# Regularizing your neural network

## Regularization

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# Logistic regression

$w \in \mathbb{R}^{n_x}$, $b \in \mathbb{R}$ 

$\lambda$ = regularization parameter

lambda lambd

$$\min_{w,b} J(w,b)$$

$$J(w,b) = \underbrace{\frac{1}{m}\sum_{i=1}^{m} \mathcal{L}(\hat{y}^{(i)}, y^{(i)})} + \frac{\lambda}{2m} \underbrace{\|w\|_2^2} \qquad \cancel{+ \frac{\lambda}{2m} b^2}$$

omit

L2 regularization $\|w\|_2^2 = \sum_{j=1}^{n_x} w_j^2 = w^T w \leftarrow$

L1 regularization $\frac{\lambda}{2m} \sum_{j=1}^{n_x} |w_j| = \frac{\lambda}{2m} \|w\|_1$

w will be sparse

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# Neural network

$$\rightarrow J(w^{[1]}, b^{[1]}, \dots, w^{[L]}, b^{[L]}) = \underbrace{\frac{1}{m}\sum_{i=1}^{m} \mathcal{L}(\hat{y}^{(i)}, y^{(i)})} + \underbrace{\frac{\lambda}{2m}\sum_{l=1}^{L} \|w^{[l]}\|_F^2}$$

$$\|w^{[l]}\|_F^2 = \sum_{i=1}^{n^{[l]}} \sum_{j=1}^{n^{[l-1]}} (w_{ij}^{[l]})^2 \qquad w^{[l]}: (n^{[l]}, n^{[l-1]})$$

"Frobenius norm" $\qquad \|\cdot\|_2^2 \qquad \|\cdot\|_F^2$

$$dw^{[l]} = \boxed{(\text{from backprop}) + \frac{\lambda}{m} w^{[l]}} \qquad \underbrace{\frac{\partial J}{\partial w^{[l]}}} = dw^{[l]}$$

$$\rightarrow w^{[l]} := w^{[l]} - \alpha \, dw^{[l]}$$

"Weight decay"

$$w^{[l]} := w^{[l]} - \alpha \left[ (\text{from backprop}) + \underline{\frac{\lambda}{m} w^{[l]}} \right]$$

$$= w^{[l]} - \frac{\alpha\lambda}{m} w^{[l]} - \alpha (\text{from backprop})$$

$$= \underbrace{\left(1 - \frac{\alpha\lambda}{m}\right)}_{<1} \underline{w^{[l]} - \alpha (\text{from backprop})}$$

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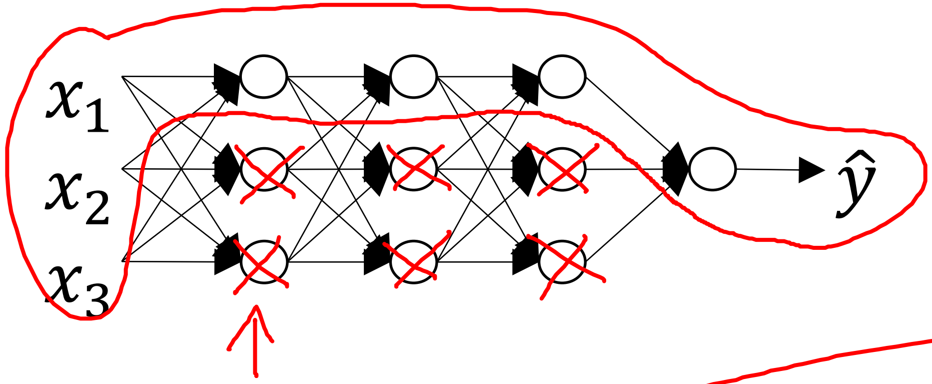
# Regularizing your neural network

