3D Computer Vision Final Project Report

Estimation of Single Camera Location

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Introduction:

In the real world, it is very important to know the 3D location of camera. For example, if we set a camera in the plane. If we knew the location of camera, then we would know the location of the plane, which is very important for successful landing. In my final project, I would introduce my own method for estimating camera location by taking a photo of special landmark in any direction and location.

Figure 1 a photo taken on the plane

My Method:

Since we want to know the location of camera, it is necessary that we have already known the size and shape of the special graph in the photo. The graph that I choose is a standard square lying on flat ground. I can use the information of three points to estimate 3D coordinates (see figure 2)

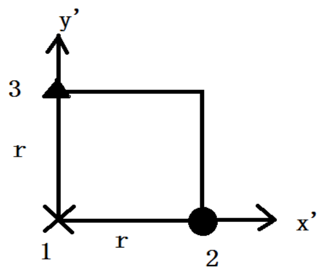


Figure 2 square

The camera should already be calibrated before we use it. During calibration, we also need to know the 3D location of camera (x_c, y_c, z_c) in calibrated coordinate system. After calibration, the next step is to take a photo of this square, so that we would know the 2D location of the three points in image. (see figure 3)

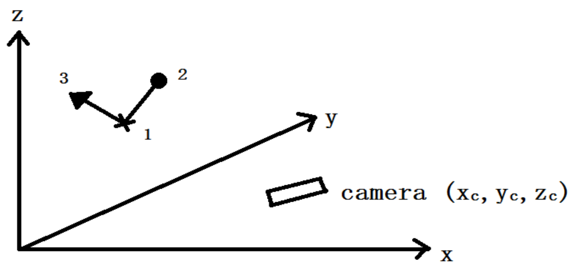


Figure 3 calibration coordinate system and square

To know the location of camera in the real world, the location of the three points in calibration coordinate system should be first calculated. We can use the calibration equations and the 2D location of three points in image to solve this problem.

Assume (u, v) is the location of one of points in image, then we have the following equations:

$$z*u=\mathbf{m_1}*\mathbf{P}$$
$$z*v=\mathbf{m_2}*\mathbf{P}$$
$$z=\mathbf{m_3}*\mathbf{P}$$

Where $\mathbf{P}$=[x y z 1]'

In these equations, x, y and z are unknowns. In this specific problem, we want to calculate nine unknowns- the coordinates of the three points(x_0, y_0, z_0), (x_1, y_1, z_1), (x_2, y_2, z_2). For each point, there are only two equations:

$$(\mathbf{m_3}*\mathbf{P})*u=\mathbf{m_1}*\mathbf{P}$$
$$(\mathbf{m_3}*\mathbf{P})*v=\mathbf{m_2}*\mathbf{P}$$

As a result there are totally six equations which make us unable to solve the equations. So there are three more equations that we need. We can use the geometry relationship of the three points to get three more equations. At first, the distance between point 2 and point 1 and the distance between point 3 and point 1 are all changeless, we can use r to represent the distance. Therefore, two more equations are provided. At last, the vector from point 1 to point 2 is perpendicular to vector from point 1 to point 3. This is another equation. So there are totally nine equations and thus we are able to calculate the unknowns.

