Digital Image Processing Homework 3

Bodong Zhang

5.

a) The Butterworth Bandreject filter has the following format

$H(u,v)=\frac{1}{1+[\frac{DW}{D^2-D_0^2}]^{2n}}$

Which eliminates the frequency near D_0, when D is close to D_0, $D^2-D_0^2$ would be small, so that $[\frac{DW}{D^2-D_0^2}]^{2n}$ would be large, as a result, the H would be small. Technically, when D=D_0, H=0.

The program is like below,

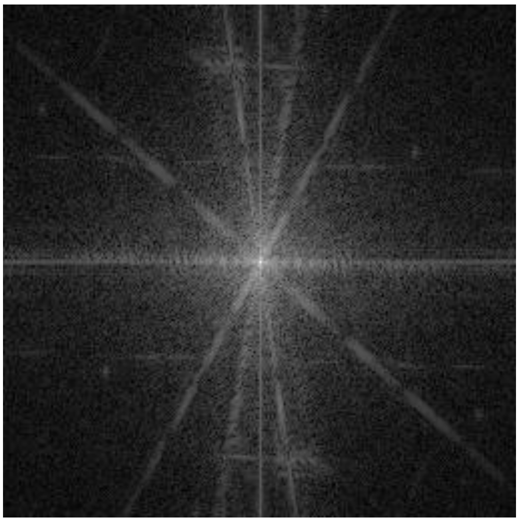
```
function Io = ButterworthBandReject (I,Do,W,n)
%This is bandreject filter, I is the image[0,1], Do is the center of
%frequency you want to reject, W is the width of filter, n is the order.Io
%is the output
```

Since we did it in the frequency domain, we first do the Fourier Transform and shift it to move frequency=0 to the center of the image. For each F(u,v), we multiply it with H(u,v), After filtering, G(u,v)=F(u,v)H(u,v). The phase would not change. Then we shift the spectrum back and do the inverse Fourier Transform.

b)First we do the Fourier Transform of the house image.

The input and spectrum image are

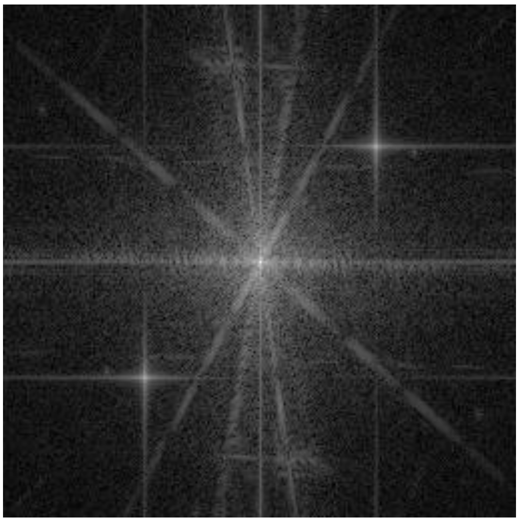
house image



spectrum

Then we do the Fourier Transform of the houseNoisy1 image.

The input and spectrum image are



houseNoisy1 image spectrum

As we can see, the houseNoisy1 image has regular periodic wave-like noise, the spectrum shows the accurate presentation of the noise. The difference between the two spectrums is that in spectrum of houseNoisy1 image, there exists a pair of high value peaks, which is the reason why the noise is like wave that has constant frequency.

c) For calculating MSE, we need to calculate $\sum_{i=1}^{i=m}\sum_{j=1}^{j=n}\left(f(i,j)-g(i,j)\right)^2/(mn)$,

After calculation, the MSE is 128.4

d) To get the minimum MSE, we need to eliminate noise frequency part as much as possible and preserve the image part as much as possible. The center of frequency to be rejected should be located at the center of the noise peak in spectrum.

The size of image is 256*256, so the point in spectrum that frequency=0 is at (129,129)

We found that in u=187, v=71, the spectrum has a peak. So we set

Do=$\sqrt{(187-129)^2+(71-129)^2}$=82.024

W means the width of the band reject area.

We try different values to calculate best W to get minimum MSE by applying hw3_5_d_find_W_new program.

Variables - save_W

output_frequency | I | I_noisy | I_fre | I_fre_shift | Ioo | save_W | MSE_way2 | I_fre_shiftI_fre_shift_filtered

2x31 double

	1	2	3	4	5	6	7	8	9	10	11	12
1	0	0.0100	0.0200	0.0300	0.0400	0.0500	0.0600	0.0700	0.0800	0.0900	0.1000	0.1100
2	81.3941	24.6272	13.5287	11.1859	10.8075	11.2092	12.0080	13.0395	14.2238	15.5190	16.8962	18.3298
3												

So the W should between 0.03 and 0.05.

