The Case of Han van Meegeren

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August 8, 2024

1 Introduction

Han van Meegeren was a famous art forger that lived in the 20th century. This report will focus on one of his most famous claimed forgings; Vermeer's *Christ and the Disciples at Emmaus.* A common way of detecting whether or not a painting is forged is through the analysis of radioactive decay. White lead, a common pigment used in painting, contains the isotope ^{210}Pb with a half-life of 22 years. Thus, by examining the concentration of ^{210}Pb in a painting can give an accurate measure of a painting's age. Additionally, the element ^{226}Ra is often present when lead is mined and decays into ^{210}Pb. However, when a pigment is created there should be no ^{226}Ra present. Given this information, we will answer questions regarding the concentration of ^{210}Pb in pigments and estimate the age of the *Christ and the Disciples at Emmaus.*

2 Math

2.1 Part A

Let $y(t)$ be the number of ^{210}Pb atoms per gram of lead in a painting. Let t_0 be the time at which the pigment was produced. Let r be the number of ^{226}Ra per gram of lead, that decay into ^{210}Pb per unit of t.

When lead is in it's ore form the rate of change for $y(t)$ looks like:

$$\frac{dy}{dt} = -\lambda y + r.$$

Where λ is some constant that measures the rate of decay for ^{210}Pb (Note that the amount of an element that decays is determined in relation to the overall amount of the element present). Hence, at each point in time, our pigment will lose a λ proportion of the overall amount of ^{210}Pb present (y). Additionally, when lead is an ore; there is ^{226}Ra present which decays into ^{210}Pb. Hence why we add this back in. After the pigment is created, there ought to be no ^{226}Ra present, hence why the rate of change becomes:

$$\frac{dy}{dt} = -\lambda y.$$

2.2 Part B

The elements ^{226}Ra and ^{210}Pb are in "radioactive equilibrium" which means that for any given moment in time, the amount of ^{226}Ra decaying into ^{210}Pb is equivalent to the amount of ^{210}Pb decaying into any other element. Hence, we can say that:

$$r = \lambda y(t_0).$$

Since $y(t_0)$ is the proportion of ^{210}Pb present in our pigment and λ is the percentage of that amount which will decay. We have already established that this quantity must be equivalent to the amount of ^{226}Ra which decays, thus our equality. Measurements from various ores have shown that r is $0-200$ per minute; therefore, we can say that:

$$r = \lambda y(t_0) = 0 - 200 \text{ per minute.}$$

2.3 Part C

We want to solve $\frac{dy}{dt} = -\lambda y$ given the initial condition $y(t_0) = \frac{r}{\lambda}$. We know that:

$$\frac{dy}{dt} = -\lambda y \implies y(t) = Ce^{-\lambda t}$$

where C is a constant dependent on the initial condition. We will solve for C by using our given initial condition:

$$y(t_0) = \frac{r}{\lambda} = Ce^{-\lambda t_0} \implies C = \frac{r}{\lambda}e^{\lambda t_0}.$$

Therefore,

$$y(t) = \frac{r}{\lambda}e^{-\lambda(t-t_0)}.$$

2.4 Part D

For the *Disciples at Emmaus* painting, it was measured that

$$-\frac{dy}{dt}(t) \cong 8.5 \text{ per minute.}$$

From Part C, we know

$$\frac{dy}{dt} = -\lambda y = -\lambda\frac{r}{\lambda}e^{-\lambda(t-t_0)} \implies -\frac{dy}{dt} = re^{-\lambda(t-t_0)}.$$

We want to use this to estimate $t - t_0$. So,

$$\begin{aligned}
-\frac{dy}{dt} &= re^{-\lambda(t-t_0)} \\
-\frac{1}{r}\frac{dy}{dt} &= e^{-\lambda(t-t_0)} \\
ln(-\frac{1}{r}\frac{dy}{dt}) &= -\lambda(t-t_0) \\
-\frac{1}{\lambda}ln(-\frac{1}{r}\frac{dy}{dt}) &= t - t_0.
\end{aligned}$$

It is given that

$$-\frac{dy}{dt}(t) \cong 8.5 \text{ per minute} \implies -\frac{dy}{dt}(t) \cong 4467600 \text{ per year.}$$

We know from Part B,

$$r = 0 \text{ - } 200 \text{ per minute} \implies r = 0 \text{ - } 105120000 \text{ per year.}$$

We know $\lambda = \frac{ln(2)}{\tau}$, where τ is the half-life. The half-life of ^{210}Pb is 22 years. Therefore, $\lambda = \frac{ln(2)}{22}$. We now have enough information to find $t - t_0$. First consider $r = 105120000$ per year:

$$\begin{aligned} t - t_0 &= -\frac{1}{\lambda} ln(-\frac{1}{r}\frac{dy}{dt}) \\ &= -\frac{22}{ln(2)} ln(\frac{1}{105120000} \cdot 4467600) \\ &\approx 100.24 \text{ years.} \end{aligned}$$

Now consider $r = 0$ per year. Since we cannot take $\frac{1}{r}$, consider taking the limit as $r \to 0$:

$$\begin{aligned} \lim_{r\to 0} t - t_0 &= \lim_{r\to 0} -\frac{1}{\lambda} ln(-\frac{1}{r}\frac{dy}{dt}) \\ &= -\frac{1}{\lambda} \lim_{r\to 0} ln(-\frac{1}{r}\frac{dy}{dt}) \\ &= -\frac{22}{ln(2)} \lim_{r\to 0} ln(\frac{1}{r} \cdot 4467600) \\ &= \infty. \end{aligned}$$

Note that $\lim_{r\to 0} t - t_0 = t - t_0$. Therefore, $t - t_0 \geq 100.24$ and it is possible for the *Disciples at Emmaus* painting to be 300 years old.

3 Conclusion

In the 1930s, *Christ and the Disciples at Emmaus* was certified as a genuine 17th century Vermeer by A. Bredius, an art historian. In 1932, van Meegeren began his experimentation with forgery using chemical and technical processes and it took several years to perfect these techniques. However, in 1945, van Meegeren claimed to be the painter of *Disciples at Emmaus*. Since the age of the painting is at least 100 years old, *Christ and the Disciples at Emmaus* is not a forgery.