

# IMO 2025 P2 Solution

BYTEDANCE SEED PROVER

17 July 2025

This is ByteDance Seed Prover team's solution to the 2025 IMO P2. The solution is generated and verified by our dedicated geometry engine SeedGeometry. However, to help human graders, ByteDance Seed Prover team translates the machine proof into a natural language (English) solution manually, with the original machine proof attached. For parts that might be confusing to follow (angle chasing, length chasing, or ratio chasing), we refer the graders to the full proof attached below (warning: very dry and boring). Please refer to the ByteDance team for any concerns and potential misinterpretations, as the full proof is considered verified by our symbolic system. Potential errors could be typos.

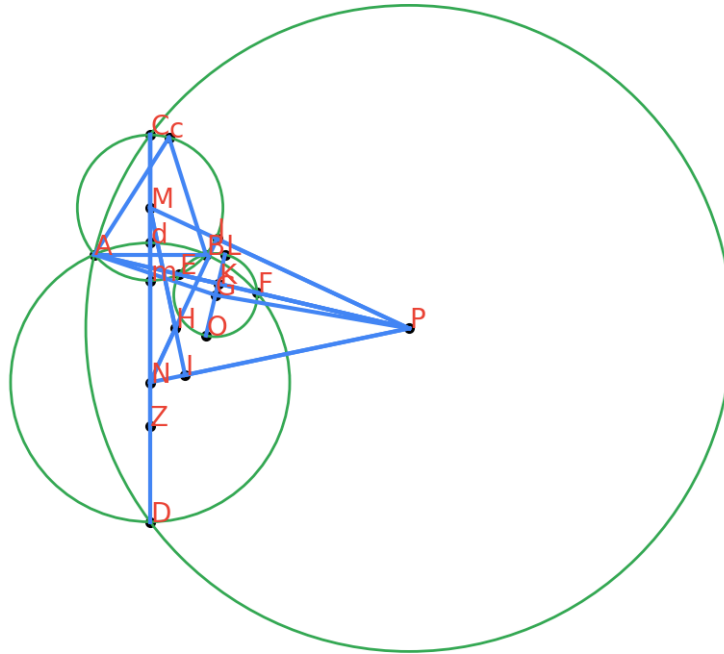
## §1 IMO 2025/2

## Problem statement

Let  $\Omega$  and  $\Gamma$  be circles with centres  $M$  and  $N$ , respectively, such that the radius of  $\Omega$  is less than the radius of  $\Gamma$ . Suppose circles  $\Omega$  and  $\Gamma$  intersect at two distinct points  $A$  and  $B$ . Line  $MN$  intersects  $\Omega$  at  $C$  and  $\Gamma$  at  $D$ , such that points  $C, M, N$  and  $D$  lie on the line in that order. Let  $P$  be the circumcentre of triangle  $ACD$ . Line  $AP$  intersects  $\Omega$  again at  $E \neq A$ . Line  $AP$  intersects  $\Gamma$  again at  $F \neq A$ . Let  $H$  be the orthocentre of triangle  $PMN$ .

Prove that the line through  $H$  parallel to  $AP$  is tangent to the circumcircle of triangle  $BEF$ .

(The *orthocentre* of a triangle is the point of intersection of its altitudes.)



The figure has a few auxiliary points: Line  $MN$  intersects the two circles at  $C, d, m$  and  $D$  in the order shown.  $\odot(BEF)$ 's center is denoted as  $G$ , with the foot of  $G$  on  $AP$  called  $K$ . Line  $KG$  intersects the  $\odot G$  at  $L$  and  $O$ . The foot of  $N$  on  $MP$  is  $J$  and that of  $M$  on  $NP$  is  $I$ . The original problem is equivalent to proving  $HO$  parallel to  $AP$ . In all of the proof below, angles are directed, denoted with  $\angle$ .

Explained in plain English, the proof is divided into two parts: proving  $\angle DAP = \angle MHO$  and  $AD \parallel HM$ .

¶  $\angle DAP = \angle MHO$

**Claim** —  $ADMP$  are concyclic.

*Proof.*

$$\angle AMD = \angle ACB = 2 \times \angle ACD = \angle APD.$$

□

Note that one can show  $ACNP$  concyclic in a similar way.  
Therefore,

$$\angle BmD = \angle BmC = \angle CBA = \angle CMP = \angle PDA = \angle DAP.$$

We now prove  $\angle BmD = \angle MHO$ , such that together we have  $\angle DAP = \angle MHO$ .  
The fact will follow if  $\triangle BDM \sim \triangle OMH$ , which can be derived from the fact that  $\angle BDM = \angle HMO$  and that  $BD/MO = DM/HM$ .

We first show  $\angle BDM = \angle HMO$ .

**Claim** —  $GM = GN$ .

*Proof.* We first show  $AN \parallel EM$ . By angle chasing, one can show  $CE \parallel AD$ . Considering the relative positioning of  $\odot M$  and  $\odot N$ , we have  $AN \parallel EM$ . By another step of angle chasing,  $\angle NMG = \angle GNM$ . □

**Claim** —  $\triangle ABE \sim \triangle NBG$  and  $\triangle ABE \sim \triangle MOG$ .

The claim goes without proving as angles and critical ratios can be easily obtained from the intermediate results above.

From the similarity of triangles, we have  $\angle BDF = \angle BAE = \angle GMO$ .

We prove  $\angle FDM = \angle HMG$  next, such that with  $\angle BDF = \angle GMO$ , we have  $\angle BDM = \angle HMO$ .

*Proof.*

$$\begin{aligned} (\angle BDM = \angle MPE) + (\angle MEP = \angle PDM) &\rightarrow \angle BDP = \angle PME \\ (\angle BDP = \angle PME) + (\angle BDF = \angle GME) &\rightarrow \angle FDP = \angle PMG \\ (\angle FDP = \angle PMG) + (\angle IMP = \angle PDM) &\rightarrow \angle FDM = \angle IMG \end{aligned}$$

□

We now show  $BD/MO = DM/HM$ .

**Claim** — The following pairs of triangles are similar:  $\triangle ABD \sim \triangle AdN$ ,  $\triangle ABD \sim \triangle ACP$ ,  $\triangle AFd \sim \triangle CDB$ ,  $\triangle Adm \sim \triangle PNM$ ,  $\triangle ACD \sim \triangle dAF$ .

The claim goes without proving as the system basically performs angle chasing and ratio chasing.

**Claim —**  $BN = MO$ .

*Proof.* One can prove  $\triangle BGN$  is congruent to  $\triangle OGM$ , because  $BG = GO$ ,  $GM = GN$  (above), and  $\angle BGN = \angle OGM$  (via angle chasing).  $\square$

From the similarity of triangles and the congruence above, we have, by ratio chasing,

$$\begin{aligned} (AC/Ad = BD/Fd) + (AC/Ad = IM/JM) &\rightarrow (BD/Fd = IM/JM) \\ (BD/Fd = IM/JM) &\rightarrow (BD/Fd = MP/HM) \\ (BD/Fd = MP/HM) + (BN/Fd = MP/Dm) &\rightarrow (BD/BN = Dm/HM) \\ (BD/BN = Dm/HM) + (BN = MO) &\rightarrow (BD/MO = Dm/HM) \end{aligned}$$

¶  $AD \parallel HM$  We now prove the other part

**Claim —** Line  $AD \parallel$  Line  $HM$

*Proof.*  $Ad \parallel IP$  because

$$\angle AdD = \frac{1}{2}\angle AND = \angle PND$$

and  $\angle DAd = \angle MIP = 90^\circ$ .  $\square$

The result  $HO \parallel AP$  follows, which completes the proof to the original problem.

