American Derivative Securities

4.1

(i) $V_0^P = \frac{116}{125}$ (ii) $V_0^C = \frac{64}{25}$ (iii) $V_0^S = \frac{412}{125}$

(iv) $At\ node\ V_1^S(T), if\ we\ strike, we\ gain\ 2; if\ not, we\ carry\ \frac{54}{25}.$
$So\ we\ choose\ not\ to\ strike; however, for\ the\ put\ option\ alone\ we\ should\ trike\ and\ for\ the\ call\ option\ alone\ we\ should\ not\ at\ this\ point. If\ we\ can\ choose\ to\ only\ strike\ the\ put\ part\ here\ then\ the\ equal\ result\ is\ true, but\ we\ cannot, which\ makes\ the\ straddle\ price\ smaller.$

4.2 $\Delta_0(S_1(T) - (1+r)S_0) + v_1(2) = (1+r)v_0 \therefore \Delta_0 = \frac{13}{30}$
$(\Delta_0 + \Delta_1)S_2 - \Delta_1(1+r)S_1(H) + v_2 = (1+r)^2(v_0 + \Delta_0 S_0)$
$\therefore \Delta_1 = -\dfrac{7}{20} \therefore At\ time\ 0, borrow\ another \dfrac{26}{15}\ to\ buy \dfrac{13}{30}\ share\ of\ stock. If\ at\ time\ 1\ it\ is\ tail, strike\ and\ receive\ 3, sell\ the\ stock\ and\ receive \dfrac{13}{15}. The\ debt\ is\ 1.25\left(\dfrac{26}{15} + 1.36\right) = \dfrac{58}{15}, which\ can\ be\ paid.$
$If\ at\ time\ 1\ it\ is\ head, do\ not\ strike\ and\ sell \dfrac{7}{20}\ share\ of\ stock,$
$receive \dfrac{14}{5}, which\ will\ be \dfrac{7}{2}\ at\ time\ 2. At\ time\ 2, if\ it\ is\ head, let\ the\ option\ expire, sell\ the\ stock\ and\ receive \dfrac{4}{3}; if\ it\ is\ tail, strike\ and\ get\ 1, sell\ the\ stock\ and\ receive \dfrac{1}{3}. The\ debt\ is\ 1.25^2\left(\dfrac{26}{15} + 1.36\right) = \dfrac{29}{6},$

$which\ can\ be\ paid.$

4.3 $v_3(32,60) = v_3(8,36) = v_3(8,24) = v_3(8,18) = v_3(2,18) = 0,$
$v_3(2,12) = 1, v_3(2,9) = \frac{4}{7}, v_3\left(\frac{1}{2}, \frac{15}{2}\right) = \frac{17}{8};$

$v_2(16,28) = v_2(4,16) = 0, v_2(4,10) = \frac{2}{3}, v_2(1,7) = \frac{5}{3};$
$v_1(8,12) = 0, v_1(2,6) = 1; v_0(4,4) = \frac{2}{5}$

4.4 $We\ should, if\ we\ do\ not\ charge\ more, the\ insider\ can$
$borrow\ 1.36\ and\ purchase\ the\ option\ that\ is\ worth\ 1.74,$
$which\ is\ an\ arbitrage.$

4.5 $\tau = 0; \tau = 1; \tau = 2; \tau = \infty;$
$\big(\tau(HH) = \tau(HT)\big) \neq \big(\tau(TH) = \tau(TT)\big);$
$\tau(HH) \neq \tau(HT) \neq \tau(TH) \neq \tau(TT)$
$\tau(HH) = \infty, \tau(HT) = 2, \tau(TH) = \tau(TT) = 1, V_0 = 1.36$

4.6

(i) $V_n = \max\big(K - S_n, E_n(V_{n+1})\big) = \max\left(K - S_n, \frac{K}{1+r} - S_n\right) = K - S_n$
$\therefore V_n > E_n(V_{n+1})\ for\ \forall n \in \{1, 2, \dots, N-1\}$
$\therefore the\ optimal\ strike\ time\ is\ time\ 0$

(ii) $If\ we\ does\ not\ strike\ before\ N:$
$K < S_N, V_N^{AP} = K - S_N + V_0^{EC} = 0;$

$If\ we\ strike\ at\ n\ before\ N$:

$K \geq S_n, V_N^{AP} = K - S_n < K - S_n + V_n^{EC} < (1+r)^n (K - S_0) + V_n^{EC}$;

$\therefore V_0^{AP} \leq K - S_0 + V_0^{EC}$

(iii) $\frac{K}{(1+K)^N} - S_0 + V_0^{EC} = V_0^{EP}$

$If\ we\ strike\ at\ time\ N, V_N^{EP} = V_N^{AP}; if\ strike\ at\ n\ before\ N$:

$$V_n^{AP} > E_n(V_{n+1}^{AP}) \geq E_n(V_N^{EP}) \therefore \frac{K}{(1+K)^N} - S_0 + V_0^{EC} = V_0^{EP} \leq V_0^{AP}$$

4.7 $V_n = \max\left(S_n - K, S_n - \frac{K}{1+r}\right) = E_n(V_{n+1})$

$\therefore we\ should\ not\ strike\ until\ N, then\ it\ is\ just\ a\ forward\ contract,$
$with\ value$ 0 $at\ time$ 1.