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A Predictive Control Approach for the Inverse Pendulum on a Cart Problem

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***Abstract* - A model-based predictive control algorithm that uses a limited number of control sequences for on-line simulation of future behaviour of the process is presented in the paper. Each control sequence used in simulation generates a predicted sequence of the output signal. The output predicted sequences are analysed and evaluated and then, using a set of rules, the 'optimal' control signal is computed. For simulating the future behaviour of the process it is used a process model and also the previous sequences of the input and output signals from the process. The way it works, the algorithm permits the utilization of a nonlinear model of the process. For exemplifying the performance of the algorithm, the well-known problem of the inverted pendulum on a cart was chosen, in the position in which the cart movement is done in a limited space. The realized algorithm balances the position of the pendulum as well as the position of the cart.**

Index Terms – Predictive Control, Inverse Pendulum on a Cart

I. INTRODUCTION

The Inverted Pendulum is a classic study and application of control theory. In the past decades, lots of researches have deal with the inverted pendulum equilibrium control and lots of control methods have been tested. Among these different approaches we can denote: linear quadratic regulator and sliding mode control [1], harmonic variation of suspension point [2], nonlinear control approach based on energy control [3] and [4], fuzzy control [5], [6], H∞ control approach [7], neural networks [8], etc. The design and control of the pendulum requires a number of stages. A mathematical model must first be obtained. From this, the control system can be designed and simulated to obtain optimum control. The mechanical design of the pendulum must be light and be connected by a joint with as little friction as possible. Once the control is designed and the apparatus built, an interface to a PC must also be built so that the positions of the cart and pendulum can be read by the PC. Finally, a software user interface needs to be constructed that will give the user flexibility in allowing them to change the state feedback gains as well as many other options.

In our work we consider the inverted pendulum process consists of a pendulum attached to a motor driven cart. The purpose of the process is to keep the pendulum in the inverted position. This is achieved by moving the cart back and forth applying the appropriate force. The inverted pendulum system has many practical applications [9]:

- to control the vertical deviation of a space shuttle during take-off;
- to balance a rocket as it is moved to the launch-pad (crawler);
- to maintain a walking biped robot in its upright position.

These are just a few of the practical applications of the inverted pendulum process. As it can be seen from the above examples, the task at hand is essentially one of altering the centre of gravity of an object, using varied techniques, to keep the mass in its desired position of unstable equilibrium. As well as keeping the pendulum in the inverted position, it is also desirable to control the horizontal position of the cart on the track. One of the main reasons the horizontal position control is required is the fact that there is a finite length of the track upon which the cart can move. Mainly, there are three modes of operation for apparatus:

- keeping the pendulum inverted and only ensuring that the cart's position does not move out of range;
- applying a step input to the cart, requesting it to move from one position to another, whilst keeping the pendulum inverted;
- a third mode called "swing-up" involves starting the pendulum from its natural pendant position, gaining momentum until it swings up to the inverted position and maintaining there.

II. A MODEL BASED PREDICTIVE ALGORITHM

A model based predictive algorithm which uses on line simulation and rule based control, designed for linear processes, is developed in [10], [11] and [12]. In the followings, taking into account a SISO process, are shortly presented the main ideas of the algorithm.

Let us consider a process described by a numerical form model. An easy approach to find at each step of the sampling the optimal value of the control signal, is the following:

- it is chosen an enough wide prediction interval (N);
- all the possible control sequences are chosen;
- for each control sequence, the control system behaviour is simulated and then the cost function is computed;
- it is chosen the optimal sequence and the first term of this sequence it is ascribed to the control signal.

For a first look, the advantages of the proposed algorithm include the following: the minimum of objective function is global, it can be applied to nonlinear processes if a nonlinear model is available, and the constraints (linear or nonlinear) can easily be implemented.

The drawback of this scheme is a very long computational time, because there are possible a lot of sequences. For example, if the control signal is applied to the process using a "p" bits digital-analog converter (DAC), the number of sequences is 2^{p*N}.

