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GLOBAL STABILIZATION OF THE INVERTED PENDULUM USING MODEL PREDICTIVE CONTROL

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Abstract: Model Predictive Control (MPC) is used to improve the performance of energy control for swinging up a pendulum. A new MPC method is developed in continuous time, but it explicitly considers its digital implementation letting the control signal be piece-wise constant. The stability properties of the algorithm are analyzed in terms of the free MPC design parameters. The achieved performance improvement is witnesses by a detailed simulation study. *Copyright © 2002 IFAC*

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1. INTRODUCTION

The inverted pendulum is a classical benchmark for nonlinear control techniques, see e.g. (Åström and Furuta, 2000), (Angeli, 2001) and the papers quoted there. In particular, (Åström and Furuta, 2000) recently proposed an almost globally stabilizing strategy based on energy control for swinging up the pendulum. Starting from their results, we suggest here to resort to Model Predictive Control (MPC) to improve the control performance in terms of a cost function suitably selected by the designer. The idea is to add an extra term computed with MPC to the control signal provided by energy control. In so doing, it is possible to use this auxiliary signal to achieve some specific goals, such as the minimization of a prescribed cost.

The natural setting of the problem and of the solution based on energy control is the continuous time, therefore also the MPC implementation proposed here is developed for continuous-time systems. However, it basically differs from the continuous time MPC algorithms for nonlinear systems previously published in the literature, see e.g. (Mayne and Michalska, 1990), (Chen and Allgöwer, 1998). In fact, these methods assume that the MPC law is continuously computed by solving at any (continuous time) instant a difficult optimization problem. This is impossible in practice, since any implementation is practically performed in digital form and requires a non-negligible computational time. The alternative method proposed here explicitly takes into account these constraints and is based on a truly digital approach relying on a continuous time problem formulation. In fact, it is assumed that the signal computed by MPC is piece-wise constant and with a limited number of free moves in the future. This leads to a discontinuous (with respect to time) control law which preserves the stability of the overall system provided that the free tuning parameters of the MPC algorithm are properly chosen, see e.g. (Mayne *et al.*, 2000). The "sampling time" between two successive solutions of the MPC problem must be selected to be greater or equal to the computational time required. However, it does not represent a critical parameter, since the auxiliary signal computed

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with MPC acts on an already almost globally stable system.

Section 2 describes in general terms the innovative MPC formulation here adopted and states the main stability result. The proposed algorithm is then used in Section 3 to globally stabilize an inverted pendulum; the results achieved are compared to those provided by energy control. Finally, some concluding remarks close the paper.

2. PROBLEM STATEMENT

Consider the nonlinear continuous-time dynamic system

$$\dot{x}(t) = f(x(t), u(t)), t \geq \bar{t} \tag{1}$$

with $x(\bar{t}) = \bar{x}$ where $x \in R^n$, $u \in R^m$, $f(\cdot,\cdot) \in C^2$, $f(0,0) = 0$, and assume that the state and control variables are restricted to fulfill the following constraints

$$x(t) \in X, \quad u(t) \in U, \quad t \geq \bar{t} \tag{2}$$

In (2), and are subset of and respectively, containing the origin as an interior point.

For system (1), assume to know a state feedback control law

(3)

which stabilizes the origin of the closed-loop system (1), (3) and define the *output admissible set* (Gilbert and Tan, 1991) as an invariant set $\bar{X}(\kappa)$ such that (2) is satisfied $\forall x \in \bar{X}(\kappa)$.