We do it more accurately,

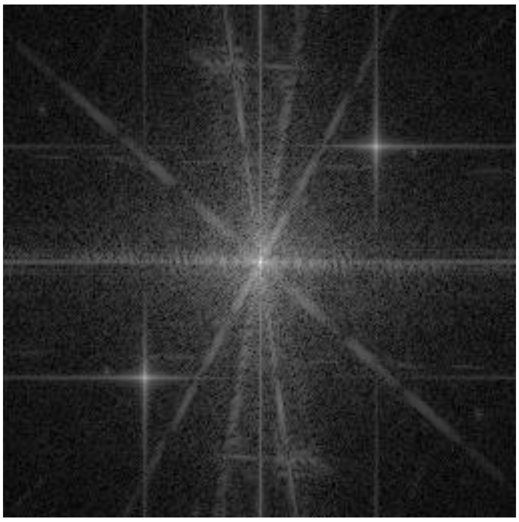
Variables - save_W

output_frequency | I | I_noisy | I_fre | I_fre_shift | Ioo | save_W | MSE_way2 | I_fre_shiftI_fre_shift_filtered

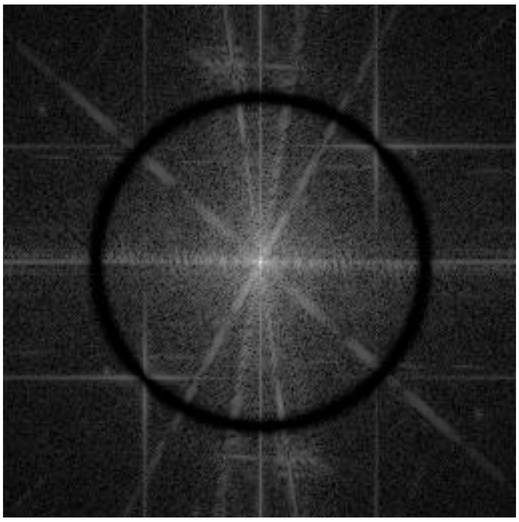
2x21 double

	5	6	7	8	9	10	11	12	13	14	15	16
1	0.0340	0.0350	0.0360	0.0370	0.0380	0.0390	0.0400	0.0410	0.0420	0.0430	0.0440	0.0450
2	10.8981	10.8586	10.8299	10.8111	10.8016	10.8006	10.8075	10.8218	10.8428	10.8702	10.9036	10.9426
3												
4												

So the W should be 0.039, the minimum MSE is 10.8006.



Frequency domain



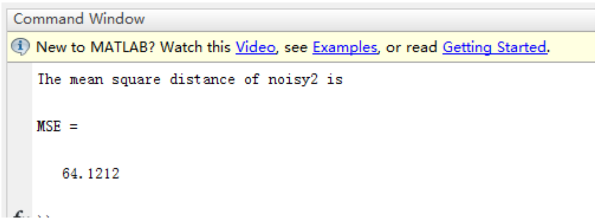
Best Do and W

Noisy1 image

Noisy1 after best butterworth

e) For calculating MSE, we need to calculate $\sum_{i=1}^{i=m}\sum_{j=1}^{j=n}\left(f(i,j)-g(i,j)\right)^2/(mn)$,

After calculation, the MSE is 64.1212

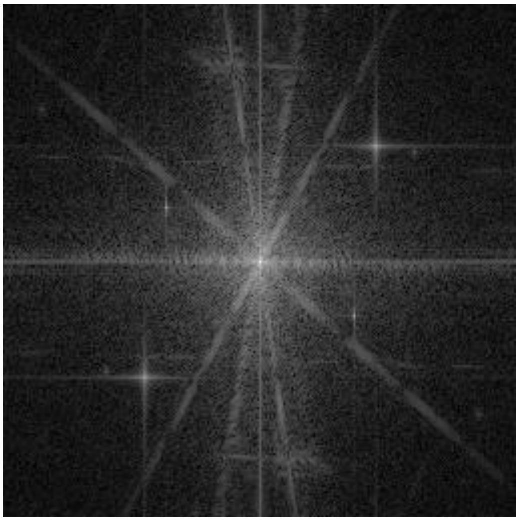


```
Command Window
New to MATLAB? Watch this Video, see Examples, or read Getting Started.
The mean square distance of noisy2 is

MSE =

   64.1212
```

The original Noisy2 image and its spectrum is

Original Noisy2 image   spectrum

As we can see, the spectrum shows that the Noisy2 image has two pairs of noise which have different frequency. One of them has large frequency, and also exists in the Noisy1 image. The second one has low frequency.

f) We can first use Butterworth filter to eliminate noise that has higher frequency, then we use the filtered image as input image and do the Butterworth filter again to eliminate another kind of noise. The two Butterworth filters have different Do and may have different W. First we choose Do and W for the first Butterworth filter to minimize MSE, after that, we choose another pair of Do and W to continue to minimize MSE.