Consequently, the number of the control sequences used in simulation must be reduced. Also, these sequences must be as much as can representative, in the way to permit the obtaining of as much as can the information on the future behaviour of the process. The sequences must be chosen in such a way that the set of rules for evaluation of the predicted input sequences and for decision taking concerning the control signal value to be easy to be conceived. Of course, choosing a limited number of sequences, the optimal solution obtained through the set of rules will be no more optimal, but the realized experiments, simulated and practical one [10],[11], shows a good behaviour of the control system.

For the linear processes having positive sign, in a first stage were chosen four control sequences:

$$\begin{aligned} u_1(t)&=\{u_{min},u_{min},..,u_{min}\},\\ u_2(t)&=\{u_{max},u_{min},..,u_{min}\},\\ u_3(t)&=\{u_{min},u_{max},..,u_{max}\},\\ u_4(t)&=\{u_{max},u_{max},..,u_{max}\} \end{aligned} \tag{1}$$

where u_{min} and u_{max} are the accepted limits of the control signal, limits imposed by the practical constraints. These values can depend on context and are function of time.

Using these control sequences, it results though simulation four sequence of the predicted output signal, denoted by $y_1(t)$, $y_2(t)$, $y_3(t)$, $y_4(t)$. The chosen set of rules uses the extreme values of the predicted errors of the output, denoted in the followings max_0, max_1, min_0, min_1. Other used notations: $e_i(t)$, $i=1..4$ are the predicted values of the errors in the four cases, d is dead time, t_0 is current time, N is the prediction interval.

It is noticed that that the $u_1(t)$ and $u_4(t)$ sequences permits the simulation of the future behaviour of the system for the extreme values of the control signal. This thing is useful especially when the error is relatively high. We can define the first two rules for this kind of situations:

Rule 1: If the sequence $u_1(t)$ leads to:

$$\max_0 = \max_{to+d<t<N}\{e_1(t)\},\ \max_0 > 0 \text{ and } e_1(t_0+d+1)<0$$

than it is chosen $u(t)=u_{min}(t)$.

Rule 2: If the sequence $u_4(t)$ leads to:

$$\min_1 = \min_{to+d<t<N}\{e_4(t)\},\ \min_1 < 0 \text{ and } e_4(t_0+d+1)>0$$

than it is chosen $u(t)=u_{max}(t)$.

With other words, in case of rule 1 in which it is used for simulation the sequence $u_1(t)$, if for $t=t_0+d+1$ the predicted error does not become positive, it is obvious to chose $u(t)=u_{min}(t)$. Similarly, for the rule 2 we choose $u(t)=u_{max}(t)$. The following two rules are made also for cases in which the error is relatively high:

Rule 3: If the sequence $u_2(t)$ leads to:

$$\max_1 = \max_{to+d<t<N}\{e_2(t)\}, \max_1 < 0$$

than it is chosen $u(t)=u_{max}(t)$.

Rule 4: if the sequence $u_3(t)$ leads to:

$$\min_0 = \min_{to+d<t<N}\{e_3(t)\}, \min_0 > 0$$

than it is chosen $u(t)=u_{min}(t)$.

With other words, in case of rule 3 in which it is used for simulation the sequence $u_2(t)$, if the predicted error does not become positive, we can choose $u(t)=u_{max}(t)$. This thing is obvious taking into account that the above choosing will lead to reducing the error, without leading to changing of the sign of error. Similarly, for the rule 4, we choose $u(t)=u_{max}(t)$.

In case in which the error is relatively low and it can not be applied none of the rules 1..4, the control signal can be obtained using a linear approximation. For example, if applying the $u_2(t)$ sequence it is obtained $max_1>0$ and if applying the $u_3(t)$ sequence it is obtained the $min_0<0$ it can be used:

Rule 5: if rules 1..4 can not be applied then it is chosen:

$$u(t) = \frac{u_{\min}(t)\max_1 - u_{\max}(t)\min_0}{\max_1 - \min_0}$$

These rules cover the most part of the possible situations and, for a lot of process category are sufficient for obtaining an acceptable behaviour of the control system.