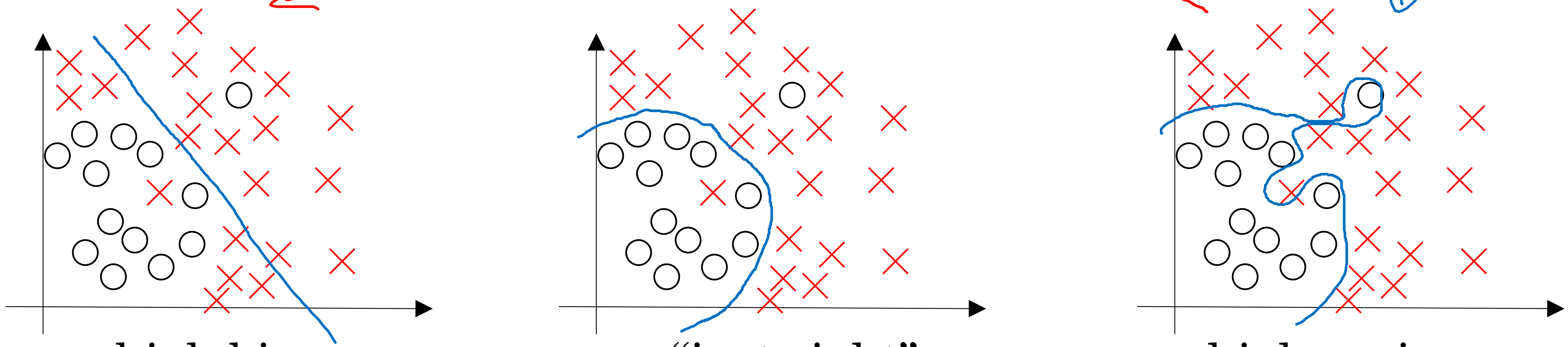
# Why regularization reduces overfitting

# How does regularization prevent overfitting?



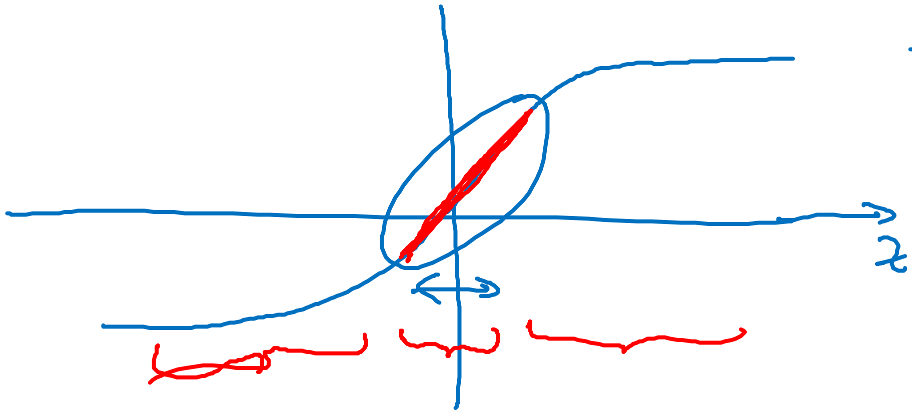

$$J(w^{[l]}, b^{[l]}) = \frac{1}{m}\sum_{i=1}^{m} \mathcal{L}(\hat{y}^{(i)}, y^{(i)}) + \frac{\lambda}{2m}\sum_{l=1}^{L} \|w^{[l]}\|_F^2$$

$$\underline{w^{[l]} \approx 0}$$



high bias      "just right"      high variance

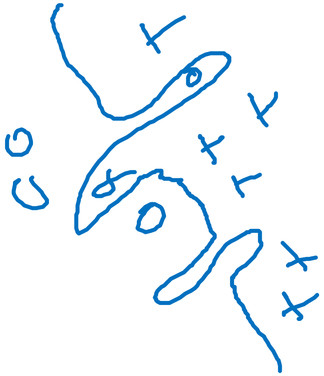
# How does regularization prevent overfitting?




tanh

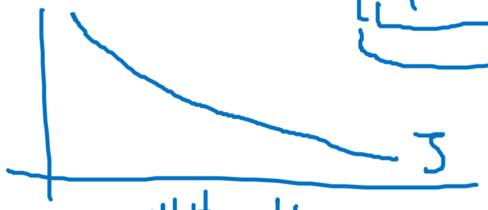
$g(z) = \tanh(z)$

$\lambda \uparrow$ $\quad W^{[l]} \downarrow$ $\quad z^{[l]} = W^{[l]} a^{[l-1]} + b^{[l]}$



Every layer ≈ linear.

$$J(\dots) = \boxed{\sum_i \mathcal{L}(\hat{y}^{(i)}, y^{(i)})} + \frac{\lambda}{2m} \sum_l \|W^{[l]}\|_F^2$$

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