Discrete Mathematics

B.Sc. in CSE level 1,term 1 final Examination Question solve – 2020

1.

a) Define Set with example. Prove that $|A_1 \cup A_2| = |A_1 + A_2| - |A_1 \cap A_2|$, where A_1 and A_2 be sets with cardinalities $|A_1|$ and $|A_2|$.

b) Explain Venn Diagram for Union and Intersection operations where A and B are two sets.

c) Write down the Laws of set theory.

d) Consider Universal set U and two sets A and B. Shade the following sets using Venn Diagram:

i) $A \cap B^C$ **ii) $(B/A)^C$**

Answer – 01

(a)
Ans.

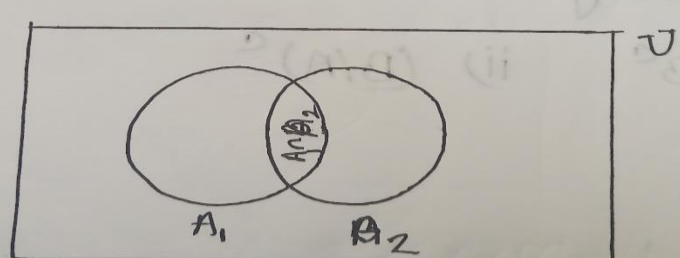
Set : Set can be defined as a collection of well-defined objects or elements.

Let, A be the set of odd numbers less than 10.

$\therefore A = \{1, 3, 5, 7, 9\}$

Here, ~~1, 2,~~ 1, 3, 5, 7, 9 are the elements of the set.

Proof of $|A_1 \cup A_2| = |A_1 + A_2| - |A_1 \cap A_2|$:



Let, A_1 and A_2 be two sets. Here, U is the universal set from the venn diagram we can write:

$$|A_1 \cup A_2| = |A_1 - A_2| + |A_2 - A_1| + |A_1 \cap A_2|$$

$$= |A_1 - A_2| + |A_1 \cap A_2| + |A_1 \cap A_2| + |A_2 - A_1|$$

$$- |A_1 \cap A_2| - \cancel{|A_1 \cap A_2|} + \cancel{|A_1 \cap A_2|}$$

$$= |A_1| + |A_2| - |A_1 \cap A_2|$$

$$\therefore |A_1 \cup A_2| = |A_1| + |A_2| - |A_1 \cap A_2|$$

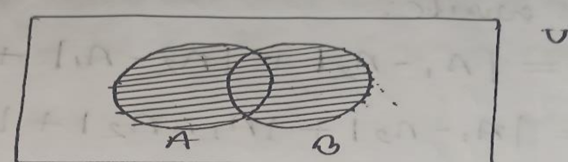
(Proved)

(b)

Ans:

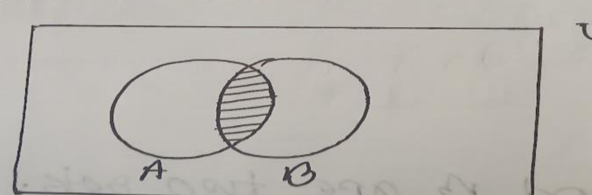
Here, A and B are two sets. Union and intersection operation for A and B sets are below:

* Union of set A and B:



Marked area in the figure is $A \cup B$. $A \cup B$ contains all the elements of set A and set B.

* Intersetion of set A and B:



Marked area in the diagram is $A \cap B$.

$A \cap B$ contains only the common elements of set A and B.

(c) Laws of set theory:

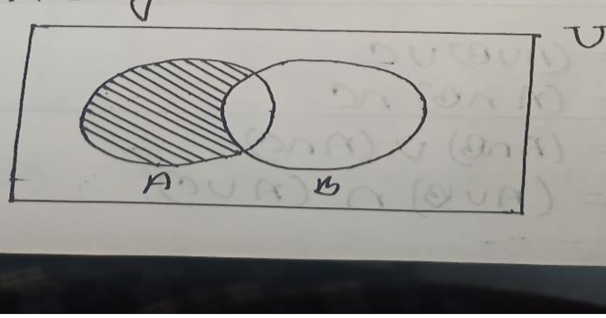
Law	Name
$A \cup \emptyset = A$ $A \cap U = A$	Identity Laws
$A \cup U = U$ $A \cap \emptyset = \emptyset$	Domination Laws
$A \cup A = A$ $A \cap A = A$	Idempotent Laws
$\overline{(\bar{A})} = A$	complementation law
$A \cup B = B \cup A$ $A \cap B = B \cap A$	Commutative Laws.
$A \cup (B \cup C) = (A \cup B) \cup C$ $A \cap (B \cap C) = (A \cap B) \cap C$	Associative Laws
$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$	Distributive Laws.
$\overline{A \cup B} = \bar{A} \cap \bar{B}$ $\overline{A \cap B} = \bar{A} \cup \bar{B}$	De Morgan's Law
$A \cup (A \cap B) = A$ $A \cap (A \cup B) = A$	Absorption Laws
$A \cup \bar{A} = U$ $A \cap \bar{A} = \emptyset$	Complement Laws