The main aspect that in some case it is not convenient is the high variation of the resulted control signal. Also, in some situations only the applying of the set of rules 1..5 may lead to the existence of a steady state error. There are many ways to solve these problems. A possibility that leads to reducing the control signal variation and also to the obtaining of a steady error practically zero, is the reducing in time of the accepted limits for control signal, reducing that is realized function of context. Obviously, if it is necessary (for example if for some reasons the error increases), these limits will increase, eventually until to the extreme values accepted at that moment.

This predictive algorithm was used in practical applications for the control of some thermo processes, displacement motion control, etc. Using the on-line identification of the model parameters the algorithm becomes of adaptive-predictive type. More information and also some demo applications concerning the proposed algorithm can be found in reference [13].

Also, this algorithm can be applied for nonlinear processes that do not change the sign. An example of application in case of the heat exchanger can be found in reference [14].

III. THE NONLINEAR MODEL OF THE PROCESS

The mathematical model is a set of dynamic equations that provide an accurate description for the motion of a particular system. This is very important when attempting to design a controller to stabilize the system. The mathematical model of the inverted pendulum is discussed in the following section. Once the mathematical model is obtained, measured values for the system can be substituted into the equation. These equations can then be used in finding the proper control gains used in the feedback of the controller. The cart-pole system considered in this paper is seen in fig. 1. It consists of a cart that moves on a horizontal track of finite length. The pole is represented by a point mass attached to the end of a massless thin rod of length l that is attached to the cart at a pivot capable of unconstrained (360°) rotation. The state space equation of this system is given by [15]:

$$\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= \frac{(M+m)g\sin x_1 - \left(mlx_2^2\sin x_1 - bx_4 + u\right)\cos x_1}{l\left(M + m\sin^2 x_1\right)} \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= \frac{-mg\sin x_1\cos x_1 + mlx_2^2\sin x_1 - bx_4 + u}{\left(M + m\sin^2 x_1\right)}
\end{aligned} \tag{2}$$

where $x_1 = \theta$ is the angle of the pendulum, $x_2 = \dot{\theta}$, $x_3 = x$ the position of the cart, $x_4 = \dot{x}$ and u is applied force to the cart. The primary control objective is to stabilize the system at $\left[x, \dot{x}, \theta, \dot{\theta}\right] = [0,0,0,0]$, starting from $[0,0,180,0]$.

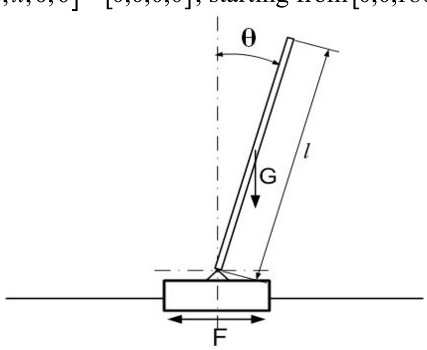

Fig.1 The inverted pendulum on a cart

This is a complicated control problem as the control is discontinuous at $\theta = \pm 90$. The system is actually not controllable at this point. The difficulties increase if the cart displacement limits are imposed.

IV. THE CONTROL OF THE INVERTED PENDULUM ON A CART

The proposed algorithm can not be applied direct to the problem of inverted pendulum on a cart, due to the nonlinearity and the complexity of the process and to the fact that in this application the sign of the process is changing. For applying this type of algorithm in this case, was necessary both the modification of the number and type of the control sequences used in simulation and also the implementation of a corresponding set of rules.

In a first variant [16] the realized algorithm permits the pendulum's elevation and speed and position stabilization of the cart without imposing other limits on the cart displacement direction. In the present paper is presented an improved variant of the algorithm that permits the fulfilling of the above conditions. The accepted limits of the cart are denoted with $x_{3min}<0$, $x_{3max}>0$ respectively.

