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Topic Hashing in Data Structure

Unit No. 04

In data structures,

- * Hashing is well known technique to search any particular element among several elements.
- * It minimizes the number of comparisons while performing the search.

Hashing Technique based on Hash Table

Advantage

Unlike other searching techniques,
(Linear and binary search)

- ① Hashing is extremely efficient.
- ② Time taken by it to perform the search does not depend on the total number of elements.
- ③ It completes the search with constant time complexity $O(1)$.

Hashing Mechanism

- ① An array data structure called as Hash Table is used to store the data items.

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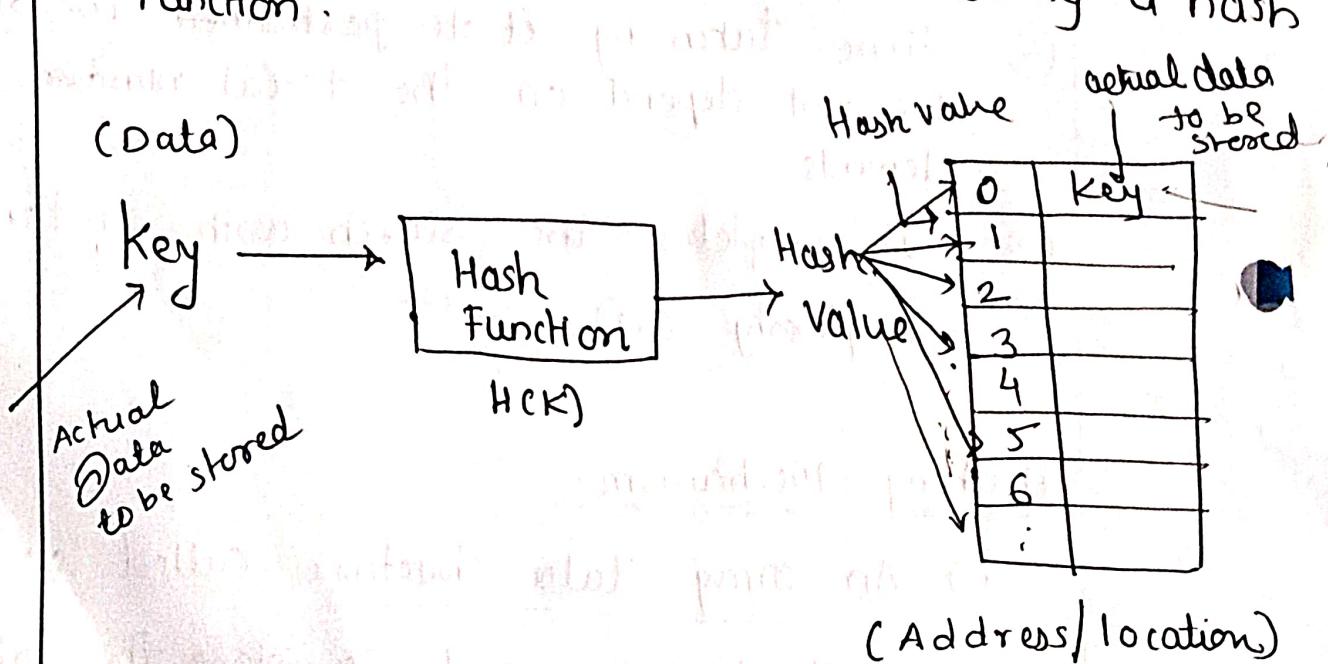
Topic

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- ② Based on the hash key value, data items are inserted into the hash table.

Hash Key Value

- ① Hash key value is a special value that serves as an index for a data item.
- ② It indicates where the data item should be stored in the hash table.
- ③ Hash key value is generated using a hash function.



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Topic Hash Function Unit No. 03

Hash Function

Hash function is a function that maps any big number or string to a small integer value.

- ① Hash function takes the data items as an input and returns a small integer value as an output.
- ② The small integer value is called as hash value.
- ③ Hash value of the data item is then used as an index for storing it into the hash table.