¶ **The original problem constructed in SeedGeometry.**

```
Action(BaseAcuteTriangle, "", "BAc")
Action(CircumscribedCircle, "ABc", "M")
Action(MidArc, "BAM", "C")
Action(ExtendEqual, "CM", "m")
Action(ExtendEqual, "Cm", "Z")
Action(AnyPoint, "mZ", "N")
Action(CenterCircle, "NA")
Action(IntersectLineCircleOff, "CMNA", "dD")
Action(CircumscribedCircle, "ACD", "P")
Action(IntersectLineCircleOn, "PAM", "E")
Action(IntersectLineCircleOn, "PAN", "F")
Action(CircumscribedCircle, "BEF", "G")
Action(PerpendicularLine, "PMN", "I")
Action(PerpendicularLine, "PNM", "J")
Action(IntersectLineLine, "MINJ", "H")
Action(PerpendicularLine, "AGP", "K")
Action(IntersectLineCircleOff, "KGGB", "LO")
```

¶ **The full proof without trimming by SeedGeometry.**

```
[1] eqcircle (M, A, M, C) because eqcircle (M, A, ~, ABC) [0] (eqcircleToEqcircle)
[2] cong (AM, CM) because eqcircle (M, A, ~, ABC) [0] (eqcircleToCong)
[3] cong (CM, Mm) because midp (M, Cm) [0] (midpToCong)
[4] eqline (CM, Cm) because midp (M, Cm) [0] (midpToEqline)
[5] eqangle (ACD, ACM) because eqline (CD, CM) [0] (eqlineToEqangle)
[6] eqangle (BCD, BCM) because eqline (CD, CM) [0] (eqlineToEqangle)
[7] cong (AP, CP) because eqcircle (P, A, ~, ACD) [0] (eqcircleToCong)
[8] cong (AP, DP) because eqcircle (P, A, ~, ACD) [0] (eqcircleToCong)
[9] cong (AM, BM) because eqcircle (M, A, ~, ABC) [0] (eqcircleToCong)
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[10] eqcircle (M, A, M, B) because eqcircle (M, A, ~, ABc)[0] (eqcircleToEqcircle)
[11] eqline (CZ, Cm) because midp (m, CZ)[0] (midpToEqline)
[12] eqangle (BmN, BmZ) because eqline (Nm, Zm)[0] (eqlineToEqangle)
[13] eqangle (AmN, AmZ) because eqline (Nm, Zm)[0] (eqlineToEqangle)
[14] eqline (NZ, Nm) because eqline (Nm, Zm)[0] (eqlineToEqline)
[15] eqratio (AC, AM, BC, AM) because cong (AC, BC)[0] (congToEqratio)
[16] cong (AN, DN) because eqcircle (N, A, ~, Add)[0] (eqcircleToCong)
[17] cong (AN, Nd) because eqcircle (N, A, ~, Add)[0] (eqcircleToCong)
[18] eqcircle (N, F, ~, Add) because eqcircle (N, A, ~, Add)[0], eqcircle (N, A, N, F)[0] (addFact)
[19] eqcircle (N, A, N, D) because eqcircle (N, A, ~, Add)[0] (eqcircleToEqcircle)
[20] eqangle (FAd, PAD) because eqline (AF, AP)[0] (eqlineToEqangle)
[21] eqline (AE, AF) because eqline (AE, AP)[0], eqline (AF, AP)[0] (addFact)
[22] eqangle (CAE, CAP) because eqline (AE, AP)[0] (eqlineToEqangle)
[23] eqline (AP, EP) because eqline (AE, AP)[0] (eqlineToEqline)
[24] eqline (AE, EP) because eqline (AE, AP)[0] (eqlineToEqline)
[25] eqangle (EAM, PAM) because eqline (AE, AP)[0] (eqlineToEqangle)
[26] cong (BG, EG) because eqcircle (G, B, ~, BEF)[0] (eqcircleToCong)
[27] cong (BG, FG) because eqcircle (G, B, ~, BEF)[0] (eqcircleToCong)
[28] eqcircle (G, B, G, E) because eqcircle (G, B, ~, BEF)[0] (eqcircleToEqcircle)
[29] eqline (AE, AK) because eqline (AE, AP)[0], eqline (AK, AP)[0] (addFact)
[30] eqline (AF, AK) because eqline (AF, AP)[0], eqline (AK, AP)[0] (addFact)
[31] eqcircle (M, E, ~, ABc) because eqcircle (M, A, ~, ABc)[0], eqcircle (M, A, M, E)[0] (addFact)
[32] eqcircle (M, E, ~, ABC) because eqcircle (M, A, ~, ABC)[0], eqcircle (M, A, M, E)[0] (addFact)
[33] cong (AM, EM) because eqcircle (M, A, M, E)[0] (eqcircleToCong)
[34] eqline (GK, GO) because eqline (GK, KO)[0] (eqlineToEqline)
[35] eqline (CD, Cd) because eqline (CD, CM)[0], eqline (CM, Cd)[0] (addFact)
[36] cong (BG, GO) because eqcircle (G, B, ~, BLD)[0] (eqcircleToCong)
[37] eqangle (IPM, NPM) because eqline (IP, NP)[0] (eqlineToEqangle)
[38] eqangle (BPI, BPN) because eqline (IP, NP)[0] (eqlineToEqangle)
[39] eqangle (GMH, GMI) because eqline (HM, IM)[0] (eqlineToEqangle)
[40] eqangle (BMH, BMI) because eqline (HM, IM)[0] (eqlineToEqangle)
[41] eqangle (MIP, PJN) because perp (NJP)[0], perp (MIP)[0] (perpAndPerpToEqangle)
[42] eqline (JM, JP) because eqline (JP, MP)[0] (eqlineToEqline)
[43] eqline (JM, MP) because eqline (JP, MP)[0] (eqlineToEqline)
[44] eqangle (BPJ, BPM) because eqline (JP, MP)[0] (eqlineToEqangle)
[45] eqline (HJ, JN) because eqline (HN, JN)[0] (eqlineToEqline)
[46] eqcircle (M, C, ~, ABC) because eqcircle (M, A, ~, ABC)[0], eqcircle (M, A, M, C)[1] (addFact)
[47] eqcircle (M, C, ~, ABc) because eqcircle (M, A, ~, ABc)[0], eqcircle (M, A, M, C)[1] (addFact)
[48] eqangle (ACM, MAC) because cong (AM, CM)[2] (congToEqangle)
[49] eqcircle (M, C, M, m) because cong (CM, Mm)[3] (congToEqcircle)
[50] eqratio (AC, CM, AC, Mm) because cong (CM, Mm)[3] (congToEqratio)
[51] eqline (CD, Cm) because eqline (CD, CM)[0], eqline (CM, Cm)[4] (addFact)
[52] eqline (CM, Mm) because eqline (CM, Cm)[4] (eqlineToEqline)
[53] eqangle (ACM, ACm) because eqline (CM, Cm)[4] (eqlineToEqangle)
[54] eqangle (BCM, BCm) because eqline (CM, Cm)[4] (eqlineToEqangle)
[55] eqline (Cm, Mm) because eqline (CM, Cm)[4] (eqlineToEqline)
[56] eqline (Cd, Cm) because eqline (CM, Cm)[4], eqline (CM, Cd)[0] (addFact)
[57] contri (AMP, CMP) because cong (AP, CP)[7], cong (AM, CM)[2] (congAndCongAndCongToContri)
[58] eqangle (ACP, PAC) because cong (AP, CP)[7] (congToEqangle)
[59] eqratio (AD, AP, AD, CP) because cong (AP, CP)[7] (congToEqratio)
[60] eqangle (ADP, PAD) because cong (AP, DP)[8] (congToEqangle)
[61] cong (CP, DP) because cong (AP, CP)[7], cong (AP, DP)[8] (addFact)
[62] contri (ACM, BCM) because cong (AC, BC)[0], cong (AM, BM)[9] (congAndCongAndCongToContri)
[63] eqcircle (M, B, M, E) because eqcircle (M, A, M, B)[10], eqcircle (M, A, M, E)[0] (addFact)
[64] eqcircle (M, B, ~, ABC) because eqcircle (M, A, M, B)[10], eqcircle (M, A, ~, ABC)[0] (addFact)
[65] eqline (Cm, Zm) because eqline (CZ, Cm)[11] (eqlineToEqline)
[66] eqangle (ANZ, ANm) because eqline (NZ, Nm)[14] (eqlineToEqangle)
[67] eqangle (BNZ, BNm) because eqline (NZ, Nm)[14] (eqlineToEqangle)
[68] eqratio (AC, CM, BC, AM) because cong (AC, BC)[0], cong (AM, CM)[2], eqratio (AC, AM, BC, AM)[15] (eqratioAndCongToEqratio)
[69] contri (ANP, DNP) because cong (AP, DP)[8], cong (AN, DN)[16] (congAndCongAndCongToContri)
