Spatial Autocorrelation

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spatial randomness

positive and negative spatial autocorrelation

spatial autocorrelation statistics

spatial weights

Spatial Randomness

- The Null Hypothesis

  spatial randomness is absence of any pattern

  spatial randomness is not very interesting

  if rejected, then there is evidence of spatial structure

- Interpreting Spatial Randomness

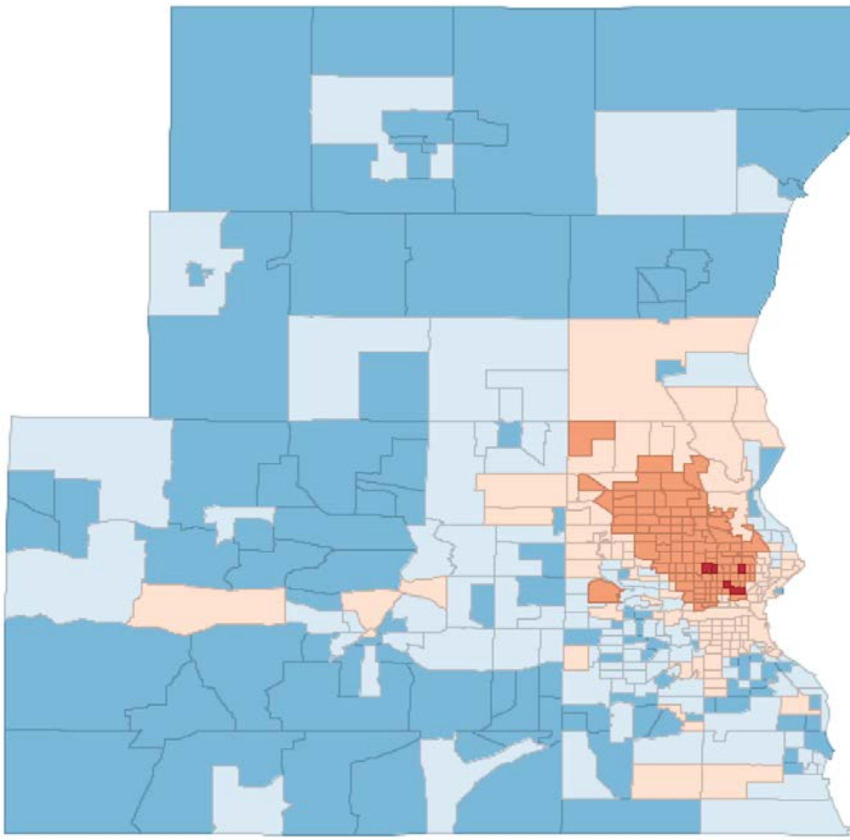
  observed spatial pattern of values is equally likely as any other spatial pattern

  value at one location does not depend on values at other (neighboring) locations

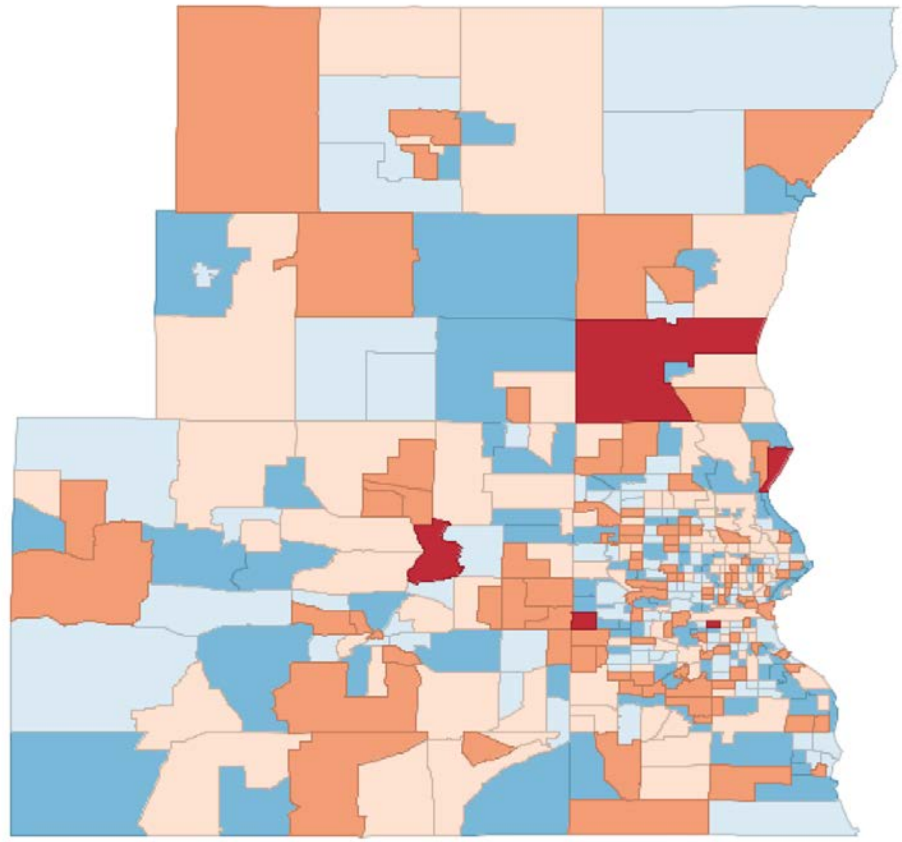
- Operationalizing Spatial Randomness

  under spatial randomness, the location of values may be altered without affecting the information content of the data

  random permutation or reshuffling of values



true map



randomly reshuffled

- Tobler's First Law of Geography

  everything depends on everything else, but closer things more so

  structures spatial dependence

  importance of distance decay

Positive and Negative Spatial Autocorrelation

- Rejecting the Null Hypothesis

rejecting spatial randomness (s.r.)

like values in neighboring locations occur more frequently than for s.r.
= positive spatial autocorrelation

dissimilar (e.g., high vs low) in neighboring locations occur more frequently than for s.r.
= negative spatial autocorrelation

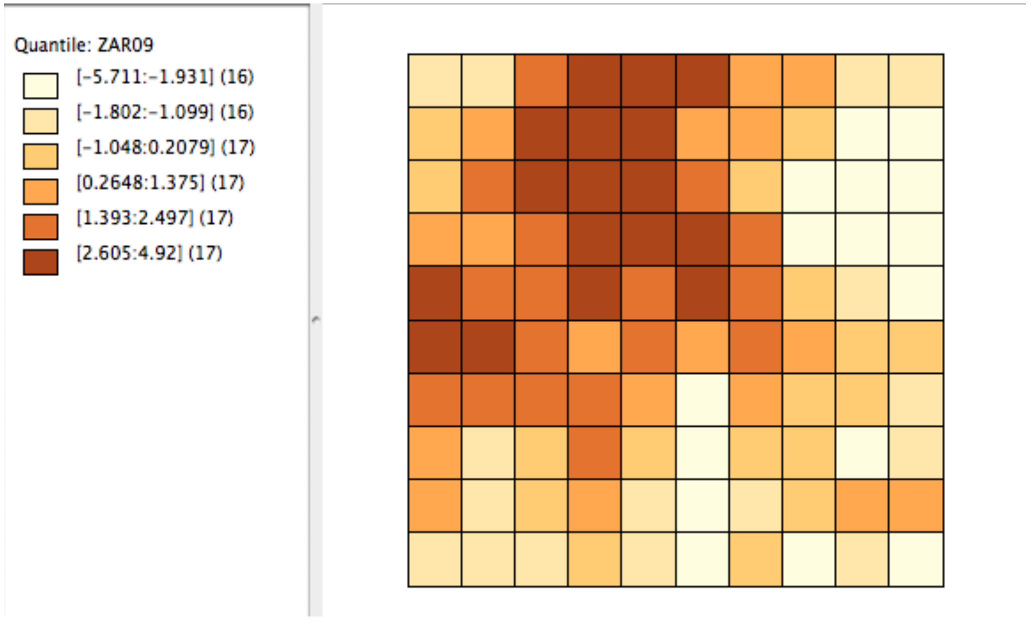
- Positive Spatial Autocorrelation

  impression of clustering

  clumps of like values

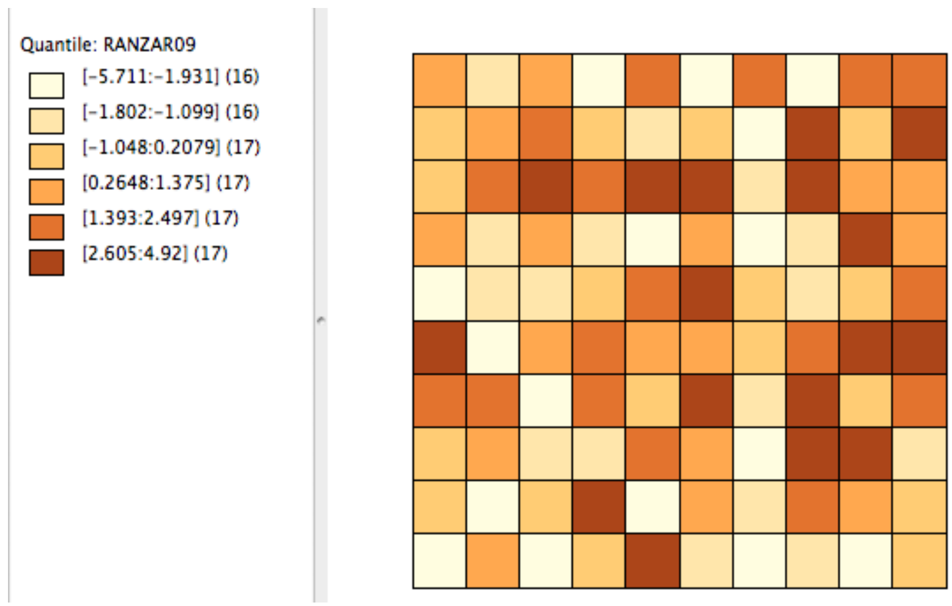
  like values can be either high (hot spots) or low (cold spots)

  difficult to rely on human perception



< positive s.a.