The problem considered in this paper is to determine an additive feedback control signal $v(t)$, such that the resulting control law is

$$u(t) = \kappa(x(t)) + v(t) \tag{4}$$

where $v(t)$ is selected to possibly enlarge the stability region and to enhance the overall control performance with the fulfillment of the constraints (2). A practicable way to solve this problem is to resort to the MPC approach applied to the closed-loop system (1), (4) described for $t \geq \bar{t}$ by

$$\dot{x}(t) = f(x(t), \kappa(x(t)) + v(t)) \tag{5}$$

with $x(\bar{t}) = \bar{x}$. Nowadays, there are many MPC techniques for nonlinear systems guaranteeing stability properties under state and control constraints, see (Mayne *et al.*, 2000). However, in the MPC algorithms for continuous time systems presented so far, see e.g. (Mayne and Michalska, 1990), (Chen and Allgöwer, 1998), the control law is obtained by continuously solving a constrained finite horizon optimization problem, which is indeed a practically impossible task. As a matter of fact, discretization is required due to the computational load involved in the minimization procedure and for the control law implementation. The sampling mechanism was explicitly considered in (Fontes, 2001) where a continuous time MPC for which a functional optimization problem is solved at each sampling time is proposed in order to stabilize some nonholonomic systems.

For this reason, and with the aim to reduce the number of optimization variables, we here propose a new MPC approach, where the signal $v(t)$ is assumed to be piece-wise constant during intervals of equal length T_s, where the "sampling time" T_s is at least equal to the time required to complete the optimization step. In this way, there are many advantages: i) the number of future "free moves" of $v(t)$ is limited, ii) the "intersample behavior" is fully considered, iii) one can make the control design in continuous time, without the need of the approximate discretization required by many MPC algorithms for nonlinear systems, see (De Nicolao *et al.*, 1998), (Magni *et al.*, 2001), iv) the sampling time T_s is a free design variable.

In order to describe the method, first define a partition of $[0, +\infty)$ as an infinite sequence π $\{t_i\}_{i \geq 0}$ consisting of numbers $t = t_0 < t_1 < t_2 < ...$ with $t_{i+1} - t_i = T_s$. Given the control sequence

$$\bar{v}_{1,N_c}(t_i) := [v_{1_{t_i}}, v_{2_{t_i}}, ..., v_{N_{ct_i}}]$$

with $N_c \geq 1$, for any $t \geq t_i$ define the associated piece-wise constant control signal

$$v(t) = \begin{cases} v_{j_{t_i}} & t \in [t_{i+j-1}, t_{i+j}),\ j = 1, ..., N_c \\ 0 & t \geq t_i + N_c T_s \end{cases} \tag{6}$$

Then, for system (5) consider the following

Finite Horizon Optimal Control Problem (FHOCP). Given the positive integers N_c and N_p, $N_c \leq N_p$ at every "sampling time" instant t_i, minimize, with respect to $\bar{v}_{1,Nc}(t_i)$, the performance index

$$J_{FH}(x_{t_i}, \bar{v}_{1,N_c}(t_i), N_c, N_p) \tag{7}$$
$$= \int_{t_i}^{t_i+N_pT_s} \psi(x(\tau), v(\tau))d\tau + V_f(x(t_i + N_pT_s))$$

where $\alpha_\psi\left(\|(x,v)\|\right)^2 \leq \psi(x,v)$, $\psi(0,0) = 0$, where α_ψ is a suitable function of class K_∞. As for the terminal penalty V_f, it is here selected as

$$V_f(x(t_i + N_pT_s)) = x(t_i + N_pT_s)'\Pi x(t_i + N_pT_s)$$

where $\Pi = \Pi' > 0$.

The minimization of (7) must be performed under the following constraints:

(i) the state dynamics (5) with $x(t_i) = x_{t_i}$;

(ii) the constraints (2), $t \in [t_i, t_i + N_pT_s)$ with u given by (4);
(iii) $v(t)$ given by (6);
(iv) the terminal state constraint $x(t_i + N_pT_s) \in X_f$, where X_f is a suitable terminal set. ■

According to the well known *Receding Horizon* approach, the state-feedback MPC control law is derived by solving the $FHOCP$ at every sampling time instant t_i, and applying the constant control signal $v(t) = v^o_{1_{t_i}}$, $t \in [t_i, t_{i+1})$ where $v^o_{1_{t_i}}$ is the first column of the optimal sequence $\bar{v}^o_{1,N_c}(t_i)$. In so doing, one implicitly defines the discontinuous (with respect to time) state-feedback control law

$$v(t) = \kappa^{RH}(x_i) \quad , \quad t \in [t_i, t_{i+1}) \tag{8}$$

Remark 1. In *FHOCP*, N_p specifies the *prediction horizon* $[t_i, t_i + N_PT_s)$, while N_c defines the *control horizon* $[t_i, t_i + N_cT_s)$. The positive integer N_c corresponds to the number of optimization variables: the bigger it is, the heavier is the computational load involved by *FHOCP*.

