

Name: Andy Chen

Email: achen81@ucsc.edu

Lab Assignment: Lab 7: Floating Point Calculations

Due Date: 3/17/18

IEEE 745 SP:	0x40c80000	0xbc4cccc	0x41233333	0xbc800000
Sign bit (0 or 1):	0	1	0	1
Sign (+/-):	+	-	+	-
Exponent bits	1000 0001	0111 1000	1000 0010	0111 1011
Real exponent factor:	2^2	2^{-7}	2^3	2^{-4}
Significand bits:	1001 0000	1001 100	0100 0110 110	0000
Significand:	1.5625	1.5938	1.2734	1.0

$$- 0x40c80000 = 0100\ 0000\ 1100\ 1000\ 0000\ 0000\ 0000\ 0000$$

$$- 0xbc4cccc = 1011\ 1100\ 0100\ 1100\ 1100\ 1100\ 1100\ 1100$$

$$120 - 127 = -7$$

$$1.1001100 = 1.59375$$

$$- 0\ 1000\ 0010\ 0100\ 0110\ 0110\ 0110\ 0110\ 011 = 41233333$$

$$130 - 127 = 3$$

$$1.01000110 = 1.2734$$

$$- 2^{-4} = 2^{123-127} : 123 = \text{exp} = 0111\ 1011$$

$$1.0 = 1.\overline{00} = \text{significand bits} = 0$$

$$1011\ 1101\ 1000\ 0000\ 0000\ 0000\ 0000\ 0000$$

1. a. $0x40866666$

$$0x40866666 = 0100\ 0000\ 1000\ 0110\ 0110\ 0110\ 0110\ 0110$$

$$\text{Sign bit} = 0 = +$$

$$\text{Exponent bit} = 1000\ 0001 = 129$$

$$\text{Exponent factor} = 2^{129-127} = 2^2$$

$$\text{Significant bits} = 0000\ \underline{1100}$$

$$\text{Denormalize: } 1.00001100110011001100110 \times 2^2$$

$$= 100.001100110011001100110$$

$$\text{Convert: } 100.001100110011$$

$$= 2^2 \cdot 0 \cdot 0 \cdot 0 \cdot 0 \cdot 2^{-3} + 2^{-4} \cdot 0 \cdot 0 \cdot 2^{-7} + 2^{-8} \cdot 0 \cdot 0 \cdot 2^{-11} + 2^{-12}$$

$$= 4 \cdot 0.125 + 0.0625 + 0.0078125 + 0.00391 + 0.0004883 + 0.000244$$

$$= \boxed{4.1999}$$

b. $0x66ff0000$

$$0x66ff0000 = 0110\ 0110\ 1111\ 1111\ 0000\ 0000\ 0000\ 0000$$

$$\text{Sign bit} = 0 = +$$

$$\text{Exponent bit} = 1100\ 1101 = 205$$

$$\text{Exponent factor} = 2^{205-127} = 2^{78}$$

$$\text{Significant bits} = 1111\ 1110\ \underline{0000}$$

$$\text{Denormalize: } 1.11111100000000000000000 \times 2^{78}$$

$$= 1111111 \times 2^{71}$$

$$\text{Convert: } 1111111 \times 2^{71}$$

$$= 2^{78} + 2^{77} + 2^{76} + 2^{75} + 2^{74} + 2^{73} + 2^{72} + 2^{71}$$

$$= \boxed{6.0210 \times 10^{23}}$$

2. a. -1600.6666

Sign: - = 1

$$\begin{array}{l}
 1600 \text{ in base } 2: 1600 = 800 \cdot 2^0 \\
 800 = 400 \cdot 2^0 \\
 400 = 200 \cdot 2^0 \\
 200 = 100 \cdot 2^0 \\
 100 = 50 \cdot 2^0 \\
 50 = 25 \cdot 2^0 \\
 25 = 12 \cdot 2^0 + 1 \\
 12 = 6 \cdot 2^0 \\
 6 = 3 \cdot 2^0 \\
 3 = 1 \cdot 2^0 + 1 \\
 1 = 0 \cdot 2^0 + 1
 \end{array}
 \Rightarrow 0110 \ 0100 \ 0000$$