[70] eqangle (ADN, NAD) because cong (AN, DN)[16] (congToEqangle)
[71] eqangle (AdN, Nad) because cong (AN, Nd)[17] (congToEqangle)
[72] eqcircle (N, F, ~, Afd) because eqcircle (N, F, ~, Add)[18] (eqcircleToEqcircle)
[73] eqangle (Add, Afd) because eqcircle (N, F, ~, Add)[18] (eqcircleToEqangle)
[74] eqcircle (N, D, ~, Add) because eqcircle (N, A, ~, Add)[0], eqcircle (N, A, N, D)[19] (addFact)
[75] eqangle (DAE, DAF) because eqline (AE, AF)[21] (eqlineToEqangle)
[76] eqline (AE, EF) because eqline (AE, AF)[21] (eqlineToEqline)
[77] eqangle (BAE, BAF) because eqline (AE, AF)[21] (eqlineToEqangle)
[78] eqangle (APM, EPM) because eqline (AP, EP)[23] (eqlineToEqangle)
[79] eqangle (AEM, PEM) because eqline (AE, EP)[24] (eqlineToEqangle)
[80] cong (EG, FG) because cong (BG, EG)[26], cong (BG, FG)[27] (addFact)
[81] eqcircle (G, E, ~, BEF) because eqcircle (G, B, ~, BEF)[0], eqcircle (G, B, G, E)[28] (addFact)
[82] eqline (AK, EK) because eqline (AE, AK)[29] (eqlineToEqline)
[83] eqline (AK, FK) because eqline (AF, AK)[30] (eqlineToEqline)
[84] cong (BM, EM) because eqcircle (M, E, ~, ABc)[31] (eqcircleToCong)
[85] eqcircle (M, E, ~, ABE) because eqcircle (M, E, ~, ABc)[31] (eqcircleToEqcircle)
[86] eqcircle (M, E, ~, ACE) because eqcircle (M, E, ~, ABC)[32] (eqcircleToEqcircle)
[87] eqangle (AEM, MAE) because cong (AM, EM)[33] (congToEqangle)
[88] eqangle (KGM, OGM) because eqline (GK, GO)[34] (eqlineToEqangle)
[89] eqangle (BGK, BGO) because eqline (GK, GO)[34] (eqlineToEqangle)
[90] eqline (CD, Dd) because eqline (CD, Cd)[35] (eqlineToEqline)
[91] eqline (Cd, Dd) because eqline (CD, Cd)[35] (eqlineToEqline)
[92] eqratio (BE, BG, BE, GO) because cong (BG, GO)[36] (congAndEqratioToEqratio)
[93] perp (MJN) because perp (NJP)[0], eqline (JM, JP)[42] (perpAndEqlineToPerp)
[94] eqangle (JMN, PMN) because eqline (JM, MP)[43] (eqlineToEqangle)
[95] eqangle (BMJ, BMP) because eqline (JM, MP)[43] (eqlineToEqangle)
[96] eqangle (IPM, NPJ) because eqangle (BPJ, BPM)[44], eqangle (BPI, BPN)[38] (eqangleAndEqangleToEqangle)
[97] eqangle (ACD, MAC) because eqangle (ACM, MAC)[48], eqangle (ACD, ACM)[5] (addFact)
[98] eqcircle (M, m, ~, ABC) because eqcircle (M, C, ~, ABC)[46], eqcircle (M, C, M, m)[49] (addFact)
[99] eqcircle (M, A, M, m) because eqcircle (M, A, M, C)[1], eqcircle (M, C, M, m)[49] (addFact)
[100] eqcircle (M, m, ~, ABc) because eqcircle (M, C, ~, ABc)[47], eqcircle (M, C, M, m)[49] (addFact)
[101] eqline (Cm, Dm) because eqline (CD, Cm)[51] (eqlineToEqline)
[102] eqline (CD, Dm) because eqline (CD, Cm)[51] (eqlineToEqline)
[103] eqline (CD, Mm) because eqline (CD, CM)[0], eqline (CM, Mm)[52] (addFact)
[104] eqangle (AMC, AMm) because eqline (CM, Mm)[52] (eqlineToEqangle)
[105] eqline (CZ, Mm) because eqline (Cm, Mm)[55], eqline (CZ, Cm)[11] (addFact)
[106] eqline (Cm, dm) because eqline (Cd, Cm)[56] (eqlineToEqline)

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[107] eqangle (AMP, PMC) because contri (AMP, CMP)[57] (contriToEqangle)  
 [108] eqangle (CDP, PCD) because cong (CP, DP)[61] (congToEqangle)  
 [109] eqangle (ACM, MCB) because contri (ACM, BCM)[62] (contriToEqangle)  
 [110] eqangle (ACB, AMC) because eqcircle (M, B,  $\sim$ , ABC)[64], eqangle (AMC, CMB)[0] (eqcircleAndEqangleToEqangleHalf)  
 [111] eqangle (BmC, BmZ) because eqline (Cm, Zm)[65] (eqlineToEqangle)  
 [112] eqangle (AmC, AmZ) because eqline (Cm, Zm)[65] (eqlineToEqangle)  
 [113] eqratio (AC, Mm, BC, AM) because eqratio (AC, CM, BC, AM)[68], eqratio (AC, CM, AC, Mm)[50] (addFact)  
 [114] eqangle (ANP, PND) because contri (ANP, DNP)[69] (contriToEqangle)  
 [115] eqangle (AEB, FEB) because eqline (AE, EF)[76] (eqlineToEqangle)  
 [116] perp (EKG) because perp (AKG)[0], eqline (AK, EK)[82] (perpAndEqlineToPerp)  
 [117] perp (FKG) because perp (AKG)[0], eqline (AK, FK)[83] (perpAndEqlineToPerp)  
 [118] contri (BGM, EGM) because cong (BG, EG)[26], cong (BM, EM)[84] (congAndCongAndCongToContri)  
 [119] eqcircle (M, B,  $\sim$ , ABE) because eqcircle (M, B, M, E)[63], eqcircle (M, E,  $\sim$ , ABE)[85] (addFact)  
 [120] eqangle (ADC, AdD) because eqline (CD, Dd)[90] (eqlineToEqangle)  
 [121] eqangle (BDC, BDd) because eqline (CD, Dd)[90] (eqlineToEqangle)  
 [122] eqangle (AdC, AdD) because eqline (Cd, Dd)[91] (eqlineToEqangle)  
 [123] perp (HJM) because eqline (HJ, JN)[45], perp (MJN)[93] (eqlineAndPerpToPerp)  
 [124] eqangle (HMJ, IMP) because eqangle (BMH, BMI)[40], eqangle (BMJ, BMP)[95] (eqangleAndEqangleToEqangle)  
 [125] simtri (IMP, JNP) because eqangle (MIP, PJN)[41], eqangle (IPM, NPJ)[96] (eqangleAndEqangleAndEqangleToSimtri)  
 [126] eqangle (DCP, PAM) because eqangle (ACP, PAC)[58], eqangle (ACD, MAC)[97] (eqangleAndEqangleToEqangle)  
 [127] eqcircle (M, m,  $\sim$ , Bcm) because eqcircle (M, m,  $\sim$ , ABC)[98] (eqcircleToEqcircle)  
 [128] eqangle (ABm, ACm) because eqcircle (M, m,  $\sim$ , ABC)[98] (eqcircleToEqangle)  
 [129] eqangle (ABC, AmC) because eqcircle (M, m,  $\sim$ , ABC)[98] (eqcircleToEqangle)  
 [130] perp (CAm) because eqcircle (M, m,  $\sim$ , ABC)[98], eqline (CM, Mm)[52] (eqcircleAndEqlineToPerp)  
 [131] perp (CBm) because eqcircle (M, m,  $\sim$ , ABC)[98], eqline (CM, Mm)[52] (eqcircleAndEqlineToPerp)  
 [132] eqangle (BAm, BCm) because eqcircle (M, m,  $\sim$ , ABC)[98] (eqcircleToEqangle)  
 [133] eqcircle (M, m,  $\sim$ , ABm) because eqcircle (M, m,  $\sim$ , ABc)[100] (eqcircleToEqcircle)  
 [134] eqangle (BmC, Bmd) because eqline (Cm, Dm)[101] (eqlineToEqangle)  
 [135] eqangle (AmC, Amd) because eqline (Cm, Dm)[101] (eqlineToEqangle)  
 [136] eqangle (ADC, Adm) because eqline (CD, Dm)[102] (eqlineToEqangle)  
 [137] eqline (DM, Mm) because eqline (CD, Mm)[103] (eqlineToEqline)  
 [138] eqline (CM, Zm) because eqline (CZ, Mm)[105] (eqlineToEqline)  
 [139] eqangle (AmC, Amd) because eqline (Cm, dm)[106] (eqlineToEqangle)  
 [140] eqangle (ABC, AMP) because eqangle (AMP, PMC)[107], eqcircle (M, A,  $\sim$ , ABC)[0] (eqangleAndEqcircleToEqangleHalf)  