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Types of Hash Functions

There are various types of hash functions available such as,

- ① Division Hash Function
- ② Mid Square Hash Function
- ③ Folding Hash Function etc.
- ④ Modulo Multiplication Hash Function

It depends on user which hash function he want to use.

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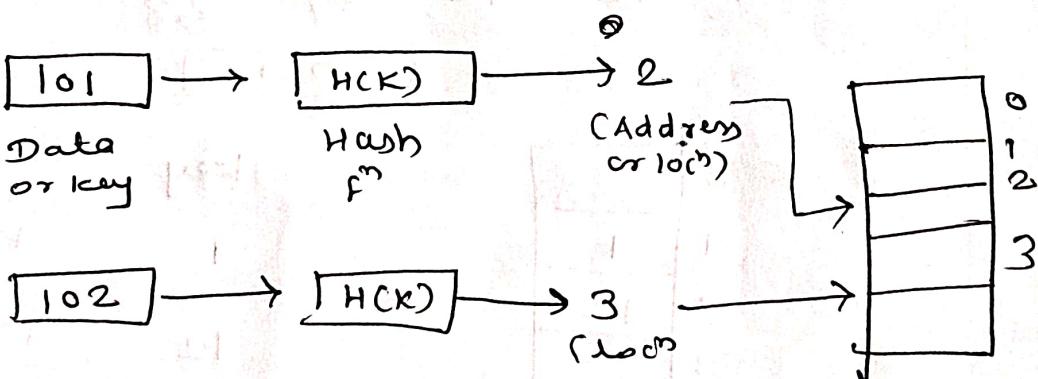
Types of Hashing Function

Direct Hashing

Algorithmic

- ① In direct hashing No algorithm calculations are required.
- ② No collision occur (Minimum collision)
- ③ It is limited
- ④ Not suitable for large key value.

Example



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(2) Module Division Hashing

- Also known as Division remainder Hashing
- Work with any size of data.

$$H(k) = k \bmod n$$

Where $k = \text{key / data}$
 $n = \text{size of list}$

Example

1, 12, 15, 27, 38,
 & $m = 10$

0	
1	1
2	12
3	
4	
5	15
6	
7	27
8	38
9	

key	location
1	$1 \% 10 = 1$
12	$12 \% 10 = 2$
27	$27 \% 10 = 7$
15	$15 \% 10 = 5$
38	$38 \% 10 = 8$

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|--|----------|
| • If list size is a prime number then less number of collision occurs. | |

③

Mid Square Hashing

- ① Middle square Hashing
- ② Key is separated and the address is selected from the middle of square numbers
- ③ problem :- Non Uniform key distribution

eg. key = 9452

$$= (9452)^2 = 89340304$$

④

After Mid square we have to take middle number that is depend on hash table size

- Example :
- ① Size is 1000 then take middle '3' number
 - ② if size is 10000 then take middle '4' number.

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Folding Hashing

Fold Shift Hashing

- ① key value is divided into ~~two~~ parts whose size matches to the size of the required address.

Example

key = 123456789

size = 3

size = 3

- ② Divide it into three parts.

$$P_1 = 123$$

$$P_2 = 456$$

$$P_3 = 789$$

- ③ Perform Addition of

$$\text{Result} = P_1 + P_2 + P_3$$

$$\begin{array}{r}
123 \\
456 \\
789 \\
\hline
1368
\end{array}$$

As size = 3, Discard the value of 1st number

key	value = 368
(Add)	

Fold Boundary Hashing

In this left & right no are folded on a fixed boundary between them & the center number is calculated.