Remark 2. It is here implicitly assumed that T_s is chosen so as to be larger than the time required to solve *FHOCP*. To this regard, recall that T_s is the "sampling time" for the MPC control law (8) acting on the (stabilized) system (1), (3) for optimization purposes. As such, there is some freedom in the selection of T_s, which can be chosen large enough. Note also that, in order to consider all the implementation aspects, one should take care of the delay of one sampling time T_s due to the measurement of $x(t_i)$, to the computation of the optimal solution of *FHOCP* and to the refresh of $v(t)$. However, this would make the analysis reported in the sequel much more involved without bringing new significant information on the approach proposed here. Then, for this aspect, the interested reader is referred to (Magni and Scattolini, 2001).

The use of the auxiliary feedback control law (8) can modify the stability properties of the closed-loop system (1), (3). Then, it is now to be verified that the proper selection of the free MPC design parameters N_p, N_c and, in particular, of the terminal cost V_f and of the terminal constraint set X_f can maintain the stability of the origin. To this regard, it is first necessary to define the resulting overall closed-loop system (1), (3), (8), given by, $\forall t \in [t_i, t_{i+1})$

$$\begin{bmatrix} \dot{x}(t) \\ \dot{x}_1(t) \end{bmatrix} = \begin{bmatrix} f(x(t), \kappa(x(t)) + \kappa^{RH}(x_1(t))) \\ 0 \end{bmatrix} \tag{9}$$
$$\begin{bmatrix} x(t_i) \\ x_1(t_i) \end{bmatrix} = \begin{bmatrix} x(t_i^-) \\ x(t_i^-) \end{bmatrix}$$

where the auxiliary state variable has been used to describe the hold mechanism producing the piecewise constant signal $v(t)$ defined through (8).

Letting $\xi = [x' \ x_1']' \in R^{2n}$ be the composite state, for system (9) we here introduce three definitions which will be used in the following analysis. First, let $\varphi^{RH}(t, \bar{t}, \bar{\xi}) \in R^{2n}$ be the movement of (9) with $\xi(\bar{t}) = \bar{\xi}$. Second, define by $X^0_\pi(N_c, N_p) \in R^n$ be the set of states x_{t_i} of system (5) in the sampling times t_i such the there exists a feasible control sequence $\bar{v}_{1,N_c}(t_i)$ for the $FHOCP$. Finally, denote by $X^0(N_c, N_p)$ the set of states of (9) at any time instant t such that a solution of $FHOCP$ will exist at the next sampling time t_i in the future, that is

$$X^0(N_c, N_p) := \left\{ \begin{array}{c} \xi \in R^{2n} : [I^n \ 0^n]\varphi^{RH}(t_i, t, \xi) \in X^0_\pi(N_c, N_p), \\ t_i = \arg \min\limits_{\tau \in \pi, \tau > t} \tau - t \end{array} \right\} \tag{10}$$

where I^n and 0^n are the identity and zero matrices of dimension n, respectively.

In order to state the main stability result, it is necessary to preliminary establish some intermediate results, which can be proven along the lines depicted in (Magni *et al.*, 2001). To this end, first let $A = \frac{\partial f}{\partial x}(0,0)$, $B = \frac{\partial f}{\partial u}(0,0)$, $K = \frac{\partial \kappa}{\partial x}(0)$, and note that, in view of the stability of the origin of system (1), (3), $A_{cl} = A + BK$ is Hurwitz.