 [141] eqangle (ACM, mCB) because eqangle (ACM, MCB)[109], eqangle (BCM, Bcm)[54] (addFact)  
 [142] eqangle (ACM, MCB) because eqangle (ACM, MCB)[109], eqangle (ACM, ACm)[53] (addFact)  
 [143] eqangle (ACM, DCB) because eqangle (ACM, MCB)[109], eqangle (BCD, BCM)[6] (addFact)  
 [144] eqangle (ACB, Amm) because eqangle (ACB, AMC)[110], eqangle (AMC, Amm)[104] (addFact)  
 [145] contri (EGK, FGK) because cong (EG, FG)[80], perp (EKG)[116], perp (FKG)[117] (eqangleAndCongAndCongToContri)  
 [146] eqangle (BGM, MGE) because contri (BGM, EGM)[118] (contriToEqangle)  
 [147] eqangle (BMG, GME) because contri (BGM, EGM)[118] (contriToEqangle)  
 [148] eqangle (HJM, MIP) because perp (HJM)[123], perp (MIP)[0] (perpAndPerpToEqangle)  
 [149] eqratio (IM, JM, MP, NP) because simtri (IMP, JNP)[125] (simtriToEqratio)  
 [150] eqangle (CDP, MAP) because eqangle (CDP, PCD)[108], eqangle (DCP, PAM)[126] (addFact)  
 [151] eqangle (ABC, Cmb) because eqcircle (M, m,  $\sim$ , Bcm)[127], eqcircle (M, A,  $\sim$ , ABC)[0], eqangle (AMC, CMB)[0] (eqcircleAndEqangleToEqangleCen)  
 [152] eqangle (ABm, ACM) because eqangle (ABm, ACm)[128], eqangle (ACM, ACm)[53] (addFact)  
 [153] eqangle (CAM, MIP) because perp (MIP)[0], perp (CAm)[130] (perpAndPerpToEqangle)  
 [154] eqangle (CAM, NJM) because perp (MJN)[93], perp (CAm)[130] (perpAndPerpToEqangle)  
 [155] eqcircle (M, A,  $\sim$ , ABm) because eqcircle (M, A, M, m)[99], eqcircle (M, m,  $\sim$ , ABm)[133] (addFact)  
 [156] eqangle (ABC, Amd) because eqangle (ABC, AmC)[129], eqangle (AmC, Amd)[135] (addFact)  
 [157] eqangle (AMD, Amm) because eqline (DM, Mm)[137] (eqlineToEqangle)  
 [158] eqline (CM, Nm) because eqline (CM, Zm)[138], eqline (Nm, Zm)[0] (addFact)  
 [159] eqangle (ABC, Amd) because eqangle (ABC, AmC)[129], eqangle (AmC, Amd)[139] (addFact)  
 [160] eqangle (AmC, PMC) because eqangle (ABC, AmC)[129], eqangle (ABC, AMP)[140], eqangle (AMP, PMC)[107] (addFact)  
 [161] eqangle (ACm, mCB) because eqangle (ACM, mCB)[141], eqangle (ACM, ACm)[53] (addFact)  
 [162] eqangle (ACD, mCB) because eqangle (ACM, mCB)[141], eqangle (ACD, ACM)[5] (addFact)  
 [163] eqangle (ACM, mAB) because eqangle (ACM, mCB)[141], eqangle (BAm, BCm)[132] (addFact)  
 [164] eqangle (ABm, MCB) because eqangle (ABm, ACm)[128], eqangle (ACm, MCB)[142] (addFact)  
 [165] eqangle (ACD, DCB) because eqangle (ACM, DCB)[143], eqangle (ACD, ACM)[5] (addFact)  
 [166] simtri (ABC, mAM) because eqratio (AC, Mm, BC, AM)[113], eqangle (ACB, Amm)[144] (eqratioAndEqangleToSimtri)  
 [167] eqangle (EGK, KGF) because contri (EGK, FGK)[145] (contriToEqangle)  
 [168] eqangle (BFE, BGM) because eqangle (BGM, MGE)[146], eqcircle (G, B,  $\sim$ , BEF)[0] (eqangleAndEqcircleToEqangleHalf)  
 [169] eqangle (BAE, BMG) because eqangle (BMG, GME)[147], eqcircle (M, B,  $\sim$ , ABE)[119] (eqangleAndEqcircleToEqangleHalf)  
 [170] simtri (HJM, PIM) because eqangle (HMJ, IMP)[124], eqangle (HJM, MIP)[148] (eqangleAndEqangleAndEqangleToSimtri)  
 [171] eqangle (ABC, Dmb) because eqangle (ABC, Cmb)[151], eqangle (BmC, Bmd)[134] (addFact)  
 [172] eqangle (AmC, Cmb) because eqangle (ABC, Cmb)[151], eqangle (ABC, AmC)[129] (addFact)  
 [173] eqangle (ABm, ACD) because eqangle (ABm, ACM)[152], eqangle (ACD, ACM)[5] (addFact)  
 [174] eqangle (AMP, Amd) because eqangle (ABC, Amd)[156], eqangle (ABC, AMP)[140] (addFact)  
 [175] eqline (CN, Nm) because eqline (CM, Nm)[158] (eqlineToEqline)  
 [176] eqline (MN, Nm) because eqline (CM, Nm)[158] (eqlineToEqline)  
 [177] eqline (CD, Nm) because eqline (CD, CM)[0], eqline (CM, Nm)[158] (addFact)  
 [178] eqline (Cd, Nm) because eqline (CM, Nm)[158], eqline (CM, Cd)[0] (addFact)  
 [179] eqline (CM, MN) because eqline (CM, Nm)[158] (eqlineToEqline)  
 [180] eqline (MN, Mm) because eqline (CM, Nm)[158] (eqlineToEqline)  
 [181] eqline (CM, CN) because eqline (CM, Nm)[158] (eqlineToEqline)  
 [182] eqline (CM, NZ) because eqline (CM, Nm)[158], eqline (NZ, Nm)[14] (addFact)  
 [183] eqangle (Amd, PMC) because eqangle (ABC, Amd)[159], eqangle (ABC, AMP)[140], eqangle (AMP, PMC)[107] (addFact)  
 [184] contri (ACm, BCm) because eqangle (ACm, mCB)[161], cong (AC, BC)[0] (eqangleAndCongAndCongToContri)  
 [185] eqangle (ABm, DCB) because eqangle (ABm, MCB)[164], eqangle (BCD, BCM)[6] (addFact)  
 [186] contri (ACD, BCD) because eqangle (ACD, DCB)[165], cong (AC, BC)[0] (eqangleAndCongAndCongToContri)  
 [187] eqratio (AB, Am, AC, Mm) because simtri (ABC, mAM)[166] (simtriToEqratio)  
 [188] eqangle (EBF, EGK) because eqangle (EGK, KGF)[167], eqcircle (G, E,  $\sim$ , BEF)[81] (eqangleAndEqcircleToEqangleHalf)  
 [189] eqangle (BFE, MGE) because eqangle (BFE, BGM)[168], eqangle (BGM, MGE)[146] (addFact)  
 [190] eqangle (BAF, BMG) because eqangle (BAE, BAF)[77], eqangle (BAE, BMG)[169] (addFact)  
 [191] eqratio (HJ, IP, JM, IM) because simtri (HJM, PIM)[170] (simtriToEqratio)  
 [192] eqratio (HJ, IP, HM, MP) because simtri (HJM, PIM)[170] (simtriToEqratio)  
 [193] eqangle (AmC, ZmB) because eqangle (AmC, Cmb)[172], eqangle (BmC, BmZ)[111] (addFact)  
 [194] eqangle (Amd, Cmb) because eqangle (AmC, Cmb)[172], eqangle (AmC, Amd)[139] (addFact)  
 [195] eqangle (Amm, APD) because eqcircle (P, A,  $\sim$ , ACD)[0], eqcircle (M, A,  $\sim$ , ABm)[155], eqangle (ABm, ACD)[173] (eqcircleAndEqangleToEqangleCir)  
 [196] eqangle (ANC, ANm) because eqline (CN, Nm)[175] (eqlineToEqangle)  
 [197] eqangle (BNC, BNm) because eqline (CN, Nm)[175] (eqlineToEqangle)  
 [198] eqangle (BNM, BNm) because eqline (MN, Nm)[176] (eqlineToEqangle)  
 [199] eqline (CN, DN) because eqline (CD, Nm)[177] (eqlineToEqline)  
 [200] eqline (CD, DN) because eqline (CD, Nm)[177] (eqlineToEqline)  
 [201] eqline (CN, Nd) because eqline (Cd, Nm)[178] (eqlineToEqline)  
 [202] eqline (Cd, Nd) because eqline (Cd, Nm)[178] (eqlineToEqline)  
 [203] eqangle (CME, NME) because eqline (CM, MN)[179] (eqangleAndEqlineToEqangle)

[204] eqline (Cd, MN) because eqline (CM, MN)[179], eqline (CM, Cd)[0] (addFact)  
