Heart Weight

Caleb Alexander, Ava Kerry, Nate Smith

July 23, 2024

1 Introduction

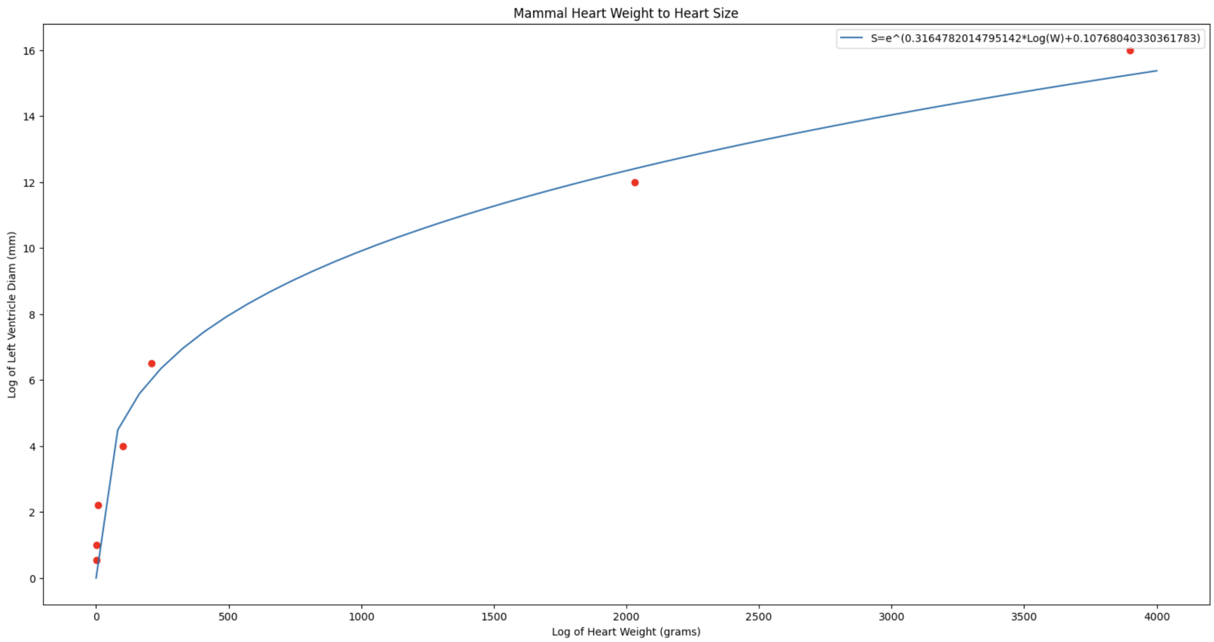
In this report we will show a relationship between heart weight and heart size in mammals, specifically the diameter of the left ventricle. As expected, this relationship is clearly positive; however, we would like to establish a more predictive relationship. Scaling laws in biology are not isometric, meaning they are not linear. Rather, allometric (nonlinear) scaling is utilized. In 1932 Max Kleiber proposed the law of quarter powers which revealed that the relationship between mammal mass M and metabolic rate Y resembles a logarithmic relation where the log of metabolic rate is equal to the log of mass multiplied by some value b plus a constant c ($\log Y = b \log M + c$). Specifically, he found this b to be roughly 0.25. So, if you plot the log of both values (mammal mass and metabolic rate); the values approach a line with a slope of 0.25. Since 1932, this "law" has been found to approximate many other biological relations, scaling to approximately the power of 0.25.

