Regression metrics:
(R)MSPE, MAPE, (R)MSLE

Plan for the video

1) **Regression**
   - MSE, RMSE, R-squared
   - MAE
   - (R)MSPE, MAPE
   - (R)MSLE
2) **Classification:**
   - Accuracy, LogLoss, AUC
   - Cohen's (Quadratic weighted) Kappa

From MSE and MAE to MSPE and MAPE

- **Shop 1**: predicted 9, sold 10, MSE = 1
- **Shop 2**: predicted 999, sold 1000, MSE = 1

It could happen that off by one error in the first case, is much more critical than in the second case. But MSE and MAE are equal to one for both shops predictions, and thus according to those metrics, these off by one errors are indistinguishable.

From MSE and MAE to MSPE and MAPE

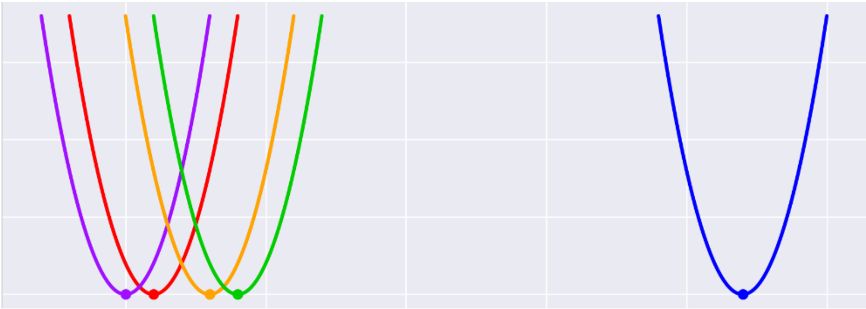
- **Shop 1**: predicted 9, sold 10, MSE = 1
- **Shop 2**: predicted 999, sold 1000, MSE = 1

- **Shop 1**: predicted 9, sold 10, MSE = 1
- **Shop 2**: predicted 900, sold 1000, MSE = 10000

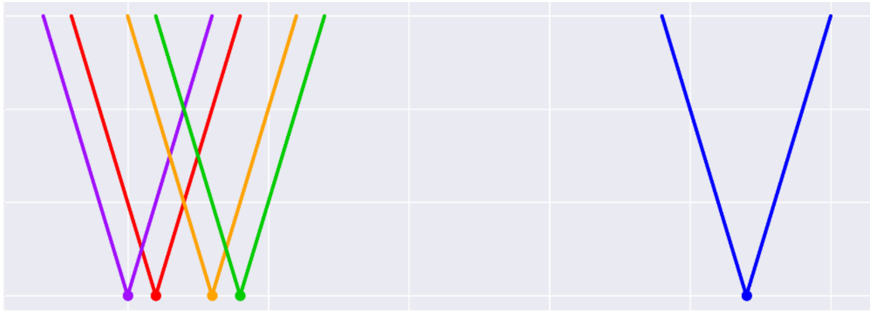
- **Shop 1**: predicted 9, sold 10, relative_metric = 1
- **Shop 2**: predicted 900, sold 1000, relative_metric = 1

From MSE and MAE to MSPE and MAPE

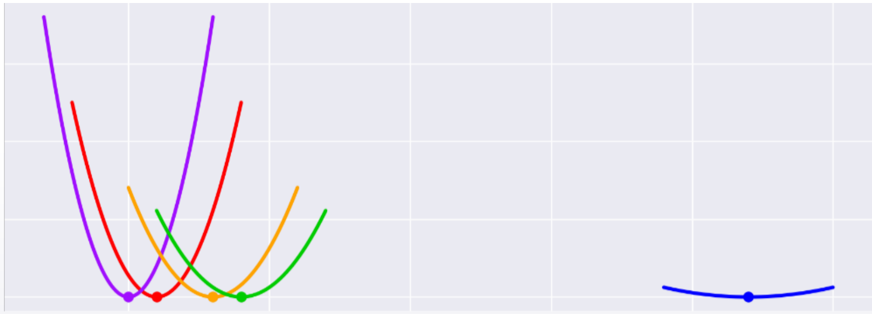
$$\mathrm{MSE} = \frac{1}{N}\sum_{i=1}^{N}(y_i - \hat{y}_i)^2$$