 [205] eqline (CD, MN) because eqline (CD, CM)[0], eqline (CM, MN)[179] (addFact)  
 [206] eqangle (CMP, NMP) because eqline (CM, MN)[179] (eqangleAndEqlineToEqangle)  
 [207] eqangle (AMN, AMm) because eqline (MN, Mm)[180] (eqlineToEqangle)  
 [208] eqline (CD, CN) because eqline (CD, CM)[0], eqline (CM, CN)[181] (addFact)  
 [209] eqangle (ACM, ACN) because eqline (CM, CN)[181] (eqlineToEqangle)  
 [210] eqline (Cd, NZ) because eqline (CM, NZ)[182], eqline (CM, Cd)[0] (addFact)  
 [211] cong (Am, Bm) because contri (ACm, BCm)[184] (contriToCong)  
 [212] cong (AD, BD) because contri (ACD, BCD)[186] (contriToCong)  
 [213] eqratio (AB, AC, Am, Mm) because eqratio (AB, Am, AC, Mm)[187] (eqratioToEqratio)  
 [214] eqangle (BEF, MGK) because eqangle (EBF, EGK)[188], eqangle (BFE, MGE)[189] (eqangleAndEqangleToEqangle)  
 [215] eqangle (AmZ, ZmB) because eqangle (AmC, ZmB)[193], eqangle (AmC, AmZ)[112] (addFact)  
 [216] eqangle (AMD, APD) because eqangle (AMD, AMm)[157], eqangle (AMm, APD)[195] (addFact)  
 [217] eqangle (ANC, AND) because eqline (CN, DN)[199] (eqlineToEqangle)  
 [218] eqangle (BNC, BND) because eqline (CN, DN)[199] (eqlineToEqangle)  
 [219] eqline (DN, Dd) because eqline (CD, DN)[200], eqline (CD, Dd)[90] (addFact)  
 [220] eqangle (ADC, ADN) because eqline (CD, DN)[200] (eqlineToEqangle)  
 [221] eqangle (BDC, BDN) because eqline (CD, DN)[200] (eqlineToEqangle)  
 [222] eqangle (ANC, ANd) because eqline (CN, Nd)[201] (eqlineToEqangle)  
 [223] eqangle (AdC, AdN) because eqline (Cd, Nd)[202] (eqlineToEqangle)  
 [224] eqline (MN, Nd) because eqline (Cd, MN)[204] (eqlineToEqline)  
 [225] eqline (DM, DN) because eqline (CD, MN)[205] (eqlineToEqline)  
 [226] eqangle (AmC, PMN) because eqangle (AmC, PMC)[160], eqangle (CMP, NMP)[206] (addFact)  
 [227] eqangle (Amd, PMN) because eqangle (Amd, PMC)[183], eqangle (CMP, NMP)[206] (addFact)  
 [228] eqangle (AMN, APD) because eqangle (AMN, AMm)[207], eqangle (AMm, APD)[195] (addFact)  
 [229] eqline (CD, Nd) because eqline (CD, CN)[208], eqline (CN, Nd)[201] (addFact)  
 [230] eqangle (ACN, mAB) because eqangle (ACM, mAB)[163], eqangle (ACM, ACN)[209] (addFact)  
 [231] eqline (NZ, Nd) because eqline (Cd, NZ)[210] (eqlineToEqline)  
 [232] eqratio (AD, AP, BD, CP) because cong (AD, BD)[212], cong (AP, CP)[7], eqratio (AD, AP, AD, CP)[59] (eqratioAndCongToEqratio)  
 [233] eqratio (AD, AN, BD, AN) because cong (AD, BD)[212] (congToEqratio)  
 [234] eqratio (AB, AC, AB, BC) because cong (AC, BC)[0], eqratio (AB, AC, Am, Mm)[213] (eqratioAndCongToEqratio)  
 [235] eqangle (AEB, KGM) because eqangle (AEB, FEB)[115], eqangle (BEF, MGK)[214] (addFact)  
 [236] eqangle (AmZ, NmB) because eqangle (AmZ, ZmB)[215], eqangle (BmN, BmZ)[112] (addFact)  
 [237] eqcircle (" , ADM, " , ADP) because eqangle (AMD, APD)[216] (eqangleToEqcircle)  
 [238] eqangle (BDN, BDd) because eqline (DN, Dd)[219] (eqangleAndEqlineToEqangle)  
 [239] eqangle (ADC, NAD) because eqangle (ADC, ADN)[220], eqangle (ADN, NAD)[70] (addFact)  
 [240] eqangle (Adm, NAD) because eqangle (ADC, Adm)[136], eqangle (ADC, ADN)[220], eqangle (ADN, NAD)[70] (addFact)  
 [241] eqangle (AdC, NAd) because eqangle (AdC, AdN)[223], eqangle (AdN, NAd)[71] (addFact)  
 [242] eqangle (ANM, ANd) because eqline (MN, Nd)[224] (eqlineToEqangle)  
 [243] eqangle (ADM, ADN) because eqline (DM, DN)[225] (eqlineToEqangle)  
 [244] eqangle (AmC, JMN) because eqangle (AmC, PMN)[226], eqangle (JMN, PMN)[94] (addFact)  
 [245] eqline (DN, Nd) because eqline (CD, Nd)[229], eqline (CD, DN)[200] (addFact)  
 [246] eqangle (ANZ, ANd) because eqline (NZ, Nd)[231] (eqlineToEqangle)  
 [247] eqangle (AEB, OGM) because eqangle (AEB, KGM)[235], eqangle (KGM, OGM)[88] (addFact)  
 [248] eqangle (AmN, NmB) because eqangle (AmZ, NmB)[236], eqangle (AmN, AmZ)[13] (addFact)  
 [249] eqangle (ADP, AMP) because eqcircle (" , ADM, " , ADP)[237] (eqcircleToEqangle)  
 [250] eqangle (DAM, DPM) because eqcircle (" , ADM, " , ADP)[237] (eqcircleToEqangle)  
 [251] eqangle (ADM, APM) because eqcircle (" , ADM, " , ADP)[237] (eqcircleToEqangle)  
 [252] eqangle (CDP, PAN) because eqangle (ADP, PAD)[60], eqangle (ADC, NAD)[239] (eqangleAndEqangleToEqangle)  
 [253] eqangle (ADD, NAD) because eqangle (ADC, AdD)[120], eqangle (ADC, NAD)[239] (addFact)  
 [254] eqangle (NAP, PDm) because eqangle (ADP, PAD)[60], eqangle (Adm, NAD)[240] (eqangleAndEqangleToEqangle)  
 [255] simtri (ACm, JNM) because eqangle (CAm, NJM)[154], eqangle (AmC, JMN)[244] (eqangleAndEqangleAndEqangleToSimtri)  
 [256] perp (DAd) because eqcircle (N, D, " , AdD)[74], eqline (DN, Nd)[245] (eqcircleAndEqlineToPerp)  
 [257] contri (ANm, BNm) because eqangle (AmN, NmB)[248], cong (Am, Bm)[211] (eqangleAndCongAndCongToContri)  
 [258] eqangle (AMP, PAD) because eqangle (ADP, PAD)[60], eqangle (ADP, AMP)[249] (addFact)  
 [259] eqangle (ADP, Amd) because eqangle (ABC, Amd)[156], eqangle (ABC, AMP)[140], eqangle (ADP, AMP)[249] (addFact)  
 [260] eqangle (MAP, PAN) because eqangle (CDP, MAP)[150], eqangle (CDP, PAN)[252] (addFact)  
 [261] eqangle (DCP, NAP) because eqangle (CDP, PCD)[108], eqangle (CDP, PAN)[252] (addFact)  
 [262] eqangle (NAP, PDd) because eqangle (ADP, PAD)[60], eqangle (Add, NAD)[253] (eqangleAndEqangleToEqangle)  
 [263] eqratio (AC, JN, Am, JM) because simtri (ACm, JNM)[255] (simtriToEqratio)  
 [264] eqangle (CBm, DAd) because perp (DAd)[256], perp (CBm)[131] (perpAndPerpToEqangle)  
 [265] eqangle (DAd, MIP) because perp (MIP)[0], perp (DAd)[256] (perpAndPerpToEqangle)  
 [266] cong (AN, BN) because contri (ANm, BNm)[257] (contriToCong)  
 [267] eqangle (ANm, mNB) because contri (ANm, BNm)[257] (contriToEqangle)  
 [268] eqangle (Bmd, DAP) because eqangle (ABC, DmB)[171], eqangle (ABC, AMP)[140], eqangle (AMP, PAD)[258] (addFact)  
 [269] eqangle (ABC, PAD) because eqangle (ABC, AMP)[140], eqangle (AMP, PAD)[258] (addFact)  
 [270] eqangle (DAM, PDm) because eqangle (ADP, Amd)[259] (eqangleToEqangle)  