$$\begin{array}{l}
 0.6667 \text{ in base } 2: 0.6667 \cdot 2 = 0.3334 \cdot 1 \\
 0.3334 \cdot 2 = 0.6668 \cdot 0 \\
 0.6668 \cdot 2 = 0.3336 \cdot 1 \\
 0.3336 \cdot 2 = 0.6672 \cdot 0 \\
 0.6672 \cdot 2 = 0.3344 \cdot 1 \\
 0.3344 \cdot 2 = 0.6688 \cdot 0 \\
 0.6688 \cdot 2 = 1.3376 \cdot 1 \\
 0.3376 \cdot 2 = 0.6752 \cdot 0 \\
 0.6752 \cdot 2 = 0.3504 \cdot 1 \\
 0.3504 \cdot 2 = 0.7008 \cdot 0 \\
 0.7008 \cdot 2 = 0.4016 \cdot 1 \\
 0.4016 \cdot 2 = 0.8032 \cdot 0
 \end{array}
 \Rightarrow .1010$$

$$\begin{aligned}
 1600.66 \text{ in base } 2 &= 0110 \ 0100 \ 0000 \ .1010_2 \\
 &= 1.10010000001010 \cdot 2^{10} \quad \text{Significand} \\
 \text{Exponent: } 10 + 127 &= 137
 \end{aligned}$$

$$\begin{array}{l}
 137 = 68 \cdot 2 + 1 \\
 68 = 34 \cdot 2 + 0 \\
 34 = 17 \cdot 2 + 0 \\
 17 = 8 \cdot 2 + 1 \\
 8 = 4 \cdot 2 + 0 \\
 4 = 2 \cdot 2 + 0 \\
 2 = 1 \cdot 2 + 0 \\
 1 = 0 \cdot 2 + 1
 \end{array}
 \Rightarrow 1000 \ 1001$$

IEEE-754 SP = |Sign| Exponent |Significand|

$$\begin{aligned}
 &= 1100 \ 0100 \ 1100 \ 1000 \ 0001 \ 0101 \ 0101 \ 0101_2 \\
 &= \boxed{0xC4C81555}
 \end{aligned}$$