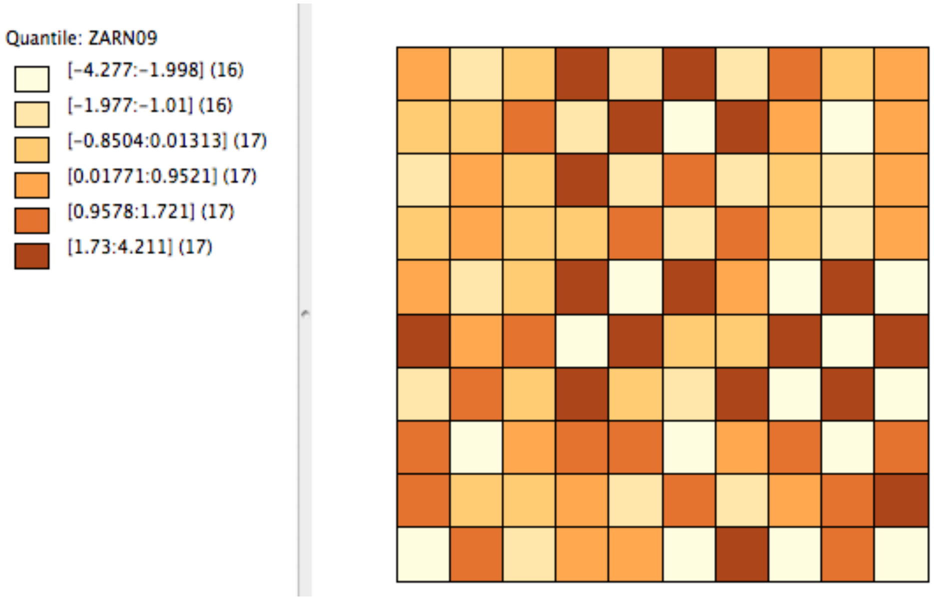
random >



- Negative Spatial Autocorrelation

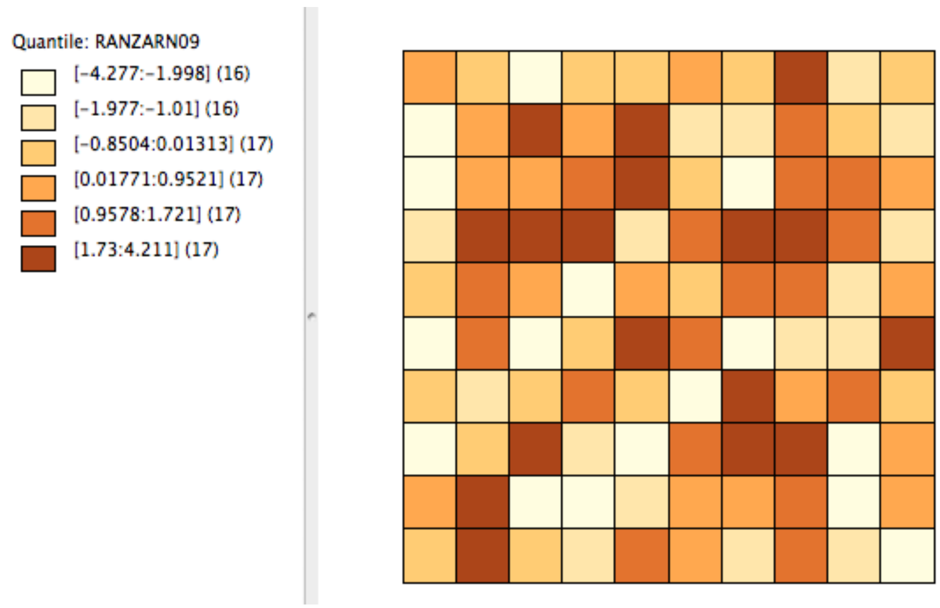
  checkerboard pattern

  hard to distinguish from spatial randomness



< negative s.a.

random >



Spatial Autocorrelation Statistics

- What is a Test Statistic?

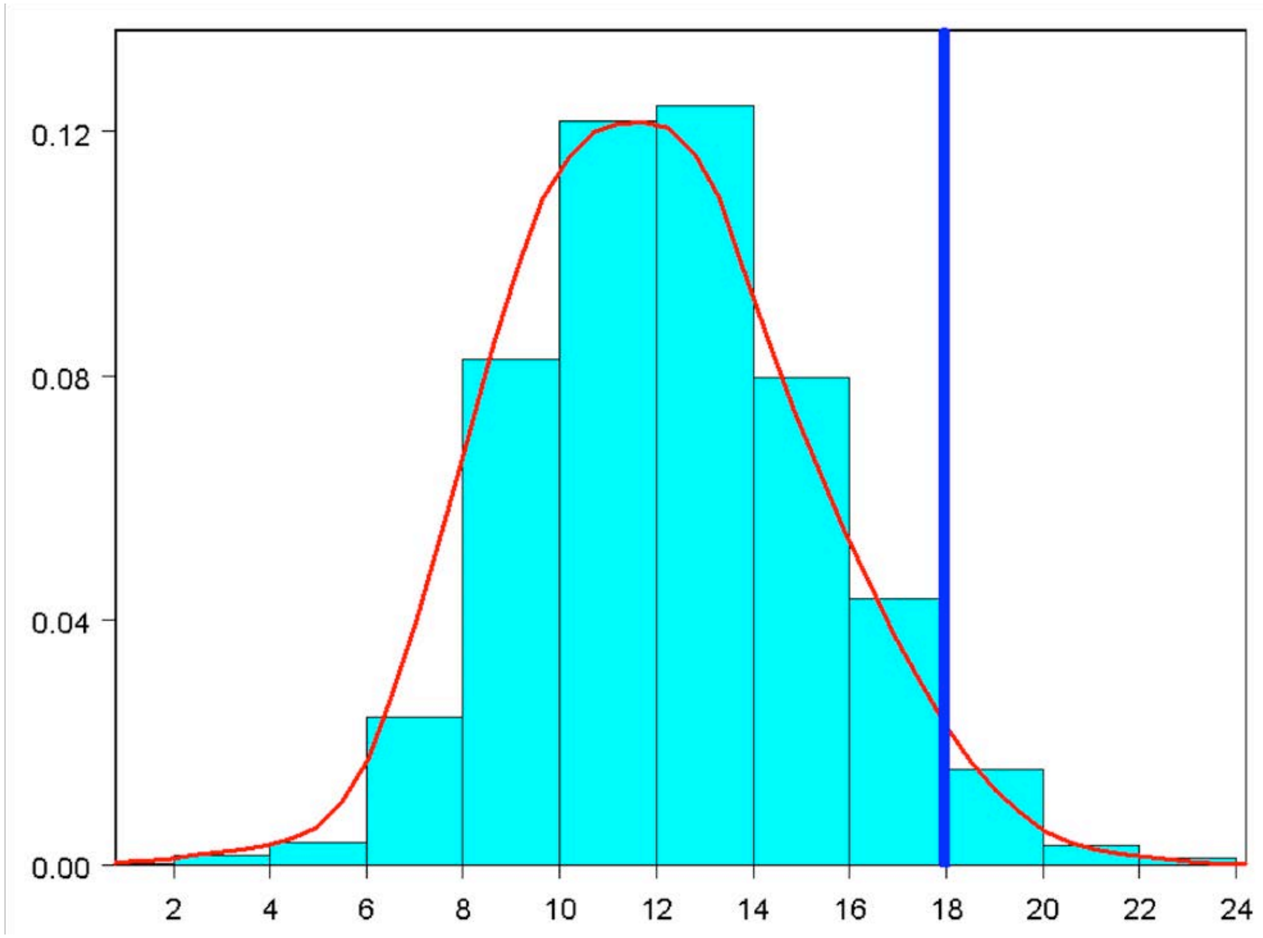
  a statistic is any value that summarizes characteristics of a distribution

  calculated from the data

  test statistic: calculated from the data and compared to a reference distribution

  how likely is the value if it had occurred under the null hypothesis (spatial randomness)

  when unlikely (low p value) the null hypothesis is rejected



compare value of test statistic
to its distribution under the null hypothesis
of spatial randomness

- Spatial Autocorrelation Statistic

captures both attribute similarity and locational similarity

how to construct an index from the data that captures both attribute similarity and locational similarity (i.e., neighbors are alike)

- Attribute Similarity

  summary of the similarity (or dissimilarity) of observations for a variable at different locations

  variable y

  locations i, j

  how to construct $f(y_i, y_j)$

- Similarity Measure
  - cross product: $y_i.y_j$

    under randomness, cross product is not systematically large or small

    when large values are systematically together, product will be larger, and vice versa

- Dissimilarity Measure
  - squared difference: $(y_i - y_j)^2$
  - absolute difference: $|y_i - y_j|$

  under randomness, difference measure will not be systematically large or small

  when small values or large values are systematically together, difference measures will be smaller

- Locational Similarity

formalizing the notion of neighbors
= spatial weights (w_{ij})

when are two spatial units i and j a priori likely to interact

not necessarily a geographical notion, can be based on social network concepts or general distance concepts (distance in multivariate space)

