Geographical Review of Japan **48—7** 485~489 1975

"STATIC" RELATIONSHIP AMONG BEACH SLOPE, SAND SIZE, AND WAVE PROPERTIES

Tsuguo SUNAMURA*

I Introduction

The problem of beach gradient in the swash zone has been discussed in connection with waves and/or the size of beach material, ignoring the effect of time on the beach slope change ; this has been done on the assumption that the slope change has quick response to waves, *i.e.* that the change becomes steady immediately after wave action. The term "static" implies such a condition.

This paper aims to elucidate a comprehensive, quantitative relation among beach slope, sand size, and wave properties, because the previous studies were incomplete and/or qualitative, as summarized below.

II Previous Studies

A study conducted along the California coast by Bascom (1951) presented that foreshore slope, $\tan\beta$, increased with increase of sand grain size, d, *i.e.*

$$\tan\beta \sim d. \tag{1}$$

The similar relation was found through a model study (Kemp, 1962) and a field investigation (McLean & Kirk, 1969).

Analyzing statistically field data having considerable scatter of points, obtained at Marsden Bay, County Duham, King (1953) concluded that the beach became flatter as wave period, T, became larger, *i.e.*

$$\tan\beta \sim 1/T. \tag{2}$$

King (1972, pp. 325~326) carried out a model test and found that the beach gradient decreased as wave length, L, increased :

$$\tan\beta \sim 1/L. \tag{3}$$

Furthermore, King (1972, pp. 325~326) confirmed the presence of this relation using field data collected on the Mediterranean Sea coast. Since L is direclty related to T, this relation is essentially the same as Eq. (2).

According to model studies by Meyers (1933), Rector (1954), and King (1972, p. 328), the beach gradient decreased as wave steepness, H/L, increased for a given sand size :

$$\tan\beta \sim 1/(H/L) \sim L/H \sim T/H. \tag{4}$$

A comparison of Eqs. (4) and (2) or (3) indicates that the effect of L or T on $\tan\beta$ is

* Coastal Engineering Laboratory, Department of Civil Engineering, University of Tokyo, Tokyo 113, Japan.

obviously reverse. This means that King's conclusions are contradictory to each other.

On the Hawaiian coast, Inman et al. (1963) investigated the beach slope-sand size relation for various wave exposure ; Wiegel (1964, p. 359) also plotted the similar relation incorporating Bascom's data. Both studies showed that the more exposed beaches had the gentler slopes, although the wave exposure was not quantitatively expressed. If the degree of exposure can be represented by wave height, H, then

$$\tan\beta \sim 1/H. \tag{5}$$

The previous studies mentioned above discuss separetely the effects of sand grain size and waves, on beach slope. But the beach gradient is determined by the interaction between sand size and waves ; therefore the problem should be investigated taking simultaneously account of these two factors. Kemp & Plinston (1968) first discussed this relation in the field and laboratory, adopting a parameter like $H_b/d^{0.5}T$, where $H_b=$ breaker height ; this parameter was not nondimensional. They did not offer the formula linking beach slope, sand size, and waves.

III Nondimensional Parameter

Considering Eqs. (1), (4), and (5), the following relation is obtained :

$$\tan\beta \sim d\,L/H. \tag{6}$$

Here, H_b and T should be substituted respectively for H and L. The reason is that (1) the former two quantities are more easily measurable by a visual observation, especially in the field, and (2) they affect greatly the beach slope change (Kemp & Plinston, 1968). Then, Eq. (6) is written as

$$\tan\beta \sim d\,T/H_b, \tag{7}$$

in which the righthand side is not a dimensionless quantity. Dimensional analysis of Eq. (7) using gravitational acceleration, g, yields the following dimensionless equation :

$$\tan\beta = \mathrm{f}_1\,(g^{0.5}\,d^{0.5}\,T/H_b). \tag{8}$$

Equation (8) can be rewritten as

$$\tan\beta = \mathrm{f}_2\,(H_b/g^{0.5}\,d^{0.5}\,T). \tag{9}$$

This is basically the same as the parameter which Kemp & Plinston (1968) used.

IV Result

In order to determine the shape of function in Eq. (9), the previous field and laboratory data were used together with the author's field data (Table 1) ; they were plotted in Fig. 1. Beach gradient was measured at the water line from beach profile record in case of lacking the actual value. Some of the previous laboratory studies did not provide information of breaker height. In this case, it was estimated using Iversen's "breaker height index" (U. S. Army Coastal Engineering Research Center, 1966, p. 102).