Unknowns: x_1 y_1 z_1 x_2 y_2 z_2 x_3 y_3 z_3

$$(\mathbf{m}_3*[x_1\ y_1\ z_1\ 1]')*u_1=\mathbf{m}_1*[x_1\ y_1\ z_1\ 1]'$$
$$(\mathbf{m}_3*[x_1\ y_1\ z_1\ 1]')*v_1=\mathbf{m}_2*[x_1\ y_1\ z_1\ 1]'$$
$$(\mathbf{m}_3*[x_2\ y_2\ z_2\ 1]')*u_2=\mathbf{m}_1*[x_2\ y_2\ z_2\ 1]'$$
$$(\mathbf{m}_3*[x_2\ y_2\ z_2\ 1]')*v_2=\mathbf{m}_2*[x_2\ y_2\ z_2\ 1]'$$
$$(\mathbf{m}_3*[x_3\ y_3\ z_3\ 1]')*u_3=\mathbf{m}_1*[x_3\ y_3\ z_3\ 1]'$$
$$(\mathbf{m}_3*[x_3\ y_3\ z_3\ 1]')*v_3=\mathbf{m}_2*[x_3\ y_3\ z_3\ 1]'$$
$$(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2=r^2$$
$$(x_3-x_1)^2+(y_3-y_1)^2+(z_3-z_1)^2=r^2$$
$$(x_2-x_1)*(x_3-x_1)+(y_2-y_1)*(y_3-y_1)+(z_2-z_1)*(z_3-z_1)=0$$

After solving those equations, we can easily know the location of camera related to the three points. The coordinate system we used before are just for calibration, so we need to change it to new coordinate system whose original point is point 1, and point 2 is (r, 0,0), point 3 is (0, r, 0). The method to calculate camera location (x', y', z') is shown in figure 4

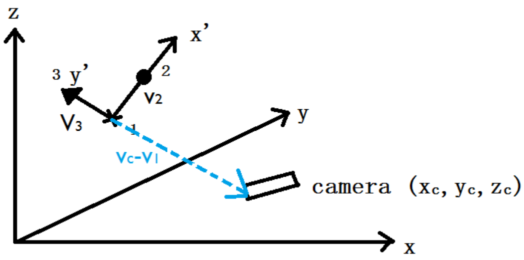


Figure 4 Calculate camera location

Set

vector v_2=point2-point1,

vector v_3=point3-point1

vector $v_c=(x_c,y_c,z_c)$

Then

coordinate x' of camera is $(v_c-v_1)\bullet v_2/r$

coordinate y' of camera is $(v_c-v_1)\bullet v_3/r$

coordinate z' of camera is $(v_c-v_1)\bullet(v_2\times v_3)/r^2$

This is the location of camera in the new coordinate system.

Experiments

In the experiment, set r=150mm, and some approaches are adopted to evaluate the results. We can move camera in three different directions and observe the change of estimated location.

First, the camera moves in the x-direction, the results are below.

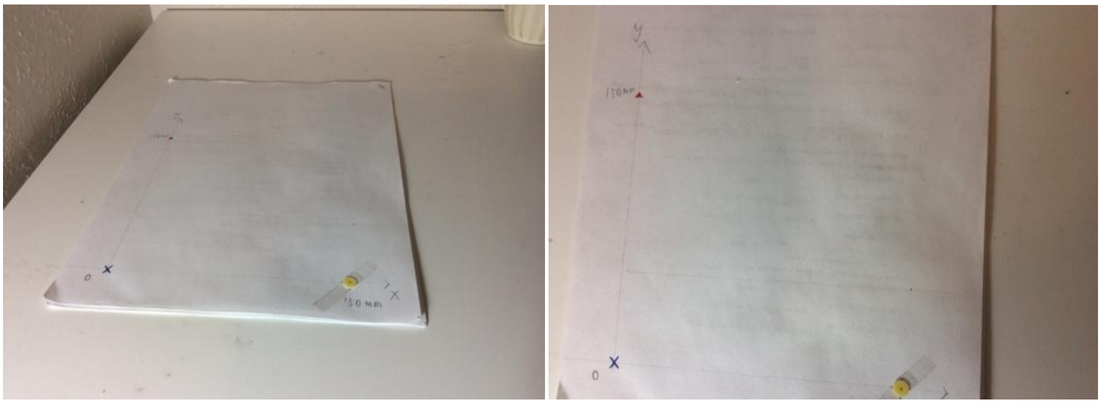
Estimation:

x'=-25.4428 y'=-209.8513 z'=407.3369 x'=304.7019 y'=-239.4790 z'=452.9168

Figure 5: move in x-direction (Unit: mm)

We can see the estimation of x increases a lot while there is a little change in y and z.