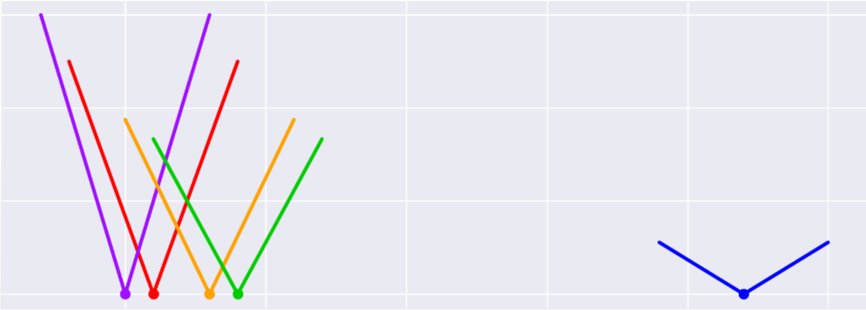
$$\mathrm{MAE} = \frac{1}{N}\sum_{i=1}^{N}|y_i - \hat{y}_i|$$



$$\mathrm{MSPE} = \frac{100\%}{N}\sum_{i=1}^{N}\left(\frac{y_i - \hat{y}_i}{y_i}\right)^2$$



$$\mathrm{MAPE} = \frac{100\%}{N}\sum_{i=1}^{N}\left|\frac{y_i - \hat{y}_i}{y_i}\right|$$