Lemma 1. Consider a positive definite matrix $\tilde{Q}$ and a real positive scalar γ such that $\gamma < \lambda_{\min}(\tilde{Q})$. Let Π be the unique symmetric positive definite solution of the following Lyapunov equation:

$$A_{cl}'\Pi + \Pi A_{cl} + \tilde{Q} = 0 \tag{11}$$

Then, there exists a constant $c \in (0, \infty)$ specifying a neighborhood $\Omega_c(\kappa)$ of the origin of the form

$$\Omega_c(\kappa) = \{x \in \Re^n \mid x'\Pi x \le c\} \tag{12}$$

such that:

(i) $x \in X, \quad \kappa(x) \in U$, for all $x \in \Omega_c(\kappa)$;
(ii) $\forall x \in \Omega_c(\kappa)$,

$$2x'\Pi f(x, \kappa(x)) \le -\gamma x'x \tag{13}$$

In view of (i), $\Omega_c(\kappa)$ is an output admissible set for (1), (3), or equivalently for (5) with $v(t) = 0$, while (ii) states that $V_L(x) = x'\Pi x$ is a Lyapunov function for the same system. Then, the following result holds.

Theorem 2. Given the $FHOCP$ with $X_f = \Omega_c(\kappa)$, where $\Omega_c(\kappa)$ and Π are derived according to Lemma 1 with γ such that $\gamma x'x > \psi(x, 0)$, $\forall x \in X_f$, the $NMPC$ control algorithm applied to (5) asymptotically stabilizes the origin of (9) with output admissible set $X^o(N_c, N_p)$.

Proof:

First we show that $X^o(N_c, N_p)$ is an invariant set for system (9). To this end, assume that $\xi(t_i) \in X^o(N \quad N \ )$; than, from (10), it results that $\varphi^{RH}(t \ \bar{t} \ \bar{\xi}) \in X \ (N_c, N_p), \forall t \in [t_i, t_{i+1})$, provided that $[I \quad 0 \ ]\varphi \quad (t_{i+1}, t_i, \xi(t_i)) \in X_\pi^0(N_c, N_p)$. To prove this, note that, letting $\bar{v}_{t_i}^o$ the optimal solution of the $FHOCP$ at time t_i, a feasible solution at time t_{i+1} for the $FHOCP$ is

$$ {}_{i} \qquad\qquad {}_{i} \tag{14}$$

Then $[\mathrm{I} \qquad]\varphi \quad (\ _{i+1}, \ _i, \xi(\ _i)) \in \ _\pi(\ _c, \ _p)$ and $\varphi^{RH}(t_{i+1}, t_i, \xi(t_i)) \in X^o(N_c, N_p)$. Moreover in view of constraints (ii) of the $FHOCP$ (2) are satisfied.

Assume that $\bar{v}_{1,N_c}^o(t_i) = [v_{1_{t_i}}^o, v_{2_{t_i}}^o, ..., v_{N_{ct_i}}^o]$ is the solution of $FHOCP$ at time t_i, and define $\bar{v}_{2,N_c}^o(t_i) = [v_{2_{t_i}}^o, ..., v_{N_{ct_i}}^o]$. Letting, $\forall t \in [t_i, t_{i+1})$

$$\begin{aligned} &V(\xi, t) \\ &:= J_{FH}(\varphi^{RH}(t_{i+1}, t, \xi(t)), \bar{v}_{2,N_c}^o(t_i), N_c, N_p) \end{aligned}$$

it is now shown that $V(\xi, t)$ is a Lyapunov function for (9), $\forall \xi \in X^o(N_c, N_p)$. To this end, first note that by definition $V(0, t) = 0 \ \forall t$, and $V(\xi, t) > 0 \ x \neq 0$. Moreover

- $\forall t \in [t_i, t_{i+1})$

$$\begin{aligned} V(\xi(t), t) &= V(\xi(t_i), t_i) \\ &\quad - \int_{t_i}^{t} \psi(x(\tau), \kappa^{RH}(x_1(\tau)) d\tau \end{aligned}$$

