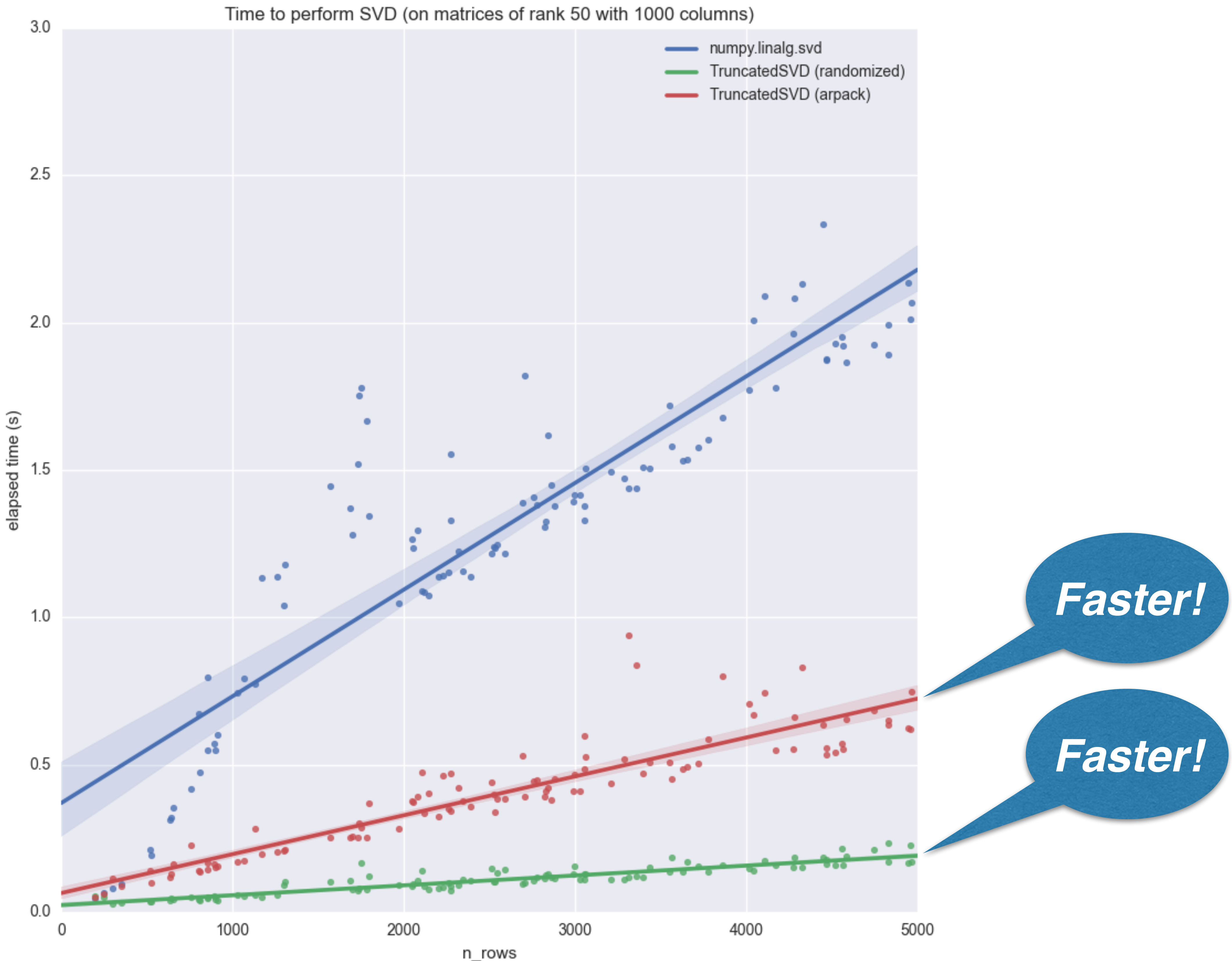
- **Only** the *t* column vectors of *U* and *t* row vectors of *V** corresponding to the *t* largest singular values S_t are calculated.
- **The rest of the matrix is discarded.**
  - much **quicker** and more economical.
- The matrix U_t is thus $m \times t$, S_t is $t \times t$ diagonal, and V_t^* is $t \times n$.

The truncated SVD is no longer an exact decomposition of the original matrix *M*, the approximate matrix *M* is the closest approximation to *M* that can be achieved by a matrix of rank *t*.

Truncated SVD



Truncated SVD



http://blog.explainmydata.com/2016/01/how-much-faster-is-truncated-svd.html