2 Math

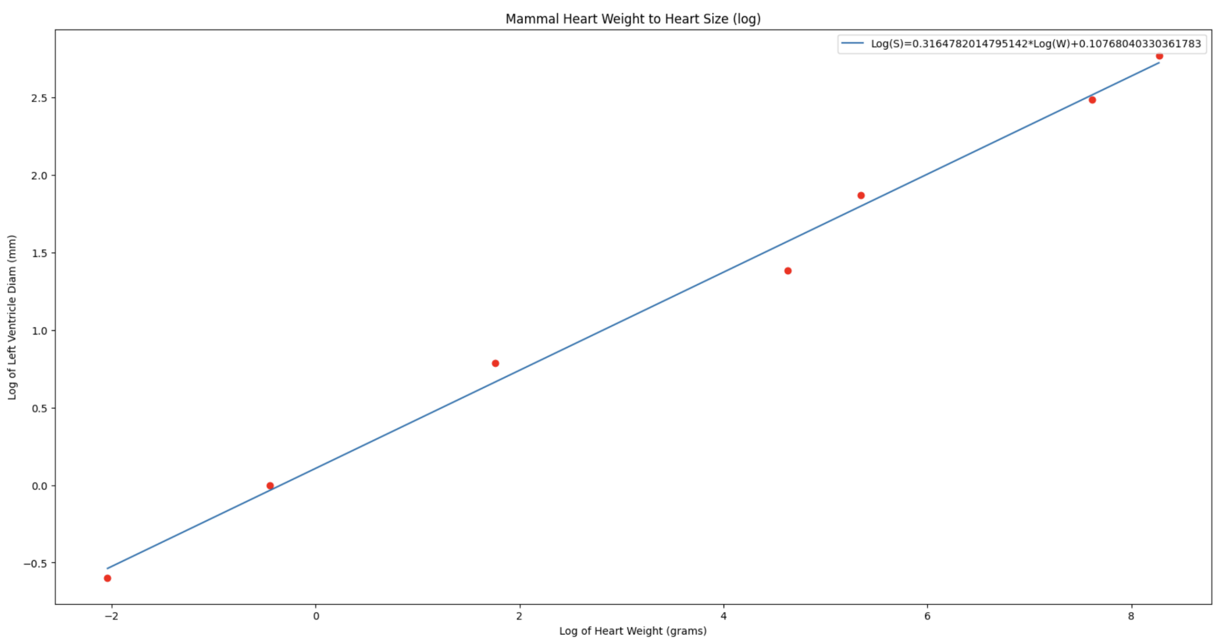
Below is a chart of Heart Weight by Left Ventricle Diameter.

Animal	Heart Weight (g)	Left Ventricle Diameter (mm)	log (Heart Weight)	log(Left Ventricle Diameter)
Mouse	0.13	0.55	-0.886	-0.260
Rat	0.64	1.0	-0.194	0
Rabbit	5.8	2.2	0.763	0.342
Dog	102	4.0	2.009	0.602
Ship	210	6.5	2.322	0.813
Ox	2030	12.0	3.307	1.079
Horse	3900	16.0	3.591	1.204

This is the graph of Heart Weight to Left Ventricle Diameter with a logarithmic trend line in blue.



If this relationship truly follows the law, we would expect the graph of the logs to resemble a line.



As you can see the graph of the logs resembles the line $\log V = 0.316 \log W + 0.108$. After some further analysis on our line of best fit we prepared the following list of common line of best fit metrics.

```
Errors are:  [-0.05983198562610981, 0.03355973627972772, 0.12445324485715303, -0.18508912002358247, 0.07187879915894269, -0.0330056148372595, 0.0480349401911333]
Deviations from mean are:  [-1.8415816107067509, -1.2437446099511305, -0.4552872495868602, 0.14254975116876012, 0.6280575669504609, 1.24116203983687, 1.5288441122
886507]
Residual Sum of Squared Errors is:  0.06301600276818939
Total Sum of Squares is:  9.438234231085827
R^2 is:  0.993323327094316
```

3 Conclusion

An R^2 of 0.99 is an extremely good score and heavily implies that this line of best fit approximate our data very closely. In this scenario the law of quarter powers roughly predicts our sample data. While the law of quarter powers is just an approximate, our data is limited by the amount of significant figures and the amount of data, thus it could be possible that our value $b = 0.316$ is exaggerated and thus the exact relationship may better approximate closer to $b = 0.25$. However, it may still be concluded that this example of allometric scaling is at least roughly approximated by the law of quarter powers. For further conclusions about this law, it should be noted that more data points would be necessary before any sweeping conclusions are made about all mammals.