It is supposed that the process model is accurate and the noise is null.

A. Initial stage of the pendulum's lifting

A first choice that must be done it refers to the pendulum's lifting strategy. For limitation the difficulties that can appear in analyzing and interpretation of the trajectories generated by the control sequences used in simulation, it was preferred the construction of some control sequences that contain only some values u_{max} and u_{min}. In the present paper it is supposed that $u_{max}= -u_{min}$. It is used the sequences (1) with the following remarks:

- it is obvious the fact that for lifting the pendulum in the vertical position it must be changed at a certain moment the force direction;
- if the displacement workspace of the cart is big enough, than, in lake of an adequate strategy it will be attained the limits x_{3min}, x_{3max} respectively; it can be searched the "optimal" moment in which should be realized the modification of the force direction which will lead to a quicker attaining of the final goal. For treating such situations a possibility is choosing the control sequences u_1 and u_4 under form:

$$\begin{aligned}
u_1(t) &= \{u_{min}, u_{min}, .., u_{min}, u_{max}, u_{max}, .., u_{max}\} \\
u_4(t) &= \{u_{max}, u_{max}, .., u_{max}, u_{min}, u_{min}, .., u_{min}\}
\end{aligned} \tag{3}$$

the force's direction is changed after $N_1<N$ time periods;

- if in simulation the accepted limits of the cart position are excelled, than the last step in simulation is repeated changing the force direction. This possibility appears if the displacement accepted space of the cart is not sufficient for lifting up the pendulum. It is possible to be necessary resumption of more steps in simulation.

The four trajectories of the pendulum obtained through simulation are analysed for obtaining the control sequence that leads most rapidly to the instable equilibrium position (or as closed as possible of it). A trajectory is characterized by the absolute minimal value of the deviation of the pendulums position toward the instable equilibrium position as well as to the time value at which is realized this minimum. It is denoted with min_1, min_2, min_3, min_4 the four minimal absolute values and with t_1, t_2, t_3, t_4 the moments at which these minimum can be attained. If during simulation, a control sequence produces a better trajectory then others that were found before, that control sequence is memorized. The operation of memorizing is necessary due to the fact that when it is adopted control sequences of type (3) it is possible to loose a founded "optimal" trajectory for a number smaller then N_1-1 sampling periods. A new trajectory obtained, for being taken into consideration it must be compared with the memorized "optimal" trajectory.

From the control sequence that generates at a certain moment the trajectory considered optimal, it is chosen the first term that it is attributed to the control signal. Since the optimal control sequence can be modified very often, especially in the initial phase of the pendulums lifting it is possible that the control signal to vary often between its accepted extreme values. This aspect must be taken into consideration for practical implementation and if it is necessary it must be modified the way of work.

As it is noticed, the set of rules used in frame of this stage corresponds to the set of rules 1-4 used for control of linear processes or some nonlinear processes that doesn't change the sign, but it is more complex. These rules are applied when the error is relatively high. In both situations the aim is to reduce this error using a control signal having the accepted extreme values. This implies the searching of that trajectory that brings the pendulum as closed as possible to the vertical position.

B. Intermediate stage of the pendulum's lifting

Finally, when the pendulum is brought in the vertical position, the control signal will be practically null. Additionally, in vertical position, the pendulum can be controlled by using smaller forces. These remarks have conducted in a first variant to choosing the following supplementary control sequences [16]:

$$u_5(\mathrm{t})=\{u_{min},0,..,0\},\quad u_6(\mathrm{t})=\{0,0,..0\},\quad u_7(\mathrm{t})=\{u_{max},0,..0\} \tag{4}$$

For the case in which the cart movement is limited in space, the following supplementary control sequences permitted the obtaining of better results:

$$u_5(\mathrm{t})=\{ku_{min},\ ku_{min}\ ,..,\ ku_{min}\},\quad u_6(\mathrm{t})=\{0,0,..0\},\quad u_7(\mathrm{t})=\{ku_{max},\ ku_{max},..\ ku_{max}\} \tag{5}$$

where $k<1$ is a parameter of the control algorithm.