- General Spatial Autocorrelation Statistic

general form

sum over all observations of an attribute similarity measure with the neighbors

$f(x_i x_j)$ is attribute similarity between i and j for x

w_{ij} is a spatial weight between i and j

statistic = $\Sigma_{ij}\ f(x_i x_j).w_{ij}$

Spatial Weights

Basic Concepts

Why Spatial Weights

formal expression of locational similarity

spatial autocorrelation is about interaction

n x (n - 1)/2 pairwise interactions but only n observations in a cross-section

insufficient information to extract pattern of interaction from cross-section

example: North Carolina has 100 counties
5,000 pairwise interactions, 100 observations

Solution

- impose structure

  limit the number of parameters to be estimated

  incidental parameter problem = number of parameters grows with sample size

  for spatial interaction, number of parameters grows with n^2

- Spatial Weights

    exclude some interactions

    constrain the number of neighbors, e.g., only those locations that share a border

    single parameter = spatial autocorrelation coefficient

- strength of interaction = combined effect of coefficient and weights

    small coefficient with large weights

    large coefficient with small weights

GeoDa

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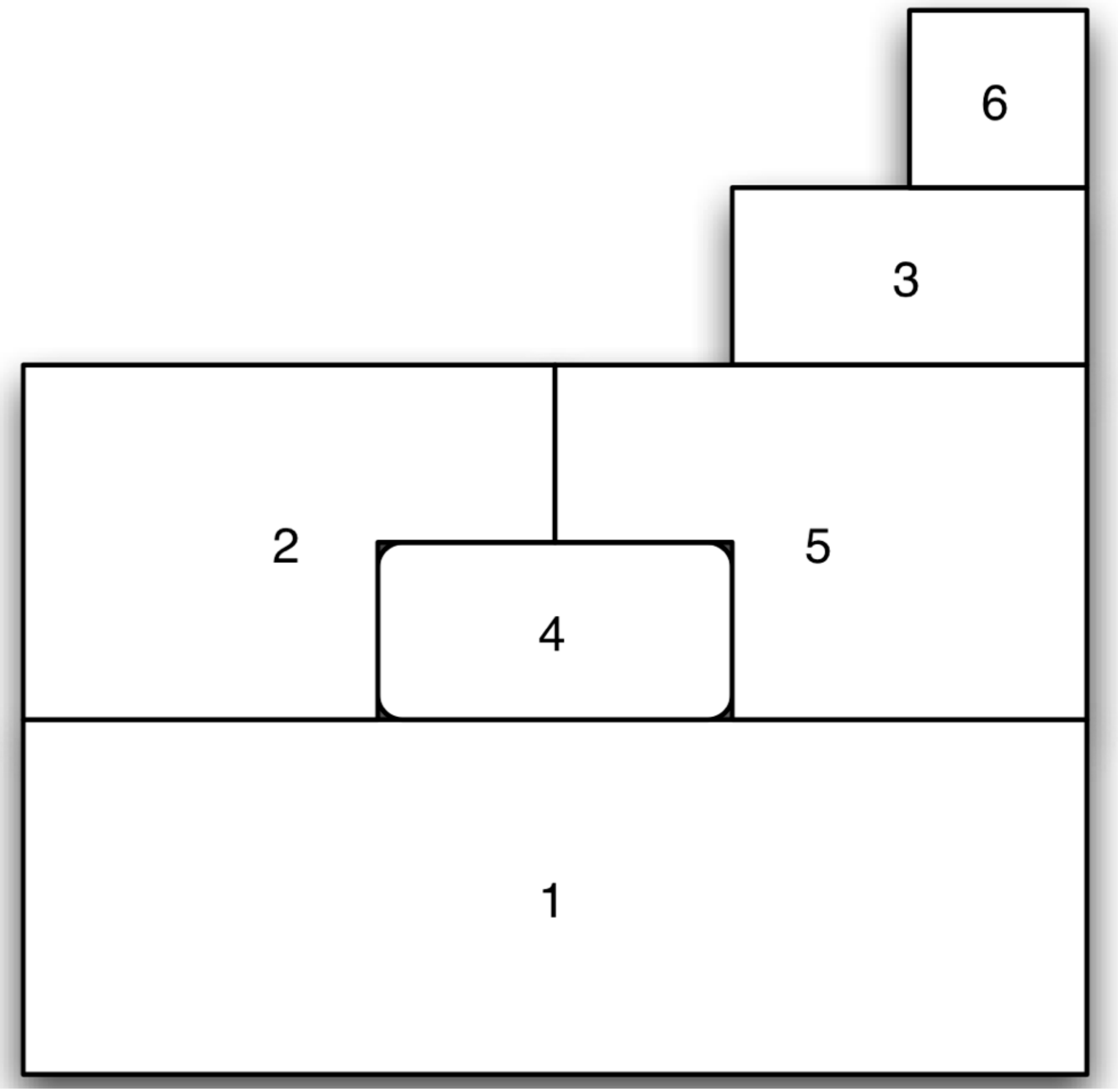
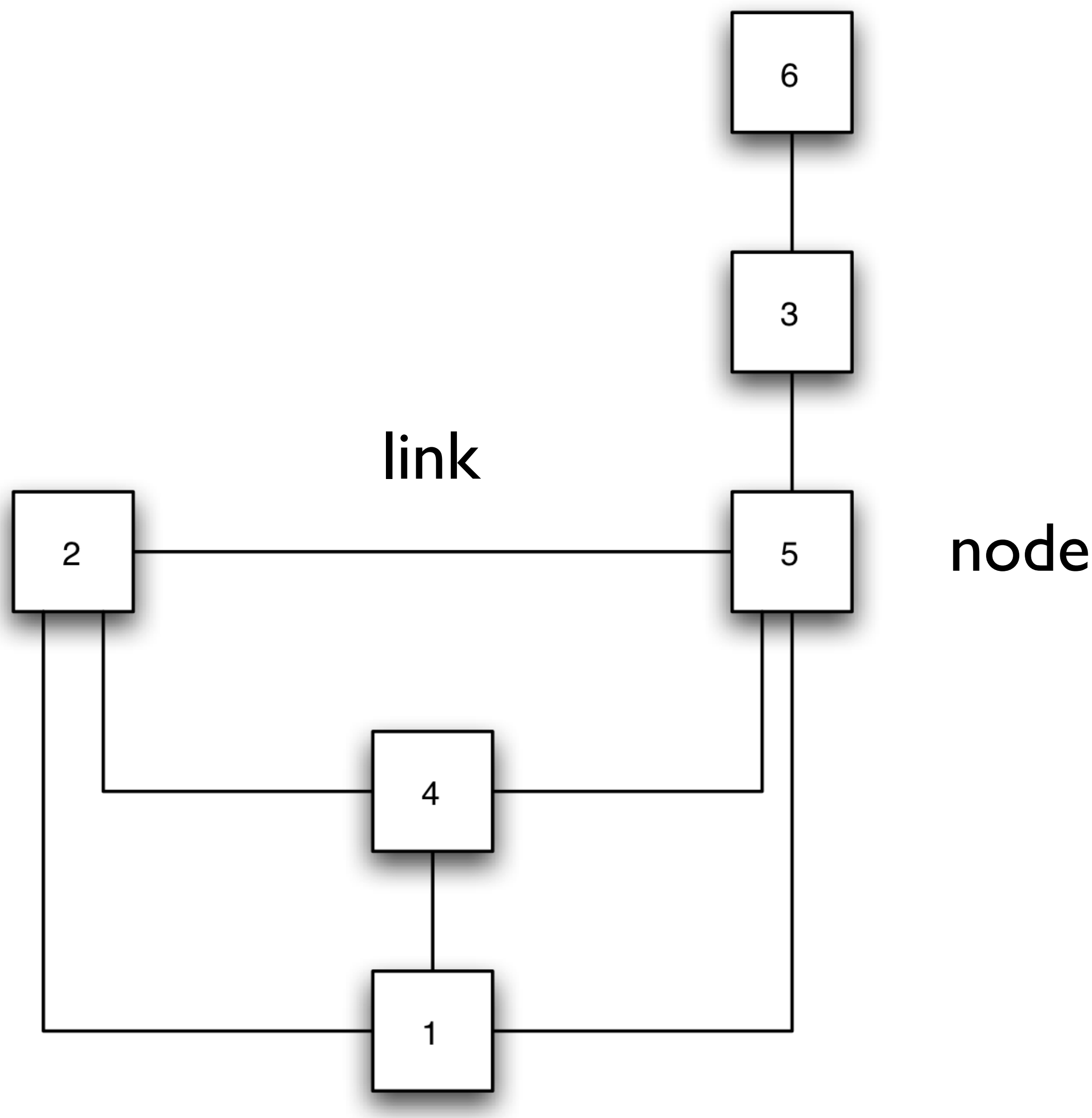


six polygons - neighbors share common border



neighbor structure as a graph

- Spatial Weights Matrix Definition

  N by N positive matrix W with elements wij

  w_{ij} non-zero for neighbors

  $w_{ij} = 0$, i and j are not neighbors

  $w_{ii} = 0$, no self-similarity

$$\mathbf{W} = \begin{bmatrix} w_{11} & w_{12} & \dots & w_{1n} \\ w_{21} & w_{22} & \dots & w_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ w_{n1} & w_{n2} & \dots & w_{nn} \end{bmatrix}$$