$$b. -1.6 \times 10^{-19}$$

$$\text{Sign} = - = 1$$

$$-1.6 \times 10^{-19} / 2^{-63} = 1.4757$$

$$1 \text{ in base } 2: 1 = 0.2 \cdot 1$$

$$\Rightarrow 0001$$

$$0.4757 \text{ in base } 2: 0.4757 \cdot 2 = 0.9515 \cdot 0$$

$$0.9515 \cdot 2 = 0.9030 \cdot 1$$

$$0.9030 \cdot 2 = 0.8059 \cdot 1$$

$$0.8059 \cdot 2 = 0.6118 \cdot 1$$

$$0.6118 \cdot 2 = 0.2237 \cdot 1$$

$$0.2237 \cdot 2 = 0.4473 \cdot 0$$

$$0.4473 \cdot 2 = 0.8947 \cdot 0$$

$$0.8947 \cdot 2 = 0.7893 \cdot 1$$

$$0.7893 \cdot 2 = 0.5786 \cdot 1$$

$$0.5786 \cdot 2 = 0.1573 \cdot 1$$

$$0.3145 \cdot 0$$

$$0.6791 \cdot 0$$

$$0.2682 \cdot 1$$

$$0.5364 \cdot 0$$

$$0.0728 \cdot 1$$

$$0.1456 \cdot 0$$

$$0.2912 \cdot 0$$

$$0.5823 \cdot 0$$

$$0.1164 \cdot 1$$

$$0.2328 \cdot 0$$

$$0.4656 \cdot 0$$

$$0.9312 \cdot 0$$

$$0.1862 \cdot 1$$

$$0.3724 \cdot 0$$

$$0.7448 \cdot 0$$

$$0.1489 \cdot 0$$

$$\text{Significant bits: } 0111\ 1001\ 1100\ 1010\ 0001\ 000$$

$$1.4757 \text{ in base } 2: 1.0111\ 1001\ 1100\ 1010\ 0001\ 000$$

$$\text{Exponent: } -63 \cdot 127 = 64$$

$$\Rightarrow 64 = 32 \cdot 2 \cdot 0$$

$$32 = 16 \cdot 2 \cdot 0$$

$$16 = 8 \cdot 2 \cdot 0$$

$$8 = 4 \cdot 2 \cdot 0$$

$$\Rightarrow 4 = 2 \cdot 2 \cdot 0$$

$$2 = 1 \cdot 2 \cdot 0$$

$$1 = 0 \cdot 2 \cdot 1$$

$$\Rightarrow 0100\ 0000$$

$$\text{IEEE-754 SP: } 1010\ 0000\ 0011\ 1100\ 1110\ 0101\ 0000\ 1000$$

$$= \boxed{0x a03ce508}$$

$$0x44800000 + 0x3f000000$$

$$0x44800000 = 0100\ 0100\ 1000\ 0000\ 0000\ 0000\ 0000\ 0000$$

$$\text{Exponent bit} = 1000\ 1001 = 137 \Rightarrow 137 - 127 = 10 = \text{exponent}$$

$$\text{Significand bits} = 0000$$

$$\text{Denormalize: } 1.0000 \times 2^{10}$$

$$0x3f000000 = 0011\ 1111\ 0000\ 0000\ 0000\ 0000\ 0000\ 0000$$

$$\text{Exponent bit} = 0111\ 1110 = 126 \Rightarrow 126 - 127 = -1 = \text{exponent}$$

$$\text{Significand bits} = 0000$$

$$\text{Denormalize: } 1.0000 \times 2^{-1}$$

$$= 0.00000000000001 \times 2^{10}$$

$$\text{Add: } 1.00000000000000 \times 2^{10}$$

$$+ 0.00000000000001 \times 2^{10}$$

$$1.00000000000001 \times 2^{10}$$

$$\text{Sign: } + = 0$$

$$\text{Exponent: } 10 + 127 = 137$$

$$\Rightarrow 137 = 68 \cdot 2 + 1$$

$$68 = 34 \cdot 2 + 0$$

$$34 = 17 \cdot 2 + 0$$

$$17 = 8 \cdot 2 + 1$$

$$8 = 4 \cdot 2 + 0$$

$$4 = 2 \cdot 2 + 0$$

$$2 = 1 \cdot 2 + 0$$

$$1 = 0 \cdot 2 + 1$$

$$\Rightarrow 1000\ 1001$$

$$\text{Significand: } 00000000000100$$

$$\text{IEEE-754 SP} = 0100\ 0100\ 1000\ 0000\ 0001\ 0000\ 0000\ 0000$$

$$= \boxed{0x44801000}$$

b. $0x3c066666 + 0x3c111111$

$0x3c066666 = 0011\ 1100\ 0000\ 1011\ 1011\ 1011\ 1011\ 1011$

Exponent bit: $0111\ 1000 = 120 \Rightarrow 120 - 127 = -7 = \text{exponent}$

Significand bits: $0001\ 0111$

Denormalize: $1.0001\ 0111 \times 2^{-7}$

$0x3c111111 = 0011\ 1100\ 0001\ 0001\ 0001\ 0001\ 0001\ 0001$

Exponent bit: $0111\ 1000 = 120 \Rightarrow 120 - 127 = -7 = \text{exponent}$

Significand bits: $0010\ 0010$

Denormalize: 1.0010×2^{-7}

Add: $1.0001\ 0111 \times 2^{-7}$

$+ 1.0010\ 0010 \times 2^{-7}$

$10.0011\ 1001 \times 2^{-7} = 1.0001\ 11001 \times 2^{-6}$

Sign: $+$

Exponent: $-6 + 127 = 121$

$\Rightarrow 121 = 60 \cdot 2 + 1 \rightarrow 7 = 3 \cdot 2 + 1$

$60 = 30 \cdot 2 + 0 \quad 3 = 1 \cdot 2 + 1$

$30 = 15 \cdot 2 + 0 \quad 1 = 0 \cdot 2 + 1$

$15 = 7 \cdot 2 + 1$

$\Rightarrow 0111\ 1001$

Significand bits: $0001\ 11001$

IEEE-754 SP = $0011\ 1100\ 1000\ 1110\ 0110\ 0110\ 0110\ 0110$

$= 0x3c8e6666$

$$C. 0x42c80000 + 0xc1f80000$$

$$0x42c80000: 0100\ 0010\ 1100\ 1000\ 0000\ 0000\ 0000\ 0000$$

$$\text{Exponent bit: } 1000\ 0101 = 133 \Rightarrow 133 - 127 = 6 = \text{exponent}$$