 [271] eqangle (MAP, mDP) because eqangle (MAP, PAN)[260], eqangle (NAP, PDm)[254] (addFact)  
 [272] eqangle (MAP, dDP) because eqangle (MAP, PAN)[260], eqangle (NAP, PDd)[262] (addFact)  
 [273] eqcircle (N, A, N, B) because cong (AN, BN)[266] (congToEqcircle)  
 [274] eqratio (AD, AN, BD, BN) because cong (AD, BD)[212], cong (AN, BN)[266], eqratio (AD, AN, BD, AN)[233] (eqratioAndCongToEqratio)  
 [275] eqangle (ANC, MNB) because eqangle (ANC, ANm)[196], eqangle (ANm, mNB)[267], eqangle (BNm, BNm)[198] (addFact)  
 [276] eqangle (ANZ, ZNB) because eqangle (ANZ, ANm)[66], eqangle (ANm, mNB)[267], eqangle (BNZ, BNm)[67] (addFact)  
 [277] eqangle (ANZ, CNB) because eqangle (ANZ, ANm)[66], eqangle (ANm, mNB)[267], eqangle (BNC, BNm)[197] (addFact)  
 [278] eqangle (ABm, PAD) because eqangle (ABC, PAD)[269], eqangle (CBm, DAd)[264] (eqangleAndEqangleToEqangle)  
 [279] eqangle (Adm, APM) because eqangle (ADP, AMP)[249], eqangle (MAP, mDP)[271] (eqangleAndEqangleToEqangle)  
 [280] eqangle (EAM, PDd) because eqangle (MAP, dDP)[272], eqangle (EAM, PAM)[25] (addFact)  
 [281] eqcircle (N, B, " , AdD) because eqcircle (N, A, N, B)[273], eqcircle (N, A, " , AdD)[0] (addFact)  
 [282] eqcircle (N, B, N, F) because eqcircle (N, A, N, B)[273], eqcircle (N, A, N, F)[0] (addFact)  
 [283] eqangle (AND, MNB) because eqangle (ANC, MNB)[275], eqangle (ANC, AND)[217] (addFact)  
 [284] eqangle (ANZ, DNB) because eqangle (ANZ, CNB)[277], eqangle (BNC, BND)[218] (addFact)  
 [285] eqangle (BCD, dAP) because eqangle (ABm, DCB)[185], eqangle (ABm, PAD)[278] (addFact)  
 [286] simtri (Adm, APM) because eqangle (Adm, APM)[279], eqangle (AMP, Amd)[174] (eqangleAndEqangleAndEqangleToSimtri)  
 [287] eqangle (MEP, PDd) because eqangle (AEM, PEM)[79], eqangle (AEM, MAE)[87], eqangle (EAM, PDd)[280] (addFact)  
 [288] eqcircle (N, B, " , ABD) because eqcircle (N, B, " , AdD)[281] (eqcircleToEqcircle)  
 [289] eqcircle (N, B, " , ABD) because eqcircle (N, B, " , AdD)[281] (eqcircleToEqcircle)  
 [290] eqangle (ABd, AdD) because eqcircle (N, B, " , AdD)[281] (eqcircleToEqangle)  
 [291] cong (BN, DN) because eqcircle (N, B, " , AdD)[281] (eqcircleToCong)  
 [292] eqangle (ABD, AdD) because eqcircle (N, B, " , AdD)[281] (eqcircleToEqangle)  
 [293] eqcircle (N, B, " , BDd) because eqcircle (N, B, " , AdD)[281] (eqcircleToEqcircle)  
 [294] eqangle (BAD, BDd) because eqcircle (N, B, " , AdD)[281] (eqcircleToEqangle)  
 [295] cong (BN, FN) because eqcircle (N, B, N, F)[282] (eqcircleToCong)  
 [296] eqangle (BCD, dAP) because eqangle (BCD, dAP)[285], eqangle (FAd, Pad)[20] (addFact)  
 [297] eqratio (AD, AP, Dm, MP) because simtri (Adm, APM)[286] (simtriToEqratio)  
 [298] eqcircle (N, A, " , ABD) because eqcircle (N, A, N, B)[273], eqcircle (N, B, " , ABD)[288] (addFact)  
 [299] eqangle (ADB, ANZ) because eqcircle (N, B, " , ABD)[288], eqangle (ANZ, ZNB)[276] (eqcircleAndEqangleToEqangleHalf)  
 [300] eqcircle (N, A, " , ABD) because eqcircle (N, A, N, B)[273], eqcircle (N, B, " , ABD)[289] (addFact)



[301] eqangle (ABd, ADC) because eqangle (ABd, AdD) [290], eqangle (ADC, AdD) [120] (addFact)

[302] eqangle (BDN, NBD) because cong (BN, DN) [291] (congToEqangle)

[303] eqangle (ABD, NAd) because eqangle (ABD, AdD) [292], eqangle (AdC, AdD) [122], eqangle (AdC, NAd) [241] (addFact)

[304] eqangle (ABD, AdN) because eqangle (ABD, AdD) [292], eqangle (AdC, AdD) [122], eqangle (AdC, AdN) [223] (addFact)

[305] perp (DBd) because eqcircle (N, B,  $\cdot$ , Bdd) [293], eqline (DN, Nd) [245] (eqcircleAndEqlineToPerp)

[306] eqangle (BAd, BDN) because eqangle (BAd, Bdd) [294], eqangle (BDC, Bdd) [121], eqangle (BDC, BDN) [221] (addFact)

[307] contri (BGN, FGN) because cong (BG, FG) [27], cong (BN, FN) [295] (congAndCongAndCongToContri)

[308] eqcircle (N, F,  $\cdot$ , ABD) because eqcircle (N, A,  $\cdot$ , ABD) [298], eqcircle (N, A, N, F) [0] (addFact)

[309] eqangle (ABD, ANP) because eqangle (ANP, PND) [114], eqcircle (N, A,  $\cdot$ , ABD) [298] (eqangleAndEqcircleToEqangleHalf)

[310] eqangle (ADB, DNB) because eqangle (ADB, ANZ) [299], eqangle (ANZ, DNB) [284] (addFact)

[311] eqangle (ADB, ANd) because eqangle (ADB, ANZ) [299], eqangle (ANZ, ANd) [246] (addFact)

[312] eqangle (ANd, APC) because eqcircle (P, A,  $\cdot$ , ACD) [0], eqcircle (N, A,  $\cdot$ , ABd) [300], eqangle (ABd, ADC) [301] (eqcircleAndEqangleToEqangleCir)

[313] eqangle (CBm, DBd) because perp (DBd) [305], perp (CBm) [131] (perpAndPerpToEqangle)

[314] eqangle (BGN, NGF) because contri (BGN, FGN) [307] (contriToEqangle)

[315] eqangle (BNG, GNF) because contri (BGN, FGN) [307] (contriToEqangle)

[316] eqangle (BAD, BFD) because eqcircle (N, F,  $\cdot$ , ABD) [308] (eqcircleToEqangle)

[317] eqangle (DAF, DBF) because eqcircle (N, F,  $\cdot$ , ABD) [308] (eqcircleToEqangle)

[318] eqcircle (N, F,  $\cdot$ , ABF) because eqcircle (N, F,  $\cdot$ , ABD) [308] (eqcircleToEqcircle)

[319] eqangle (ADD, PND) because eqangle (ABD, AdD) [292], eqangle (ABD, ANP) [309], eqangle (ANP, PND) [114] (addFact)

[320] eqangle (ADN, NBD) because eqangle (BDN, NBD) [302], eqangle (ADB, DNB) [310] (eqangleAndEqangleToEqangle)

[321] simtri (ABD, AdN) because eqangle (ABD, AdN) [304], eqangle (ADB, ANd) [311] (eqangleAndEqangleAndEqangleToSimtri)

[322] eqangle (ANC, APC) because eqangle (ANC, ANd) [222], eqangle (ANd, APC) [312] (addFact)

[323] eqangle (ANM, APC) because eqangle (ANM, ANd) [242], eqangle (ANd, APC) [312] (addFact)

[324] eqangle (ADC, AFd) because eqcircle (N, F,  $\cdot$ , AFd) [72], eqcircle (P, A,  $\cdot$ , ACD) [0], eqangle (ANd, APC) [312] (eqcircleAndEqangleToEqangleCen)

[325] eqangle (ADB, APC) because eqangle (ADB, ANd) [311], eqangle (ANd, APC) [312] (addFact)

[326] eqangle (Add, CDB) because eqangle (CBm, DBd) [313], eqangle (Amd, Cmb) [194], eqangle (ACD, mCB) [162] (eqangleAndEqangleAndEqangleToEqangleIsotriquad)

[327] eqangle (BEF, BGN) because eqangle (BGN, NGF) [314], eqcircle (G, B,  $\cdot$ , BEF) [0] (eqangleAndEqcircleToEqangleHalf)