spatial weights matrix W with elements w_{ij}

Geography-Based Spatial Weights

- Binary Contiguity Weights

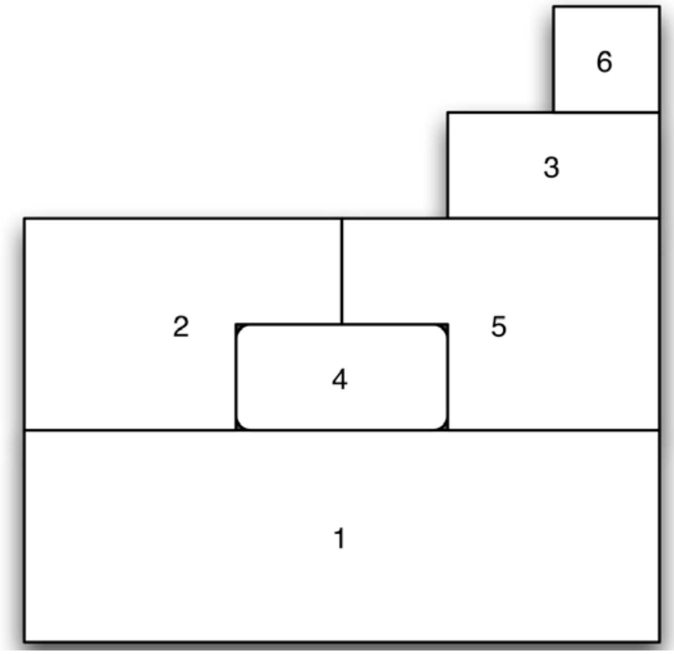
  contiguity = common border

  i and j share a border, then $w_{ij} = 1$

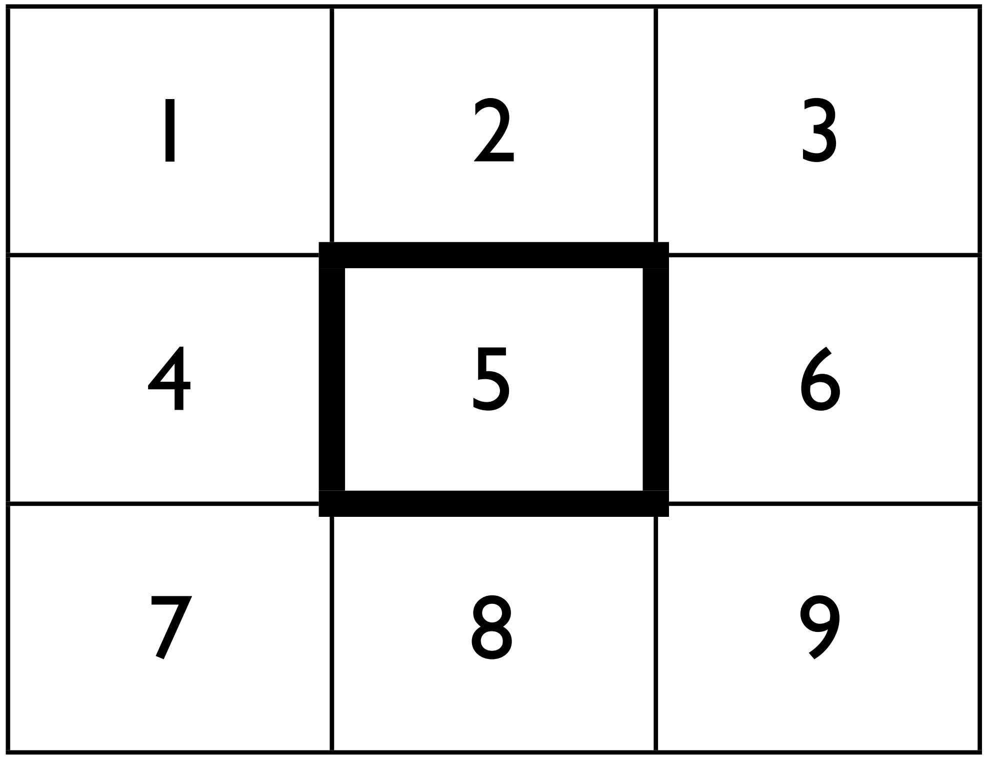
  i and j are not neighbors, then $w_{ij} = 0$

  weights are 0 or 1, hence binary

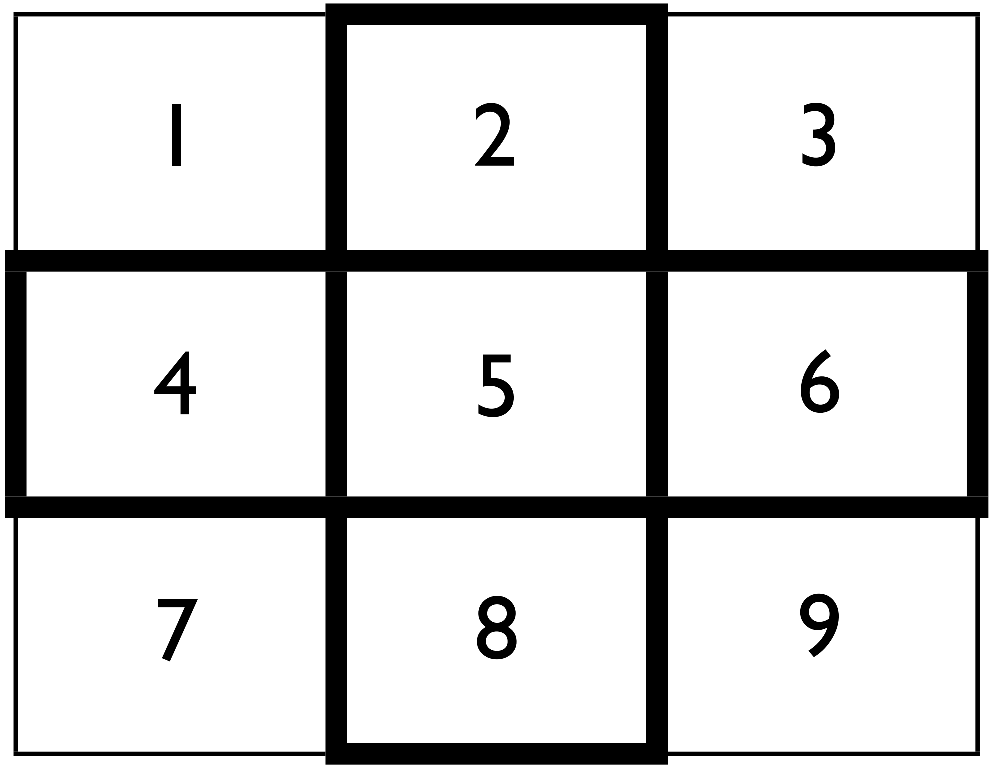
$$\mathbf{W} = \begin{bmatrix} 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}.$$



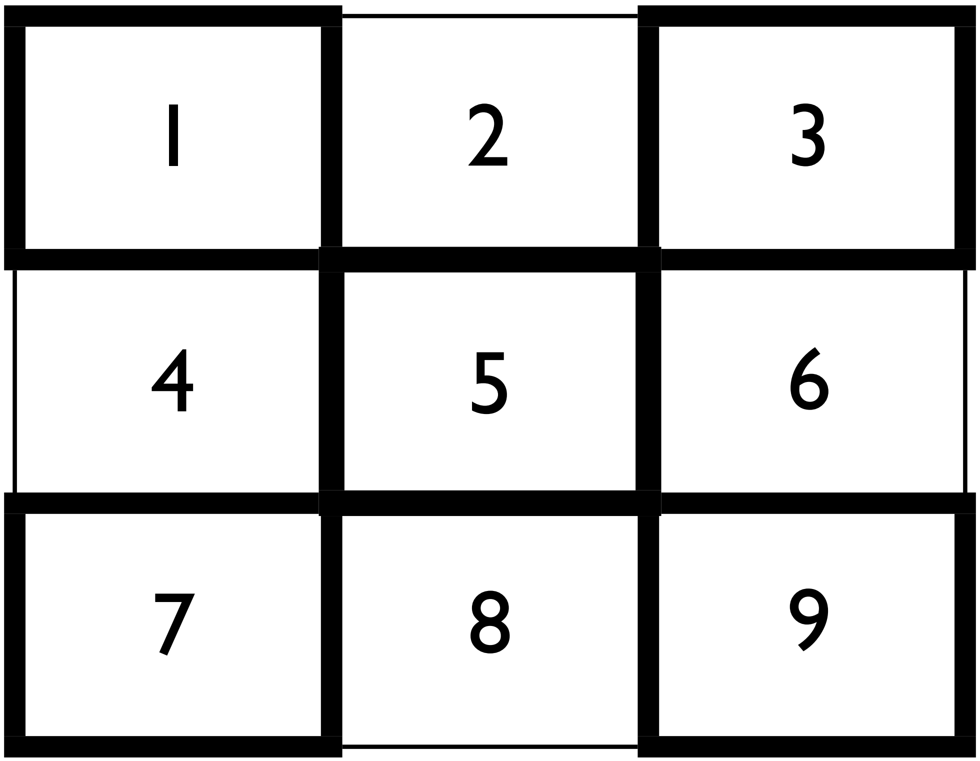
binary contiguity weights matrix for six-region example



contiguity on a regular grid - different definitions



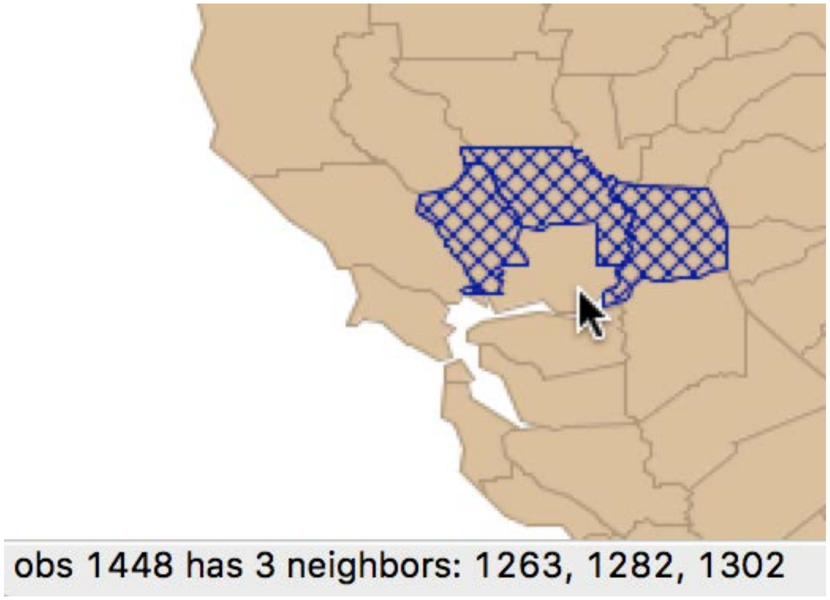
rook contiguity - edges only
2, 4, 6, 8 are neighbors of 5



