1 PROOFS

1.1 Proposition 3.1

Proof. First, consider the trend component $t_i = \text{median}\left(\left[x_{i+j};\ \lfloor -\frac{m-1}{2} \rfloor \le j \le \lfloor \frac{m-1}{2} \rfloor\right]\right)$. We define $A_k = \left[x_{k+l};\ \lfloor -\frac{m-1}{2} \rfloor \le l \le \lfloor \frac{m-1}{2} \rfloor\right]$ as the trend estimation vector of the original series with error x_i, where x_k is the k-th element of the series. Similarly, we define $B_k = \left[x_{k+l};\ \lfloor -\frac{m-1}{2} \rfloor \le l \le \lfloor \frac{m-1}{2} \rfloor \wedge l \ne i, x_i^*\right]$ as the trend estimation vector of the clean series. Since $x_i^* \ge \max(x_1, \cdots, x_{i-1}, x_{i+1}, \cdots, x_n)$, x_i^* is the largest number in B_k. Further, since $x_i > x_i^*$, x_i is also the largest number in A_k. Considering that the median is the value separating the higher half from the lower half of the data, and the order of the elements in A_k and B_k is the same, the median median A_k and median B_k is also the same, i.e., the trend component on $[x_1, \cdots, x_i, \cdots, x_j, \cdots, x_n]$ with errors is exactly the same as that on the clean series $[x_1, \cdots, x_i^*, \cdots, x_j^*, \cdots, x_n]$.

Then consider the seasonal component $s_i = \text{median}\left(\left[d_j;\ 1 \le j \le n \wedge (j-i) \mid m\right]\right)$. We define $C_k = \left[d_l;\ 1 \le l \le n \wedge (l-k) \mid m\right]$ as the trend estimation vector of the original series with error x_i, and $B_k = \left[d_l;\ 1 \le j \le n \wedge (l-k) \mid m \wedge l \ne i, x_i^*\right]$ as the trend estimation vector of the clean series. Similarly to the trend component, since $x_i^* \ge \max(x_1, \cdots, x_{i-1}, x_{i+1}, \cdots, x_n)$, x_i^* is the largest number in D_k. Further, since $x_i > x_i^*$, x_i is also the largest number in C_k. Therefore, the order of the elements in C_k and D_k is the same. Then the median median C_k and median D_k is also the same, i.e., the seasonal component on $[x_1, \cdots, x_i, \cdots, x_j, \cdots, x_n]$ with errors is exactly the same as that on the clean series $[x_1, \cdots, x_i^*, \cdots, x_j^*, \cdots, x_n]$. Since the trend term and the seasonal term of the clean sequence and the error sequence are equal, considering that $r_i = x_i - t_i - s_i$, the residual component is also equal.

For the minimum value x_j, there is a similar conclusion. In summary, the error-tolerant decomposition by Formulas 5 and 6 on $[x_1, \cdots, x_i, \cdots, x_j, \cdots, x_n]$ with errors is exactly the same as that on the clean series $[x_1, \cdots, x_i^*, \cdots, x_j^*, \cdots, x_n]$.

□

1.2 Proposition 3.3

Proof. Let $W_k = \{x_l;\ ((l-k) \mid m) \wedge (l \ne k)\} \cup \{x_l;\ 1 \le l \le m\}$ for $1 \le k \le m$ in a original series, and $Y_k = W_k - \{x_k\} + \{x_k^*\}$ for $1 \le k \le m$ in a clean series. Since component extension $t_k = \text{median}(\{x_l; 1 - \lfloor -\frac{m-1}{2} \rfloor \le l \le k + \lfloor \frac{m-1}{2} \rfloor\} \cup \{t_{1+\lfloor \frac{m-1}{2} \rfloor} + s_l; m + k - \lfloor -\frac{m-1}{2} \rfloor \le l < m + 1 - \lfloor -\frac{m-1}{2} \rfloor\})$, t_k is obtained by calculating the median of the elements in W_k multiple times. Since $x_i^* \ge \max(W_k)$, x_i^* is the largest number in Y_k. Further, since $x_i > x_i^*$, x_i is also the largest number in W_k. Considering that the median is the value separating the higher half from the lower half of the data, and the order of the elements in W_k and Y_k is the same, the median in W_k and Y_k is also the same, i.e., the trend extension on $[x_1, \cdots, x_i, \cdots, x_j, \cdots, x_n]$ with errors is exactly the same as that on the clean series $[x_1, \cdots, x_i^*, \cdots, x_j^*, \cdots, x_n]$. For the minimum value x_j, there is a similar conclusion.

□

1.3 Proposition 3.6

Proof. If $x_i^* \ge \max(x_1, \cdots, x_{i-1}, x_{i+1}, \cdots, x_n)$ is the only error and $x_i > x_i^*$, by Proposition 3.1, x_i can not affect the result of error tolerant decomposition. Furthermore, the repair value of x_i, i.e., x_i' is calculated from x_k which satisfies the residual constraint, so x_i' also satisfies the residual constraint. Therefore, the series after repairing x_i (1) has the same result of error tolerant decomposition, and (2) satisfies the residual constraint. As the result, the seasonal repair x' generated by Formulas 9 and 10 is always the optimal repair. For the minimum value x_j, there is a similar conclusion. In summary, for any errors $x_i > x_i^*$ and $x_j < x_j^*$ occurring on these two values, the seasonal repair x' generated by Formulas 9 and 10 is always the optimal repair with cost $\Delta(x, x') = 2$. □

1.4 Proposition 3.9

Proof sketch of Proposition 3.9. If only one cycle violates the residual constraint, then the components of the remaining data points with the same phase in the other cycles all satisfy the residual constraint. The seasonal repair generation takes the median of these residual components that satisfy the residual constraint, adds this to the seasonal and trend components which have the least influence from errors. This intuitively produces a repair that satisfies the residual constraint.

□