① Apply hw3_5_f_step1_new to eliminate noise with higher frequency

First, the Do is still Do=$\sqrt{(187-129)^2+(71-129)^2}$=82.024.

Then we try different W to get different MSE

Variables - save_W

+5 | output_frequency | I | I_noisy | I_fre | I_fre_shift | Ioo | save_W

2x30 double

	1	2	3	4	5	6	7	8	9
1	0.0100	0.0200	0.0300	0.0400	0.0500	0.0600	0.0700	0.0800	0.090
2	39.2608	37.5151	37.9572	38.9204	40.0931	41.3589	42.6556	43.9439	45.194
3									

So W should between 0.01 and 0.03.

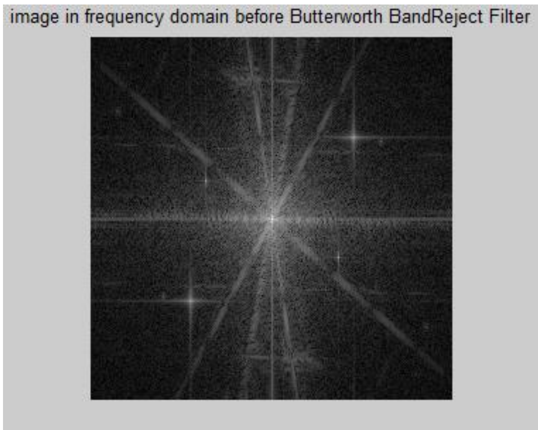
Variables - save_W

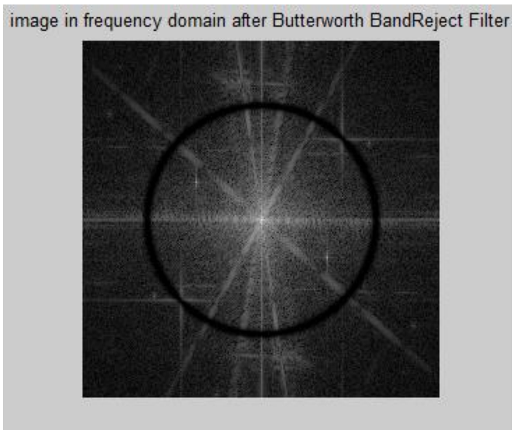
+5 | output_frequency | I | I_noisy | I_fre | I_fre_shift | Ioo | save_W

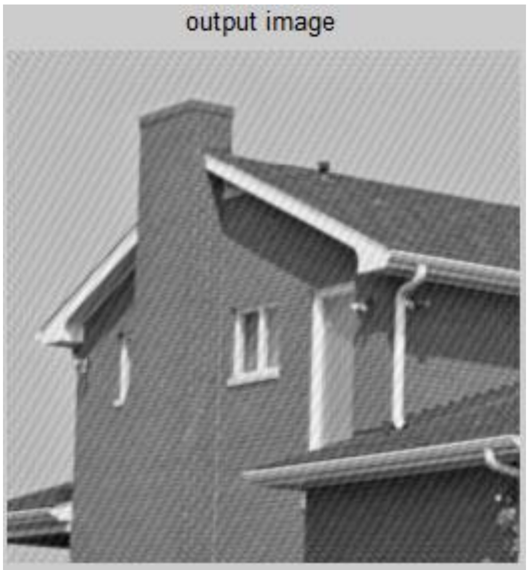
2x21 double

	7	8	9	10	11	12	13	14	15
1	0.0160	0.0170	0.0180	0.0190	0.0200	0.0210	0.0220	0.0230	0.0240
2	37.7149	37.6283	37.5691	37.5325	37.5151	37.5140	37.5268	37.5516	37.5868
3									
4									
5									

So W should be 0.021.