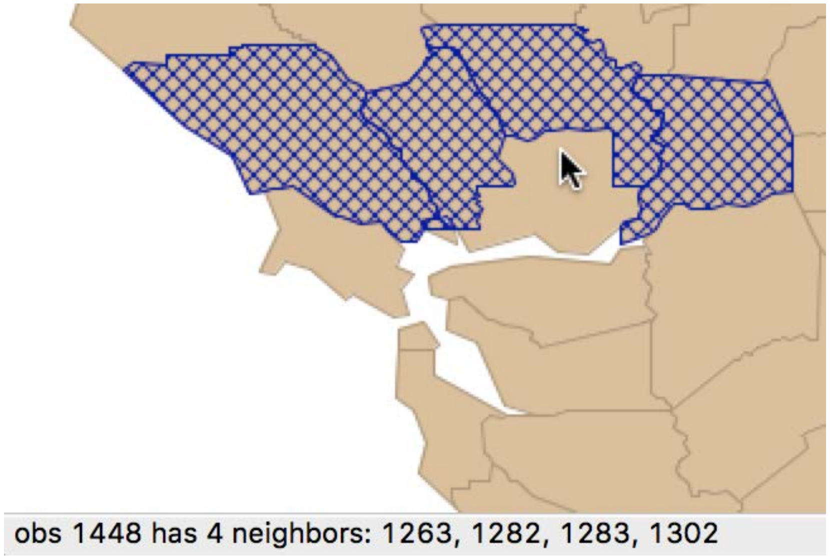
bishop contiguity - corners only
1, 3, 7, 9 are neighbors of 5

1	2	3
4	5	6
7	8	9

queen contiguity - edges and corners
5 has eight neighbors



rook



queen

Solano county, CA contiguity

- Distance-Based Weights

  distance between points

  distance between polygon centroids or central points

  in general, can be any function of distance that implies distance decay, e.g., inverse distance

  in practice, mostly based on a notion of contiguity defined by distance

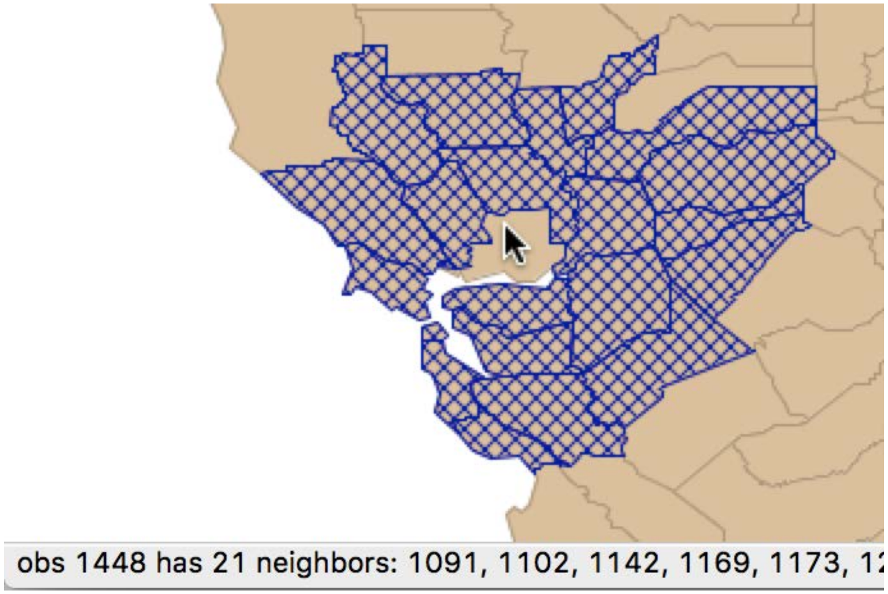
- Distance-Band Weights

  w_{ij} nonzero for $d_{ij} < d$
  less than a critical distance d

  potential problem: isolates = no neighbors

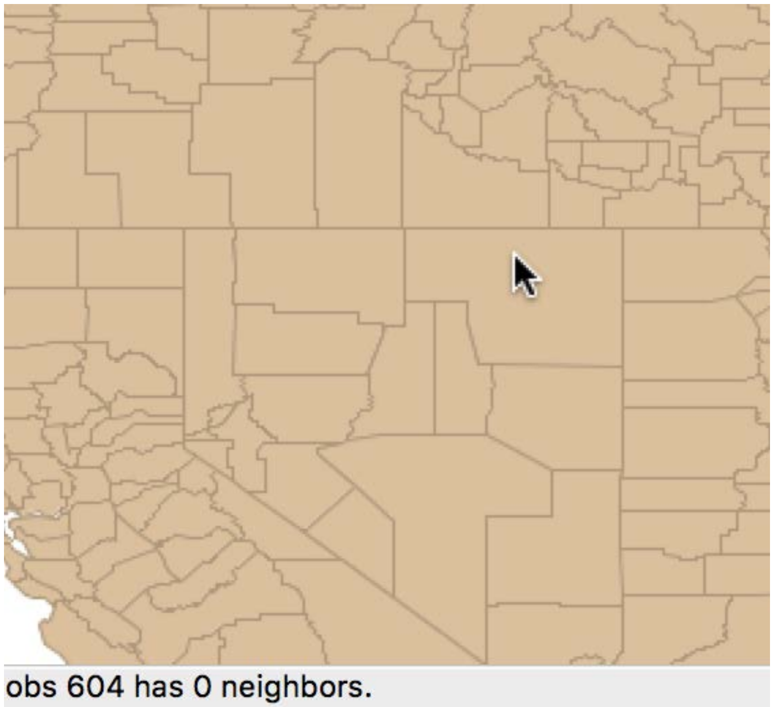
    make sure critical distance is max-min, i.e., the largest of the nearest neighbor distance for each observation