Example 123456789

$$\text{① } 123 \text{ Folded to} = 321$$

$$\& 789 \text{ Folded to} = 987$$

middle No (asked) = 456

- ② Perform the Add'.

$$\begin{array}{r}
321 \\
+ 987 \\
+ 456 \\
\hline
1764
\end{array}$$

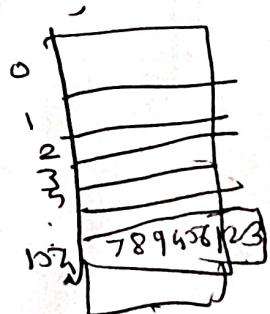
- ③ Now, Discard the number ①

∴ Key value = 764

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Topic	Unit No.
(N) <u>Pseudo Random Hashing</u>	
(4) <u>Digit Extraction Method</u>	$y = 0x + c$ $y = \text{Random NO}$, $c = \text{Coefficient}$ $h(x) = y \bmod n$ (Less chance of collision)
	$p_1 = 1000$
①	$\text{Key} = \frac{789}{p_1} \frac{456}{p_2} \frac{123}{p_3}$ [as size $\rightarrow M = 1000$] [Take '3' digits] For group/ Extract.
②	perform Add ⁿ of p_1, p_2, p_3 $= p_1 + p_2 + p_3 = 789 + 456 + 123$
③	key = <u>1368</u> as. size of Hash table = 1000 if $1368 > 1000$ so again Apply digit Extraction Method, $\therefore \text{key} = \frac{1368}{p_1 p_2} =$ $= p_1 + p_2$ $= 136 + 8$ $= \underline{\underline{154}}$ [Now stored at 154 loc]
	
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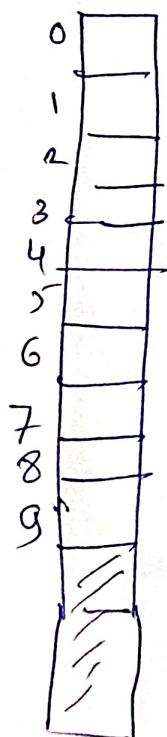
Open Hashing
Chaining

Load fa

keys :

25, 97, 80, 65, 38, 40, 59, 75, 17, 99,
 35, 23

Let, $h(k) = k \bmod m$



Load Factor

Each chain \equiv $\frac{n}{m}$ (no. of elements)
 (Avg) m (size of hash fⁿ)

Load factor

$$\boxed{\alpha = \frac{n}{m}}$$

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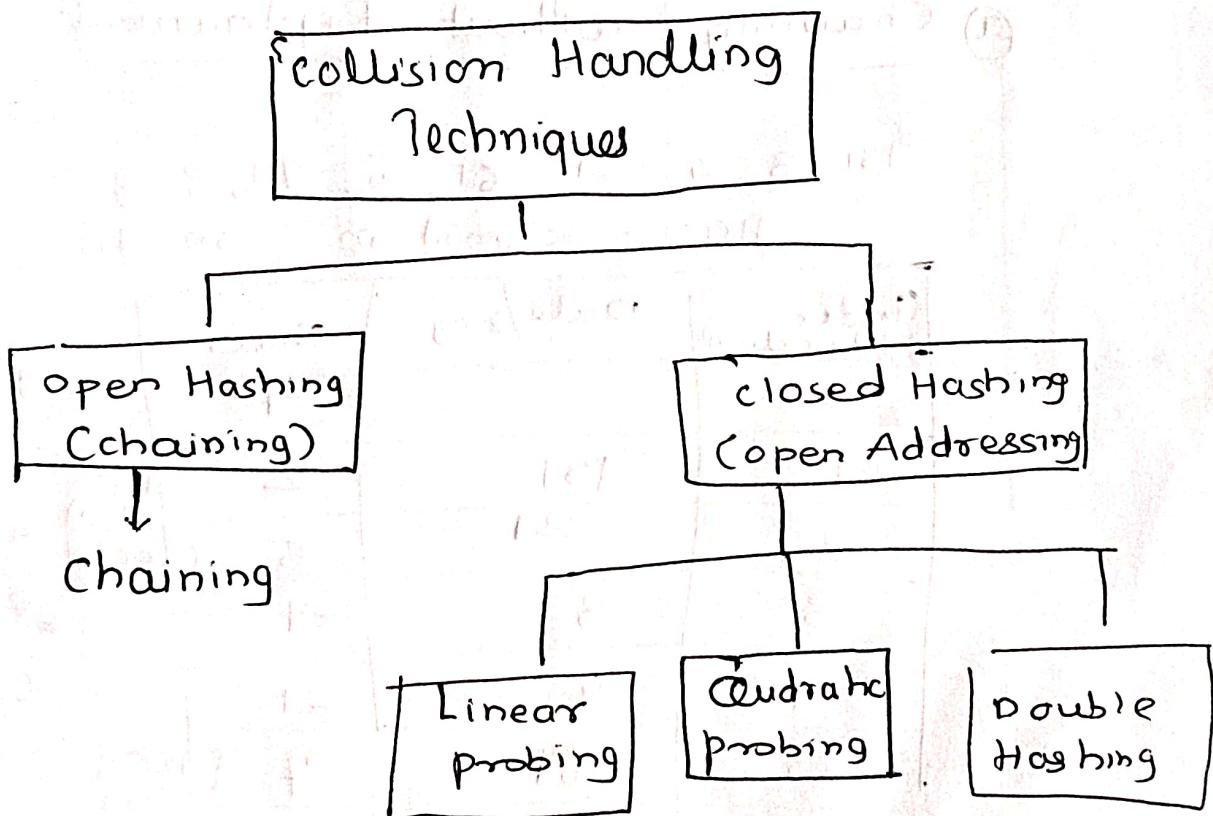
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Collision Resolution Technique

