STAT 205A (= MATH 218A): Probability Theory (Fall 2014)

NEWS: (8/27): Note corrected times for class and for office hours.

IMPORTANT. The best reference, and some of the homeworks, are from R. Durrett *Probability: Theory and Examples* **4th Edition**.

Instructor: David Aldous

Teaching Assistant (GSI): Wenpin Tang.

Class time: TuTh 2.00 - 3.30 in room 70 Evans

Optional Lab section: F 1.00-2.00 in 330 Evans

This is the first half of a year course in mathematical probability at the measure-theoretic level. It is designed for students whose ultimate research will involve rigorous proofs in mathematical probability. It is aimed at Ph.D. students in the Statistics and Mathematics Depts, but is also taken by Ph.D. students in Computer Science, Electrical Engineering, Business and Economics who expect their thesis work to involve probability.

In brief, the course will cover

- Sketch of pure measure theory (not responsible for proofs)
- Measure-theoretic formulation of probability theory
- Classical theory of sums of independent random variables: laws of large numbers
- Technical topics relating to proofs of above: notions of convergence, a.s. convergence techniques
- Conditional distributions, conditional expectation
- Discrete time martingales
- Introduction to Brownian motion

This roughly coincides with Chapters 1, 2, 5 and (first half of) 8 in Durrett's book.

Weekly schedule

Week	dates	topics	Billingsley	Durrett
1/2	Aug 28, Sep 2/4	Fields, sigma-fields, measurable functions, measures, Lebesgue measure, distribution functions, coin-tossing, abstract integration	2,10,13; 3,12,15	1.1, 1.2
3	Sep 9/11	Probability spaces, random variables, expectation, inequalities	4,5,20	1.3 - 1.6
4	Sep 16/18	Independence, WLLN, Bernstein's theorem, Borel-Cantelli lemmas, 4'th moment SLLN, Glivenko-Cantelli, gambling on favorable game.	6,20	2.1, 2.2
5	Sep 23/25	a.s. limit theorems for maxima, 2nd moment SLLN, modes of convergence, dominated convergence, maximal inequality, convergence of random series, 1st moment SLLN	6,21	2.3, 2.4
6	Sep 30/Oct 2	variant SLLNs, Fatou, Renewal SLLN. Stopping times, Wald's equation, Kolmogorov 0-1 law; Radon-Nikodym; Cantor measure, decomposition of measures on R.	22	2.4, 2.5

7	Oct 7	Large deviations	22, 9	4.1, 2.6
7/8	Oct 9/14/16	joint distributions correspond to marginals and a kernel. Product measure, Fubini's theorem and examples. Kolmogorov consistency theorem.	32, 33	5.1, A4
9	Oct 21/23	Conditional expectation. Definition and examples of martingales. Convexity.	18, 34, 36	1.7, 5.1
10	Oct 28/30	Doob decomposition, martingale transforms, stopping times, bounded version of Optional Sampling Theorem. Maximal and upcrossing inequalities.	35	5.2
11	Nov 4/6	MG convergence theorems, Levy 0-1 law, L_p convergence, conditional Borel-Cantelli, Kakutani's theorem, general form of optional sampling, MG analog of Wald.	35	5.3, 5.7
12/13	Nov 13/18/20	Boundary crossing examples. Patterns in coin-tossing. MG proof of Radon-Nikodym theorem. Azuma's inequality; examples. Reversed MGs and SLLN. Exchangeability and de Finetti's theorem. Kolmogorov consistency theorem.	35	5.3, 5.4, 5.5, 5.7
14/15	Nov 25, Dec 2/4	Brownian motion. Existence and path continuity. Invariance properties. Path non-differentiability. Associated martingales and their use in finding distributions, e.g. of hitting time for BM with drift. Reflection principle and formulas derived from it. Mention bridge, excursion, meander.	37	8.1 - 8.5
		Take-home final. 2.00pm Thursday 12/4 -- 2.00pm Monday 12/8		

Prerequisites

Ideally

- Upper division probability - familiarity with calculations using random variables.
- Upper division analysis, e.g. uniform convergence of functions, basics of complex numbers. Basic properties of metric spaces helpful.

If you haven't seen any measure theory it is helpful to read a little before the start of the course, for instance from the Billingsley or Leadbetter et al books below.

Books

R. Durrett *Probability: Theory and Examples (4th edition)* is the required text, and the single most relevant text for the whole year's course. The style is deliberately concise. Quite a few of the homework problems are from there,

P. Billingsley *Probability and Measure (3rd Edition)*. Chapters 1-30 contain a more careful and detailed treatment of some of the topics of this semester, in particular the measure-theory background. Recommended for students who have not done measure theory.

R. Leadbetter et al *A Basic Course in Measure and Probability: Theory for Applications* is a new book giving a careful treatment of the measure-theory background.

There are many other books at roughly the same ``first year graduate" level. Here are my personal comments on some.

D. Khoshnevisan *Probability* is a well-written concise account of the key topics in 205AB.

K.L. Chung *A Course in Probability Theory* covers many of the topics of 205A: more leisurely than Durrett and more focused than Billingsley.

D. Williams *Probability with Martingales* has a uniquely enthusiastic style; concise treatment emphasizes usefulness of martingales.

Y.S. Chow and H. Teicher *Probability Theory: Independence, Interchangeability, Martingales* . Uninspired exposition, but has useful variations on technical topics such as inequalities for sums and for martingales.

R.M. Dudley *Real Analysis and Probability*. Best account of the functional analysis and metric space background relevant for research in theoretical probability.

B. Fristedt and L. Gray *A Modern Approach to Probability Theory*. 700 pages allow coverage of broad range of topics in probability and stochastic processes.

L. Breiman *Probability*. Classical; concise and broad coverage.

O. Kallenberg *Foundations of Modern Probability*. Quoting an amazon.com reviewer: ``.... a compendium of all the relevant results of probability ..... similar in breadth and depth to Loeve's classical text of the mid 70's. It is not suited as a textbook, as it lacks the many examples that are needed to absorb the theory at a first pass. It works best as a reference book or a "second pass" textbook."

John B. Walsh *Knowing the Odds: An Introduction to Probability*. New in 2012. Looks very nice -- concise treatment with quite challenging exercises developing part of theory.

George Roussas An Introduction to Measure-Theoretic Probability. Recent treatment of classical content.

Santosh Venkatesh *The Theory of Probability: Explorations and Applications*. Unique new book, intertwining a broad range of undergraduate and graduate-level topics for an applied audience.

Jim Pitman has his very useful lecture notes linked to the Durrett text; these notes cover more ground than my course will! Also some lecture notes by Amir Dembo for the Stanford courses equivalent to our 205AB.

HOMEWORK

Here are the 11 weekly homework assignments, due in class on Tuesdays. You can pick up the graded homeworks at GSI office hours. Homework solutions will be posted here.

Final

There will be a take-home final exam .

Grading 60% homework, 40% final.

Office Hours

David Aldous (aldous@stat) Thursdays 11.00 - 1.00 in 351 Evans

Wenpin Tang (x09@berkeley.edu) Mondays 3.30 - 5.00 in 447 Evans. TBA

if you email us put "STAT 205A" in subject.