distance-band weights

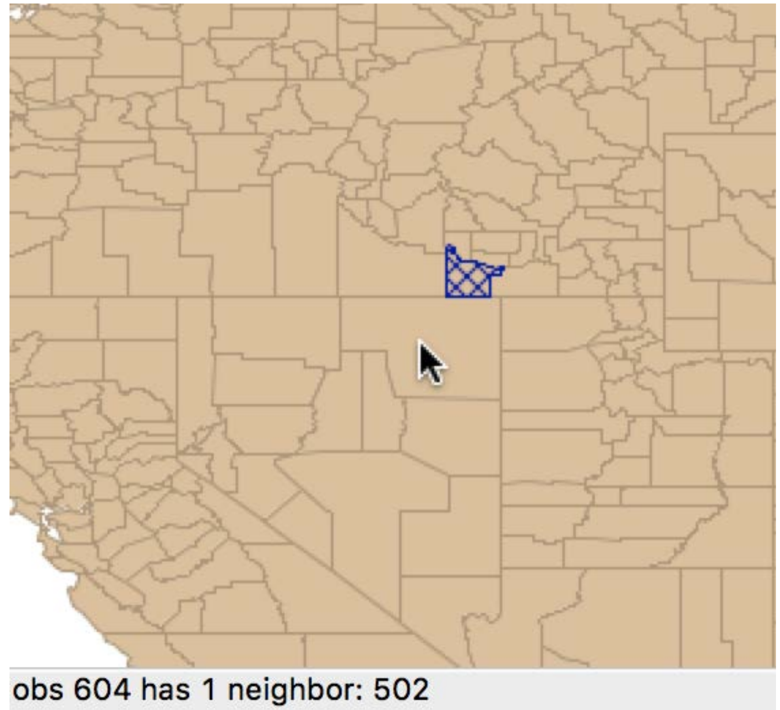


Solano county, CA, distance-band neighbors
d = 90 mi

distance-band weights



d = 80 mi
no neighbors

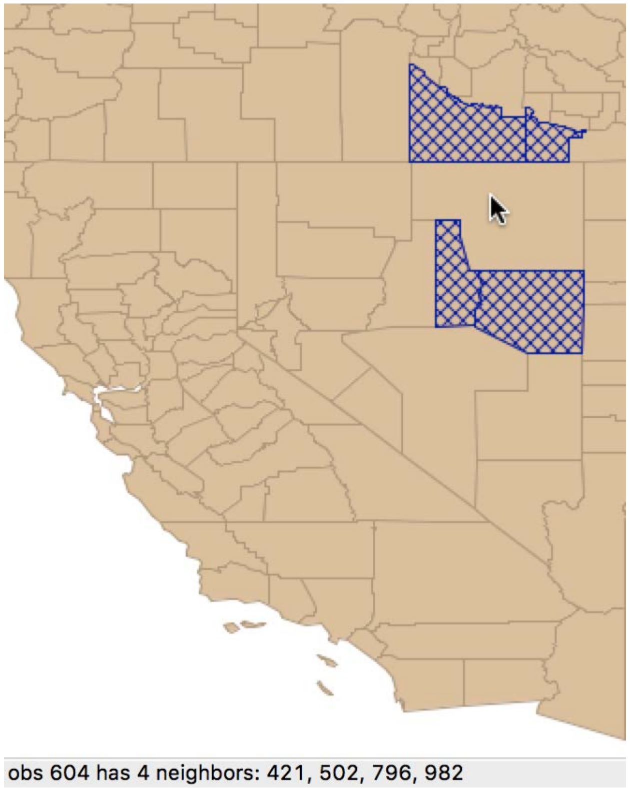


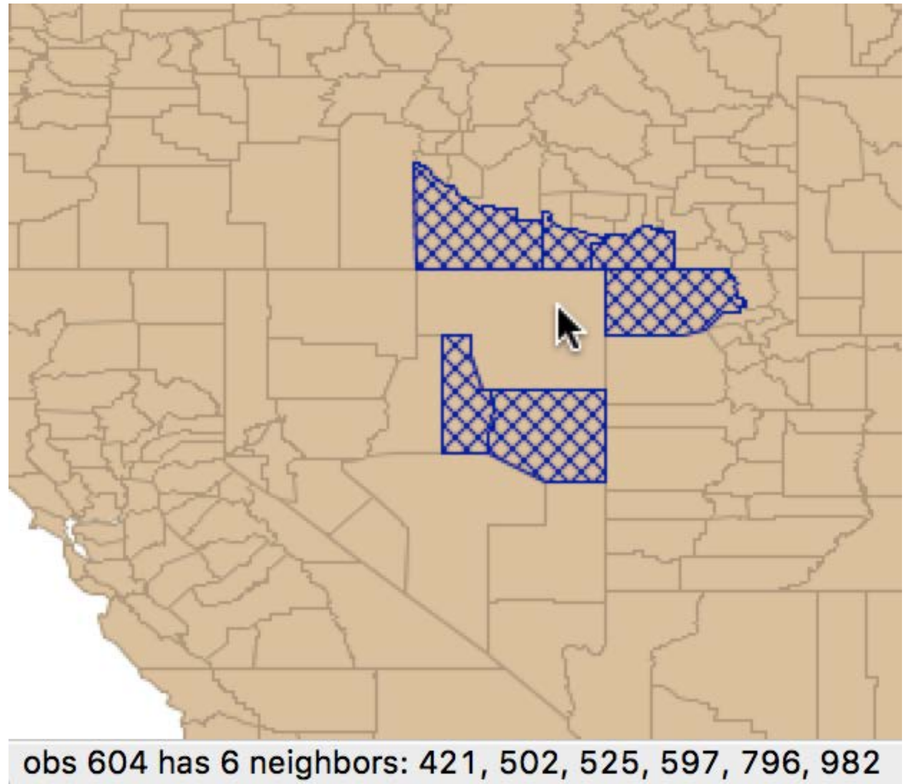
d = 90 mi
one neighbor

Elko county, NV, distance band neighbors

- k-Nearest Neighbor Weights
  - k nearest observations, irrespective of distance
    - fixes isolates problem for distance bands
  - same number of neighbors for all observations
  - in practice, potential problem with ties
    - needs tie-breaking rule (random, include all)

k-nearest neighbor weights





k = 4

k = 6

Elko county, NV, k-nearest neighbors

Table 1. Spatial weights formats supported by PySAL.

Type	File extension
Sparse contiguity (SpaceStat, GeoDa, R spdep, etc.)	GAL
Sparse general weights (SpaceStat, GeoDa, R spdep, etc.)	GWT
ArcGIS text weights	TXT
ArcGIS dbf weights	DBF
ArcGIS swm weights	SWM
Matlab spatial weights (old version)	DAT
Matlab spatial weights (new version)	MAT
Lotus weights	WK1
GeoBUGS weights	TXT
Stata weights	TXT
MatrixMarket weights	MTX

many spatial weights file formats

Spatial Weights Transformations

- Row-Standardized Weights

  rescale weights such that $\Sigma_j \, w_{ij} = 1$

  $w_{ij}^* = w_{ij} / \Sigma_j \, w_{ij}$

  constrains parameter space

  makes analyses comparable

  spatial lag = average of the neighbors

$$\mathbf{W}^* = \begin{bmatrix} 0 & 1/3 & 0 & 1/3 & 1/3 & 0 \\ 1/3 & 0 & 0 & 1/3 & 1/3 & 0 \\ 0 & 0 & 0 & 0 & 1/2 & 1/2 \\ 1/3 & 1/3 & 0 & 0 & 1/3 & 0 \\ 1/4 & 1/4 & 1/4 & 1/4 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}.$$

row-standardized weights matrix

- Stochastic Weights

  double standardization

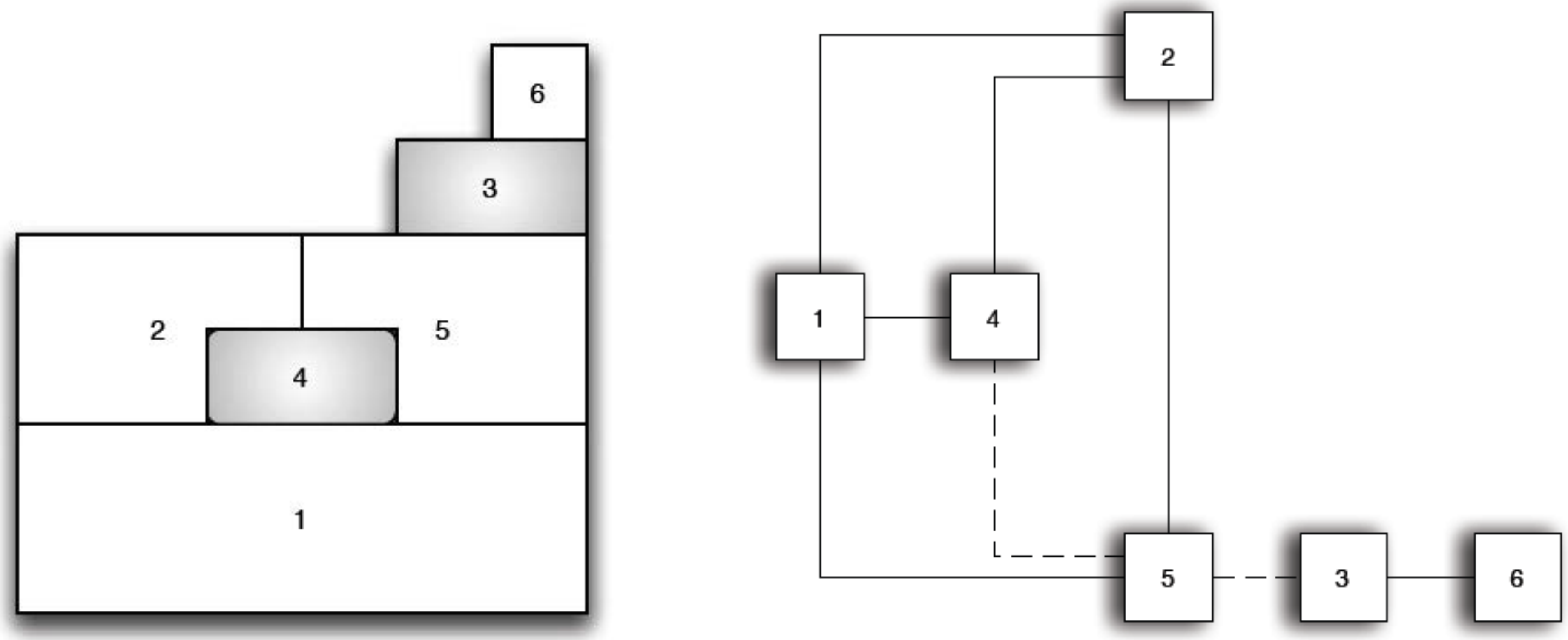
  $w_{ij}^* = w_{ij} / \Sigma_i \Sigma_j w_{ij}$

  rescaled such that $\Sigma_i \Sigma_j w_{ij} = 1$

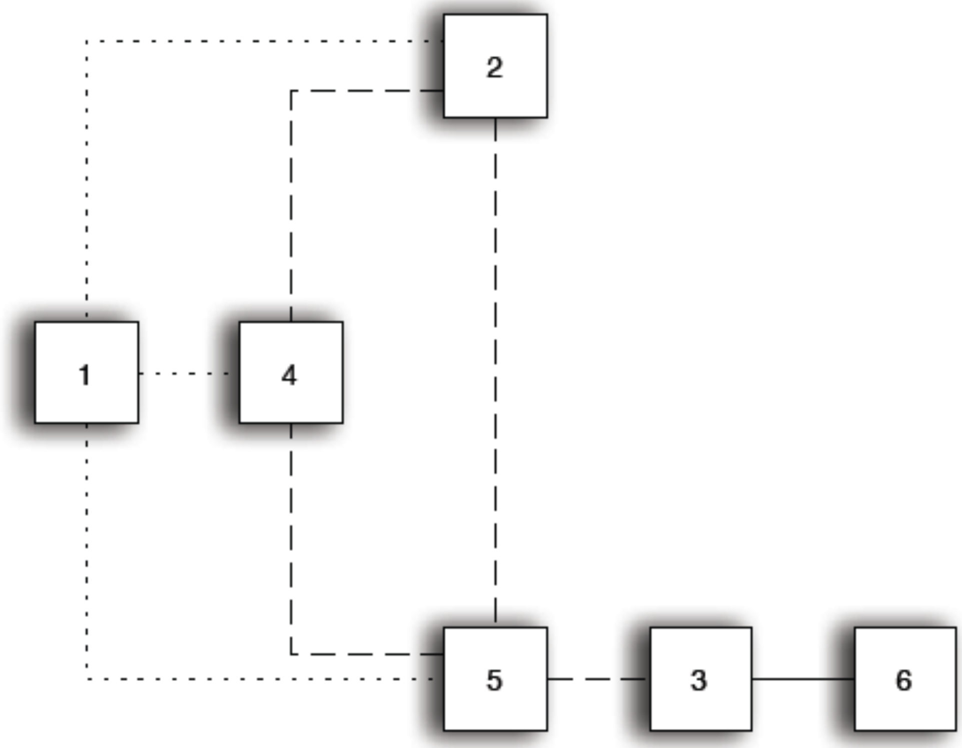
  similar to probability

$$
\mathbf{W}^* = \begin{bmatrix} 0 & 1/16 & 0 & 1/16 & 1/16 & 0 \\ 1/16 & 0 & 0 & 1/16 & 1/16 & 0 \\ 0 & 0 & 0 & 0 & 1/16 & 1/16 \\ 1/16 & 1/16 & 0 & 0 & 1/16 & 0 \\ 1/16 & 1/16 & 1/16 & 1/16 & 0 & 0 \\ 0 & 0 & 1/16 & 0 & 0 & 0 \end{bmatrix}.
$$