(d)

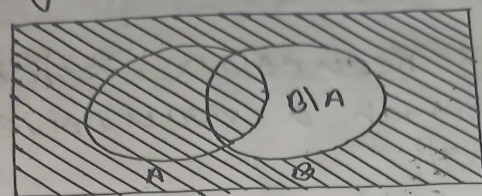
Ans:

Let, U be the universal set and A, B are two sets.

i) Venn diagram of $A \cap B^c$:



(ii) venn diagram of $(B/A)^c$:



2.

a) **What is recursion? A function f is defined recursively by f(0) = 3, f(n+1) = 2f(n)+3; Find the value of f(4).**

b) **What is recurrence relation? Obtain the recurrence relation for:**

$$G(k) = 2.4^k - 5.(-3)^k$$

c) **Find the solution to the recurrence relation: $a_n = 6a_{n-1} - 11a_{n-2} + 6a_{n-3}$ with the initial conditions $a_0=2$, $a_1=5$ and $a_2=15$**

<u>Answer -02</u>

Ans. to the ques. no: 02

(a)

Recursion: Recursion refers to a process in which a recursive process repeats itself.

(b) Here,

$$G(k) = 2\cdot 4^k - 5\cdot(-3)^k$$

$$\therefore G(k-1) = 2\cdot 4^{(k-1)} - 5\cdot(-3)^{(k-1)}$$

$$= 2\cdot 4^k \cdot \frac{1}{4} - 5\cdot(-3)^k\cdot\frac{1}{(-3)}$$

$$= \frac{4^k}{2} + \frac{5}{3}(-3)^k$$

$$G(k-2) = 2\cdot 4^{(k-2)} - 5\cdot(-3)^{(k-2)}$$

$$= 2\cdot \cancel{2} 4^k\cdot\frac{1}{4^2} - 5\cdot(-3)^k\cdot\frac{1}{(-3)^2}$$

$$= \frac{4^k}{8} - 5\cdot\frac{(-3)^k}{9}$$

$$= \frac{1}{8}\cdot 4^k - \frac{5}{9}(-3)^k$$

Now,

$$G(k) - G(k-1) = 2\cdot 4^k - 5\cdot(-3)^k - \frac{1}{2}\cdot 4^k - \frac{5}{3}(-3)^k$$

$$\therefore G(k) - G(k-1) = \frac{3}{2}4^k - \frac{20}{3}(-3)^k$$

Now,

$$G(k-2) = \frac{1}{8}4^k - \frac{5}{9}(-3)^k$$

$$= \frac{3}{2}\cdot\frac{1}{4\cdot 3}4^k - \frac{20}{3}\cdot\frac{1}{3\cdot 4}(-3)^k$$

$$= \frac{1}{12}\left\{\frac{3}{2}4^k - \frac{20}{3}(-3)^k\right\}$$

$$\therefore 12G(k-2) = \frac{3}{2}4^k - \frac{20}{3}(-3)^k$$

$$\therefore G(k) - G(k-1) - 12G(k-2) = \frac{3}{2}4^k - \frac{20}{3}(-3)^k - \frac{3}{2}4^k + \frac{20}{3}(-3)^k$$

$$\Rightarrow G(k) - G(k-1) - 12G(k-2) = 0$$

(Ans.)