Figure 1 shows that beaches in laboratory are generally steeper than those in nature for $H_b/g^{0.5}\,d^{0.5}\,T > 1$. One reason for this difference would probably be scale effect. The

Table 1 Data collected on the Pacific coasts of Japan.

locations	$\tan\beta$	d (mm)	H_b (m)	T (sec)	$H_b/g^{0.5}\,d^{0.5}\,T$
Kujukuri (1)	0.017	0.2	1.1	8	3.11
(2)	0.013	0.2	1.6	6	6.02
(3)	0.011	0.2	1.3	10	2.94
(4)	0.032	0.2	1.5	10	3.39
(5)	0.037	0.2	1.5	10	3.39
Katsuura (1)	0.12	0.50	0.8	10	1.14
(2)	0.13	0.38	0.8	10	1.31
(3)	0.10	0.42	1.0	10	1.56
(4)	0.11	0.27	1.0	10	1.94
(5)	0.066	0.31	0.8	10	1.45
(6)	0.10	0.30	0.8	10	1.48
Sagami Bay	0.15	1.0	1.1	12	0.93

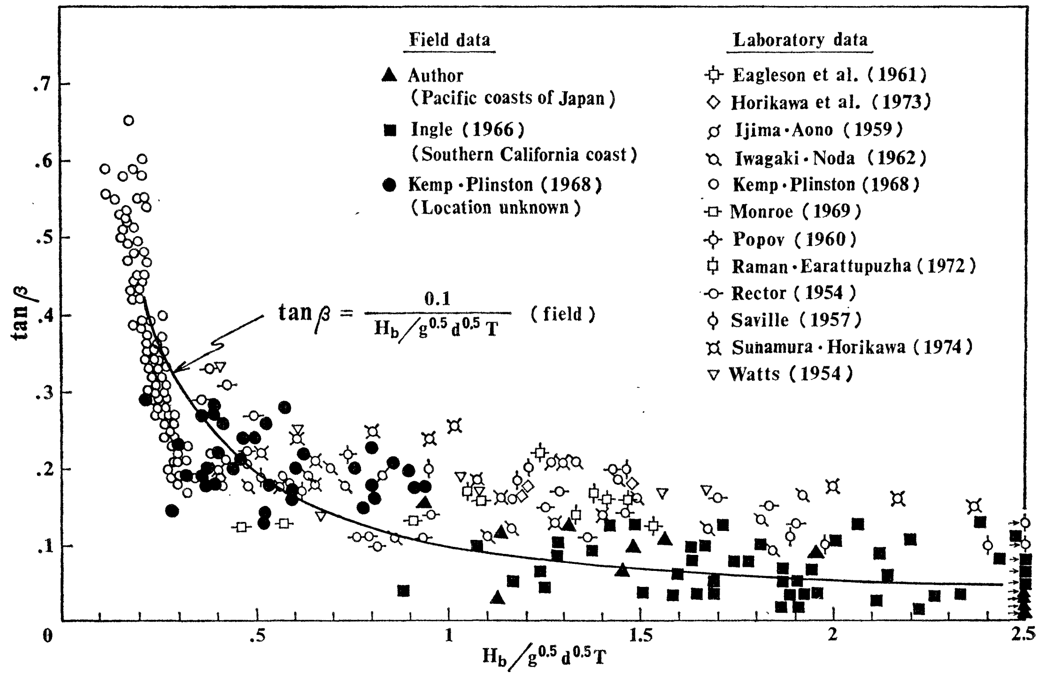

Fig. 1 Relation of beach slope, beach material size, and wave properties.

field data indicate the following relation, although the data points are scattered :

$$\tan\beta = 0.10\; g^{0.5}\, d^{0.5}\; T/H_b. \qquad (10)$$

Komar & Gaughan (1972) gave the following equation to relate breaker height to deep water wave properties :

$$H_b = 0.39\; g^{0.2}\,(T\, H_o^2)^{0.4}, \qquad (11)$$

where H_o=wave height in deep water.

Transformation of Eq. (10) using Eq. (11) and $L_o=(g/2\,\pi)\,T^2$ reduces to

$$\tan\beta = 0.45\,(d/H_o)^{0.5}\,(H_o/L_o)^{-0.3}, \qquad (12)$$

where L_o=wave length in deep water. This equation shows that the beach slope in nature can be expressed by such dimensionless parameters as d/H_o and H_o/L_o ; the first parameter denotes the relative magnitude of beach material size to wave height, and the second shows deep water wave steepness.

The scatter of field data in Fig. 1 is due mainly to the following reasons : (1) the effect of time was not completely eliminated, *i.e.* all the data were not perfectly in "static", (2) the definition of beach gradient was somewhat different according to the researchers, (3) the data were not always obtained in the two-dimensional environment, and (4) the influence of sorting of sediment on beach gradient (McLean & Kirk, 1969) was involved.

For "dynamic" approach to the beach-slope problem, it is necessary to conduct precise, temporal measurements of beach change and swash characteristics (Waddell, 1973a, b), and also to investigate physical properties of beach material such as permeability and suspension characteristics.

V Conclusion

The natural beach slope in "static" condition was given by the following dimensionless empirical equations :

$$\tan\beta = 0.10\, g^{0.5}\, d^{0.5}\, T/H_b$$

or

$$\tan\beta = 0.45\,(d/H_o)^{0.5}\,(H_o/L_o)^{-0.3},$$

where $\tan\beta$=beach slope, g=acceleration of gravity, d=grain size of beach material, T=wave period, H_b=breaker height, H_o=deep water wave height, and L_o=deep water wave length.