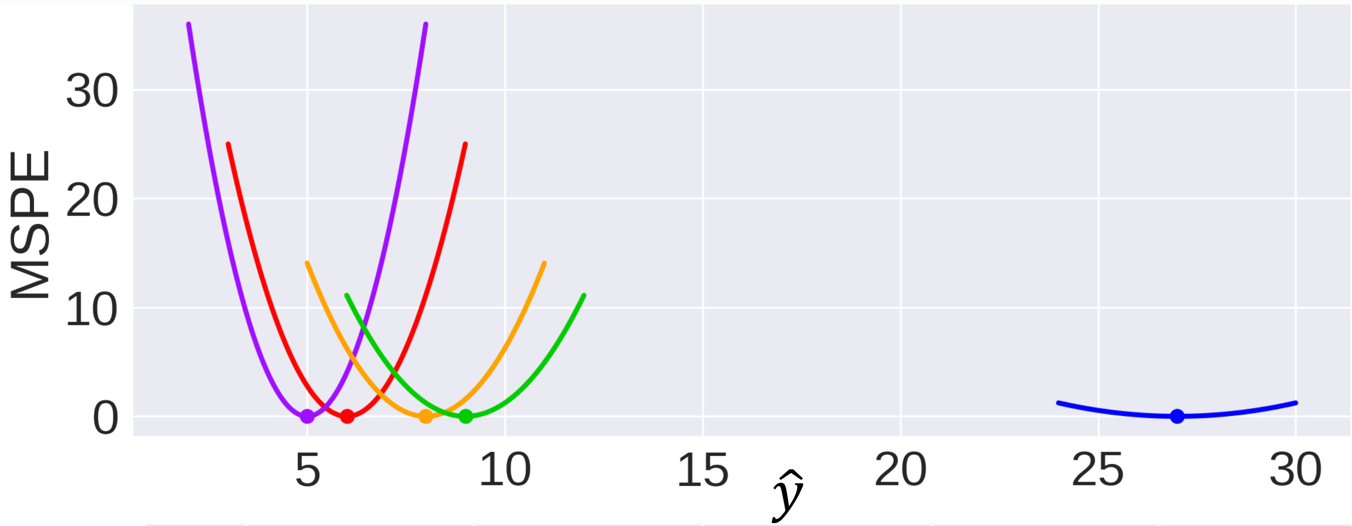
MSPE: constant

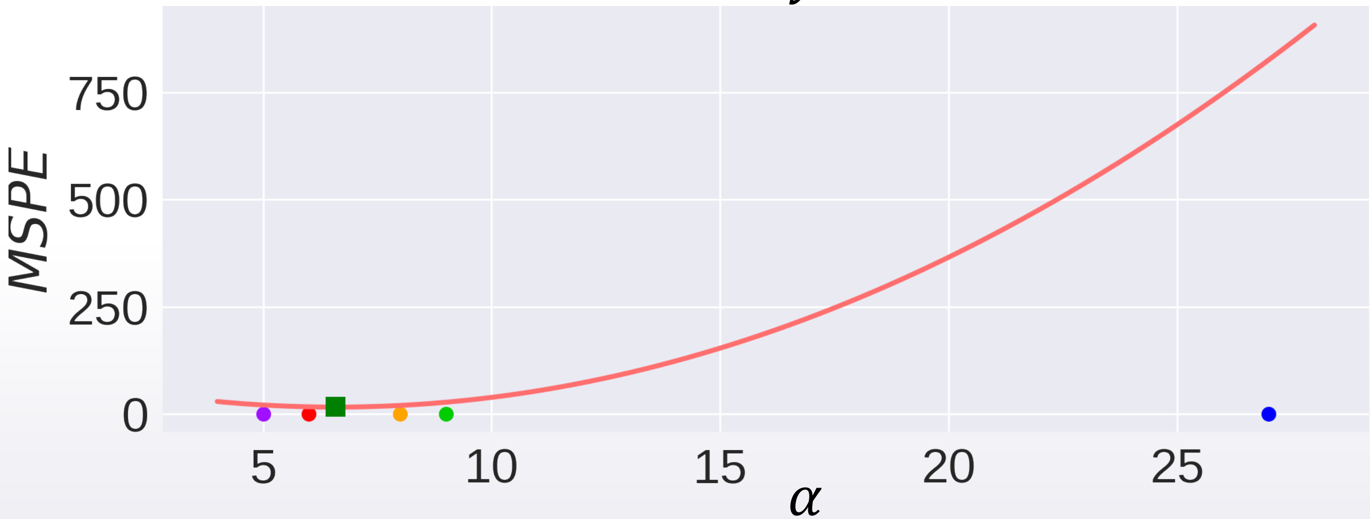
$$\text{MSPE} = \frac{100\%}{N} \sum_{i=1}^{N} \left(\frac{y_i - \alpha}{y_i} \right)^2$$

Best constant:
weighted target mean

Data:

X	Y
-1	4
1	3
-2	6
3	7
3	25





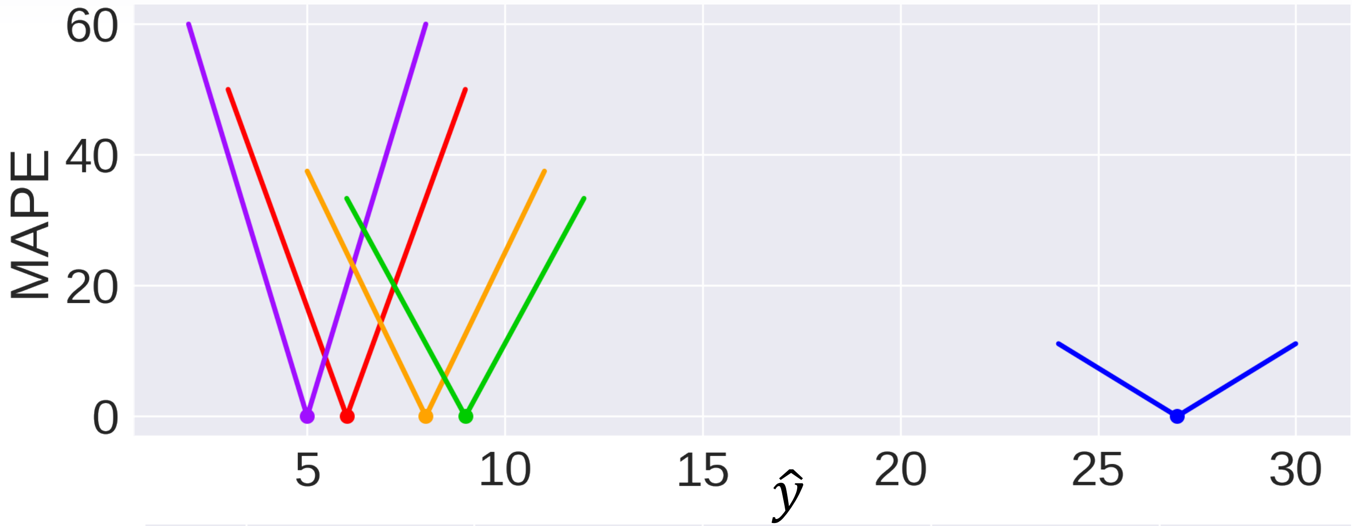
MAPE: constant

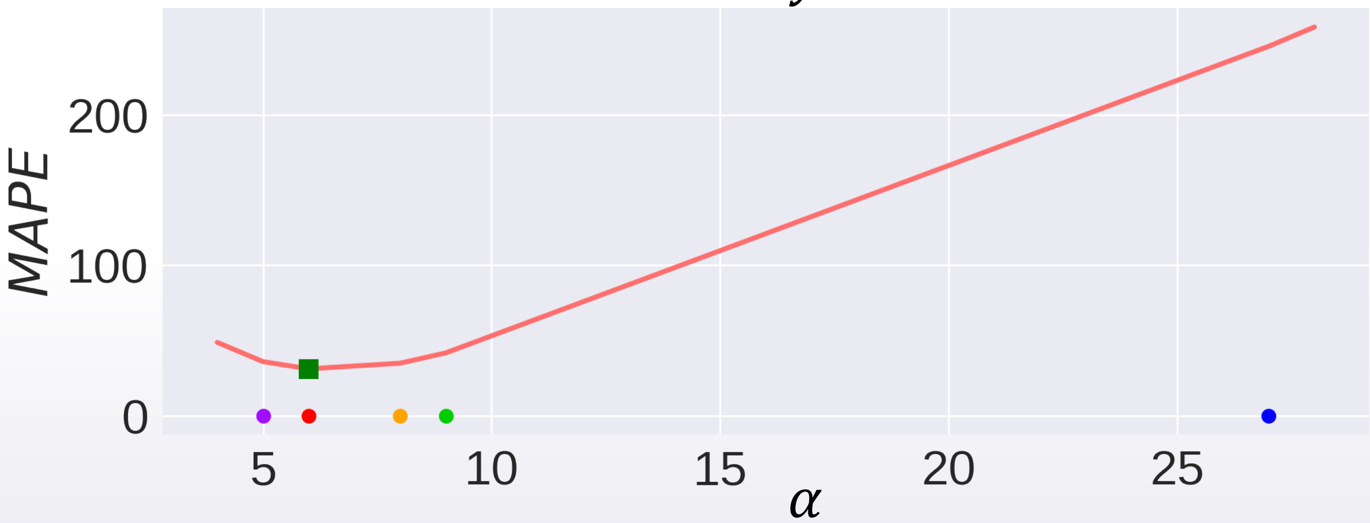
$$\text{MAPE} = \frac{100\%}{N} \sum_{i=1}^{N} \left| \frac{y_i - \alpha}{y_i} \right|$$

Best constant:
weighted target median

Data:

X	Y
-1	4
1	3
-2	6
3	7
3	25





(R)MSLE: Root Mean Square Logarithmic Error

$$\text{RMSLE} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\log(y_i+1)-\log(\hat{y}_i+1)\right)^2} =$$