[328] eqangle (DAE, DBF) because eqangle (DAE, DAF) [75], eqangle (DAF, DBF) [317] (addFact)

[329] eqcircle (N, B,  $\cdot$ , ABF) because eqcircle (N, B, N, F) [282], eqcircle (N, F,  $\cdot$ , ABF) [318] (addFact)

[330] eqangle (BME, BNF) because eqcircle (N, F,  $\cdot$ , ABF) [318], eqcircle (M, B,  $\cdot$ , ABE) [119], eqangle (BAE, BAF) [77] (eqcircleAndEqangleToEqangleCir)

[331] para (Ad, NP) because eqangle (Add, PND) [319], eqline (DN, Dd) [219] (eqangleAndEqlineToPara)

[332] eqangle (ADM, dDB) because eqangle (ADM, AdN) [243], eqangle (ADN, NDB) [320], eqangle (BDN, Bdd) [238] (addFact)

[333] eqangle (ADM, dAB) because eqangle (ADM, AdN) [243], eqangle (ADN, NDB) [320], eqangle (BAd, BDN) [306] (addFact)

[334] eqratio (AB, Ad, AD, AN) because simtri (ABD, AdN) [321] (simtriToEqratio)

[335] eqcircle ( $\cdot$ , ACN,  $\cdot$ , ACP) because eqangle (ANC, APC) [322] (eqangleToEqcircle)

[336] eqangle (CPD, MAN) because eqangle (AMN, APD) [228], eqangle (ANM, APC) [323] (eqangleAndEqangleToEqangle)

[337] simtri (ABD, ACP) because eqratio (AD, AP, BD, CP) [232], eqangle (ADB, APC) [325] (eqratioAndEqangleToSimtri)

[338] eqangle (AFd, CDB) because eqangle (Add, CDB) [326], eqangle (Add, AFd) [73] (addFact)

[339] eqangle (AEB, NGB) because eqangle (AEB, FEB) [115], eqangle (BEF, BGN) [327] (addFact)

[340] eqangle (EBF, MGN) because eqangle (BEF, BGN) [327], eqangle (BFE, BGM) [168] (eqangleAndEqangleToEqangle)

[341] eqangle (BAE, BDF) because eqangle (BAD, BFD) [316], eqangle (DAE, DBF) [328] (eqangleAndEqangleToEqangle)

[342] eqangle (BAF, BNG) because eqangle (BNG, GNF) [315], eqcircle (N, B,  $\cdot$ , ABF) [329] (eqangleAndEqcircleToEqangleHalf)

[343] para (Ad, IP) because eqline (IP, NP) [0], para (Ad, NP) [331] (eqlineAndParaToPara)

[344] eqangle (APM, dDB) because eqangle (ADM, dDB) [332], eqangle (ADM, APM) [251] (addFact)

[345] eqangle (APM, dAB) because eqangle (ADM, dAB) [333], eqangle (ADM, APM) [251] (addFact)

[346] eqratio (AB, Ad, BD, BN) because eqratio (AD, AN, BD, BN) [274], eqratio (AB, Ad, AD, AN) [334] (addFact)

[347] eqangle (ACP, ANP) because eqcircle ( $\cdot$ , ACN,  $\cdot$ , ACP) [335] (eqcircleToEqangle)

[348] eqangle (CAN, CPN) because eqcircle ( $\cdot$ , ACN,  $\cdot$ , ACP) [335] (eqcircleToEqangle)

[349] eqangle (ACN, APN) because eqcircle ( $\cdot$ , ACN,  $\cdot$ , ACP) [335] (eqcircleToEqangle)

[350] eqangle (CPM, DAN) because eqangle (DAM, DPM) [250], eqangle (CPD, MAN) [336] (eqangleAndEqangleToEqangle)

[351] eqratio (AB, AC, AD, AP) because simtri (ABD, ACP) [337] (simtriToEqratio)

[352] simtri (AFd, CDB) because eqangle (BCD, dAF) [296], eqangle (AFd, CDB) [338] (eqangleAndEqangleAndEqangleToSimtri)

[353] eqangle (FGK, NGM) because eqangle (EGK, KGF) [167], eqangle (EBF, EGK) [188], eqangle (EBF, MGN) [340] (addFact)

[354] eqangle (BDF, GME) because eqangle (BAE, BDF) [341], eqangle (BAE, BMG) [169], eqangle (BMG, GME) [147] (addFact)

[355] eqangle (BAE, BNG) because eqangle (BAE, BAF) [77], eqangle (BAF, BNG) [342] (addFact)

[356] eqangle (BMG, GNF) because eqangle (BAF, BMG) [190], eqangle (BAF, BNG) [342], eqangle (BNG, GNF) [315] (addFact)

[357] para (Ad, IM) because eqangle (DAd, MIP) [265], para (Ad, IP) [343] (eqangleAndParaToPara)

[358] eqangle (Bdd, MPE) because eqangle (APM, dDB) [344], eqangle (APM, EPM) [78] (addFact)

[359] eqangle (ANP, PAC) because eqangle (ACP, PAC) [58], eqangle (ACP, ANP) [347] (addFact)

[360] eqangle (ACP, NAd) because eqangle (ABD, NAd) [303], eqangle (ABD, ANP) [309], eqangle (ACP, ANP) [347] (addFact)

[361] eqangle (APN, mAB) because eqangle (ACN, mAB) [230], eqangle (ACN, APN) [349] (addFact)

[362] eqangle (CAD, MPN) because eqangle (CAN, CPN) [348], eqangle (CPM, DAN) [350] (eqangleAndEqangleToEqangle)

[363] eqratio (AB, BC, AD, AP) because eqratio (AB, AC, AB, BC) [234], eqratio (AB, AC, AD, AP) [351] (addFact)

[364] eqratio (AF, CD, Fd, BD) because simtri (AFd, CDB) [352] (simtriToEqratio)

[365] eqratio (AD, BC, Fd, BD) because simtri (AFd, CDB) [352] (simtriToEqratio)

[366] eqangle (BGM, NGK) because eqangle (FGK, NGM) [353], eqangle (BGN, NGF) [314] (eqangleAndEqangleToEqangle)

[367] simtri (ABE, NBG) because eqangle (AEB, NGB) [339], eqangle (BAE, BNG) [355] (eqangleAndEqangleAndEqangleToSimtri)

[368] para (AD, HM) because eqline (HM, IM) [0], para (AD, IM) [357] (eqlineAndParaToPara)

[369] eqangle (BDP, PME) because eqangle (Bdd, MPE) [358], eqangle (MEP, Pdd) [287] (eqangleAndEqangleToEqangle)

[370] eqangle (ABD, PAC) because eqangle (ABD, ANP) [309], eqangle (ANP, PAC) [359] (addFact)

[371] eqangle (ACD, PAd) because eqangle (ACP, NAd) [360], eqangle (DCP, NAP) [261] (eqangleAndEqangleToEqangle)

[372] eqangle (MPN, mAd) because eqangle (APM, dAB) [345], eqangle (APN, mAB) [361] (eqangleAndEqangleToEqangle)

[373] eqangle (CAD, MPI) because eqangle (CAD, MPN) [362], eqangle (IPM, NPM) [37] (addFact)

[374] eqratio (AB, BC, Dm, MP) because eqratio (AB, BC, AD, AP) [363], eqratio (AD, AP, Dm, MP) [297] (addFact)

[375] eqratio (AB, BC, Fd, BN) because eqratio (Ad, BC, Fd, BD) [365], eqratio (AB, Ad, BD, BN) [346] (eqratioAndEqratioToEqratio)

[376] eqangle (BGN, MGO) because eqangle (BGK, BGO) [89], eqangle (BGM, NGK) [366] (eqangleAndEqangleToEqangle)

[377] eqratio (AE, GN, BE, BG) because simtri (ABE, NBG) [367] (simtriToEqratio)

[378] eqratio (AB, BN, AE, GN) because simtri (ABE, NBG) [367] (simtriToEqratio)

[379] eqangle (FDP, PMG) because eqangle (BDF, GME) [354], eqangle (BDP, PME) [369] (eqangleAndEqangleToEqangle)

[380] eqangle (ABD, EAC) because eqangle (ABD, PAC) [370], eqangle (CAE, CAP) [22] (addFact)

[381] eqangle (ACD, FAd) because eqangle (ACD, PAd) [371], eqangle (FAd, PAd) [20] (addFact)

[382] simtri (Adm, PMN) because eqangle (Amd, PMN) [227], eqangle (MPN, mAd) [372] (eqangleAndEqangleAndEqangleToSimtri)

[383] eqangle (DAm, IMP) because eqangle (CAm, MIP) [153], eqangle (CAD, MPI) [373] (eqangleAndEqangleToEqangle)