(c)

Here,

$a_n = 6a_{n-1} - 11a_{n-2} + 6a_{n-3}$

$\Rightarrow a_n - 6a_{n-1} + 11a_{n-2} - 6a_{n-3} = 0$

$\Rightarrow r^3 - 6r^2 + 11r - 6 = 0$

$\Rightarrow r^3 - r^2 - 5r^2 + 5r + 6r - 6 = 0$

$\Rightarrow r^2(r-1) - 5r(r-1) + 6(r-1) = 0$

$\Rightarrow (r-1)(r^2 - 5r + 6) = 0$

$\Rightarrow (r-1)(r^2 - 3r - 2r + 6) = 0$

$\Rightarrow (r-1)\{r(r-3) - 2(r-3)\} = 0$

$\Rightarrow (r-1)(r-3)(r-2) = 0$

$r = 1, 2, 3$

$\therefore r_n = 1^n \cdot A + 2^n \cdot B + 3^n \cdot C$ — (1)

Now,

$a_0 = A + B + C$

$\Rightarrow A + B + C = 2$ — (1)

$a_1 = A + 2B + 3C$

$\Rightarrow A + 2B + 3C = 5$ — (2)

$a_2 = A + 4B + 9C$

$\Rightarrow A + 4B + 9C = 3$ — (3)

From no. (1) and (2),

$2 - B - C + 2B + 3C = 5$

$\Rightarrow B + 2C = 3$ — (4)

from no. (1) and (3),

$2 - B - C + 4B + 9C = 3$

$\Rightarrow 3B + 8C = 1$ — (5)

From, (5) − (4)×3 :

$$3B + 8C = 1$$
$$\underline{(-)3B \mp 6C = 9}$$
$$2C = -8$$
$$\Rightarrow C = -4$$

$\therefore B = 3 - 2C$ (From no.4)

$$= 3 - 2\cdot(-4)$$
$$= 3 + 8 = 11$$

$\therefore A + 11 - 4 = 2$

$\Rightarrow A = 2 + 4 - 11 = -5$

$\therefore r_n = -5\cdot 1^n + 11\cdot 2^n - 4\cdot 3^n$ (Ans.)

3.

a) Define reflexive, symmetric and transitive relation.

b) Consider the following five relations on the set A = {1, 2, 3, 4}:

R = {(1, 1), (1, 2), (2, 3), (1, 3), (4, 4)}
S = {(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (4, 4)}
T = {(1, 3), (2, 1)}
U = A × A, the universal relation V =
∅, the empty relation

Determine whether or not each of the above relations on A is

i) **Reflexive ii) Symmetric iii) Transitive**

c) Define Equivalence relation with example.

d) Let A = {1,2,3,4}, B = {4,5,6} and C = {6,7,8}. R is a relation from A to B and S is a relation from B to C, which is given by

R = {(x,y): x + y = 7}
S = {(x,y): y - x = 1}

Determine R and S.

Answer-03

Ans. to the ques. no: 03

(a)

Reflexive Relations: A relation R on a set A is called reflexive if $(a,a) \in R$ for every element $a \in A$.

Consider a Relation R on $A = \{1,2,3,4\}$

$R = \{(1,1), (1,2), (2,2), (3,3), (4,1), (4,4)\}$, Here R is a reflexive relation.

Symmetric Relations: A Relation R on a set A is called symmetric if $(b,a) \in R$ whenever $(a,b) \in R$, for all $a,b \in A$.

If $A = \{1,2,3,4\}$ and

$R = \{(1,1), (1,2), (2,1)\}$, Then R is symmetric.

Transitive Relations: A relation R on a set A is called transitive if whenever $(a,b) \in R$ and $(b,c) \in R$ then $(a,c) \in R$, for all $a,b,c \in A$.

If $A = \{1,2,3,4\}$ and $R = \{(2,1), (3,1), (3,2), (4,1), (4,2), (4,3)\}$, Then R is transitive.

(b)

(i)
Here, S and U are Reflexive, because they consists $(a,a) \in R$ for every element $a \in A$.

(ii) S and U are Reflexive because they consists $(b,a) \in R$ whenever $(a,b) \in R$ for all $a,b \in A$.

(iii)

Here, S is transitive for the elements (1,2) and (2,1).

The universal relation U is also transitive as it ~~contains~~ satisfies the conditions of transitive relation.

(c)

Equivalance Relation: A relation R on a set A is said to be an equivalance relation if and only if the relation R is reflexive, symmetric and transitive. An equivalance relation is represented by the symbol "~".

If $A = \{1, 2, 3, 4\}$ And $R = A \times A$

Then, R is an Equivalance Relation.

Ans. to the ques. no: 03

(d) Here,

$A = \{1, 2, 3, 4\}$

$B = \{4, 5, 6\}$

R is a relation from A to B.

$x + y = 7$

$\therefore y = 7 - x$

Now,

x	1	2	3	4
y	6	5	4	3

But ~~3&~~ $3 \notin B$. So, $(4, 3) \notin R$

$\therefore R = \{(1, 6), (2, 5), (3, 4)\}$

(Ans.)