Simple & Direct
with double hashing

- Collision occur then it should be handled by applying some techniques, such techniques are called collision handling techniques.



a) Chaining without Replacement

131, 3, 4, 21, 61, 6, 71, 8, 9

$$H(K) = K \bmod m, m = 10$$

Index (Location)	Data/Key	Chain
0		-1
1	131	2 (locn of 21)
2	21	5 (locn of 61)
3	3	-1
4	4	-1
5	61	7 [locn of 71]
6	6	-1
7	71	-1
8	8	-1
9	9	-1

$$131 \% 10 = 1$$

$$3 \% 10 = 3$$

$$4 \% 10 = 4$$

$$21 \% 10 = 1 \text{ (Collision)}$$

$$61 \% 10 = 1 \text{ (Collision)}$$

$$6 \% 10 = 6 \text{ (Collision)}$$

$$71 \% 10 = 1 \text{ (Collision)}$$

$$8 \% 10 = 8$$

$$9 \% 10 = 9$$

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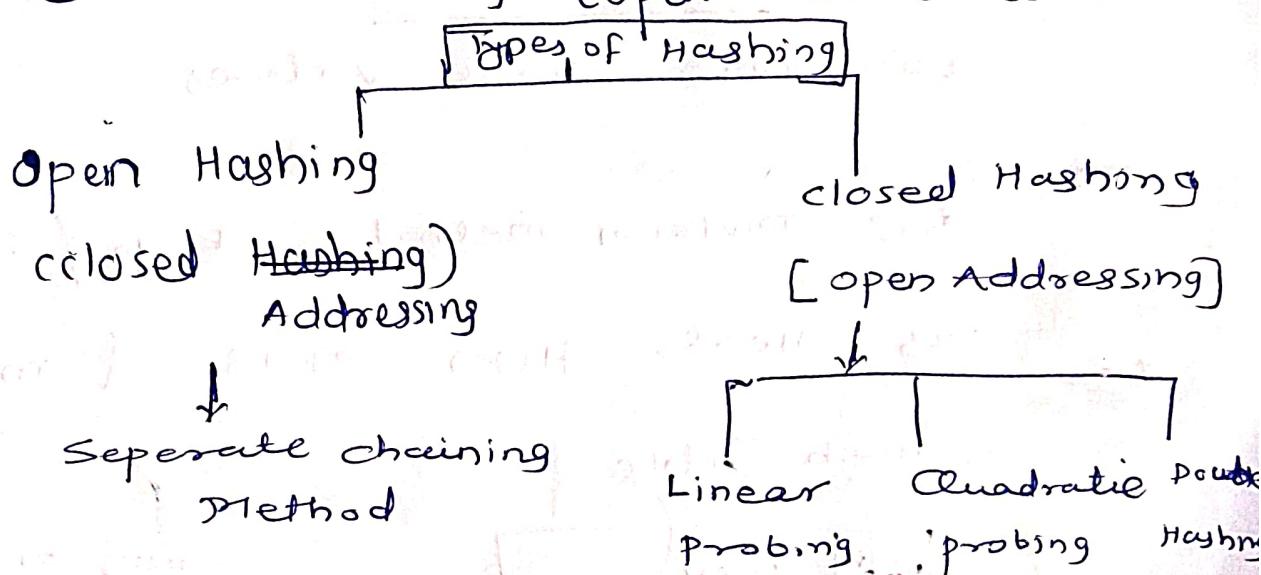
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03