Result three other output predicted sequences $y_5(t)$, $y_6(t)$, $y_7(t)$ and corresponding predicted errors $e_5(t)$, $e_6(t)$, $e_7(t)$.

These sequences can be modified function of the context. So, if in simulation the accepted limits of the cart position (x_{3min}, x_{3max}) are excelled, than the actual sense of the control signal is changed in this way:

- if at the simulation time t_0 the right limit is attained (x_{3max}), then it is chosen $u_5(t_0)=u_{min}$, $u_6(t_0)=u_{min}$, $u_7(t_0)=u_{min}$, $u_5(t_0+1)=ku_{min}$, $u_6(t_0+1)=0$, $u_7(t_0+1)=ku_{min}$ etc.;

- if at the time t_0 the left limit is attained (x_{3min}) than is chosen $u_5(t_0)=u_{max}$, $u_6(t_0)=u_{max}$, $u_7(t_0)=u_{max}$, $u_5(t_0+1)=ku_{max}$, $u_6(t_0+1)=0$, $u_7(t_0+1)=ku_{max}$ etc.;

So, sequences (5) doesn't have a fix construction; a term of a sequence may take values in the $\{u_{min},\ ku_{min},\ 0,\ ku_{max},\ u_{max}\}$ domain. The final aim is finding of some control sequences of type (5) that permits the pendulum to come to the instable equilibrium position, without exceeding the limits imposed to the cart. Is important that in this simulation both the pendulum and the cart must respect the imposed conditions, because the calculus of the control signal will use data in this manner obtained. The sequences (5) permit approaching to the instable equilibrium position using smaller forces. These become usefully after that the trajectory generated by them intersect this position of instable equilibrium. It can be made a lot of strategies of usage of these trajectories for computing the control signal inclusive through the usage of some optimization techniques.

Most of the time the simple crossover with the instable equilibrium position is not sufficient because it is possible that the pendulum cannot be maintained in that position.

It was chosen the next strategy: if after the usage of a control signal (5) it is attained the position of instable equilibrium, than for maintaining the pendulum as much as possible in that position, in the sequence are used only the values $\{u_{min},\ u_{max}\}$. As a result it can be obtained the following information: the time at which it takes place the attaining of the instable equilibrium position, time duration in which the pendulum can be maintained in that position, time at which the cart exceeds the imposed limits (x_{3min},x_{3max})etc. The information obtained from the realized simulations with the sequences (5) is used by the set of rules based on which the actual control signal is computed. From the viewpoint of the used rules, this stage is similar to the previous stage but there are used values more reduced of the control signal.

C. Final stage of pendulum stabilization

In the first two stages of pendulum lifting, the control signal can vary very much between the accepted extreme values $\{\ u_{min},\ u_{max}\ \}$. In the final stage it is necessary as much as possible that this variation to be a lot reduced.

For simulations, are used the not modified sequences (5) that permits (in simulation) the limits (x_{3min}, x_{3max}) surpassing. Also the pendulum is not maintained (in simulation) in the position of unstable equilibrium. The aim of using these sequences is first of all to find an "optimal" control signal whose final value to be null and that will bring the pendulum in position of instable equilibrium. Of course, is necessary to respect the imposed limits for cart displacement. From this reason, the passing to this stage of stabilization of the pendulum position is done after the analysis of the actual and estimated positions of the pendulum and cart. For obtaining the control signal a variant is using a linear approximation:

Rule 5.1: If $e_5(N_2)e_7(N_2)<0$, Then

$$u(t)=\frac{u_{min}(t)e_7(N_2)-u_{max}(t)e_5(N_2)}{e_7(N_2)-e_5(N_2)}$$ Else **Rule 5**

where N_2 is a parameter of the control algorithm and e_5(t) and e_7(t) are the errors obtained through utilization of the sequences u_5(t) and u_7(t).