$$= RMSE\left(\log(y_i+1), \log(\hat{y}_i+1)\right) =$$

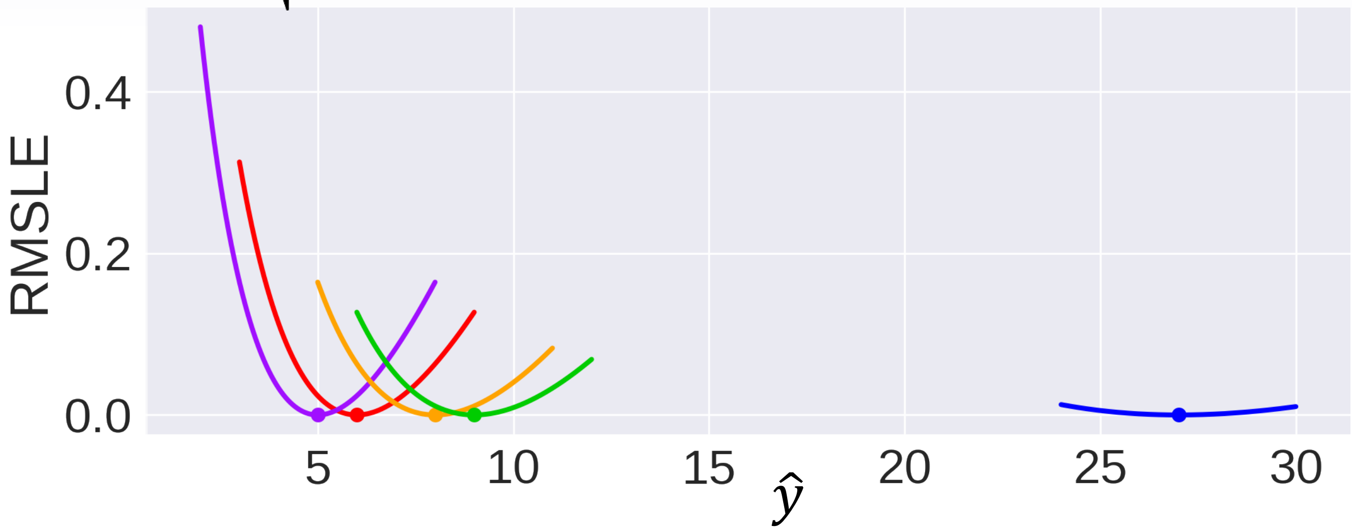
$$= \sqrt{MSE\left(\log(y_i+1), \log(\hat{y}_i+1)\right)}$$

(R)MSLE: Root Mean Square Logarithmic Error

$$\text{RMSLE} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\log(y_i+1) - \log(\hat{y}_i+1)\right)^2}$$

Data:

X	Y
-1	4
1	3
-2	6
3	7
3	25



(R)MSLE: constant

$$\text{RMSLE} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\log(y_i+1)-\log({\color{red}\alpha}+1)\right)^2} =$$

$$= RMSE\left(\log(y_i+1), \log({\color{red}\alpha}+1)\right) =$$

$$= \sqrt{MSE\left(\log(y_i+1), \log({\color{red}\alpha}+1)\right)}$$

(R)MSLE: constant

$$\text{RMSLE} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\log(y_i+1) - \log(\alpha+1)\right)^2} =$$

$$= RMSE\left(\log(y_i+1), \log(\alpha+1)\right) =$$

$$= \sqrt{MSE\left(\log(y_i+1), \log(\alpha+1)\right)}$$

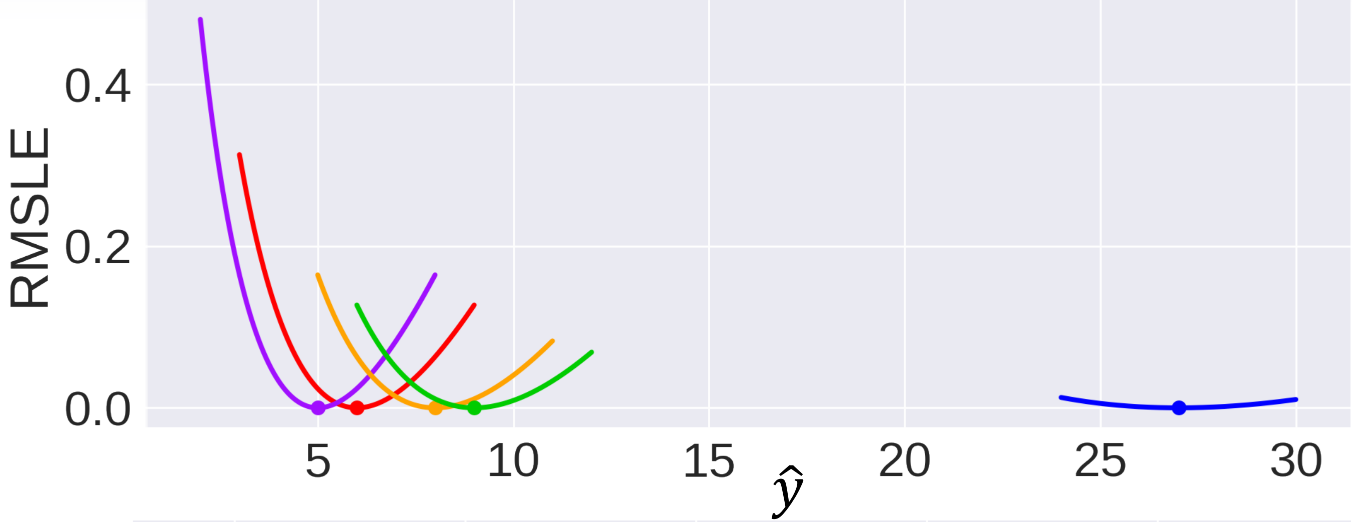
- Best constant in log space is a mean target value
- We need to exponentiate it to get an answer

