Image Smoothing using Wavelet Transform

1 Introduction

This project aims to recover noisy black and white images by using wavelet transform-based denoising. A Daubechies wavelet is used to decompose the image, suppress noise through thresholding of detail coefficients, and reconstruct a cleaner version of the original image. Gaussian noise with user-defined intensity is added to simulate real-world degradation.

2 Main Content

2.1 Wavelet Decomposition

The 2D Discrete Wavelet Transform (DWT) decomposes the image into approximation and detail coefficients:

$$A[n] = \sum_k f[k] \cdot h[2n - k] \tag{1}$$

where $f[k]$ is the original signal, h is the low-pass Daubechies filter, and n is the index of the output signal.

In 2D, the image is convolved first row-wise and then column-wise, resulting in four subbands: Approximation (LL), Horizontal detail (LH), Vertical detail (HL), and Diagonal detail (HH).

2.2 Noise Estimation & Thresholding

Noise is simulated by adding zero-mean Gaussian noise:

$$f_{\text{noisy}}(x, y) = f(x, y) + \eta(x, y), \quad \eta \sim \mathcal{N}(0, \sigma^2) \tag{2}$$

The noise level is estimated using Median Absolute Deviation (MAD):

$$\hat{\sigma} = \frac{\text{median}(|D^D|)}{0.6745} \tag{3}$$

Soft-thresholding is applied to detail coefficients:

$$\tilde{w} = \begin{cases} \text{sign}(w)(|w| - T) & \text{if } |w| > T \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

Universal threshold:

$$T = \hat{\sigma}\sqrt{2 \log N} \tag{5}$$

2.3 Reconstruction

After thresholding, the inverse DWT is used to reconstruct the denoised image:

$$\hat{f}(x, y) = \text{IDWT}(A_j, \tilde{D}_j^H, \tilde{D}_j^V, \tilde{D}_j^D) \tag{6}$$

This reconstruction combines the approximation coefficients with the thresholded detail coefficients to produce the final smoothed image.

2.4 Results and Observations

The denoising process effectively reduces Gaussian noise while preserving key features of the image. The choice of wavelet and threshold significantly affects the outcome. Daubechies wavelets provide good balance between smoothness and localization.

3 Conclusion

Wavelet-based denoising is a powerful tool for image restoration. By decomposing an image into multiple frequency bands and applying adaptive thresholding, it is possible to suppress noise while retaining important image features.