- At time $t = t_{i+1}$, $\tilde{v}_{t_{i+1}}$ given by (14) is a (sub-optimal) feasible solution for the new $FHOCP$ so that

$$\begin{aligned} &V(\xi(t_{i+1}), t_{i+1}) \\ &\leq J_{FH}(\varphi^{RH}(t_{i+1}, t_i, \xi(t_i)), \tilde{v}_{1,N_c}(t_{i+1}), N_c, N_p) \\ &= V(\xi(t_{i+1}^-), t_{i+1}^-) \\ &\quad + \int_{t_i+N_pT_s}^{t_{i+1}+N_pT_s} \psi(x(\tau), 0) d\tau \\ &\quad + x(t_{i+1} + N_pT_s)'\Pi x(t_{i+1} + N_pT_s) \\ &\quad - x(t_i + N_pT_s)'\Pi x(t_i + N_pT_s) \end{aligned}$$

and from Lemma 1

$$\begin{aligned} V(\xi(t_{i+1}), t_{i+1}) &\leq V(\xi(t_{i+1}^-), t_{i+1}^-) \\ &\quad + \int_{t_i+N_pT_s}^{t_{i+1}+N_pT_s} (\psi(x(\tau), 0) - \gamma x(\tau) \ x(\tau)) \, d\tau \\ &\leq V(\xi(t_{i+1}^-), t_{i+1}^-) \end{aligned}$$

In conclusion

$$\begin{aligned} &V(\xi(t), t) - V(\xi(\bar{t}), \bar{t}) \\ &\leq - \int^{t} \psi(x(\tau), \kappa^{RH}(x_1(\tau)) d\tau \quad , t \geq \bar{t} \end{aligned}$$

and, in view of the assumption on $\psi(\cdot, \cdot)$, $\lim_{t \to \infty} x(t) = 0$, from (9) $\lim_{t \to \infty} \xi(t) = 0$ so that $\xi = 0$ is an asymptotically equilibrium point for (9).

3. GLOBAL STABILIZATION OF A PENDULUM

The equation of motion of a pendulum, written in normalized variables (Åström and Furuta, 2000), is

$$\ddot{\theta}(t) - \sin \theta(t) + u(t) \cos \theta(t) = 0, \tag{15}$$

where θ is the angle between the vertical and the pendulum, assumed to be positive in the clockwise direction, and u is the normalized acceleration, positive if directed as the positive real axis. The system has two state variables, the angle θ and the rate of change of the angle $\dot{\theta}$ (i.e. $x = [\theta \ \dot{\theta}]'$), defined taking θ modulo 2π, with two equilibria, i.e. $u = 0$, $\theta = 0$, $\dot{\theta} = 0$, and $u = 0$, $\theta = \pi$, $\dot{\theta} = 0$. Moreover, it is assumed that $|u| \leq n$.

The normalized total energy of the uncontrolled system ($u = 0$) is

$$E_n(t) = \frac{1}{2}\dot{\theta}^2(t) + \cos \theta(t) - 1$$

Consider now the energy control law

$$u(t) = sat_n(k_u E_n(t) sign(\dot{\theta}(t) \cos \theta(t))) \tag{16}$$

where sat_n is a linear function which saturates at n. In (Åström and Furuta, 2000) it is shown that the control law (16) is able to bring the pendulum at the upright position provided that its initial condition does not coincide with the download stationary position (in fact, with $\theta = \pi$, $\dot{\theta} = 0$, (16) gives $u = 0$ so that the pendulum remains in the download equilibrium). However, the upright equilibrium is an unstable saddle point. For this reason, when the system approaches the origin of the state space, a different strategy is used to locally stabilize the system. In the reported simulations, a linear control law computed with the LQ method applied to the linearized system has been used. This switching strategy, synthetically called in the sequel again "energy control", is described by the control law (3) with

$$\begin{aligned} &\kappa(x) \\ &= \begin{cases} sat_n(k_u E_n sign(\dot{\theta} \cos \theta)) & if \ x_{2\pi} \notin \Omega(K) \\ -K x_{2\pi}' & if \ x_{2\pi} \in \Omega(K) \end{cases} \end{aligned} \tag{17}$$

where $x_{2\pi} := [mod_{2\pi}(\theta) \ \dot{\theta}]$, K is the gain of the locally stabilizing LQ control law and $\Omega(K)$ is an associated output admissible set.