Here,

$B = \{4, 5, 6\}$

$C = \{6, 7, 8\}$

S is a relation from B to C.

Now,

$y - x = 1$

$\therefore y = x + 1$

x	4	5	6
y	5	6	7

Here, $5 \notin C$. So, $(4,5) \notin$ ~~B~~ S.

$\therefore S = \{(5,6), (6,7)\}$ (Ans)

4. a) State the converse, contrapositive and inverse of the following conditional statements:

   b) If it snows tonight, then I will stay at home.

   c) I go to the beach whenever it is a sunny summer day.

   b) State and proof the Principle of Inclusion and Exclusion.

   c) Let A = {1,2,3} and R be the relation, R = {(1,2), (1,3), (2,2), (2,3), (3,3)}. Draw the diagraph by using this relation.

   d) Define tautology and contradiction with examples. Verify that $(p \wedge q) \wedge \neg (p \vee q)$ is a contradiction by using truth table.

Answer-04

Ans. to the ques. no: 04

(a)

(i)

Converse: I will stay at home, if it snows tonight.

Inverse: If it does not snow tonight, I will not stay at home.

Contrapositive:
I will not stay at home, if it does not snow tonight.

(ii)

Converse:
Whenever it is a sunny summer day, I go to the beach.

Inverse:
I do not go to the beach whenever it is not a sunny summer day.

Contrapositive:
Whenever it is not a sunny summer day, I do not go to the beach.

(b)

Principal of inclusion and exclusion is an approach which derives the method of finding the numbers of elements in the union of two finite sets.
Considers two finite sets are A and B. We can denote the principal

of inclusion and exclusion by-

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

(b)

Statement:

If $A_1, A_2, \ldots, A_n$ are finite sets, then

$$|A_1 \cup A_2 \cdots \cup A_n| = \sum |A_i| - \sum |A_i \cap A_j| + \sum |A_i \cap A_j \cap A_k| - + \cdots + (-1)^{n+1} |A_1 \cap A_2 \cdots \cap A_n|$$

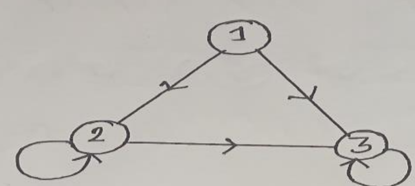
Proof:

(c)

Here, $A = \{1, 2, 3\}$

and $R = \{(1,2), (1,3), (2,2), (2,3), (3,3)\}$.

Diagraph for the Relation R:



(d)

Tautology: A compound proposition that is always true, no matter what the truth values of the propositional variables that occur in it, is called a tautology.

Contradiction: A compound proposition that is always false is called a contradiction.

Example of a Tautology and a contradiction:

p	$\neg p$	$p \vee \neg p$	$p \wedge \neg p$
T	F	T	F
F	T	T	F

Here, $p \vee \neg p$ is a Tautology and $p \wedge \neg p$ is a contradiction.

5. a) **Define with figure:**

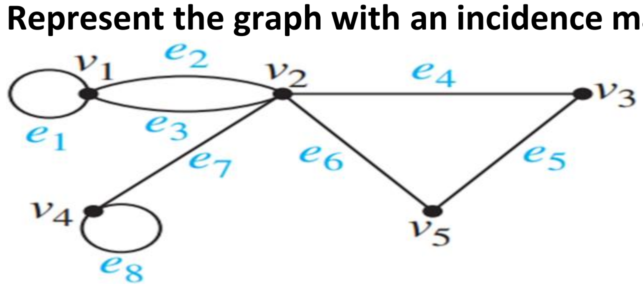
   b) **Graph**
   c) **Subgraph**
   d) **Bipartite graph**
   e) **Complete graph**
   f) **Directed graph**

Draw the graph *G* corresponding to the adjacency matrix:

$$A = \begin{bmatrix} 1 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix}$$

b) **What is isomorphism? Draw two graphs that are isomorphic and describe briefly.**

c) **Represent the graph with an incidence matrix.**

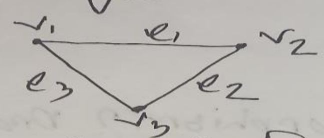


Answer-05

(a)

Ans:

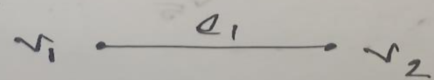
(i) Graph: A graph $G = (V, E)$ consists of V, a nonempty set of vertices (or nodes) and E, a set of edges.