$$\text{Significant bits: } 1001\ 0000$$

$$\text{Denormalize: } 1.1001\ 0000 \cdot 2^6$$

$$0xc1f80000: 1100\ 0001\ 1111\ 1000\ 0000\ 0000\ 0000\ 0000$$

$$\text{Exponent bit: } 1000\ 0011 = 131 \Rightarrow 131 - 127 = 4 = \text{exponent}$$

$$\text{Significant bits: } 1111\ 0000$$

$$\text{Sign bit: } 1 = -$$

$$\text{Denormalize: } -1.111\ 0000 \cdot 2^4$$

$$= -0.0111\ 10000 \cdot 2^6$$

$$\text{Add: } 1.1001\ 0000 \cdot 2^6$$

$$+ 0.0111\ 10000 \cdot 2^6$$

$$1.0001\ 0100 \cdot 2^6$$

$$\text{Exponent: } 6 + 127 = 133$$

$$\Rightarrow 133 = 66 \cdot 2 + 1$$

$$66 = 33 \cdot 2 + 0$$

$$33 = 16 \cdot 2 + 1$$

$$16 = 8 \cdot 2 + 0$$

$$1 = 0 \cdot 2 + 1$$

$$\Rightarrow 1000\ 0101$$

$$\text{Sign: } + = 0$$

$$\text{Significant bit: } 0001\ 0100$$

$$\text{IEEE-754 SP: } 0100\ 0010\ 1000\ 1010\ 0000\ 0000\ 0000\ 0000$$

$$= 0x428a0000$$

Denormalize: $1.1101\ 0111 \cdot 2^{-9}$

Denormalize: $1.1110\ 0000 \cdot 2^6$

$$= -1.01100111111111110101 \cdot 2^{-2}$$

$\Rightarrow 01111101$

"0xbedcffff



b. $0x44000000 * 0x3b000000$

$$0x44000000 = 0100\ 0100\ 0000\ 0000\ 0000\ 0000\ 0000\ 0000$$

Sign bit: $0 = +$

Exponent bit: $1000\ 1000 = 136 \Rightarrow 136 - 127 = 9$

Significand bits: 0000

Denormalize: $1.\underline{000} * 2^9$

$$0x3b000000 = 0011\ 1011\ 0000\ 0000\ 0000\ 0000\ 0000\ 0000$$

Sign bit: $0 = +$

Exponent bit: $0111\ 0110 = 118 \Rightarrow 118 - 127 = -9$

Significand bits: 0000

Denormalize: $1.\underline{0000} * 2^{-9}$

$$\begin{array}{r} \text{Multiply: } 1.\underline{0000} * 2^9 \\ \quad \times 1.\underline{0000} * 2^{-9} \\ \hline 1.0000 \dots 0 * 2^0 \\ = 1.\underline{0000} \end{array}$$

Sign bit: $0 = +$

Exponent bit: $0 + 127 = 127$

$$\Rightarrow 127 = 63 \cdot 2 + 1 \quad \rightarrow 7 = 3 \cdot 2 + 1$$

$$63 = 31 \cdot 2 + 1 \quad \left. \begin{array}{l} 3 = 1 \cdot 2 + 1 \\ 1 = 0 \cdot 2 + 1 \end{array} \right\}$$

$$31 = 15 \cdot 2 + 1$$

$$15 = 7 \cdot 2 + 1$$

$$\Rightarrow 0111\ 1111$$

Significand bit: 0000

IEEE-754 SP: $0011\ 1111\ 1000\ 0000\ 0000\ 0000\ 0000\ 0000$

$0x3f800000$