The $NMPC$ control algorithm described in the previous section has been applied to the closed-loop system (15), (17), with the aim of enhancing the performance provided by (17) in terms of the energy required to swing up the pendulum and of the time required to reach the upright position.

For this reason, the stage-cost of the $FHOCP$ is given by

$$\psi(x, u) = \phi(\theta)E_n^2 + (1 - \phi(\theta))V_n \qquad (18)$$

where

$$V_n = k_{v1}\frac{1}{2}\sin^2\left(\frac{\theta}{2}\right) + \frac{1}{2}\dot{\theta}^2 \qquad (19)$$

and

$$\phi(\theta) = \frac{\beta\tan^2\left(\frac{\theta}{2}\right)}{1 + \beta\tan^2\left(\frac{\theta}{2}\right)} \qquad (20)$$

The function V_n given by (19) penalizes the state deviation from the origin, while $\phi(\theta)$ allows to balance the need to reduce the total energy applied and to bring the state to zero. The dependence of $\phi(\theta)$ from the parameter β is shown in Fig. 1.

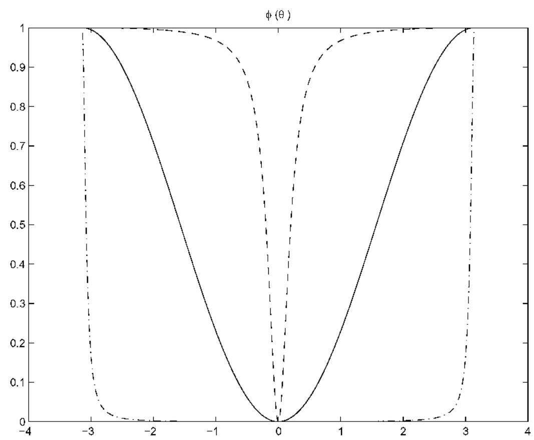

Fig. 1. $\phi(\theta)$ with $\beta = 0.01$ (dash-dot line), $\beta = 1$ (continuous line), $\beta = 100$ (dashed line)

In the following simulation examples the saturate value is $n = 0.29$, the $FHOCP$ is solved every $T_s = 0.1$ sec and the following parameters are used to synthesize the $NMPC$ control law.

- Auxiliary control law (17): $k_u = 100$, K is the LQ control gain with state penalty matrix Q di $\{2\ 5\ 1\}$, and control penalty matrix $R = 1$; $\Omega(K)$ is given by

$$\Omega(K) = \{x \in \Re^n \mid x'_{2\pi}Px_{2\pi} \leq c\}$$

  where P is the solution of the Riccati equation for the solution of the LQ control problem and $c = 0.001$.
- $FHOCP$: $N_p = 2500$, $k_{v1} = 10$, $\Omega_c(\kappa) = \Omega(K)$, $\tilde{Q} = Q + K'RK$, $\gamma = \lambda_{\min}(\tilde{Q})/20$.

For different choices of the tuning parameters, and starting from the initial condition $[\pi, 0]$, the results summarized in the Table have been obtained. In the table, J_{IH} is the infinite horizon performance index with stage cost (18) and % is the variation with respect to the performance provided by the "energy control" strategy. Note that for N_c 0, when the "energy control" strategy is used, a numerical error is sufficient to move the pendulum in the output admissible set guaranteed by the energy control strategy. On the contrary, with $N_c \geq 1$, the MPC control law guarantees the global stabilization of the inverted pendulum. Moreover note that the best improvement is obtained with a low β because in this case the energy is not considered in the cost function. In Fig. 2-4 the movement of the angle position, velocity and of the control signal are reported for the control strategies with β 0 and with different control horizons N_c.