Collision Resolution Technique

Types of Hashing

- ① Open Hashing (closed Addressing)
- ② closed Hashing (Open Addressing)



① open Hashing (closed Addressing)

- In this separate chaining method is used to ^{solve} avoid the collision
- In separate chaining method separate chain is maintained along with hash Table whenever collision occurs.

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Unit No. 3

Topic

* open Hashing (closed Addressing)

Example key = { 3, 2, 9, 6, 11, 13, 7, 12 }

$$H(K) = 2K + 3, \quad m = 10,$$

use division method & closed addressing
to store these values.

So, Division method = $\text{key} \% m$

as Here, $H(K) = 2K + 3 \quad \& \quad m = 10$

∴ Hash Table

Locn	Key
0	
1	9
2	
3	
4	
5	
6	
7	2
8	
9	3

Key	Location	Locn
3	$= (3 \times 2 + 3) \% 10$	9
2	$= (2 \times 2 + 3) \% 10$	7
9	$= (2 \times 9 + 3) \% 10$	1
6	$(2 \times 6 + 3) \% 10$	5
11	$(11 \times 2 + 3) \% 10$	5 C
13	$(13 \times 2 + 3) \% 10$	9 C
7	$(2 \times 7 + 3) \% 10$	7 C
12	$(2 \times 12 + 3) \% 10$	7 C

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Unit No.

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Topic

Prepare the Hash Table

Loc' / Address	Key
0	9
1	9
2	
3	
4	
5	6
6	
7	2
8	
9	3

Diagram illustrating the Hash Table:

```

graph LR
    Loc0[Loc 0] --> Key9[9]
    Loc1[Loc 1] --> Key9[9]
    Loc2[Loc 2] --- Loc3[Loc 3]
    Loc3[Loc 3] --- Loc4[Loc 4]
    Loc4[Loc 4] --- Loc5[Loc 5]
    Loc5[Loc 5] --> Key6[6]
    Loc6[Loc 6] --- Loc7[Loc 7]
    Loc7[Loc 7] --> Key2[2]
    Loc8[Loc 8] --- Loc9[Loc 9]
    Loc9[Loc 9] --> Key3[3]
    Key6[6] --> Node11[11]
    Key6[6] --> Node12[12]
    Key6[6] --> Node13[13]
    Key2[2] --> Node17[17]
    Key3[3] --> Node13[13]
    
```

Advantages:-

- ① Simpler to implement
- ② Less sensitive to Hash function
 - even hash fn is not good or poor collision are managed efficiently by using linked list (Chaining)
- ③ Dynamic memory allocation

Disadvantages

- ① Extra space - Required additional space to store linked list
- ② Poor cache performance
- ③ Worst case mapping - if all elements are mapped with same index

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* Open addressing (Closed Hashing)

* Linear Probing \Rightarrow Here whenever collision occurs search the next ^{First} empty Locⁿ is Example : Hash Table to store the key (value)

$$\text{key} = \{ 3, 2, 9, 6, 11, 13, 7, 12 \}$$

$$H(K) = (2K + 3)$$

$$m = 10$$

use division method for open addressing to store the value.