Using the above relation will lead only to the obtaining of the vertical position of the pendulum (x_1=0, x_2=0). For obtaining also the position stabilization and cart velocity (x3=0, x4=0) it is used the following supplementary rule:

Rule 0: If $e_5(N_2)e_7(N_2)<0$ and $e(t)<\theta_p$

Then {If $e_6(N_2)e_5(N_2)<0$

$$\text{Then} \quad u(t)=\frac{u_{min}(t)e_6(N_2)}{e_6(N_2)-e_5(N_2)}$$

$$\text{Else} \quad u(t)=\frac{u_{max}(t)e_6(N_2)}{e_6(N_2)-e_7(N_2)}$$

$u(t) \leftarrow u(t)+k_p x$ }

Else If $e(t)<\theta_q$ and $e_5(N_2)e_7(N_2)>0$

Then If $e_6(N_2)>0$ Then $u(t)=u_{max}(t)$

Else $u(t)=u_{min}(t)$

Else **Rule 1, 2, 3, 4**

In this paper it is used next values of the parameters [15]: M=1.378 is the mass of the cart, m=0.0551 is the mass of the pendulum, l=0.325 is the length of the pendulum, g=9.81 is the gravity, b=12.98 is the coefficient of viscous friction for motion of the cart. The parameters θ_p, θ_q, N_2, k_p, can be chosen in large limits. For next examples the sample period is 0.1s, θ_p =80, θ_q =25, N_2=10, k_p=10, k=0.1, set point is changed from 180 to 0 at t=1s and the cart limits are x_{3min}=-4.9, x_{3max}=4.9.

First it is presented a case (fig. 2) for initial stage of pendulum's lifting if u_{max}=70, u_{min}=-70. At t=6.7s is marked the 'optimal' trajectory which was founded through simulations.

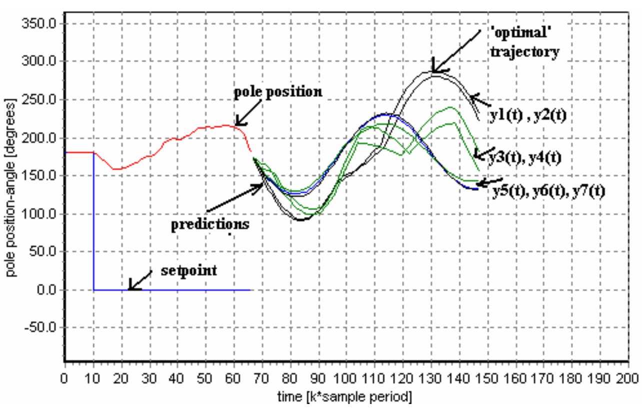

Fig.2 Example: Initial stage of pendulum's lifting

In fig. 3 is presented a case for the beginning of the intermediate stage of pendulum's lifting. At t=10.5s, one of the output sequences cross the setpoint. The simulation shows that this event will be at t=16.9s.