$\beta = 0$				
N_c	0	2	4	8
J_{IH}	73.9	70.0	68.7	67.0
%	0	-5.3	-7.0	-9.4
$\beta = 0.01$				
J_{IH}	64.7	60.6	61.0	58.7
%	0	-6.3	-5.7	-9.2
$\beta = 1$				
J_{IH}	37.0	35.9	35.5	35.5
%	0	-3.2	-4.2	-4.2
$\beta = 100$				
N_c	0	2	4	8
J_{IH}	26.4	26.2	25.9	25.7
	0	-0.6	-1.8	-2.3

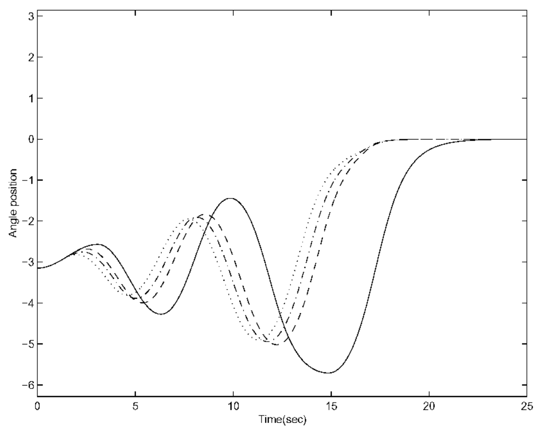

Fig. 2. Angle position movement with "energy control" (continuous line), MPC with β 0 and $N_c = 2$ (dashdot line), $N_c = 4$ (dashed line) and $N_c = 8$ (dotted line)

4. CONCLUSIONS

The study reported in this paper refers to the problem of swinging up a pendulum, which represents a classical benchmark for the analysis of nonlinear control

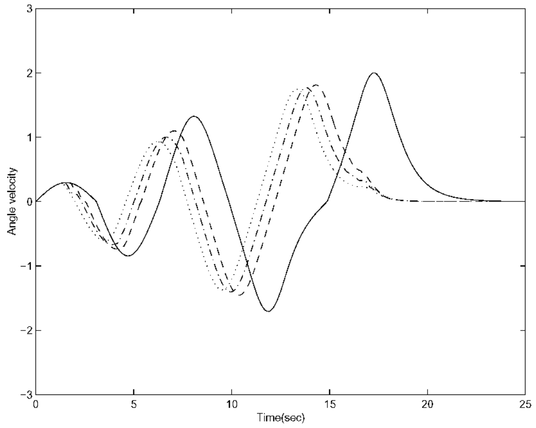

Fig. 3. Angle velocity movement with "energy control" (continuous line), MPC with $\beta = 0$ and $N_c = 2$ (dashdot line), $N_c = 4$ (dashed line) and $N_c = 8$ (dotted line)

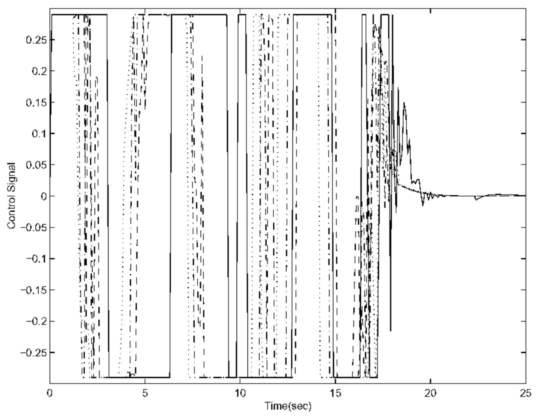

Fig. 4. Control signal with "energy control" (continuous line), MPC with $\beta = 0$ and $N_c = 2$ (dashdot line), $N_c = 4$ (dashed line) and $N_c = 8$ (dotted line)

techniques. It is believed that two ideas developed for the solution of this problem are of general validity and can represent a significant step towards the application of nonlinear MPC techniques. They are:

(1) the use of the MPC method to improve the control performance of an already stabilized system. In the worst case, MPC does not provide any extra benefit, but can not deteriorate the performance already achieved. In general, it allows to achieve specific goals.
(2) The formulation of the method is carried out in continuous time, but explicitly takes into account its intrinsic digital implementation. This is achieved by forcing the control variable to be constant between two successive sampling times. The proposed solution has a twofold advantage: first it avoids the approximate discretization of the continuous time plant model, which is usually required by the most popular MPC algorithms for nonlinear systems. Second, it allows to take care of many significant implementation aspects, such as the computational time required by the solution of the optimization problem.

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