~~Graph G~~ Figure: G

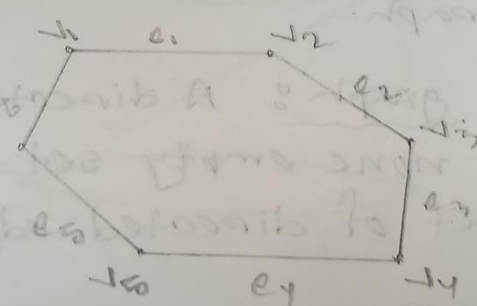
Here, G is a graph having v_1, v_2, v_3 vertices and e_1, e_2, e_3 edges.

(ii) Sub graph: A graph whose vertices and edges are subset of another graph, is called the subgraph of the first graph.

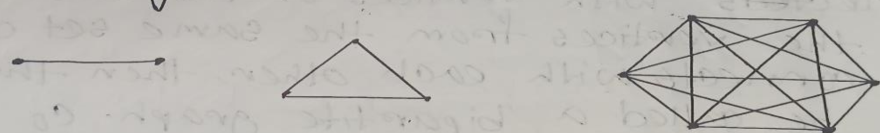


$v_1 v_2$ graph is ~~the~~ a subgraph of graph G.

(iii) Bipartite graph: If ~~a grap~~ the vertices of a graph can be divided into two sets and the ~~vertices~~ vertices from the first set ~~is~~ connecteds with vertices of the second set but the vertices from the same set does not connects with each other, then the graph can be called a bipartite graph. C_6 is a perfect ~~exap~~ example of a bipartite set.

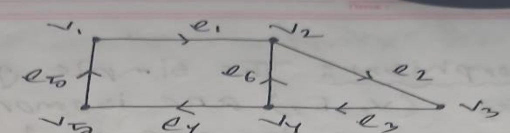


(iv) Complete graph: A complete graph is a graph ~~is a graph~~ in which every vertex has an edge to all other vertices is called a complete graph in other words, each pair of veratices is connected by an edge.



All graphs in ~~figur~~ the figure are complete graph.

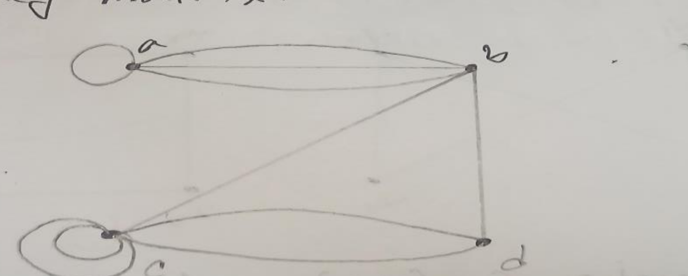
(v) Directed graph: A directed graph (V, E) consists of none empty set of vertices V and a set of directed edges E.



(b)

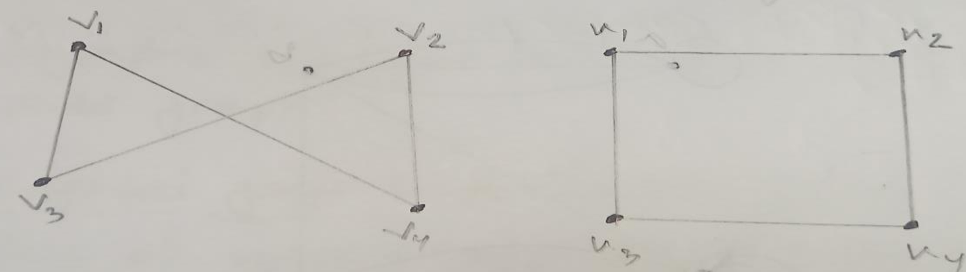
Ans. ~~The adjacency mattrix for~~

The corresponding graph for the given adjacency mattrix:



(c)

Isomorphism: The simple graph $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ are isomorphic if there exists a one-to-one and onto function f from V_1 to V_2 with the property that a and b are adjacent in G_1 if and only if $f(a)$ and $f(b)$ are adjacent in G_2, for all a and b in V_1. Such a function f is called isomorphism.



The graphs $G = (V, E)$ and $H = (W, F)$, displaye in the figure are isomorphic.

(d) The incidence matrix for the graph is:

$$
\begin{array}{c c}
 & \begin{array}{cccccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 & e_8 \end{array} \\
\begin{array}{c} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} &
\begin{bmatrix}
1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 \\
0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\
0 & 0 & 0 & 0 & 1 & 1 & 0 & 0
\end{bmatrix}
\end{array}
$$