\Rightarrow Here we can used linear probing method.

$$H(K) = (2K + 3), m = 10$$

$$\therefore \text{Division method} = H(K) \% m$$

Key	$H(K) = (2K + 3) \% 10$	Loc ⁿ	probe
3	$(2 \times 3 + 3) \% 10 = 9$	9	1
2	$(2 \times 2 + 3) \% 10 = 7$	7	1
9	$(2 \times 9 + 3) \% 10 = 1$	1	1
6	$(2 \times 6 + 3) \% 10 = 5$	5	1
11	$(2 \times 11 + 3) \% 10 = 5$	(5)	2
13	$(2 \times 13 + 3) \% 10 = 9$	(9)	2
7	$(2 \times 7 + 3) \% 10 = 7$	(7)	2
12	$(2 \times 12 + 3) \% 10 = 7$	(7)	6

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Now, Let us prepare the Hash Table,

Loc ⁿ	key
0	13
1	9
2	12
3	-
4	-
5	6
6	11
7	2
8	7
9	3

Order of Elements in Hash Table,

$$= \{ 13, 9, 12, - , - , 6, 11, 2, 7, 3 \}$$

$$\text{Avg Probe} = \frac{16}{8} = 2$$

$\boxed{\text{Avg. Probe} = 2}$

C: 0. 4.
- 2. 3.

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Unit No. 3

Topic

Quadratic Probing

(Open Addressing)

(2)

$$(u+i^2) \% m$$

Here we are moving in quadratic way.

$$\text{key} = \{3, 2, 9, 6, 11, 13, 7, 12\}$$

$$H(k) = (2k+3), m=10$$

use division method

⇒

key	$H(k) = (2k+3) \% m$	$\frac{10}{10}$	probe
3	$(2 \times 3 + 3) \% 10 = 9$	9	1
2	$(2 \times 2 + 3) \% 10 = 7$	7	1
9	$(2 \times 9 + 3) \% 10 = 1$	1	1
6	$(2 \times 6 + 3) \% 10 = 5$	5	1
11	$(2 \times 11 + 3) \% 10 = 5$	5	2
13	$(2 \times 13 + 3) \% 10 = 9$	9	2
7	$(2 \times 7 + 3) \% 10 = 7$	7	2
12	$(2 \times 12 + 3) \% 10 = 7$	7	5

$$\text{Avg probe} = 15/8 = 1.8825$$

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Ques, for Key = 11 collision is occur

$$(u+i^2) \% m, i=0 \text{ to } m-1$$

$$u = 5$$

$$\text{for } i=0, (5+0^2) \% 10 = 5$$

$$\text{for } i=1, (5+(1)^2) \% 10 = 6$$

Ques for Key = 13 collision is occur

$$(u+i^2) \% m, i=0 \text{ to } m-1$$

$$u = 9, m = 10$$

$$i=0, (9+0^2) \% 10 = 9$$

$$i=1, (9+(1)^2) \% 10 = 0$$

Ques for Key = 7, collision is occur

$$(u+i^2) \% m, i=0 \text{ to } m-1$$

$$u = 7, m = 10$$

$$\text{for } i=0, (7+0^2) \% 10 = 7$$

$$i=1, (7+(1)^2) \% 10 = 8$$

I. U. I
4. 0.
3 0

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Now, for key = 12 collision is occur

$$\therefore u = (v + i^2) \% m \quad m = 10, \quad i = 0 \text{ to } m-1$$

$$u = 7$$

$$\text{for } i = 0, \quad u = (7 + (0)^2) \% 10 = 7 - c$$

$$i = 1 \quad u = (7 + (1)^2) \% 10 = 8 - c$$

$$i = 2 \quad u = (7 + (2)^2) \% 10 = 1 - c$$

$$i = 3 \quad u = (7 + (3)^2) \% 10 = 6 - c$$

$$i = 4 \quad u = (7 + (4)^2) \% 10 = 3$$

Now prepare the Hash Table

Locn/Add	key
0	13
1	9
2	
3	12
4	
5	6
6	11
7	2
8	7
9	3

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Order of Element stored in Hash Hash

Table

$$= \{ 13, 9, 12, 6, 11, 2, 7, 3 \}$$

Order of element stored in Hash Hash

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Topic

Double Hashing Technique

Here whenever collision is occurs two hash function is used to resolve the collision