References

Bascom, W. N. (1951) : The relationship between sand size and beach-face slope. *Trans. Am. Geophys. Union,* **32,** 866～874.

Eagleson, P. S., Glenne, B., and Dracup, J. A. (1961) : Equilibrium characteristics of sand beaches in the off-shore zone. *Beach Erosion Board, Tech. Memo.,* **126,** 66p.

Horikawa, K., Sunamura, T., and Kitoh, H. (1973) : A study of beach transformation by wave action. *Proc. 20th Jap. Conf. Coastal Eng.,* 357～363. (in Japanese).

Ijima, T. and Aono, H. (1959) : Effect of sand grain size on beach configuration. *Proc. 6th Jap. Conf. Coastal Eng.,* 49～56. (in Japanese).

Ingle, J. C., Jr. (1966) : *The Movement of Beach Sand.* Elsevier, Amsterdam, 221p.

Inman, D. L., Gayman, W. R., and Cox, D. C. (1963) : Littoral sedimentary processes on Kauai, a sub-tropical high island. *Pacific Sci.,* **17,** 106～130.

Iwagaki, Y. and Noda, H. (1962) : Laboratory study of scale effects in two-dimensional beach processes. *Proc. 8th Conf. Coastal Eng.,* 194～210.

Kemp, P. H. (1962) : A model study of the behaviour of beaches and groynes. *Proc. Inst. Civil Eng.,* **22,** 191～210.

Kemp, P. H. and Plinston, D. T. (1968) : Beaches produced by waves of low phase difference. *Jour. Hydr.*

Div., Proc. Am. Soc. Civil Eng., **94**, 1183～1195.
King, C. A. M. (1953) : The relationship between wave incidence, wind direction and beach changes at Marsden Bay, County Durham. *Trans. Inst. Brit. Geogr.*, **19**, 13～23.
King, C. A. M. (1972) : *Beaches and Coasts* (2nd ed.). Arnold, London, 570p.
Komar, P. D. and Gaughan, M. K. (1972) : Airy wave theory and breaker height prediction. *Proc. 13th Conf. Coastal Eng.*, 405～418.
McLean, R. F. and Kirk, R. M. (1969) : Relationships between grain size, size-sorting, and foreshore slope on mixed sand-shingle beaches. *New Zealand Jour. Geol. Geophys.*, **12**, 138～155.
Meyers, R. D. (1933) : A model of wave action on beaches. *Univ. of California, Unpub. M. Sc. Thesis.*
Monroe, F. F. (1969) : Oolitic aragonite and quartz sand : laboratory comparison under wave action. *Coastal Eng. Res. Center, Misc. Paper*, **1-69**, 29p.
Popov, I. J. (1960) : Experimental research in formation by waves of stable profile of upstream face of earth dams and reservoir shores. *Proc. 7th Conf. Coastal Eng.*, 282～293.
Raman, H. and Earattupuzha, J. J. (1972) : Equilibrium conditions in beach wave interaction. *Proc. 13th Conf. Coastal Eng.*, 1237～1256.
Rector, R. L. (1954) : Laboratory study of equilibrium profiles of beaches. *Beach Erosion Board, Tech. Memo.*, **41**, 38p.
Saville, T., Jr. (1957) : Scale effects in two dimensional beach studies. *Proc. 7th Meeting Int. Assoc. Hydr. Res.*, (A 3) 1～8.
Sunamura, T. and Horikawa, K. (1974) : Two-dimensional beach transformation due to waves. *Proc. 14th Conf. Coastal Eng.* 920～938.
U. S. Army Coastal Engineering Research Center (1966) : Shore protection, planning and design (3rd ed.). *Tech. Rep.*, **4**, 401p.
Waddell, E. (1973a) : Dynamics of swash and implication to beach response. *Coastal Studies Inst., Louisiana State Univ., Tech. Rep.*, **139**, 49p.
Waddell, E. (1973b) : A field investigation of swash characteristics. *Coastal Eng. Japan*, **16**, 61～71.
Watts, G. M. (1954) : Laboratory study of effect of varying wave periods on beach profiles. *Beach Erosion Board, Tech. Memo.*, **53**, 19p.
Wiegel, R. L. (1964) : *Oceanographical Engineering.* Prentice-Hall, Englewood Cliffs, N. J., 532p.
(Received Feb., 28, 1975 ; accepted Mar., 14, 1975)

海浜勾配，海浜堆積物の粒径および波浪特性の間に成立する“静的”関係

砂　村　継　夫*

時間の影響を無視できる，いわゆる“静的”な状態においては，自然海浜の前浜勾配は次の無次元経験式で与えられる．

$$\tan\beta = 0.10\, g^{0.5}\, d^{0.5}\, T/H_b$$

あるいは

$$\tan\beta = 0.45(d/H_o)^{0.5}(H_o/L_o)^{-0.3},$$

ここに，$\tan\beta$=前浜勾配，g=重力の加速度，d=海浜堆積物の粒径，T=波の周期，H_b=砕波波高，H_o=深海における波高，L_o=深海における波長，である．

地理学評論　48—7　485～489　1975
*　東京大学工学部土木工学科港湾研究室