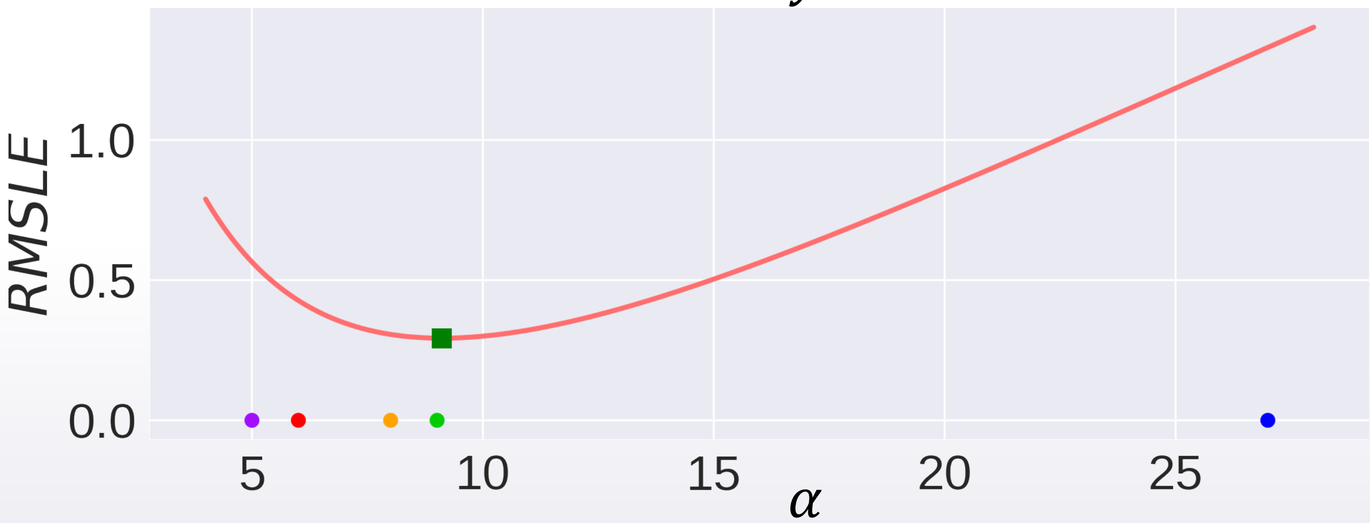
(R)MSLE: constant

$$\text{RMSLE} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\log(y_i+1) - \log(\alpha+1)\right)^2}$$

Data:

X	Y
-1	4
1	3
-2	6
3	7
3	25





Compare the constants

Metric	Constant
MSE	11
RMSLE	9.11
MAE	8
MSPE	6.6
MAPE	6

Conclusion

Discussed the metrics, sensitive to relative errors:

- **(R)MSPE**
  - Weighted version of MSE
- **MAPE**
  - Weighted version of MAE
- **(R)MSLE**
  - MSE in log space