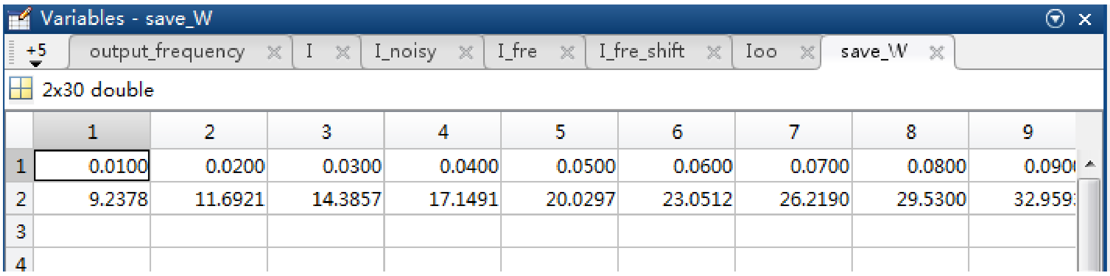


As shown above, one of the wave has disappeared.

② Apply hw3_5_f_step2_new to eliminate noise with lower frequency

First, the Do is Do=$\sqrt{(102-129)^2+(82-129)^2}$=54.2033.

Then we try different W to get different MSE



	1	2	3	4	5	6	7	8	9
1	0.0100	0.0200	0.0300	0.0400	0.0500	0.0600	0.0700	0.0800	0.090(
2	9.2378	11.6921	14.3857	17.1491	20.0297	23.0512	26.2190	29.5300	32.959:
3									
4									

So W should between 0 and 0.02.

Variables - save_W

2x21 double

	2	3	4	5	6	7	8	9	10
1	00e-03	0.0020	0.0030	0.0040	0.0050	0.0060	0.0070	0.0080	0.0090
2	9.1209	9.3165	9.0295	8.6436	8.5113	8.5485	8.6652	8.8287	9.0234
3									

Variables - save_W

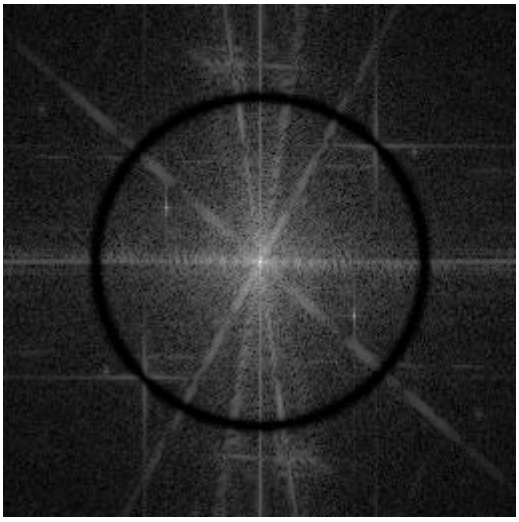
2x21 double

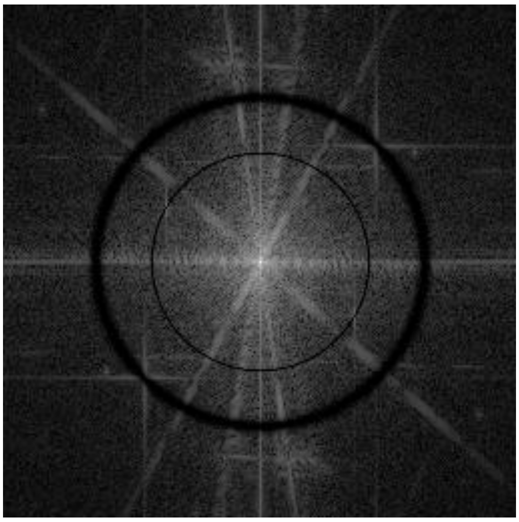
	9	10	11	12	13	14	15	16	17
1	0.0048	0.0049	0.0050	0.0051	0.0052	0.0053	0.0054	0.0055	0.0056
2	8.5199	8.5147	8.5113	8.5096	8.5093	8.5104	8.5128	8.5163	8.5209
3									
4									
5									

So W is 0.0052.

Input image(filtered once)

Output image(filtered twice)





Spectrum of input image Spectrum of output image

g) In Butterworth band reject filter, all the areas with same frequency(same distance to frequency=0) are treated equally. This results in that some points in spectrum that have information of image are eliminated too. So the best solution is to just eliminate the some circle area around the noise point in spectrum. So we can use Notch reject filters to realize this.

We can set point (u_0,v_0) in spectrum which represent the point of noise, the closer to this point in spectrum, the less response of the filter.

The Spectrum response of filter could be like H(u,v)=$\frac{1}{1+(\frac{W^2}{(u-u_0)^2+(v-v_0)^2})^{2n}}$,

where W is the width, n is the order.

If there are many noise points, we can change value of (u_0,v_0), and do the notch reject filtering many times.