[384] eqratio (BN, Fd, MP, Dm) because eqratio (AB, BC, Dm, MP) [374], eqratio (AB, BC, Fd, BN) [375] (addFact)

[385] eqangle (AND, EMC) because eqcircle (M, E,  $\cdot$ , ACE) [86], eqcircle (N, A,  $\cdot$ , ABD) [298], eqangle (ABD, EAC) [380] (eqcircleAndEqangleToEqangleCir)

[386] simtri (ACD, dAF) because eqangle (ACD, FAd) [381], eqangle (ADC, AFd) [324] (eqangleAndEqangleAndEqangleToSimtri)

[387] eqratio (Ad, NP, Am, MP) because simtri (Adm, PMN) [382] (simtriToEqratio)

[388] eqangle (IMP, Pdm) because eqangle (DAm, Pdm) [270], eqangle (DAm, IMP) [383] (addFact)

[389] eqangle (BNM, CME) because eqangle (AND, MNB) [283], eqangle (AND, EMC) [385] (addFact)

[390] eqratio (AC, Ad, CD, AF) because simtri (ACD, dAF) [386] (simtriToEqratio)

[391] eqratio (Ad, Am, NP, MP) because eqratio (Ad, NP, Am, MP) [387] (eqratioToEqratio)

[392] eqangle (Fdm, IMG) because eqangle (IMP, Pdm) [388], eqangle (FDP, PMG) [379] (eqangleAndEqangleToEqangle)

[393] eqangle (BNM, NME) because eqangle (BNM, CME) [389], eqangle (CME, NME) [203] (addFact)

[394] eqratio (AC, Ad, BD, Fd) because eqratio (AC, Ad, CD, AF) [390], eqratio (AF, CD, Fd, BD) [364] (addFact)

[395] eqratio (Ad, Am, JN, IM) because eqratio (Ad, Am, NP, MP) [391], eqratio (IM, JN, MP, NP) [149] (addFact)

[396] eqangle (Fdm, HMG) because eqangle (Fdm, IMG) [392], eqangle (GMH, GMI) [39] (addFact)

[397] eqangle (BMN, MNF) because eqangle (BME, BNF) [330], eqangle (BNM, NME) [393] (eqangleAndEqangleToEqangle)



[398]  $\text{eqratio}(\text{Ad}, \text{JN}, \text{Am}, \text{IM})$  because  $\text{eqratio}(\text{Ad}, \text{Am}, \text{JN}, \text{IM})$  [395] ( $\text{eqratioToEqratio}$ )  
 [399]  $\text{eqangle}(\text{GMN}, \text{MNG})$  because  $\text{eqangle}(\text{BMG}, \text{GNF})$  [356],  $\text{eqangle}(\text{BMN}, \text{MNF})$  [397] ( $\text{eqangleAndEqangleToEqangle}$ )  
 [400]  $\text{eqratio}(\text{AC}, \text{Ad}, \text{IM}, \text{JM})$  because  $\text{eqratio}(\text{AC}, \text{JN}, \text{Am}, \text{JM})$  [263],  $\text{eqratio}(\text{Ad}, \text{JN}, \text{Am}, \text{IM})$  [398] ( $\text{eqratioAndEqratioToEqratio}$ )  
 [401]  $\text{cong}(\text{GM}, \text{GN})$  because  $\text{eqangle}(\text{GMN}, \text{MNG})$  [399] ( $\text{eqangleToCong}$ )  
 [402]  $\text{eqratio}(\text{BD}, \text{Fd}, \text{IM}, \text{JM})$  because  $\text{eqratio}(\text{AC}, \text{Ad}, \text{BD}, \text{Fd})$  [394],  $\text{eqratio}(\text{AC}, \text{Ad}, \text{IM}, \text{JM})$  [400] ( $\text{addFact}$ )  
 [403]  $\text{eqratio}(\text{AE}, \text{GM}, \text{AE}, \text{GN})$  because  $\text{cong}(\text{GM}, \text{GN})$  [401],  $\text{eqratio}(\text{AB}, \text{BN}, \text{AE}, \text{GN})$  [378] ( $\text{eqratioAndCongToEqratio}$ )  
 [404]  $\text{contri}(\text{BGN}, \text{OGM})$  because  $\text{eqangle}(\text{BGN}, \text{MGD})$  [376],  $\text{cong}(\text{BG}, \text{GO})$  [36],  $\text{cong}(\text{GM}, \text{GN})$  [401] ( $\text{eqangleAndCongAndCongToContri}$ )  
 [405]  $\text{eqratio}(\text{BD}, \text{Fd}, \text{MP}, \text{HM})$  because  $\text{eqratio}(\text{BD}, \text{Fd}, \text{IM}, \text{JM})$  [402],  $\text{eqratio}(\text{HJ}, \text{IP}, \text{JM}, \text{IM})$  [191],  $\text{eqratio}(\text{HJ}, \text{IP}, \text{HM}, \text{MP})$  [192] ( $\text{addFact}$ )  
 [406]  $\text{eqratio}(\text{AE}, \text{GM}, \text{BE}, \text{BG})$  because  $\text{eqratio}(\text{AE}, \text{GN}, \text{BE}, \text{BG})$  [377],  $\text{eqratio}(\text{AE}, \text{GM}, \text{AE}, \text{GN})$  [403] ( $\text{addFact}$ )  
 [407]  $\text{cong}(\text{BN}, \text{MO})$  because  $\text{contri}(\text{BGN}, \text{OGM})$  [404] ( $\text{contriToCong}$ )  
 [408]  $\text{eqratio}(\text{BD}, \text{BN}, \text{Dm}, \text{HM})$  because  $\text{eqratio}(\text{BD}, \text{Fd}, \text{MP}, \text{HM})$  [405],  $\text{eqratio}(\text{BN}, \text{Fd}, \text{MP}, \text{Dm})$  [384] ( $\text{eqratioAndEqratioToEqratio}$ )  
 [409]  $\text{eqratio}(\text{AE}, \text{GM}, \text{BE}, \text{GO})$  because  $\text{eqratio}(\text{AE}, \text{GM}, \text{BE}, \text{BG})$  [406],  $\text{eqratio}(\text{BE}, \text{BG}, \text{BE}, \text{GO})$  [92] ( $\text{addFact}$ )  
 [410]  $\text{eqratio}(\text{BD}, \text{BN}, \text{BD}, \text{MO})$  because  $\text{cong}(\text{BN}, \text{MO})$  [407] ( $\text{congAndEqratioToEqratio}$ )  
 [411]  $\text{simtri}(\text{ABE}, \text{MOG})$  because  $\text{eqangle}(\text{AEB}, \text{OGM})$  [247],  $\text{eqratio}(\text{AE}, \text{GM}, \text{BE}, \text{GO})$  [409] ( $\text{eqangleAndEqratioToSimtri}$ )  
 [412]  $\text{eqratio}(\text{BD}, \text{MO}, \text{Dm}, \text{HM})$  because  $\text{eqratio}(\text{BD}, \text{BN}, \text{Dm}, \text{HM})$  [408],  $\text{eqratio}(\text{BD}, \text{BN}, \text{BD}, \text{MO})$  [410] ( $\text{addFact}$ )  
 [413]  $\text{eqangle}(\text{BAE}, \text{GMO})$  because  $\text{simtri}(\text{ABE}, \text{MOG})$  [411] ( $\text{simtriToEqangle}$ )  
 [414]  $\text{eqangle}(\text{BDF}, \text{GMO})$  because  $\text{eqangle}(\text{BAE}, \text{BDF})$  [341],  $\text{eqangle}(\text{BAE}, \text{GMO})$  [413] ( $\text{addFact}$ )  
 [415]  $\text{eqangle}(\text{BDm}, \text{HMO})$  because  $\text{eqangle}(\text{BDF}, \text{GMO})$  [414],  $\text{eqangle}(\text{FDm}, \text{HMG})$  [396] ( $\text{eqangleAndEqangleToEqangle}$ )  
 [416]  $\text{simtri}(\text{BDm}, \text{OMH})$  because  $\text{eqangle}(\text{BDm}, \text{HMO})$  [415],  $\text{eqratio}(\text{BD}, \text{MO}, \text{Dm}, \text{HM})$  [412] ( $\text{eqangleAndEqratioToSimtri}$ )  
 [417]  $\text{eqangle}(\text{BmD}, \text{MHO})$  because  $\text{simtri}(\text{BDm}, \text{OMH})$  [416] ( $\text{simtriToEqangle}$ )  
 [418]  $\text{eqangle}(\text{DAP}, \text{MHO})$  because  $\text{eqangle}(\text{BmD}, \text{DAP})$  [268],  $\text{eqangle}(\text{BmD}, \text{MHO})$  [417] ( $\text{addFact}$ )  
 [419]  $\text{para}(\text{AP}, \text{HO})$  because  $\text{eqangle}(\text{DAP}, \text{MHO})$  [418],  $\text{para}(\text{AD}, \text{HM})$  [368] ( $\text{eqangleAndParaToPara}$ )