Insert K, at first free place,
whenever collision is occurs,

$$= (u + i v) \% m \quad ; \quad i = 0 \text{ to } m-1,$$

u = calculated using First Hashing
 f^n i.e. $H_1(K)$

v = calculated using second Hashing f^n i.e. $H_2(K)$

Example :..

$$\text{key} = \{ 3, 2, 9, 6, 11, 13, 7, 12 \}$$

$$H_1(K) = (2K+3), \quad m=10$$

$$H_2(K) = (3K+1)$$

\Rightarrow

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Key

	$u = H_1(k) = 2k + 3 \mod 10$	$v = H_2(k) = (3k+1)^q$	10 entries	probe
3	$= (2 \times 3 + 3) \mod 10 = 9$		9	1
2	$= (2 \times 2 + 3) \mod 10 = 7$		7	1
9	$= (2 \times 9 + 3) \mod 10 = 1$		1	1
6	$= (2 \times 6 + 3) \mod 10 = 5$		5	1
11	$= (2 \times 11 + 3) \mod 10 = 5$	$v = (3 \times 11 + 1) \mod 10 = 4$	3	3
13	$= (2 \times 13 + 3) \mod 10 = 9$	$v = (3 \times 13 + 1) \mod 10 = 0$	0	-
7	$= (2 \times 7 + 3) \mod 10 = 7$	$v = (3 \times 7 + 1) \mod 10 = 2$	2	-
12	$= (2 \times 12 + 3) \mod 10 = 7$	$v = (3 \times 12 + 1) \mod 10 = 7$	4	2

$$\text{Avg probe} = \frac{9}{6} = 1.5$$

Now, for key = 11, collision occurs

$$u = 5, v = 4, i = 0 \text{ to } m-1$$

$$= \frac{(u+i)v}{m}$$

$$i=0, = (5+0*4) \mod 10 = 5 - e$$

$$i=1, = (5+1*4) \mod 10 = 9 - c$$

$$i=2, = (5+2*4) \mod 10 = 3 \checkmark$$

: 0. 4. 0. 4
2 3 0

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Topic

for key = 13, collision is occur

$$u = 9, v = 0, i = 0 \text{ to } m-1$$

Double Hashing = $(u + iv) \% 10$

$$\text{as } i=0, = (9 + 0*0) \% 10 = 9$$

$$i=1, = (9 + 1*0) \% 10 = 9$$

$$i=2, = (9 + 2*0) \% 10 = 9, \text{ so we can}$$

insert 13 in the hash table

for key = 7, collision is occur,

$$u = 7, v = 2, i = 0 \text{ to } m-1, m = 10$$

Double Hashing = $(u + iv) \% m$

$$i=0, = (7 + 0*2) \% 10 = 7 - c$$

$$i=1, = (7 + 1*2) \% 10 = 9 - c$$

$$i=2, = (7 + 2*2) \% 10 = 1 - c$$

$$i=3, = (7 + 3*2) \% 10 = 3 - c$$

$$i=4, = (7 + 4*2) \% 10 = 5 - c$$

$$i=5, = (7 + 5*2) \% 10 = 7 - c$$

$$i=6, = (7 + 6*2) \% 10 = 9 - c$$

$$i=7, = (7 + 7*2) \% 10 = 1 - c$$

$$i=8, = (7 + 8*2) \% 10 = 3 - c$$

$$i=9, = (7 + 9*2) \% 10 = 5 - c$$

so it is not possible to store key = 7 in Hash table

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Now for key = 12

$$u = 7, v = 7, \quad i = 0 \text{ to } m-1,$$

$$m = 10$$

$$\text{Double Hashing} = (u + iv) \% 10$$

Hashing

$$\begin{aligned} i=0, \quad & (7 + 0*7) \% 10 = 7 - c \\ i=1, \quad & (7 + 1*7) \% 10 = 4 \end{aligned}$$

Now Hash Table

Loc	Key
0	
1	9
2	
3	11
4	12
5	6
6	
7	2
8	
9	3

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Topic	Unit No.
Order of Element in Hash Table	
= { - , 9, - , 11, 12, 6, - , 2, - , 3 }	
Hash Table	
Hash Function	