stochastic weights matrix



second order contiguity: neighbor of neighbor



redundancy in higher order contiguity
paths of lenght 2 between 1 and other cells

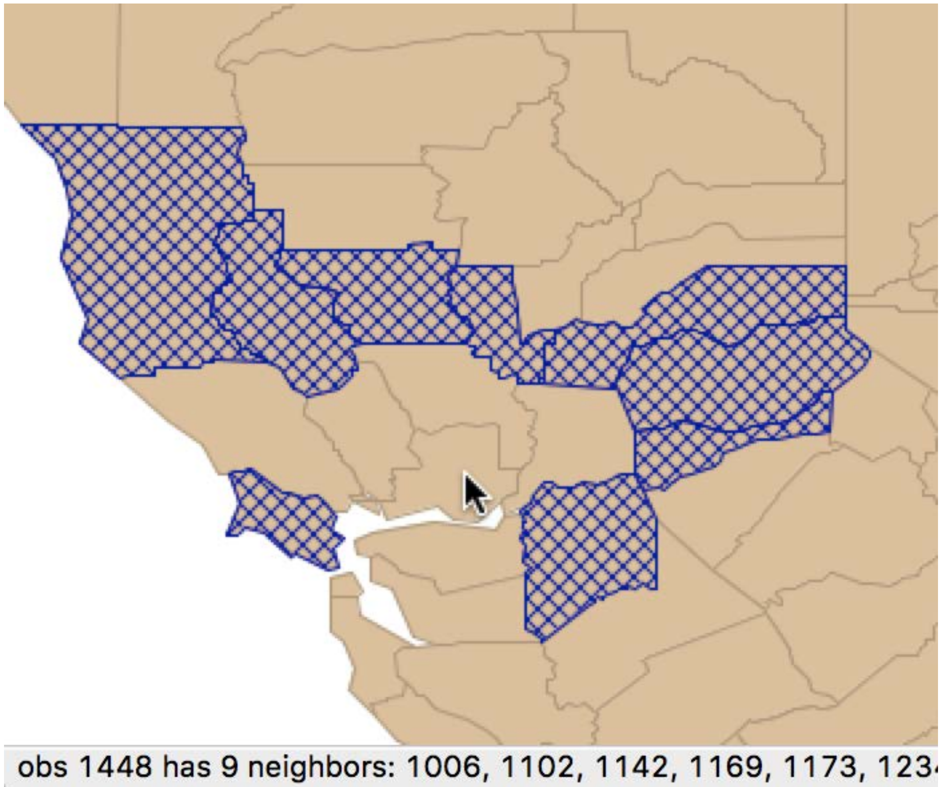
- Higher Order Weights

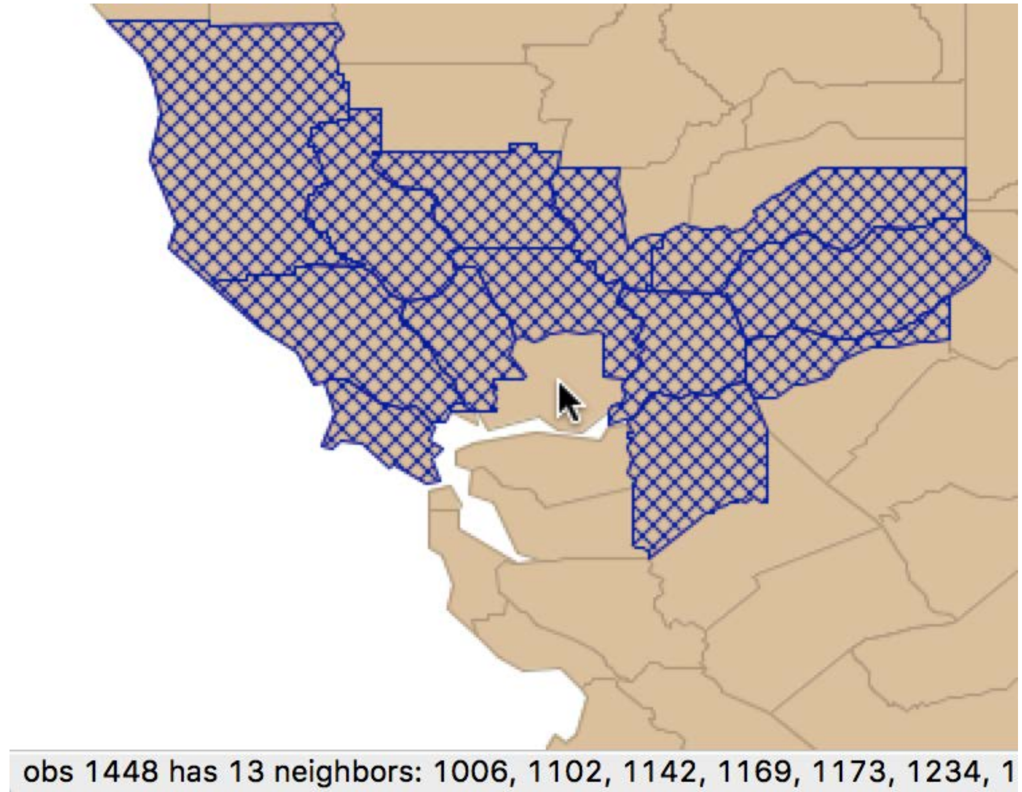
  recursive definition

  - k-th order neighbor is first order neighbor of (k-1)th order neighbor

  avoid duplication, only unique neighbors of a given order (not both first and second order)

  pure contiguity or cumulative contiguity, i.e., lower order neighbors included in weights