Also, the camera could move in y and z direction.



Estimation:

x'=153.2318   y'=-220.2165   z'=94.4576   x'=188.4209   y'=-69.4520   z'=108.9650

Figure 6: move in y-direction (Unit: mm)

Estimation:

x'=156.2066   y'=-150.8822   z'=367.8098   x'=113.0218   y'=-137.7923   z'=528.7847

Figure 7: move in z-direction (Unit: mm)

Another way to evaluate the result is that we compare the estimation results with ground truth.

Estimation:

x'=140.2158   y'=-183.9099   z'=280.8362   x'=134.4734   y'=-218.8294   z'=67.9697

Ground truth:

x'=160   y'=-160   z'=160   x'=130   y'=-220   z'=210

Figure 8 Comparison with ground truth (Unit: mm)

As we can see, the estimation of x and y is accurate, but there is a large difference in z. One of the

possible explanations is that the three points all locate at plane z=0.
The next step is to find out how to get accurate z. The first method I tried is to put another point above original point, so the location is x'=0, y'=0, z'=150mm. During estimation, we would use this points to estimate z.

Figure 9 use four points to estimate location

In the experiment, the ground truth is x'=220mm, y'=-200mm, z'=210mm. The original estimation is x'=209mm, y'=-208mm, z'=361mm

```
camera_real_location_x =

  209.6158


camera_real_location_y =

 -208.3718


camera_real_location_z =

  361.4674
```

The new estimation using four points are x'=211mm, y'=-206mm, z'=349mm

```
camera_real_location_x =

  211.3289


camera_real_location_y =

 -206.2113


camera_real_location_z =

  349.0536
```

So the results have not been solved by this solution. By doing more experiments, this phenomenon still exists. The next explanation that comes to my mind is that the position of camera is involved. When we estimate X-O-Y plane, the bottom of the camera is close to plane, when we estimate Y-O-Z plane, the left side of the camera is close to plane. So the new possible resolution is that the image should be rotated before estimating z.
First, we still use three points.

The estimation result is x'=209mm, y'=-208mm, z'=232mm if we rotate image clockwise.

```
camera_real_location_x =

  209.6158


camera_real_location_y =

 -208.3718


camera_real_location_z =

  232.7658
```

The results show that the z is much more accurate.

If we use four points and rotate image, the result is x'=211mm, y'=-208mm, z'=-193mm

```
camera_real_location_x =

  211.2583


camera_real_location_y =

 -208.3342


camera_real_location_z =

 -193.6577
```

By doing more experiments, if we use four points and rotate image, the result is bad.

In conclusion, the best option is **choosing three points and rotating image before calculating z and after calculating x and y**.

More experiments have been done to test the results.

Figure 10 Ground truth: x'=300mm, y'=-120mm, z'=210mm

Estimation: x'=309mm, y'=-128mm, z'=230mm

```
camera_real_location_x =

  308.7439


camera_real_location_y =

 -128.3039


camera_real_location_z =

  230.4083
```

Figure 11 Ground truth: x’=90mm, y’=-210mm, z’=250mm
Estimation: x’=71mm, y’=-198mm, z’=301mm

Figure 12 Ground truth: x’=550mm, y’=-100mm, z’=270mm
Estimation: x’=563mm, y’=-118mm, z’=303mm

Figure 13 Ground truth: x’=365mm, y’=65mm, z’=270mm
Estimation: x’=376mm, y’=46mm, z’=339mm

Figure 14 Ground truth: x’=280mm, y’=190mm, z’=200mm
Estimation: x’=250mm, y’=-177mm, z’=221mm

The results are all close to the ground truth, so this problem has been successfully solved.

Conclusion

In this project, we created a new topic- use single calibrated camera and take one photo to estimate its 3D location. The algorithm is to utilize calibration equations and geometry constraints. The result shows that the estimation is precise in x and y direction. Then we find another way to solve the problem in z by rotating the image that we took. We still only need three points and one photo to estimate. After this modification, the result is much better than before.