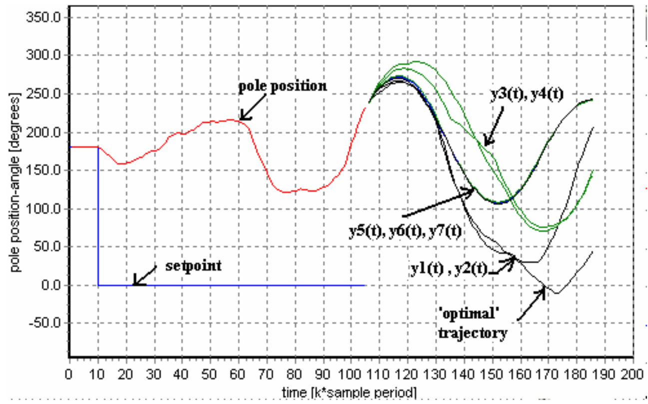

Fig.3 Example: Intermediate stage of pendulum's lifting

In what it follows are presented the results obtained using the above data and also: u_{max}=300, u_{min}=-300, the pendulums evolution for t=12s is presented in fig. 4. The cart evolution (speed and position) as well as the predicted position of the cart for t=12s is presented in fig. 5. The signal control (force) evolution that controls these movements is presented in fig. 6. As it can be seen, the pendulum is not brought up direct in vertical position, but through balancing. In case that are not imposed limits to the cart displacement, the pendulum can be brought without balancing at the vertical position. It can be distinguished the three stages of bringing the pendulum in the vertical position: initial stage for 1<t<4.1 in which are used the extreme values u_{max}, u_{min} of the control signal, the intermediate stage for 4.1<t<5.8 in which are used values from range {u_{min}, ku_{min}, 0, ku_{max}, u_{max}} and final stage in which the control signal can take any values from range [u_{min}, u_{max}] but finally, can take eventually with some oscillations, the control signal value will tend to 0.

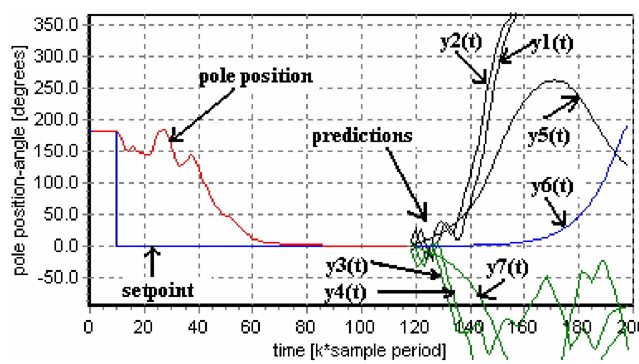

Fig.4 Pole position and predictions

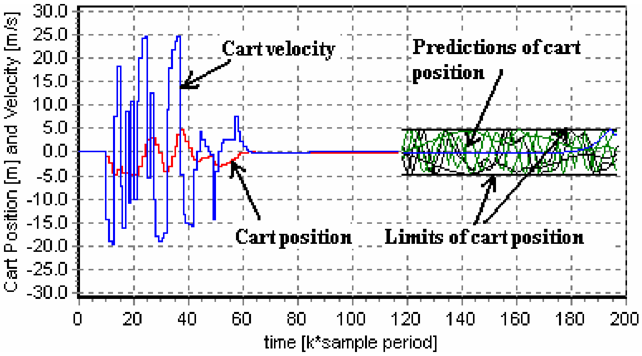

Fig.5 Cart position and predictions

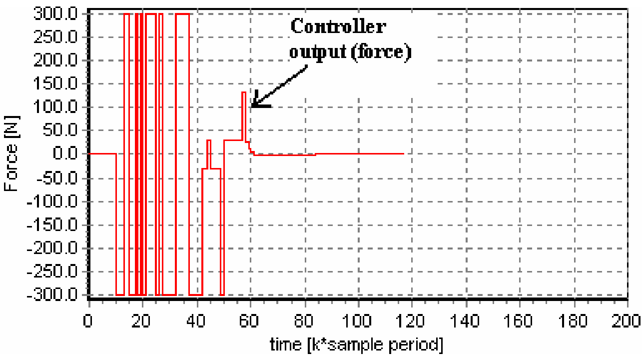

Fig.6 The controller output (force)

V. Conclusions

The proposed algorithm uses a number of control sequences for simulating the future behavior of the system. Through analysis of a set of rules of predicted evolutions, it is obtained the control signal considered optimal. Of course, since in simulation it is used a small number of control sequences, the respective optimum is not global, but it is obtained the advantage of time computing much smaller. If the control sequences and set of rules for analyzing and interpretation of the predicted evolutions are chosen accordingly, the control system behavior is good even in the case of a constraint nonlinear process.

A demo application (realized in Delphi) that implements the proposed algorithm can be downloaded from the proposed link (see reference [13]).

In the future, starting from the proposed algorithm, the work will focus on: the optimal chosen of the control parameters, the study of other set of control sequences, the study of other set of control rules, adaptive case and practical implementation.

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