exclusive of first order

inclusive of first order

Solano county, CA, second order contiguity

Properties of Weights

- Connectivity Histogram

  histogram of number of neighbors

  neighbor cardinality

  diagnostic for “isolates” or neighborless units

  assess characteristics of the distribution

- Things to Watch for

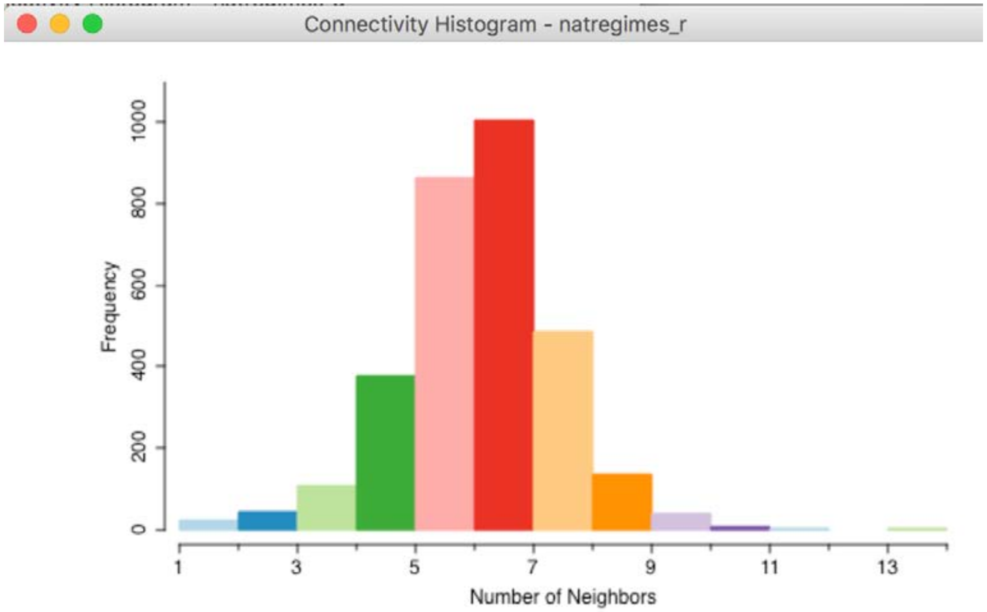
  isolates

  need to be removed for proper spatial analysis

  do not need to be removed for standard analysis

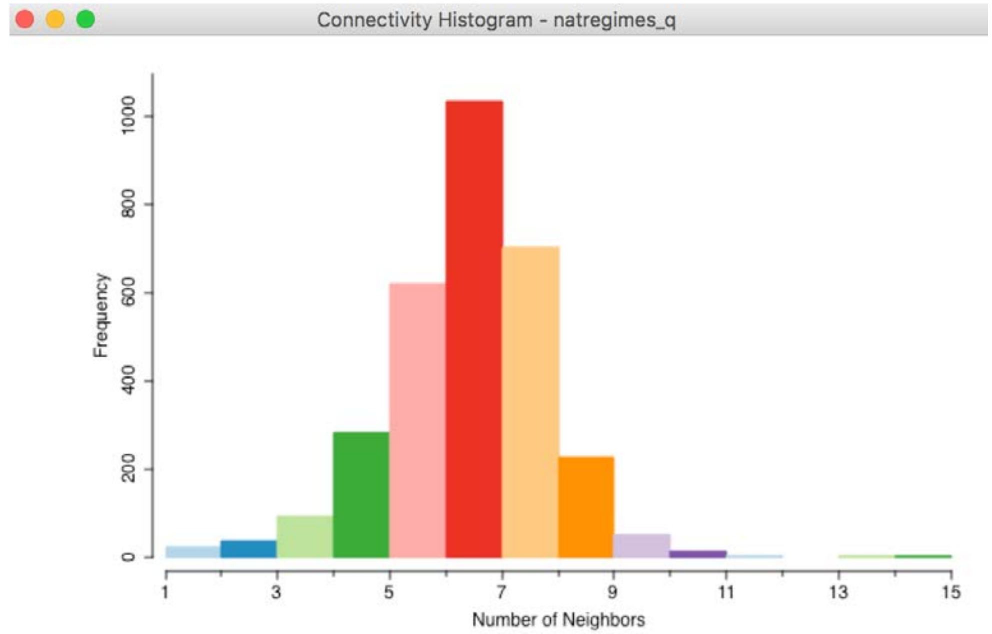
  very large number of neighbors

  bimodal distribution



from	1	2	3	4	5	6	7	8	9	10	11	12	13
to	2	3	4	5	6	7	8	9	10	11	12	13	14
#obs	24	41	108	377	863	1007	484	136	37	6	1	0	1
% of total	0.778	1.33	3.5	12.2	28	32.6	15.7	4.41	1.2	0.194	0.0324	0	0.0324
sd from mean	-2.66	-1.91	-1.17	-0.425	0	0.319	1.06	1.81	2.55	3.3	4.04	4.79	5.53

min: 1, max: 13, median: 6, mean: 5.57147, s.d.: 1.34313, #obs: 3085



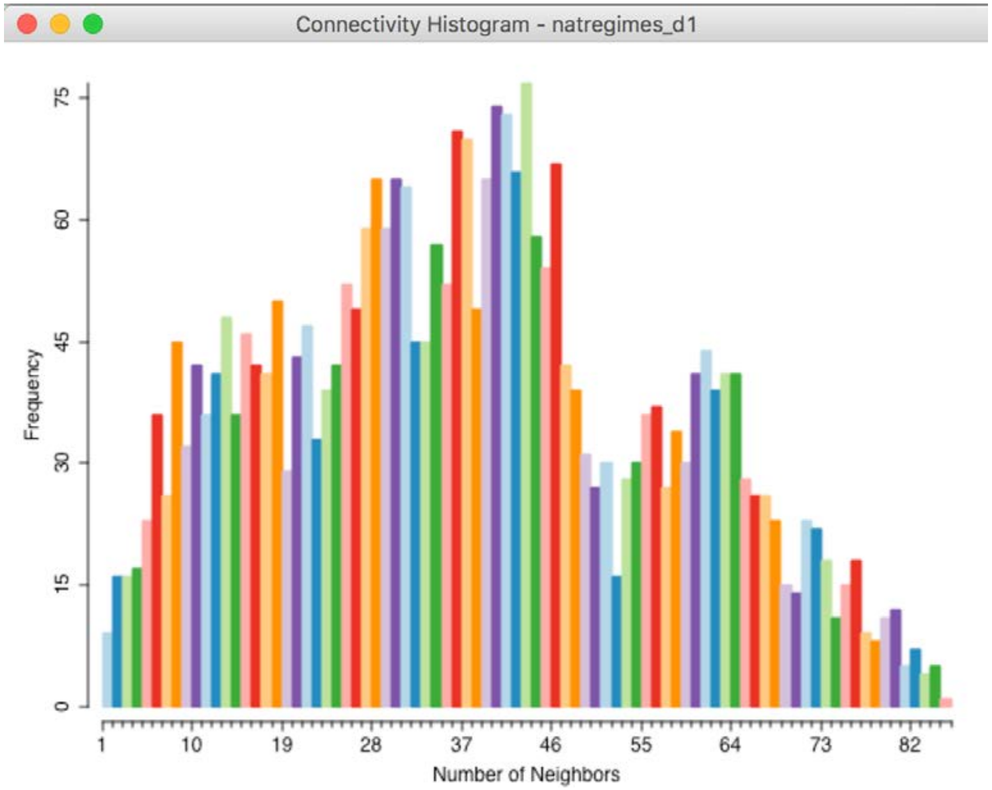
from	1	2	3	4	5	6	7	8	9	10	11	12	13	14
to	2	3	4	5	6	7	8	9	10	11	12	13	14	15
#obs	24	36	91	281	620	1037	704	227	50	11	2	0	1	1
% of total	0.778	1.17	2.95	9.11	20.1	33.6	22.8	7.36	1.62	0.357	0.0648	0	0.0324	0.0324
sd from mean	-2.76	-2.05	-1.34	-0.632	0	0.0788	0.79	1.5	2.21	2.92	3.63	4.34	5.06	5.77

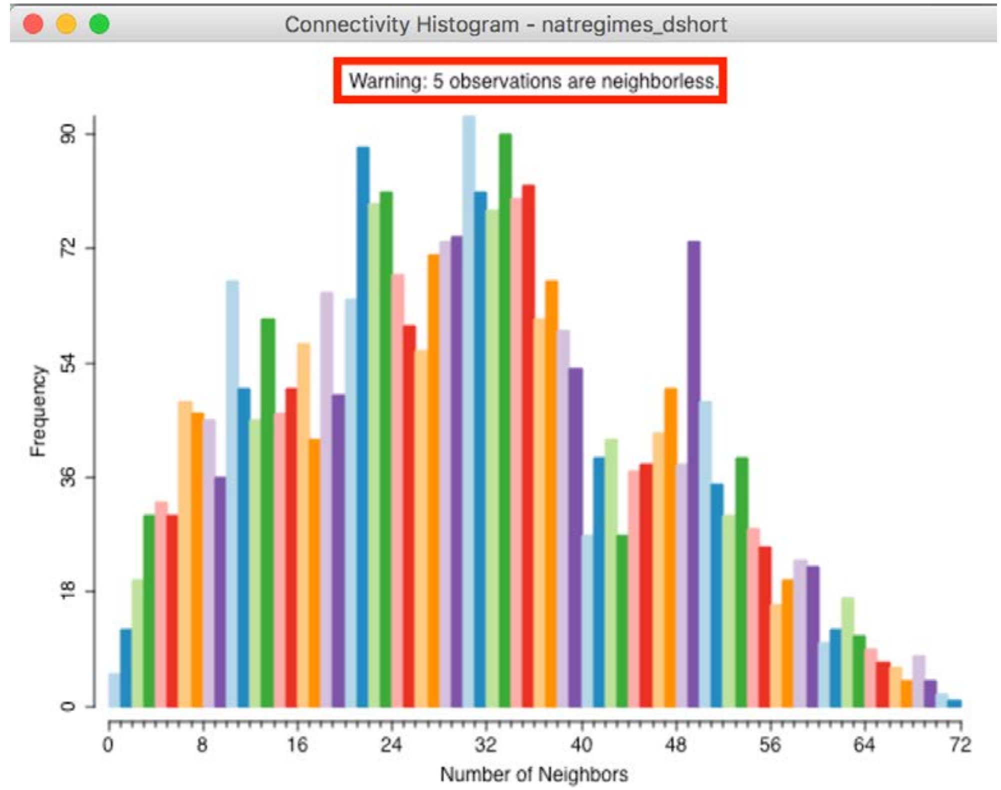
min: 1, max: 14, median: 6, mean: 5.88914, s.d.: 1.40664, #obs: 3085

rook

queen

connectivity histogram - contiguity weights
U.S. counties

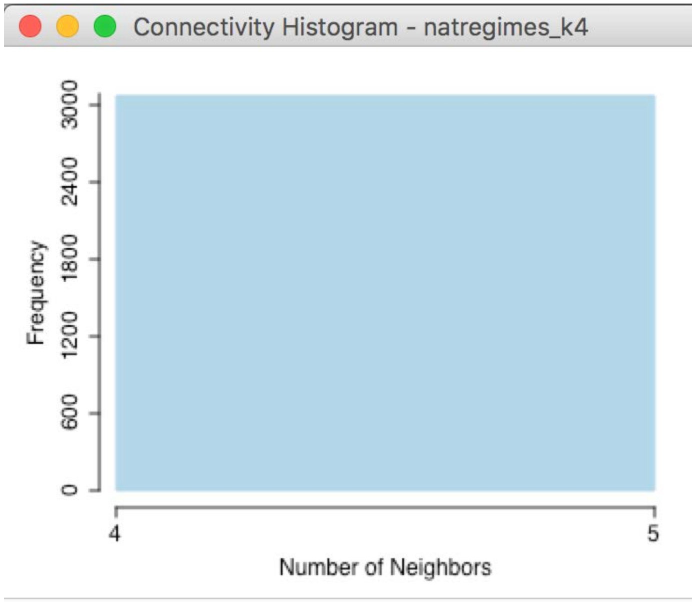




default distance 90 mi

critical distance 80 mi

connectivity histogram - contiguity weights
U.S. counties



contiguity histogram for k-nearest neighbors
or, what did you expect?